Principles and Techniques Of Modern Radar Systems
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Lecture - 36
Detection in Radar Receiver (Contd.)

Key Concepts: Analysis of matched filter on band pass signal processing, matched filter analysis for stationary target, moving target

Welcome to this NPTEL lecture on Principles and Techniques of Modern Radar System, we were discussing match filter. In earlier classes we have seen that yes, what is match filter then we have seen that what is the value of SNR for a given type of noise characteristic and we have seen that that is the maximum SNR possible by any filter. Then we have seen that not only that the match filter output is nothing, but the cross correlation between the received echo signal and the transmitted signal delayed by a round trip delay and also some casual filter delay. So, a delayed version of that. That we have seen.

Now, we know that signal is not low pass signal because received signals are band pass. So, we should see the match filter output what the complex envelope of the match filter output how it looks like. Because if it was a low pass filter then we need not have taken a low pass signal, then we have would not have continued this discussion, but we are continuing these because the actual signal is an RF signal. So, it is a band pass signal. So, that is why we need to go to complex envelope domain.

And we are finding that in complex envelope domain is the match filter output is keeping that characteristic that it is the correlation between the received echo signal and the delayed time delayed version of the transmitted signal whether that is true or not. So, that we are discussing for stationary target we have come up to certain point, we are doing mainly mathematics now, but physics I already said that what is the match filter output complex response that we are trying to determine.

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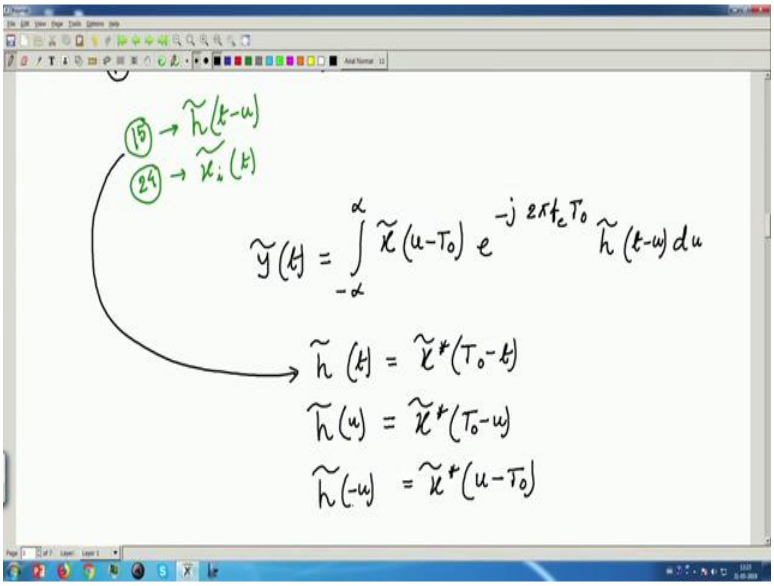

So, we have seen h t, but we will have to make this in terms of this because you see that running variable is this, so this should be inverted and given advancement of t. So, that we will now do mathematically; so, these are a few mathematical details, but I think within your limits because these have been already done in your only I am specializing that to match filter. So, if h t I will have to go to you because running variable is u. So, h u will be what? It will be T naught minus u.

$$\tilde{h}(u) = \tilde{x}^*(T_0-u)$$

Now, what will be minus u?

$$\tilde{h}(-u) = \tilde{x}^*(u-T_0)$$

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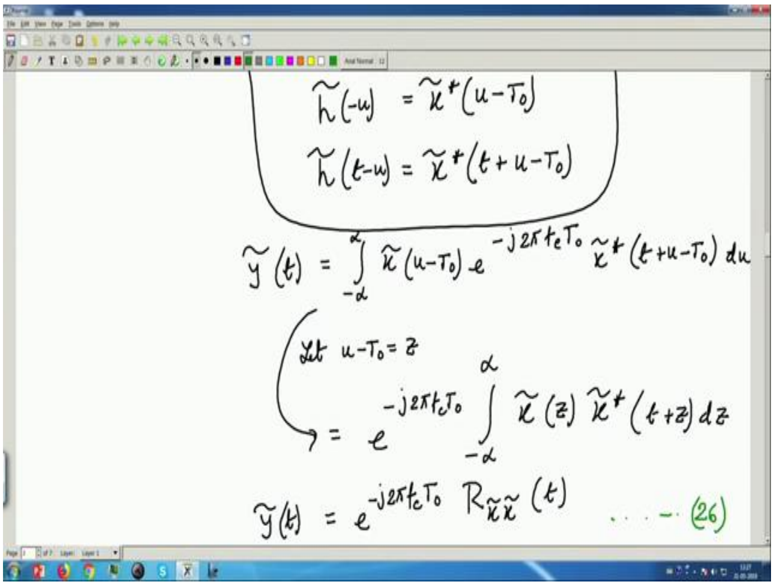

So, I think you will agree that it will be u minus T naught and if you agree then I can now give that shift that it is what will be t minus u that is nothing, but t plus u minus T naught that is all.

$$\tilde{h}(t-u) = \tilde{x}^*(t+u-T_0)$$

So, I can now write; so I will say that this part is my background thing. So, again I am writing what is y t; y t is nothing, but minus infinity to infinity now I will put those values x u minus T naught e to the power minus j 2 pi f c T naught, then I will put this thing that x star t plus u minus T naught, then I should have the integration du, ok.

$$\tilde{y}(t) = \int_{-\infty}^{\infty} \tilde{x}(u-T_0)\, e^{-j2\pi f_c T_0}\, \tilde{x}^*(t+u-T_0)\, du$$

Let $u - T_0 = z$

Now, to simplify this you can those who are experts they can do, but let us say that for everyone is not expert. So, let us say let make an substitution that make this u minus T naught is equal to z. So, that it will become x z ok. So, this is a change of variable and so, d z and d u are same, so we can make that we can continue that y t will be; you can do this and it will be e to the power minus j 2 pi f c T naught because it is nothing to do with the u or z. So, it I am taking it out of the integral and this becomes x also the limits would not change because T naught is the constant finite constant; so, x z and x star t plus z d z.

$$\tilde{y}(t) = \int_{-\infty}^{\infty} \tilde{x}(u-T_0)\, e^{-j2\pi f_e T_0}\, \tilde{x}^*(t+u-T_0)\, du$$

Let $u - T_0 = z$

$$= e^{-j2\pi f_c T_0} \int_{-\infty}^{\infty} \tilde{x}(z)\, \tilde{x}^*(t+z)\, dz$$

So, what it is? It is not only cross correlation now it turns out that for stationary target it is basically the auto correlation of the transmitted signal because x is transmitted signal this is the thing. So, I can write that it is e to the power this part is as it is this part I know because career frequency I know. So, this phase term is ok, but or I will not write it directly in terms of auto correlation I can write that this is R x x of what I will put t time function.

$$= e^{-j2\pi f_c T_0} \int_{-\infty}^{\infty} \tilde{x}(z)\, \tilde{x}^*(t+z)\, dz$$

$$= e^{-j2\pi f_c T_0} R_{\tilde{x}\tilde{x}}(t)$$

So, this is a remarkable thing in this side let me write that what is the match filter output complex envelope that is the auto correlation function of the transmitted waveform. So, beautiful thing you see that if your received signal is really coming from the transmitted signal the auto correlation will indicate you now this I will have to give a name because this is a. So, I think it will be 26.

So, beautiful thing, but I have made a simplified assumption that target is stationary now I will make it moving target. So, all these steps will help me going there.

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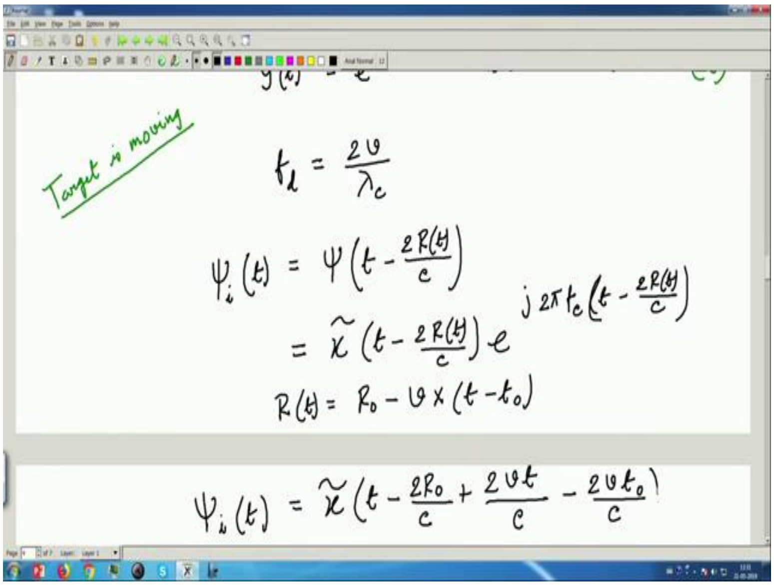

So, let me now remove that restriction of stationary. So, I can write that target is moving. So, no problem this is the general case, so I can say that and if it is having a velocity. So, we know that in that case there will be Doppler shift. So, that we will do that what is the Doppler in this case. So; that means, the received echo is not only delayed in time by t 0, but it also has a Doppler shift. So, that Doppler shift for uniform velocity we know the expression, v is the radial velocity lambda or lambda c whatever you can say lambda c let us say.

Now, what is the pre-envelope of the received signal? So, if pre-envelope of the received signal; received signal that is why I am writing i; that means, input it will be actually input to the match filter. So, this immediately I can write this is from the pre-envelope of the transmitted signal it is the delayed version.

So, t minus 2 R t by c we say this R is not a fixed now for stationary target it was fixed. So, this we have seen can be written as x t minus 2 R t by c and e to the power j 2 pi f c t minus 2 R t by c, ok.

$$\psi_i(t) = \psi\left(t - \frac{2R(t)}{c}\right)$$
$$= \tilde{x}\left(t - \frac{2R(t)}{c}\right) e^{j2\pi f_c\left(t - \frac{2R(t)}{c}\right)}$$

And again here R t value you can put because we know in this case what is the value of R t; R t is R naught minus v into t minus t naught because it is a moving target. So, t minus t naught;

$$R(t) = R_0 - v \times (t - t_0)$$

so, I can continue I can put these values here and continue that what is the pre-envelope, it is x t minus 2 R naught by c plus 2 v t by c minus 2 v t 0 by c. So, this is the time part.

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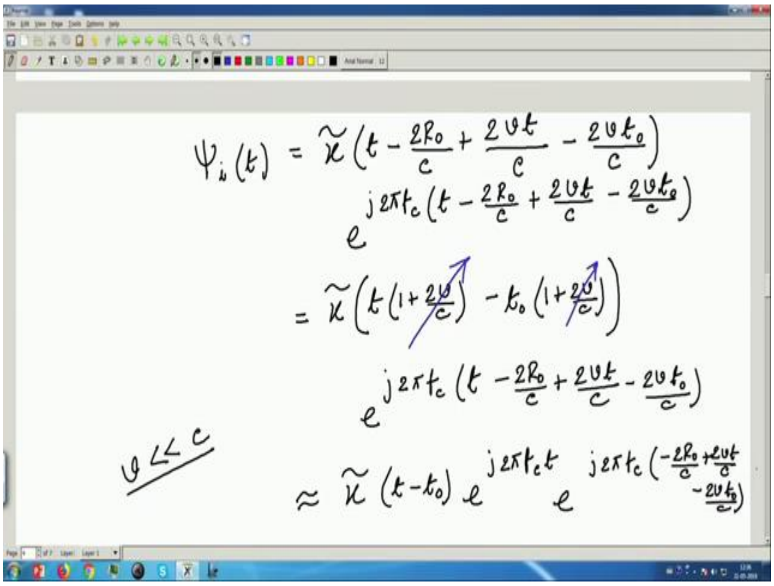

And in phase this is e to the power j 2 pi f c, then t minus this thing 2 R naught by c plus 2 v t by c minus 2 v t naught by c.

$$\psi_i(t) = \tilde{x}\left(t - \frac{2R_0}{c} + \frac{2vt}{c} - \frac{2vt_0}{c}\right) e^{j2\pi f_c\left(t - \frac{2R_0}{c} + \frac{2vt}{c} - \frac{2vt_0}{c}\right)}$$

So, phase will also change; now let us go on manipulating this. So, this is x t, I can take you see there are two time a thing. So, if I take t common 1 plus 2 v by c minus 2 minus again I can take t naught common minus t naught and then that will be again 1 plus 2 v by c. So, this is the time delay part and in the phase part what I can write, I want modified phase part or I will not any part common because that does not give anymore insight into the thing. So, I am writing it as it is, ok.

Now, actually this you see the time actually time is getting scaled. So, instead of time 1 it is actually the scaling factor is 1 plus 2 v by c actually this is the Doppler because scaling in time means nothing, but Doppler, but that is in passing. So, we can write here the in terms of scaling factor you can write, but one thing is that if you generally the for targets this v the velocity of the target that is much much less than c. Even today the fighter aircrafts etcetera they can go up to 4, 5, 6 machs; one mach is sound speed; sound speed is 330 meter per second whereas the, a thing is three into 10 to the power 8 meter per second.

So, you see that generally till now the radar target its velocity is much less than see this is a thing. So, if that is so, then v by c is a small thing, so in the this time delay portion I can remove that. So, I can now put that this part I can neglect similarly here also I can neglect because these, but that I cannot do here actually that I already proved that in phase part that can give raise to lot of phase a thing.

So, that is why with this thing we can say that for realistic targets, we can say that what is the a thing it is x t minus t naught, but I will keep it in the phase; but this time let me take it out. So, e to the power j 2 pi f c t because this is a pre-envelope. So, to getting complex envelope I will have to put in this from the rest I am putting together that is j 2 pi f c into minus 2 r naught by c plus 2 v t by c minus 2 v t naught by c, ok.

$$= \tilde{x}\left(t\left(1+\cancel{\frac{2v}{c}}\right) - t_0\left(1+\cancel{\frac{2v}{c}}\right)\right) e^{j2\pi f_c\left(t - \frac{2R_0}{c} + \frac{2vt}{c} - \frac{2vt_0}{c}\right)}$$

$$\approx \tilde{x}(t-t_0)\, e^{j2\pi f_c t}\, e^{j2\pi f_c\left(-\frac{2R_0}{c} + \frac{2vt}{c} - \frac{2vt_0}{c}\right)}$$

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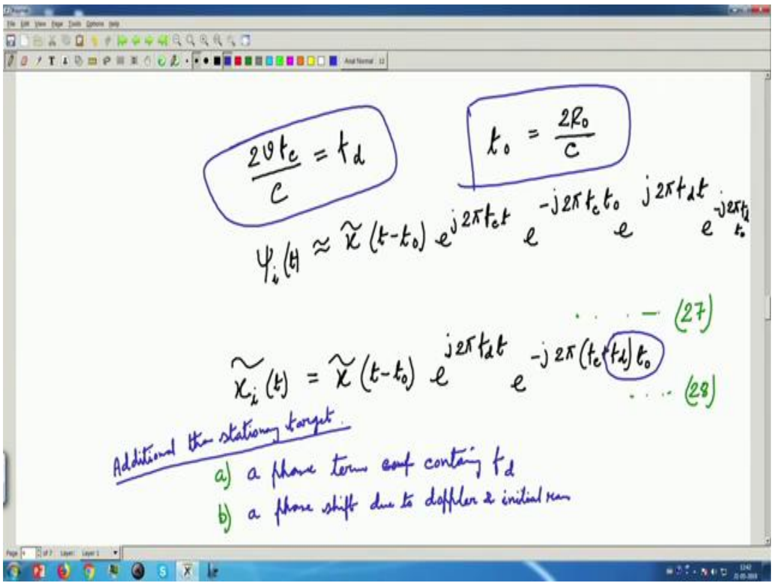

So, now, I will be and here I can easily recognise that you see we have previously seen that 2 this f c if I get multiplied 2 v f c by c, what is this thing? This is nothing, but f d. So, I should put it at we should recognise that this is there and also there is a t naught term this t naught. So, what is the value of t naught? That we will come as another thing that t naught is equal to 2 R naught by c.

So, recognising these two things I can go back that this is x t minus t naught, then e to the power j 2 pi f c t, then here I can easily write that e to the power minus j 2 pi f c 2 pi minus j 2 pi f c t naught e to the power j 2 pi f d t and then another term will come minus j 2 pi f d t naught.

$$\boxed{\frac{2 v f_c}{c} = f_d} \qquad \boxed{t_0 = \frac{2R_0}{c}}$$

$$\psi_i(t) \approx \tilde{x}(t-t_0)\, e^{j2\pi f_c t}\, e^{-j2\pi f_c t_0}\, e^{j2\pi f_d t}\, e^{-j2\pi f_d t_0}$$

So, now I can write the; so this thing I can again since this is the complex first write complex envelope.

So, phi i t that and let me give it an equation number that this is the, this thing. So, now, we can write what is the complex envelope of the received signal. So, x i t that will be x transmitted signal delayed by t naught, then e to the power j 2 pi f d t, then e to the power minus j 2 pi, I can write f c plus f t t naught.

$$\tilde{x}_i(t) = \tilde{x}(t-t_0)\, e^{j2\pi f_d t}\, e^{-j2\pi (f_c+f_d) t_0} \quad \ldots (28)$$

So, this complex envelope that is why, so I have got the complex envelope of the received signal.

So, the; so we can say that look at equation 28 the complex envelope of the received signal for a moving target; the target is moving with constant velocity so, what are the things that it is the delayed version time delayed version of the transmitted signal, but it also have various phase terms and where the information’s are. So, let me read that so, information’s are, a that a phase term additionally what it has? A phase term containing the Doppler frequency.

So, you see that in the stationary case it was not there, but now it has an additional phase term which is containing the Doppler information, then also a phase shift due to a phase shift of. This was there minus 2 pi f c t naught was there in the stationary case where due to the second term is due to Doppler and the initial range; Doppler and initial range this term. So, this is the additional than stationary target.

So, for moving target these are also present. Now, our job is not stopping here we should see. So, this is the new thing that the received signal it is complex envelope has these informations. Now, let us proceed to find what is the what will be the complex envelope of the match filter.

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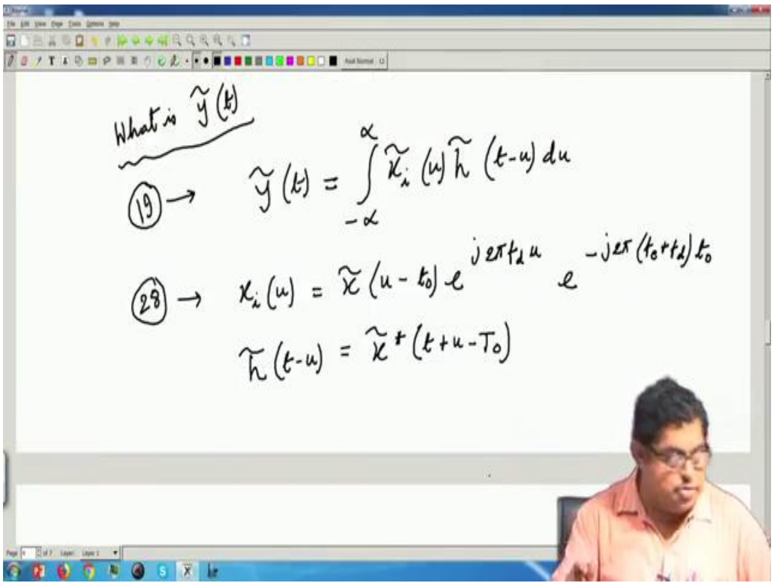

So, for match filter again. So, I am writing what is y t this is we are now again coming back. So, equation 19 will give us again the match filter output what is that we know it in stationary case also we have used the same expression it is minus infinity to infinity x i u h then t minus u d u.

$$\tilde{y}(t) = \int_{-\infty}^{\infty} \tilde{x}_i(u)\,\tilde{h}(t-u)\,du$$

So, this is equation 19 just for repetition I am writing because this will be the starting point. So, now this one what was; so, x i u, so you see x i t I know; so equation 28 I will put here that is ok, but this thing that also we have earlier derived that in. So, I can write those things that equation, from equation 28 I will put x i u value, equation 28 is in terms of x i t I will just specialise that.

So, that will be nothing, but x u minus t 0 e to the power j 2 pi f d u e to the power minus j 2 pi f c plus f d t naught

$$x_i(u) = \tilde{x}(u-t_0)\, e^{j2\pi f_d u}\, e^{-j2\pi (f_c+f_d) t_0}$$

and earlier we have seen that from stationary case expression we know what is h t minus u that can I write it x star t plus u minus T naught.

$$\tilde{h}(t-u) = \tilde{x}^*(t+u-T_0)$$

Please see that in the, I think I can show you where I have derived that, yes this one. So, h t minus u is this; this I am writing because in moving case this is not changing the match filter is not getting changed because the transmitted signal is same.

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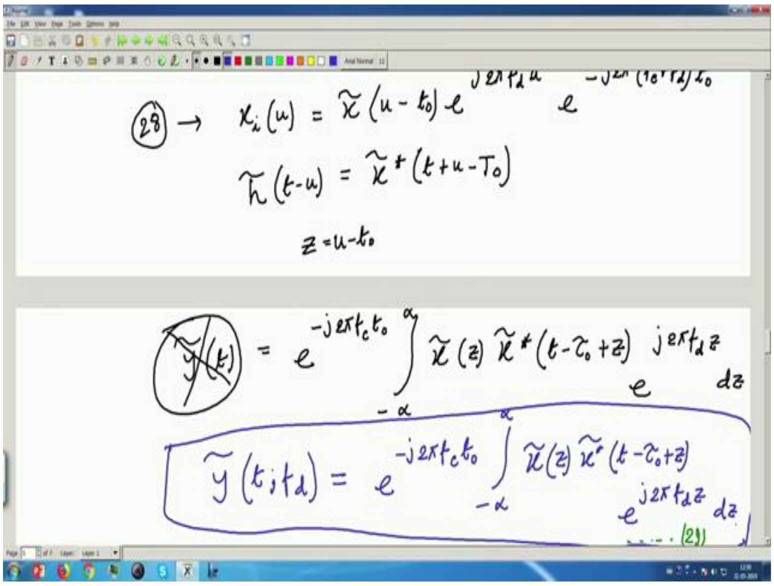

So, I will you can put this and write it as the match filter output y t. So, this will be writing you put that and again you apply the change of variable given in the stationary case that is change of variable again is you make z is equal to u minus t naught. So, what you will get is the thing I am writing the final thing it is very easy because in stationary case we have done that. So, it will be j 2 pi f c t naught this is constant out then minus infinity to plus infinity x z x star z till there then t minus t naught plus z e to the power j 2 pi f d z d z.

$$\tilde{y}(t) = e^{-j2\pi f_c t_0} \int_{-\alpha}^{\alpha} \tilde{x}(z)\, \tilde{x}^*(t-\tau_0+z)\, e^{j2\pi f_d z}\, dz$$

So, again you see that the output is again I can say something like the correlation, but there is the problem that in the phase term there is another Doppler related term. So, I cannot straight away say that it is the auto correlation of x; auto correlation of transmitted signal it has a additional phase term, but also you see that actually this whole thing I should not write is as a function of t only because t is here, but also the f d this term is has come.

So, this f d I cannot say that because it is also a variable. So, actually this thing I should change instead of calling it a 1 dimensional function I should call it a 2 dimensional function. So, I should more precisely write it as y t f d it is a 2 dimensional function and that is nothing but e to the power minus j 2 pi f c t naught then minus infinity to plus infinity x z x star t minus tau naught plus z e to the power j 2 pi f t z d z.

$$\tilde{y}(t;f_d) = e^{-j2\pi f_c t_0} \int_{-\alpha}^{\alpha} \tilde{x}(z)\, \tilde{x}^*(t-\tau_0+z)\, e^{j2\pi f_d z}\, dz \quad \ldots\ldots (29)$$

So, this I should give a equation number because this is a very very important thing that match filter output is not like stationary case this is a moving case. So, it is something related to the auto correlation of the thing, but there is a additional phase term.

So, now, we will actually this was what we are asking, but we will have to discuss something more actually this will give us the concept of resolution etcetera, but to do it actually this thing is called this equation 29 it is a very very important equation in terms of radars. And this actually later we will say that this function is called radar ambiguity function, why it is called ambiguity etcetera.

And also there is a misnomer there, but it is something related to correlation, but we are trying to prove it later we will again come back to this 29 and all the answers of what is the resolution of the radar. That means, if I have very 2 close targets either in range direction; that means, radial direction or the 2 targets velocity are very close will the radar be able to detect that there are two separate targets that is the concept of either range resolution or velocity resolution or Doppler resolution. And all that lies in this equation 29, this radar ambiguity function.

I am not defining now radar ambiguity function that we will do in a future class, but actually we made all these exercise of mathematics to come here that this an unique expression that matched filter output not only just maximises SNR. It also has this detection makes easier or gives us clue that what type of wave form we will choose to have a good detection unambiguous detection. It was not available in communication in radar technology this came, this radar ambiguity function will have further discussion and we will further define what is range resolution, what is velocity resolution etcetera and what is the effect result of this ambiguity function on that, those we will be seeing in the next class.

Thank you.