Principles and Techniques Of Modern Radar Systems
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Lecture - 38
Detection in Radar Receiver (Contd.)

Key Concepts: Concept of velocity resolution, analytical expression for velocity resolution, range-velocity resolution, definition of radar ambiguity function

Welcome, to this NPTEL lecture on Principles and Techniques of Modern Radar System. We are continuing our discussion on resolution ambiguity etcetera. So, we have seen range resolution in the last class, today we will see the velocity resolution. Actually you know that velocity is measured by Doppler shift so, sometimes it also is called the Doppler resolution, but I prefer this term velocity resolution because actually you are measuring velocity.

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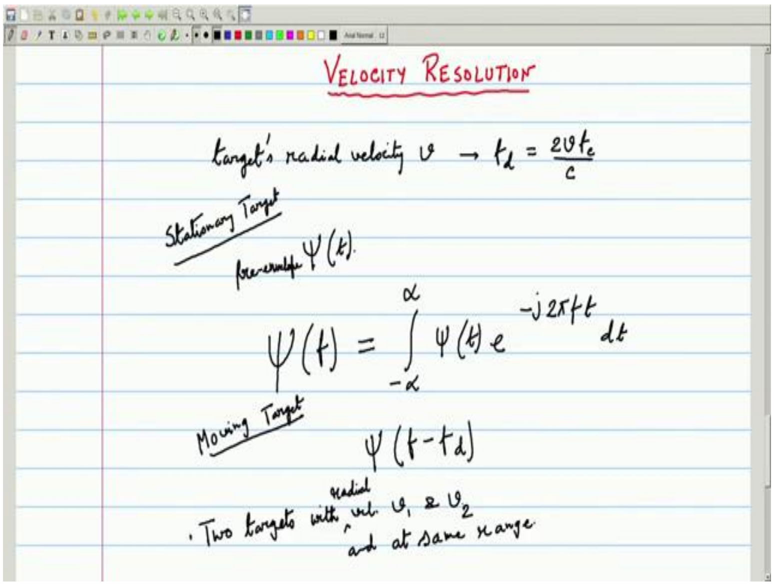

So, let us say for we know target radial velocity if it is v I am not writing v r. So, the we know that Doppler shift is 2 v f c by c.

$$\text{target's radial velocity } v \rightarrow f_d = \frac{2 v f_c}{c}$$

So, now, let us say again I have a stationary target stationary target and let us say that it's pre-envelope is phi t. So, let me write because it will be easier for you pre-envelope is psi t for this. And, so, it's Fourier transform let us take it's Fourier transform because since the concept of is frequency is coming so, unless and until we go to the Fourier domain or frequency domain we would not be able to see the Doppler shift. So, let us make the Fourier transform is psi f.

Now, this is psi t, actually I should have taken a different symbol for denoting the Fourier transform because this thing and Fourier transform are completely different functions, but if I do that then the that this is the Fourier transform of this time function that sometimes gets obscured that is why we continue that, but please remember that this psi and this psi they are not same. Though we are using the same symbol, but actually there are different things because this thing may be real, but this is the complex. So, this is a completely different function and it is basically integral thing of this.

So, all those things all of you know, but sometimes we forget that is why that I am remembering. So, what is this? This I can write from my knowledge of Fourier transform that this is psi t e to the power minus j 2 pi f t this is dt ok.

$$\Psi(t) = \int_{-\alpha}^{\alpha} \psi(t)\, e^{-j2\pi f t}\, dt$$

Now, now instead of stationary target let me go to moving target. So, in case of moving target, the received signal spectrum will be shifted by f d. So, the pre envelope becomes that time psi f minus f d ok. This is.

Now, let us consider as before that I have two targets with velocities v 1 radial velocities with radial velocities v 1 and v 2 I am not writing v r, but obvious it is radial velocity v 1 and v 2 and they are at same range because when I am studying the velocity thing I should make the range equal otherwise that will complicate. Later we will remove that restriction and make everything may change so and at same range.

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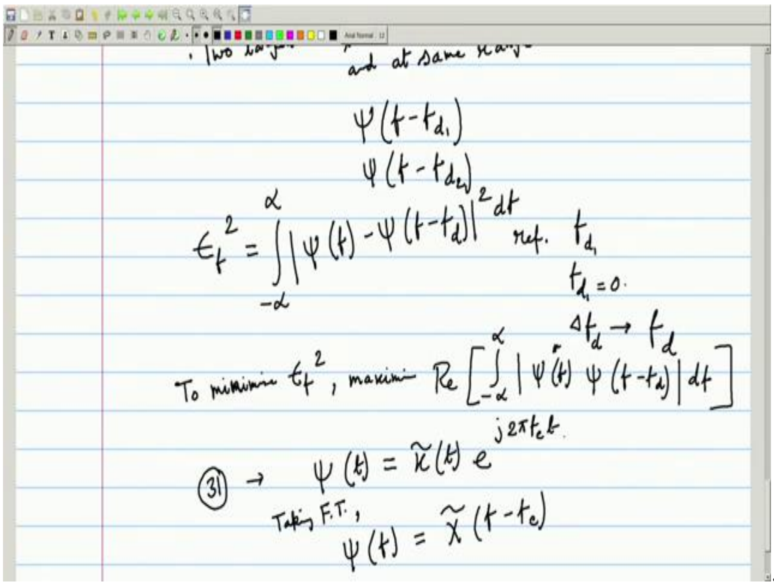

So, the returns the pre-envelopes for them one I will be calling or let me call like this that. So, one will give me return f minus f d 1 another will give me f minus f d 2 because v 1 will give me f d 1, v 2 will give me f d 2. So, I can now form that integral square error, but to remember that this is in the frequency domain between two such Fourier transform things. So, I am writing f as a subscript and I can form again this difference integral square error. So, this is phi. So, again I can make any of them as a reference.

So, let us say that our reference is f d 1; f d 1 is a let us say reference and then without using generality I can make this f d 1 to 0. So, basically we will be digging with their difference. So, that is delta f, but to say that it is the Doppler difference I will use the symbol f d. Again, I am repeating that so, I will get pre-envelopes like this, but this f d 1 let us take f d 1 as a reference and then the difference between the two in Doppler will be delta f or delta f d you can say and that I am calling f d.

So, that means, in our now f d means the difference of Doppler between the two echoes. So, I can form this thing; that means, from this first target I am getting the pre-envelope psi f and from the second target I am getting f minus f d and now I am forming their integral error like this as before. So, this thing I am not going into the details the same way you can manipulate. So, we can finally, I can say that to have this square minimum I should maximize to minimize the square error ultimately I will have to maximize real part of minus infinity to plus infinity psi f star psi f minus f d this d f, then real part. So, it is this.

$$\psi(t - t_{d_1})$$

$$\psi(t - t_{d_2})$$

$$\epsilon_t^2 = \int_{-\infty}^{\infty} \left| \psi(t) - \psi(t - t_d) \right|^2 dt \quad \text{ref. } t_{d_1}$$

$$t_{d_1} = 0.$$

$$\Delta t_d \rightarrow t_d$$

To minimize ϵ_t^2, maximize $\mathrm{Re}\left[\int_{-\infty}^{\infty} \left| \psi^*(t)\, \psi(t - t_d) \right| dt \right]$

Now, please see equation 13 because that was the definition for this thing that relation between the pre-envelope and the complex envelope. So, equation 13, I am just again writing you need not. So, you can refer to your equation this thing. This is the definition of pre envelope relation between pre envelope and complex envelope, but this is in time domain here I have got this psi f type of thing. So, I need to take Fourier transform of this. So, I can say taking Fourier transform I can write psi of f will be f minus f c this is a shifting property, I think all of you know.

$$\psi(t) = \tilde{x}(t)\, e^{j 2\pi f_c t}$$

Taking F.T.,

$$\psi(t) = \tilde{X}(t - t_c)$$

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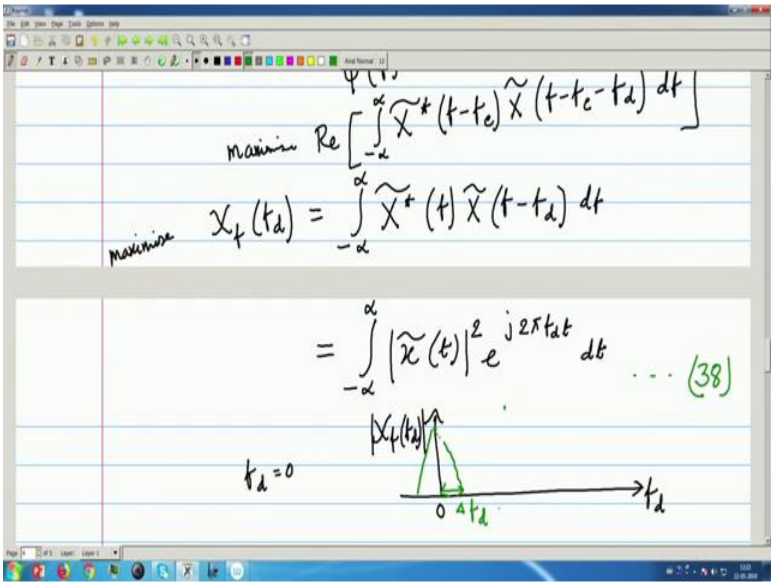

So, this thing I can write that we need to maximize real part of minus infinity to infinity, psi star f or I can directly put in terms of X. So, maximize X star f minus f c, then if f minus f c minus f d then d f.

$$\text{maximize} \quad \mathrm{Re}\left[\int_{-\infty}^{\infty} \tilde{X}^*(f-f_c)\,\tilde{X}(f-f_c-f_d)\,df\right]$$

So, this thing you see it is called actually this is again we are coming to a correlation type of thing because this is nothing, but a correlation function. So, that is called I define it like this X f f d is minus infinity to infinity and this thing if we doing time domain then you will get it simply that.

$$X_f(f_d) = \int_{-\infty}^{\infty} \tilde{X}^*(f)\,\tilde{X}(f-f_d)\,df$$

$$= \int_{-\infty}^{\infty} |\tilde{x}(t)|^2 e^{j2\pi f_d t}\,dt$$

So, this is an important thing that and this function is called frequency correlation function. So, you should give it again in equation number 38. So, what we see that this I will have to maximize this correlation function to resolve or not to resolve; that means, again the ideal thing is if this correlation function goes to peak at f d is equal to 0; that means, no relative Doppler shift between the two targets. It should go to very high value and near that it should fall again, ideally delta function, but again that is not possible in nature.

So, I should again require that if I have this plot of this frequency correlation function magnitude, then that should have a peak actually the peak is also here. So, here I am plotting the relative Doppler shift and near that it should have a peak something and then I do not know the shape, but it should fall very fast and it should fall with a high roll off so that I get a very sharp a thing. So, this the falling that width that will be called the resolution. So, if I call it delta f d that is the resolution. So, from the graph again the same concept.

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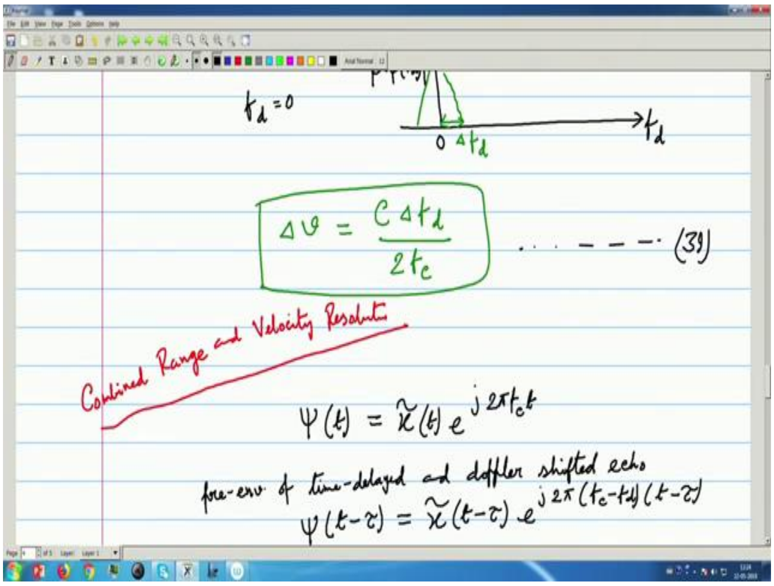

And, so, we can get that delta v the velocity resolution that will be c delta f d by 2 f c.

$$\Delta v = \frac{c \Delta f_d}{2 f_c}$$

So, this is our velocity resolution ultimately what we are looking for. So, I think we should give it a number equation number 39. So, again the Doppler resolution so, it will be determined by a thing. So, now, you can choose because you see everything depends on the shape of X t. So, X t designers can choose, so that to give both this range ambiguity function and frequency correlation function a sharp peak at the respective 0 values and a very high roll off so that the width is.

So, now, we will make that combined we will combine these two because while making the understanding that range ambiguity function we made the Doppler shift 0, that means, the velocity 0, while making velocity zero we made the range 0, now we should combine them. So, I call that the combined range and Doppler velocity let us say. So, this will be now. So, again I will now hurriedly go that the pre-envelope envelop of the transmitted waveform let us say it is x t as before and then what will be the pre envelope of the time delayed.

$$\psi(t) = \tilde{x}(t)\, e^{j 2\pi f_c t}$$

So, pre-envelope of time-delayed and Doppler shifted echo. So, that will be psi t minus tau that is x t minus tau e to the power j 2 pi f c minus f d and t minus tau, as simple as this.

$$\psi(t-\tau) = \tilde{x}(t-\tau)\, e^{j 2\pi (f_c - f_d)(t-\tau)}$$

So, again utilizing the previous two concepts that. So, now, I can form again this integral square error.

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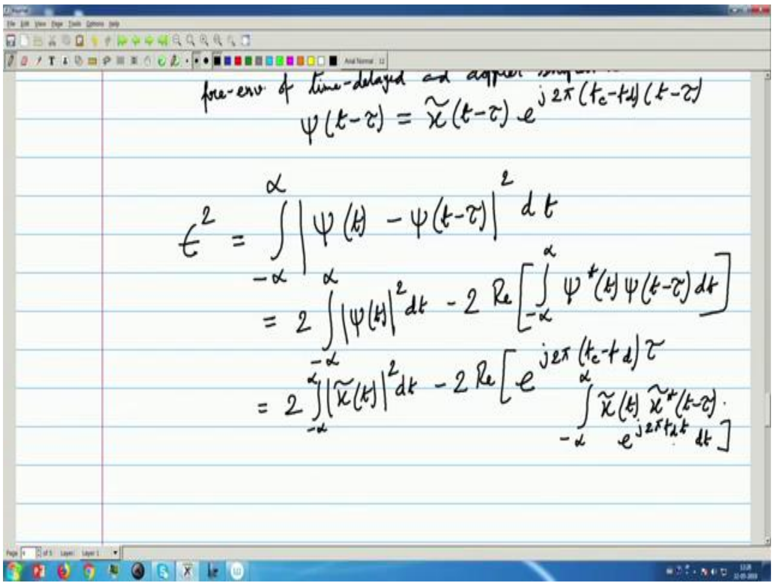
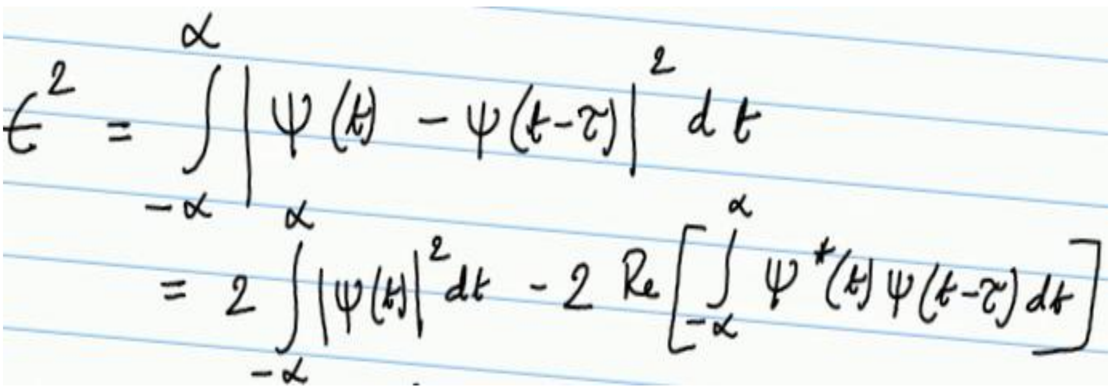


This I am not giving any subscript now, because it is the general thing. So, epsilon square that will be minus infinity to infinity; the first one minus the second one, this is the return from the second echo this is the first echo, appropriately we have taken the references. In one case, the first one range the that has been taken as reference or the first one time delay that has been taken as reference; in the frequency also the first Doppler shift has been taken reference and both of them that means, the first time delay for the first one and the Doppler shift of the first one they have been without loss of generality assumed as 0.

So, I am writing like this and t minus tau square d t. So, this can be again you can easily manipulate that this I can write directly as 2 minus infinity to infinity psi t square dt minus 2 real t minus tau dt.



Now, you can put those values as given here and if you manipulate you will see that here a j 2 pi f c minus f d tau into this integral term that x t x star t minus tau e to the power j 2 pi f d t dt, should close this ok.

$$= 2\int_{-\alpha}^{\alpha} \left|\tilde{x}(t)\right|^2 dt - 2\,Re\left[e^{j2\pi(f_c - f_d)\tau} \int_{-\alpha}^{\alpha} \tilde{x}(t)\, \tilde{x}^*(t-\tau) \cdot e^{j2\pi f_d t}\, dt\right]$$

So, you see that it is again the correlation function, but there is a this Doppler shift here and here previously it was just a phase dependent or tied to carrier frequency, now it is f c minus f d. So, again you need to maximize this. So, to have good both the good range resolution and good the velocity resolution then again we require that this integral function the function under this integration that should be maximized and then we will have a.

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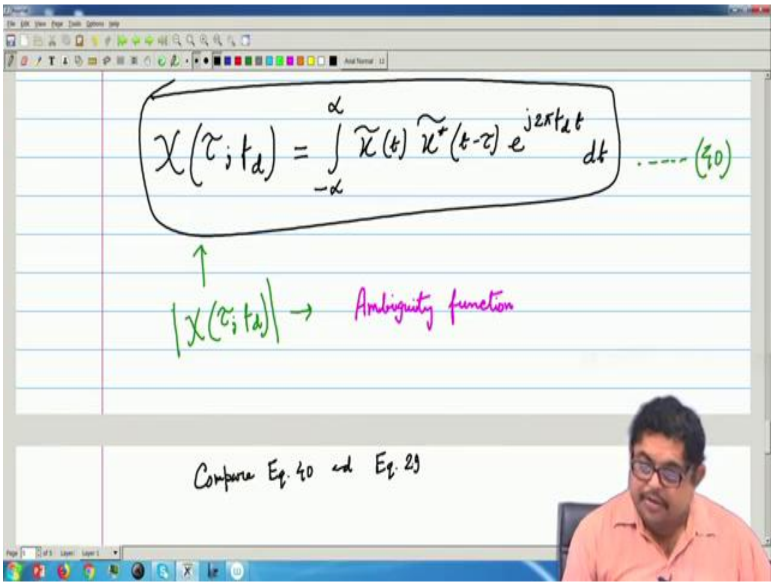

So, now, is the time for defining a function for this that we define it very important function and you see this function is a 2-dimensional function because it depends on both tau and f d. So, that is why it is a 2-dimensional function, a very important function in radar detection we define it as and this should be called equation 40.

$$\chi(\tau; f_d) = \int_{-\infty}^{\infty} \tilde{x}(t)\, \tilde{x}^*(t-\tau)\, e^{j2\pi f_d t}\, dt$$

Now, this actually the this is called the ambiguity function, no pre-qualification of range or etcetera.

So, this is the sometimes called radar ambiguity function actually the magnitude part is generally called radar ambiguity function because this whole integral has a phase and that phase have some information, but we do not process that information. So, basically the magnitude of this function is called radar ambiguity function and so, I can say that to have both this resolution because actually in measurement you want both the resolutions to be good. So, I will say that this is given a name ambiguity function; ambiguity function and you can easily see that this is nothing, but the match filter output.

So, if we just look at the match filter output for a moving target I get this thing. So, this should be by seeing these, by seeing the peak of this we understand that there is a target and detecting the next peak we understand the that there is another target. So, that is detection part, resolution part we want that this ambiguity function should have a very sharp roll off after the peak. So, that is the choice and for that we will see what are the candidates for that.

And, so, I will say that please compare please compare equation 40 and equation 29 because equation 29 I remember actually I think we saw in the last class now last to last class that it is the match filter output for a moving target, compare this and that will show that except for a phase term due to range you have the equation 40 and equation 29 are similar. So, we can that is why the amplitude of the ambiguity function is taken that both the amplitudes of match filter output and amplitudes of ambiguity function they are similar. In phase they may be different that is why we take generally the magnitude of this.

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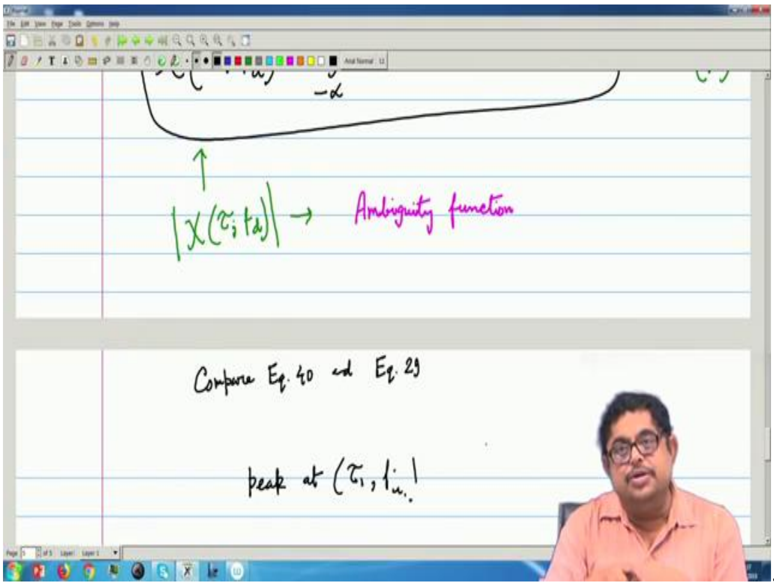

So, a peak at the output of the match filter at let us say that if you look at the match filter output and get a peak at since it is a 2-dimensional function. So, suppose tau 1 and f d 1 you get a peak. So, this means that you have a target at a delay tau 1 and Doppler shift this therefore, the you can easily measure the range and velocity of the target. Now, all this I am saying that we have made our journey almost through that we have seen what is the best filter for beating noise that we found match filter.

But, we have found that it is not only that it also help us to choose the waveform because that helps us to have the resolution that is why I said that the choice what is the waveform choice that depends on what resolution you demand. So, we will further see that how to make this resolutions good and actually there we will see that there is a problem that if I want very good resolution in one dimension the other dimension suffers; that means, if I want to have a very good range resolution the Doppler resolution gets poorer or velocity resolution gets poorer the other way round.

Then, we will find some technique where combinely you can go as you like that will be our next journey. But, another thing I want to say that throughout this though match filter we have assumed that there is a noise because it is against a noise we have found that against a particular type of noise we have found the match filter's SNR, but the whole this ambiguity thing that thing gets affected because there will be noise.

So, according to my transmitted signal what is the ambiguity function then noise will change that. So, that is why the peak of the ambiguity function should be sufficiently high from the nulls of the function; if we do not have that even if we have sharper roll off; sharper roll off improves the resolution, but you should have a sufficient peak higher than the that what we can call the noise floor of the ambiguity function so that even if noise gets increased you have sufficient margin and the target will be detected.

So, but before going there actually we will see that what are the ambiguity functions of various common radar pulses and from that we will try to find out that how we can play with the wave form shape or how you can select the wave form shape, so that you can get a very good resolution that we will take up next. So, we will start from here, actually the theoretical derivation is up to this; from here will now apply one by one those things.

And also after some time we will also see what are the properties of this ambiguity function because that also helps sometimes to get an insight that what type of waveform should be chosen etcetera.

Thank you.