Principles and Techniques Of Morden Radar Systems
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Lecture - 37
Detection in Radar Receiver (Contd.)

Key Concepts: Range resolution of radar from matched filter output (range ambiguity function): analytical expressions

Welcome to this NPTEL lecture on Principles and Techniques of Modern Radar Systems. We were seeing the Detection theory; the first thing that we saw was match filter, in the last class we have seen that match filter output actually is something as a correlation function and for moving target and for RF signals we have found that expression. So, today we will discuss the ambiguity in range and Doppler measurement.

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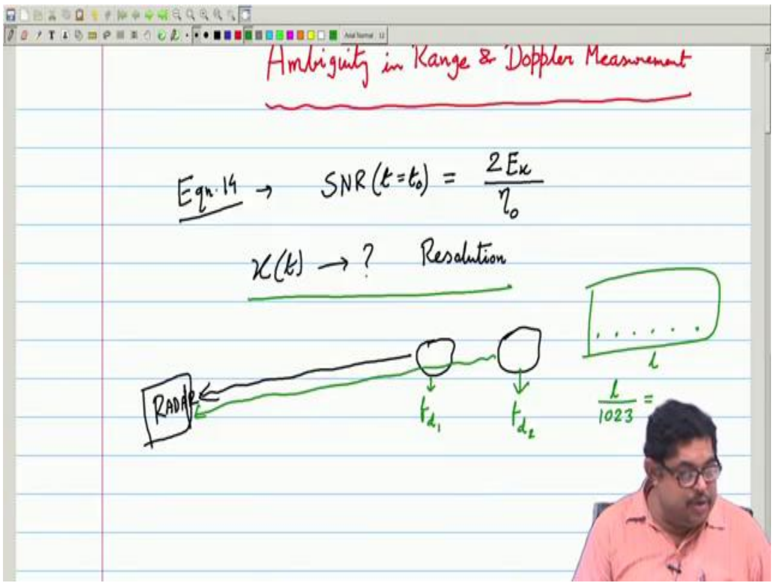

Now, to do that actually we will go back to equation 14, where we have said; so equation 14 if you look back again, so that told us that SNR of the match filter at a time t is equal to t 0 that was given by for Gaussian white noise, 0 mean Gaussian white noise it was given by 2 E x, but divided by the double sided power spectral density of noise.

$$SNR(t=t_0) = \frac{2E_x}{\eta_0}$$

So, then the question naturally is, ok that so that means, wave shape that is not at all important for maximizing SNR and if we keep the E x at a fixed value and take any shape that will do, that will give us that maximum SNR. So, but then; that means, the shape of the wave can be chosen by the Radar designer. So, question is what is the best choice for transmitted waveform? So, since match filter is being used. So, we can say that as long as the E x is same, we can choose anything; but then the designers choose it at anything. So, one of the popular choice for wave shape; that means, the transmitted wave X t. So, the question is what is the best choice? So, that actually is chosen according to the resolution concept.

So, what is the concept of resolution? Resolution means, that if we have two targets, suppose this target and this target and this is suppose the Radar. So, Radar is getting echo from this as well as echo from this. Now at the receiver, the Radar will have to take a decision that whether there are two targets or one target. So, now, suppose I go on decreasing this thing, decreasing that range or difference in any other distance among them. So, how close the two targets can be and still the radar will be able to tell that there are two distinct different targets not same target.

So, the minimum separation of the two target for which Radar is able to distinguish them as two different targets is called the range resolution. So, similarly Doppler resolution, what is the resolution in Doppler? That suppose this target has a Doppler frequency, its return echo creates a Doppler frequency f d 1 and this target its Doppler frequency created is f d 2. So, what is the minimum distance or minimum difference between f d 1 and f d 2? So, that the radar will be able to say that there are two different targets with velocity Doppler frequencies f d 1 and f d 2.

So, similarly this is the concept of resolution. As you see you come across that when you go and buy an camera you say resolution; resolution in case of optical images it is in terms of pixels. So, you say that I want in the linear or length direction I want 1024 dots and in width direction I want 760 dots; that means, that between two dots; that means, if you divide the whole page between 1024 dots, a length you divide between 1024 dots.

So, suppose this dot. So, how many dots you can accommodate here, if you call 1024 that; that means, if the length is l, you can say that l by 1023 this is your resolution; that means, minimum separation between them that the camera will be able to detect as two separate dots. So, this is the concept of resolution. So, Radar also Radar designers chooses this X t from this standpoint and we will see that match filter again helps us to answer that what is the resolution and also that whether there are two targets or one target.

So, match filter is a beautiful thing, it maximizes SNR as per equation 14; but now we have, last time we have come to equation 29 where we have seen that the output of the match filter that actually is some sort of correlation with some phase thing for a moving target. Now we will further proceed So, first we will discuss the concept of range resolution.

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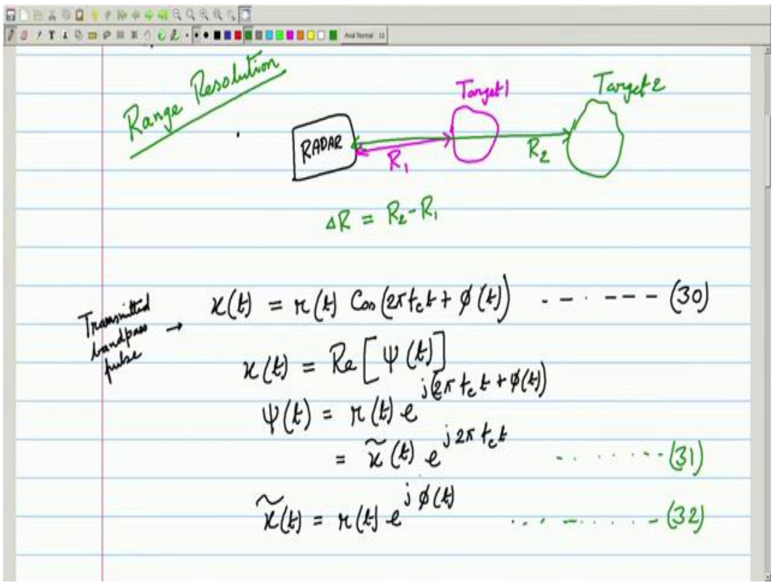

Actually Doppler is making two measurements, the velocity and range. Now, range measurement is direct; that means, from the round trip time it measures range and from the Doppler shift it measures velocity. So, first we will see the range resolution concept; because what I described that is a quantitative description. But finally, we require some quantitative things because we engineers always believe in some numerical values, so we will have to say that, what is the resolution of the Radar sensor?

So, for that we are proceeding. So, what we are doing, first let us say that let us consider first a two echos from, suppose I have a Radar and let me give some different colours; that suppose this is target 1 and let us say this is target 2, the green one target 2. And let us say that both of them are stationary, so they are not moving with time; but from the

Radar their ranges are or suppose this distance is R 1 and this distance is R 2 ok, the two ranges and since they are stationary there is their Doppler shift is 0, for both of them.

So, we can say that the delta R between them is R 2 minus R 1

$$\Delta R = R_2 - R_1$$

and we are trying to find out what is the value of this delta R in terms of the Radars various parameters. So, particularly we are trying to choose an waveform which can give us the best thing. So, for to answer this question, we will have to again see that we are going back to our pulse Radar and saying that the Radar transmitted band pass. So, radar has, so I am saying transmitted signal, transmitted band pass pulse is we know that it is a carrier modulated thing.

So, we can express it as r t cos 2 pi f c t plus phi t.

$$x(t) = r(t) \cos(2\pi f_c t + \phi(t))$$

So, another one. So, let us call that this is our starting point today, we have seen up to equation 29. So, today we are seeing equation 30. So, what is r t? r t is the, if any amplitude modulation is there in the carrier and phi t is the phase modulation in the carrier. So, correspondingly we have seen that, we can write the pre envelope and the relation is that; since this is RF signals we will have to come to complex envelope. So, the intermediate route is the pre envelope. So, we can we know that, this is real part of the pre envelope psi t.

Now, so, what is this pre envelope psi t that we know by now, that I can write it as r t e to the power j 2 pi f c t plus phi t, ok.

$$x(t) = \text{Re}[\psi(t)]$$
$$\psi(t) = r(t)\, e^{j(2\pi f_c t + \phi(t))}$$

And that we know that now the complex envelope will come that x t e to the power j 2 pi f c t. So, this let me call this equation as 31 ok.

$$\psi(t) = r(t)\, e^{j(2\pi f_c t + \phi(t))}$$
$$= \tilde{x}(t)\, e^{j 2\pi f_c t}$$

So, the what is the complex envelope value then, that I can write as the complex envelope x t is nothing, but r t e to the power j phi t. So, this is our equation 32.

$$\tilde{x}(t) = r(t)\, e^{j\phi(t)}$$

Now, so this is the Radar is transmitting and again see that two closely separated targets and in response to this x t, they are the received echos.

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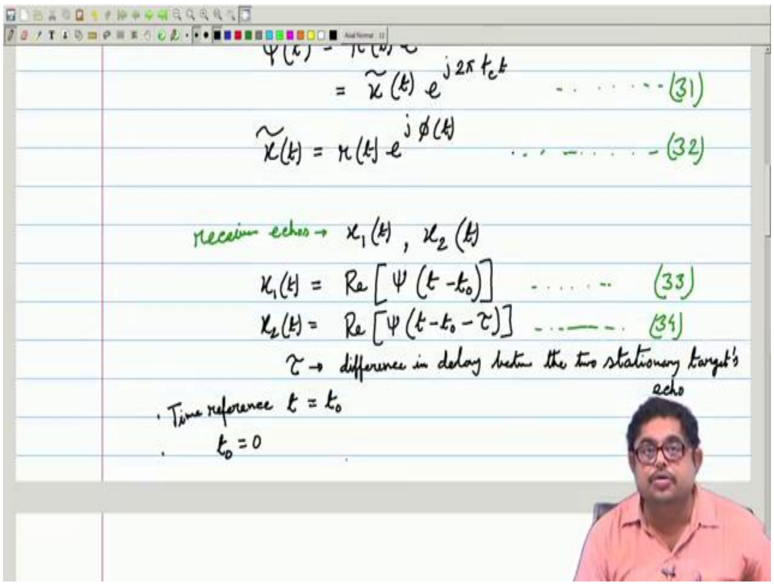

Let me call them, previously I was calling x i t now instead of calling x i 1, I am calling x 1 t and another near closely spaced target, so x 2 t, the complex envelope of the received echos. So or let me say that, this x 1 t, x 2 t is the received echos. We will later put their rating. So, then we can find their pre envelopes. So, we can write that x 1 t, its pre envelope I can call psi 1. So, real psi or psi is the for transmitted signal.

So, it is I can write t minus t 0. So, this thing; that means, it is let us say delayed by a round trip delay t naught. Previously we are calling it x i t, now I am calling x 1 t that is the only difference. So, and I can write x 2 t that is real part of psi t minus t 0 minus let us say tau.

$$x_1(t) = Re[\psi(t - t_0)] \quad \ldots\ldots \quad (33)$$

$$x_2(t) = Re[\psi(t - t_0 - \tau)] \quad \ldots\ldots \quad (34)$$

So, what is tau? Tau is the difference in delay between the two stationary targets echo as simple as that. Now, suppose we make that time reference for my whole case is at t is equal to t naught and without any loss of generality, I may say that let us make t naught is equal to 0; otherwise every time I will have to go on writing t naught.

So, now the question is, that these two echos, you see that the question is these two echo from that the radar will have to say that there are two different targets. So, the question is two targets, what is the smallest value of this difference between their range delta R, so that they will be said by the radar that they are two distinct echos. Now to answer that, let us again find the square error. So, actually we have seen t hat the two echos has pre envelope like this, actually the radar will process those complex envelopes.

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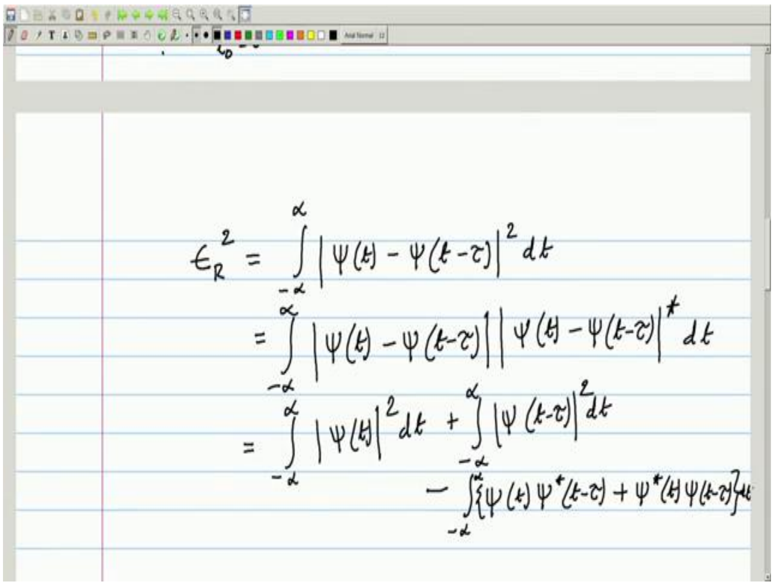

So, let us form the error square error, because x 1 or x 2 anyone may be at greater range. So, you are making square error. So, that is, that minus infinity to infinity psi t minus psi of t minus tau, you see tau naught I have made 0 without loss of generality. So, this, so let me see this error.

$$\epsilon_R^2 = \int_{-\infty}^{\infty} \left| \psi(t) - \psi(t-\tau) \right|^2 dt$$

Now; obviously, if this error is very small, then I will say that no this is the same targets' return ; and if it has a substantial value, then we will say that these are two different things, this is obvious from our common sense we can say.

So, now let us manipulate this square error. Actually this is one of the measure people found that ok, in the pre envelopes there are difference, the magnitude part of that difference and then let us square that because which one is larger that we do not know. So, let us square them and integrate that for all time and then say that ok; if that error is substantial or below a threshold you can say ok, they are same coming from the same target and; obviously, if they are from different targets then the error will be substantial. So, that is ok.

So, let us now manipulate this, that minus infinity to infinity. So, this is a complex number. So, magnitude square, so we know that this we can write as, psi t minus psi t minus tau, then the complex conjugate. So, psi t psi t minus tau star d t and if we go further. So, I can open up now and I think you will agree that I can write it as ok;

$$\epsilon_R^2 = \int_{-\infty}^{\infty} \left| \psi(t) - \psi(t-\tau) \right|^2 dt$$

$$= \int_{-\infty}^{\infty} \left| \psi(t) - \psi(t-\tau) \right| \left| \psi(t) - \psi(t-\tau) \right|^* dt$$

$$= \int_{-\infty}^{\infty} \left| \psi(t) \right|^2 dt + \int_{-\infty}^{\infty} \left| \psi(t-\tau) \right|^2 dt$$

$$- \int_{-\infty}^{\infty} \left\{ \psi(t)\psi^*(t-\tau) + \psi^*(t)\psi(t-\tau) \right\} dt$$

and if I see again this psi t things I think you will agree that magnitude of this square; that means, there, so that can be related to the x t or where is our x t. So, psi t is equal to x t to the power j phi t. So, psi t it's magnitude square can I say that it is equal to x tilde t; that means, complex envelope magnitude square, because this term e to the power term that will have a magnitude of 1.

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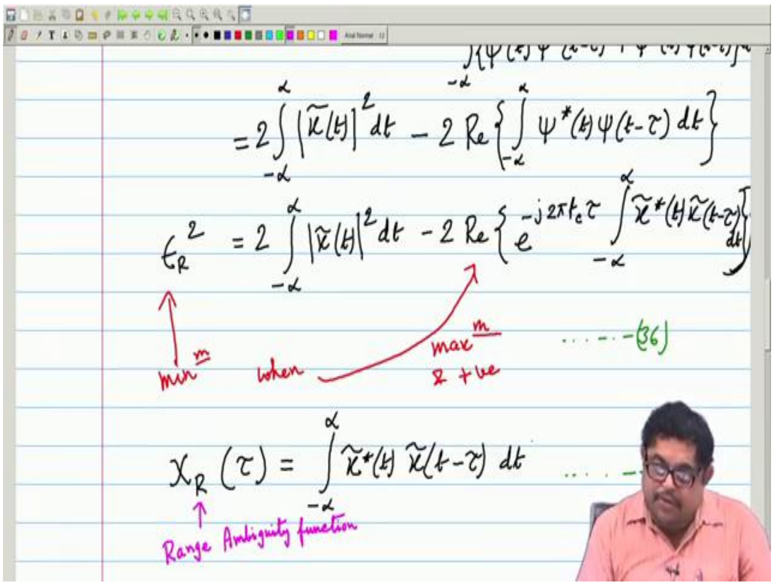

So, I can write this to be same as x t squared d t plus that also will come same, so I can write 2 of these and then this thing as it is minus this; this also you can say a b star plus a star b we know that is 2 of real part, so for complex number. So, I can write minus 2 real then or I have put x here. So, basically this t minus tau d t ok. So, this bracket should be closed and if I change to from psi to the our… this is complex envelope I will have to write.

So, 2 we know the relation between pre envelope and complex envelope. So, I can write it in terms of complex envelope, that it will be actually if you do there will be a phase term coming, simple mathematics. So, e to the power minus j 2 pi f c tau then the integral will come, you can simply do this; because we have the expression you put it, you can get that x star t x t minus tau d t ok.

$$= 2\int_{-\infty}^{\infty} |\tilde{x}(t)|^2 dt - 2\,\mathrm{Re}\left\{\int_{-\infty}^{\infty} \psi^*(t)\psi(t-\tau)\, dt\right\}$$

$$= 2\int_{-\infty}^{\infty} |\tilde{x}(t)|^2 dt - 2\,\mathrm{Re}\left\{e^{-j2\pi f_c \tau}\int_{-\infty}^{\infty} \tilde{x}^*(t)\tilde{x}(t-\tau)\, dt\right\}$$

So, this is our next equation that will be equation 36 ok. Now, the question is, so what is this side let me write it that our that squared error. So, this is equation 36.

Now, you see this x t square; that means, whatever is my transmitted signal waveform, it is nothing but the energy of that; because complex envelope will be the energy of the complex envelope we will be the same as actual transmitted signal. So, this part is constant when a for a given wave form this is constant. So, the square error that will be minimum when this integral part, this second term in the RHS, that is maximum. So, also that needs to be positive. So, I can say that this is minimum when this is maximum and positive.

So, this is the requirement that. Now, what is this second term? It is a varying function of tau; you see actually I should write it this thing is d t and this is this. So, this is an integration of in running variable is t. So, actually this thing will be tau and then there is a phase modulation tied to carrier frequency. You see both these are functions of tau; tau is the delay between the two echos from the two tau now. So, this integral let us define that, this integral, so let me give it a symbol some zeta R, because it is concerned with the ranges.

So, that x R tau, it is a function of tau; what is this, this is that integral portion x star t x t minus tau d t this is our equation 37.

$$x_R(\tau) = \int_{-\infty}^{\infty} \tilde{x}^*(t)\,\tilde{x}(t-\tau)\, dt$$

And this equation is called, this function is called range ambiguity function. Actually it should not be said ambiguity function it is a misnomer; actually it is opposite of ambiguity, it helps to resolve ambiguity. What is ambiguity that; that thing I was saying, that if this goes very high; that means, actually you can see this what is this, the right hand side is basically correlation of the transmitted signal with it is some delayed version.

So, if that is high then we will say that is from the same target; that means, the peak in this ambiguity function that tells us that there is only one target. Whereas, if I get a very small value of this correlation, I say there are two distinct values. So, it helps to do that that is why it is range ambiguity function, later we will understand it more; but now the name is this.

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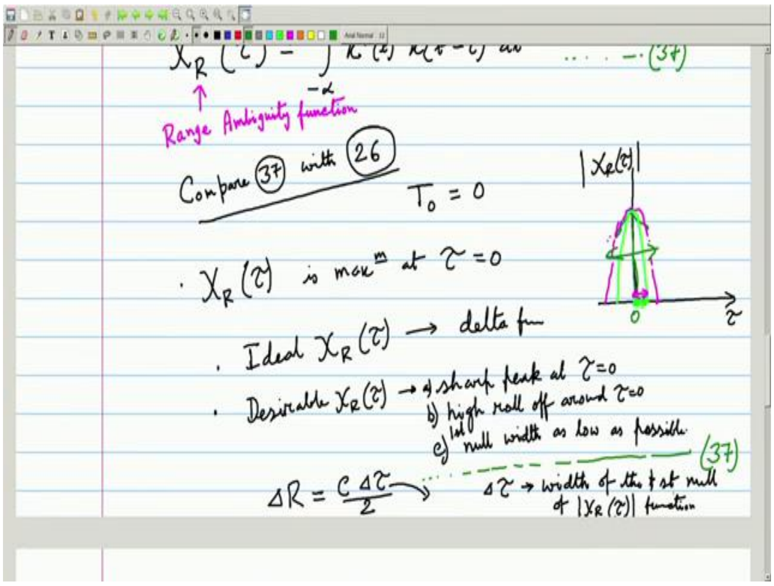

Now you can please compare this with, compare 37 with our, that output of match filter for stationary target; because this is for stationary target we have seen with 26, 26 was the match filter output for stationary target.

And you can see that, compare this you can see, if I put in that equation 26 T 0 which was the total delay that to 0; then output of match filter for a stationary target at 0 range is similar to this, is similar to the range ambiguity function. So, we will see that they have the same envelope, but different phase; this indicates that match filter not only provides maximum SNR at output also is preserves the range resolution characteristic, because I can resolve range by this I can separate two targets by this etcetera.

And also I can say that the, this X R t, it is nothing but a, I can auto correlation function of x, complex envelope of x. So; obviously, for tau is equal to 0 it will have maximum. So, I can say this that, the ambiguity function is maximum at; this we know from our undergraduate knowledge that autocorrelation function for 0 delay that will be maximum. So, when this will come I will say only one target is there; that means, these two targets they are same. So, even if they come from different I will my inference will be, that they are from the same target because autocorrelation is high; but also you see now we can answer that question that what will be the shape of x t.

If this autocorrelation function have a sharp drop, then slightly away from that if there is a target the autocorrelation will be very low; autocorrelation at that time it will be called cross correlation will be very low and I will be saying no, there are two targets. So, ideal I can now say that, ideal this range ambiguity function is a delta function; that means, it will be have a peak, if I plot it at versus tau versus this X R tau magnitude; actually generally in ambiguity function only the magnitude part is plotted, because phase part is complicated and we do not do that.

So, if I get a delta function type of thing, that at tau is equal to 0 I have a peak and then there after nothing, then every range will be resolvable; because just away from that I know that it the autocorrelation will give 0. So, I will be saying no there is another target, but; obviously, in life you cannot have this type of autocorrelation function. So, we require, that it should fall very quickly from 0, so; that means, actually the in real life you do not have that, but what we want, whatever shape I am not saying that, but the closer it comes to 0 that is better.

That means, suppose I have one such shape and another shape is this, so; obviously, this green one I will prefer, because here the autocorrelation function is coming closer here; and resolution is higher for this, because the width is less. Actually this width is the main thing; that means, in till this time that this width; that means, from 0 to coming again to the next null, this width is now the resolution of tau. So, if we put it back into our range resolution equation, then we will get that. So, most desirable shape I can say that practically. So, what we have discussed is desirable X R tau is that sharp peak at tau is equal to 0 and high roll off around tau is equal to 0 and then the null or first null width as low as possible.

Now, here drawing I have drawn this shape it can be anything it could have been triangular, it could have been rectangular anything; but the most important thing is within these zones. So, when I am at this green one, you see that anything separated with by this sorry; anything which has a separation of this any two targets whose separation between them is within these, I will be always saying no it is from the same target. So, the width, null width is the main thing. So, as from the peak; that means, 0 range; that means, 0 separation to the this width of the first null up to that I will say that it is from the one target.

So, I can easily now say that, what is now I can answer that final question, that what is my range resolution? We have said that minimum separation between the two targets that is C delta tau by 2;

$$\Delta R = \frac{C \Delta \tau}{2}$$

where what is this delta tau this is the, this is important understanding this; delta tau is the width of the first null of the range ambiguity function, X R tau function. So, if we can easily find out this correlation function, if I have a given shape of the signal then I can find out that first null and I immediately get the range resolution, I should give it a number this equation.

So, this equation is, I think it is our equation 37. So, you see that from the match filter output, we have got the range resolution equation we have quantified as we said at the start of the lecture, we can quantify; and the range resolution is nothing but C delta tau by 2 where that is the, delta tau is the first null width of the thing.

Now, null is a choice, generally we choose null; but you can have that from the peak you can have 3 dB, 6 dB whatever, what is practical what for that application. So, under something that width will now matter, if it is 3 dB, may be at this point, so this will be then the thing. But, so I will not say null is always width of the first null or width of a threshold level; width at a threshold level that will give us the resolution.

So, I end it here, next day we will see the Doppler resolution concept similar to here; and from there we will come to the concept of ambiguity function in general. Because it should be a 2 dimensional thing, I have both these measurement to make. So, we will come out with the 2 dimensional ambiguity functions.

Thank you.