Principles and Techniques of Modern Radar Systems

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Lecture - 33
Detection in Radar Receiver

Key Concepts: Introduction to target detection theory, Matched filter, Analytical model for matched filter

Welcome to this NPTEL lecture on Principles and Techniques of Modern Radar System. We have discussed in previous lectures we have seen the tracking radar.

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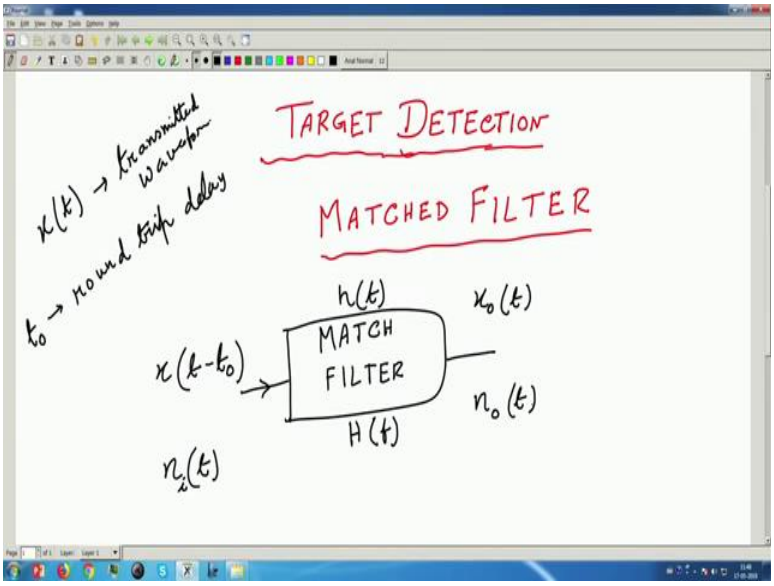

Now, today we will start our discussion with target detection basically the detection theory we will see.

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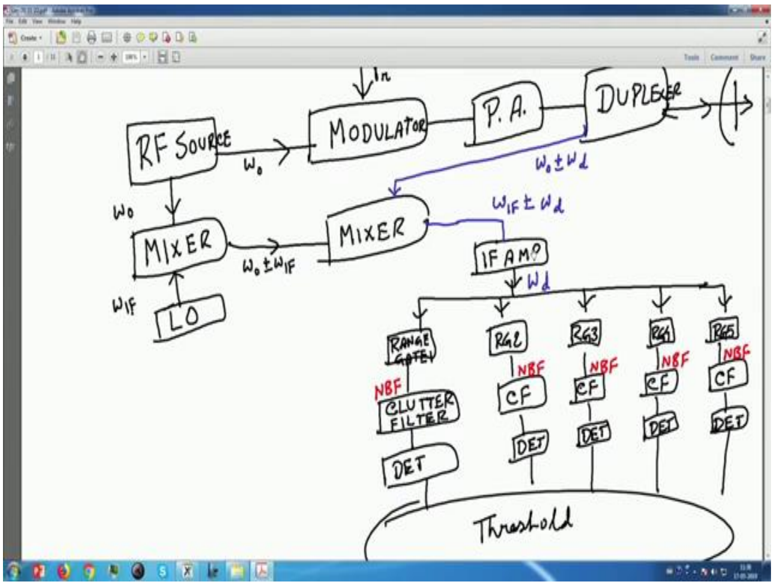

And to start with this discussion I refer you to the previous block diagram of pulse radar pulse radar or pulse Doppler radar.

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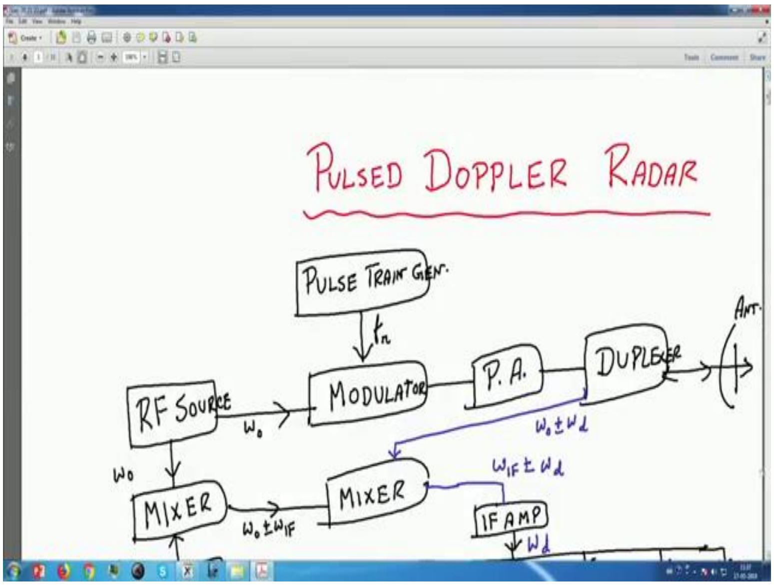

So, this was the block diagram of pulse radar, the same thing was there in the pulse radar. So, there you see that the received signal after the antenna receives it. So, we get it here the here you see the antenna. So, through duplexer it is coming and it is put to mixer. So, RF mixer mixes that and makes the IF signal. Now this IF amplifier we used to say today we will see that what is the IF amplifier implementation actually radar people call these match filter.

So, we will start our discussion from that, that what this IF amplifier is. So, we can say that when the received radar echo signal is put to RF mixer we get IF signal, these IF signal is basically fed to a match filter. Today we will discuss these match filter most of these thing may be known to you from your communication classes, but actually this match filter was invented by radar people because for radar the echo signal that is much weaker compared to the communication signal that goes to the receiver.

Because it is a two way journey for radar where in case of communication it is a simple one way from transmitter to receiver journey. So, that is why we are we know that the signal in the open space that get the it is power gets reduced by 1 by r square inversely to the square of the thing and for radar we have seen that, actually that target's distance if the range of the target is r, then the signal that is received that power is proportional to 1 by r to the power 4.

So; that means, which are very weak signal. So, that is why radar people early radar researchers who were designing that radar receiver, they were bothered with the question that how to detect such in weak signal. So, what is the best possible IF amplifier that we can design and in that process they find out that, match filter is the optimum it is the best possible IF filter because it maximizes the SNR there cannot be any filter better than that.

So, we will start the discussion today the matched filter what is a matched filter and. So, it is we will first see the theory of matched filter, then we will see that how this match filter not only maximizes SNR, it also helps for detection. So, it actually from this analysis we will also get the clue that, how we can make the detection very reliable and also what is the best possible how we can improve the resolution of the this detection thing range resolution. So, those all those things will come from this analysis.

So, I will start the this discussion of matched filter. So, let us say this is the match filter and so, we know that any in electronics if there is a black box we can characterize that; for LTI systems we can characterize it either in the time domain of a characterize that is called the it is impulse response h t, in frequency domain if we do the same thing it is the transfer function H f.

So, let us say the matched filter our matched filter is this characterized in time domain by h t or in frequency domain by H f, we will find out what is the value of this h t or H f ok. So, what is the input to the matched filter? As I said that the mixer output. So, that is here we are assuming that the thing is let us say. Actually there are signal and noise both are there because the transmitted pulse that has gone through the free space to the target and from target again it came back. So, in the process it got corrupted by noise. So, what we get is the this we generally assume in our models that noise is additive. So, weak signal it gets added, but for analysis actually we do it separately because the signal and noise their characteristics are different.

Signal is a deterministic signal. So, it has a functional representation where as noise does not have noise is a random signal. So, it does not have that characterization also in communication you have seen that, various random processes from which random signal get created. So, actually from that random signal I should not say that I can represent noise by any function, but what we can say that at any instant I am receiving a noise.

Let us at our in time instant let us call that value, but; obviously, any random signal at particular time has a value. I cannot know what value it will take, also for signal I do not know what value it will take, but I can describe that either by a function or by a table for noise I cannot do, but if I say that this is n t is not the function representing noise, but it is the sample value at that point in at time t.

So, in that sense we write that. But obviously, this signal deterministic signal that has a we can define power etcetera for that, for noise type of random signals we cannot define power, but we know that from if we know various sample values from that we can find the correlation function and from that there is a Wiener-Khintchine theorem by which we can find it is power spectral density.

So, we will treat them differently in the analysis, but let us say that the in the input I have the signal part of the received thing as if I can separate, but in reality I cannot separate, this is for analysis this is the abstraction that signal part is x t and it is n t. But here I want to say that actually I am assuming that x t is the signal part that was transmitted by the radar transmitter I know that what signal got transmitted. For pulse radar we know it is a RF carrier signal, but it is envelope amplitude envelop is something like pulse whatever. So, transmitted waveform I can say.

And so, I am getting the received signal that I am assuming will be delayed. So, I will have to put that delay. So, it will be let us say the round trip delay what is t 0? t 0 is the round trip delay; round trip delay. So, the signal I am getting is x t minus t naught and here I am assuming that the amplitude is same, but in actual reality that is not same definitely due to this there are we have seen heavy loss in the just in the signal transmission also the path loss is there etcetera. So, the amplitude part is will be definitely. So, this x t and this x t minus t naught they are different in amplitude.

But now I am not going into that details, when we will take this two together that time will take care of that. So, I said that I have received the signal part as this and added with this sample value at that time, when the matched filter will see the this thing that is n t let us say. So, this or I will more specifically call this; this is the output, sorry input. So, n i t this is a sample value of the input noise part that is coming. So, actually the input to the matched filter is this signal part plus the noise part okay and this x t minus t naught. So, this is an analytic signal deterministic signal and this is a sample value of the noise that I am calling sample function and I am writing like this.

So, at the output due to this I will get a signal part will be changed to x output o t and the noise part after passing through this matched filter that will generate a noise like n naught t. So, this is my picture. So, we will now find out what is the expression for this h t or H f. Anyone if I do we can we know Fourier transform. So, we can do that.

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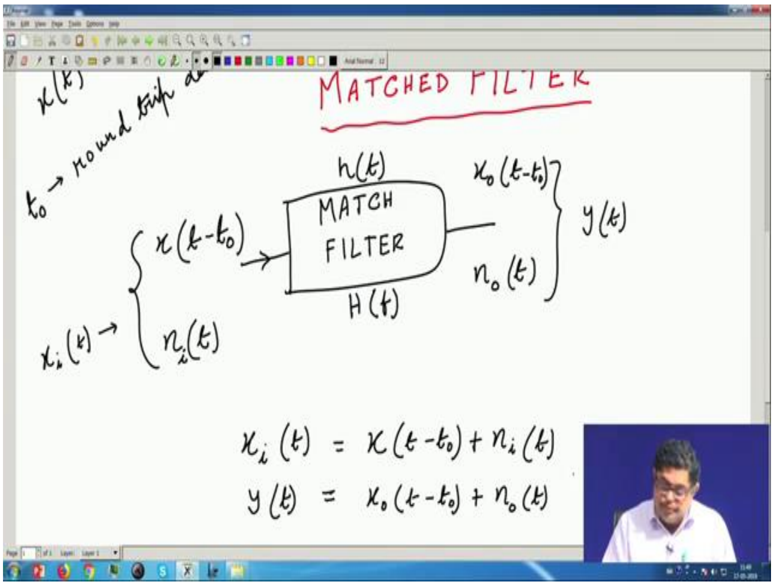

So, I can say that the what is the input signal at x i t so; that means, these two together I will call x i t that is actually x t minus t naught plus n i t. Now, so, and this output; so, this is x t is the input and these two again together will actually will be a sum. So, I am calling it y t. So, that is the output of the match filter y t. So, y t is x naught. So, this will be also delayed because x t is given. So, I should write that also as t minus t naught. So, it is t minus t naught plus the sample value of n that I am calling n naught t.

$$x_i(t) = x(t - t_0) + n_i(t)$$
$$y(t) = x_0(t - t_0) + n_0(t)$$

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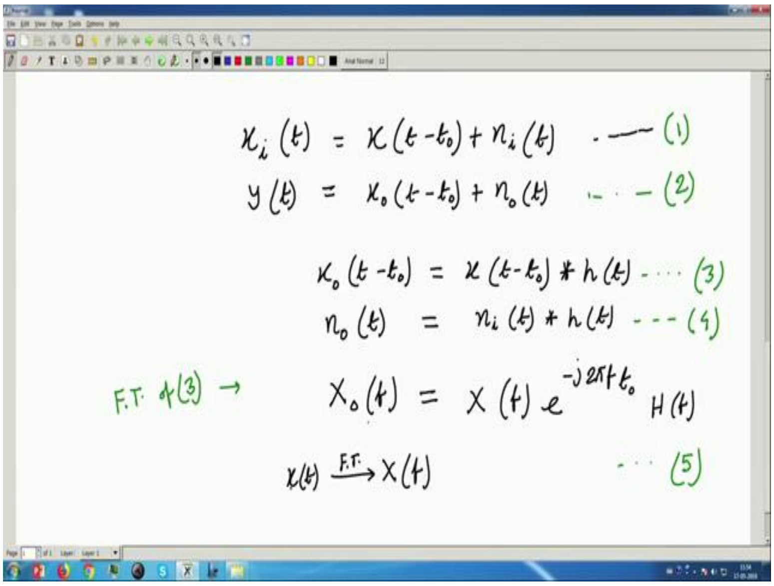

So, now you see that I know one thing if the impulse response is h t of the matched filter. So, from our electronics circuit knowledge, I can say that what is the output this x naught t minus t naught that is nothing, but x t minus t naught convolved with h t and also if I take that liberty that sample values will be also definitely this correlation thing. So, this is I can say n i t convolved with h t.

$$x_0(t - t_0) = x(t - t_0) * h(t)$$
$$n_0(t) = n_i(t) * h(t)$$

So, let me name the equations because some mathematical derivations will come. So, let us make this, this first one let me call equation number 1, the and this one let us say equation number 2. You see I am not writing this because actually in the strict functional sense I cannot write this two sorry this way n naught t is equal to this, actually this does not have any meaning just I am writing that to show that okay something noise values. So, if I take those values and do the convolution I will get the output thing ok.

So, that now what is my sorry this one let me call 2 and this one let me call 3. So, this is my 4. Now I can take the Fourier transform of equation 3. So, Fourier transform of 3. So, what I get? I know that I will get and X o f and that will be Fourier transform of that. So, let us say that we say this the Fourier transform of or here I will write sorry.

So, if I take Fourier transform I will get X naught f and that will be nothing, but X f e to the power minus j 2 pi f t naught then H f.

$$X_0(f) = X(f)\, e^{-j 2\pi f t_0}\, H(f)$$

H t has Fourier transformation H f what is X f? X f is the Fourier transform. So, that sorry x t if I take Fourier transform I get X f. Generally this thing we follow that this X is capital, this is a because their functional relationship is different this x and X so, that is why we make capital that it is a different function that you know.

So, x t X f h t H f I have already said. So, I can write this let me call this equation as 5; that means, we know that if I have a delay. So, in the Fourier transform domain this goes like this. So, now, I can find out that what is the total signal power here because ultimately we are bothered about the after match filter we will find the SNR. So, SNR means signal power divided by noise power. So, at the output we are bothered with that.

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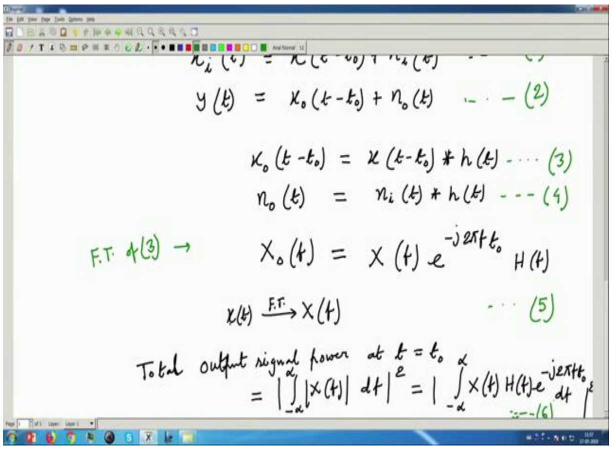

So, what is the total output signal power? Total sorry total output signal power. Now we know that there is a theorem called Parseval's theorem by which we know that if we know the Fourier transform; that means, if I know X f from X f also I can find the power. So, total output signal power at t is equal to t 0. So, that by this I think all of you are familiar. So, that will be nothing, but this minus infinity to plus infinity X f square df or to be more precise I should not say this let us say this square. So, that is if I put the what I got from equation 5, then this is X f H f e to the power minus j 2 pi f t 0 df; df then all this thing and then square.

$$\left|\int_{-\infty}^{\infty} |X(f)|\, df\right|^2 = \left|\int_{-\infty}^{\infty} X(f)\, H(f)\, e^{-j2\pi f t_0}\, df\right|^2$$

So, this I know and let me call this equation as equation number 6 ok.

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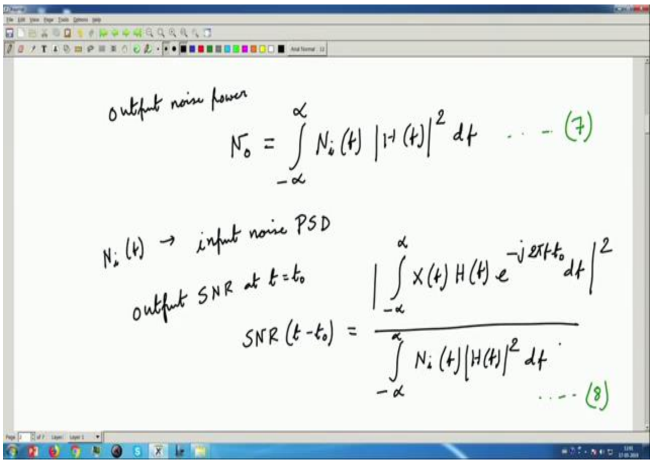

Now, I am interested about the SNR at the output. So, I will also find what is the output noise power that noise power I can write that output. Because noise we can define power through the power spectral density, we know the power spectral density. So, in the frequency domain we can integrate and find out that output noise power that if I call N naught then it is minus infinity to infinity N i f, then it has passed through the matched filter. So, I will write H f square df.

$$N_0 = \int_{-\infty}^{\infty} N_i(f)\, |H(f)|^2\, df$$

Where what is N i f? N i f is the I can say input noise power spectral density this is an analytic thing and I am writing this. Please note that this input noise PSD I am not assuming any characteristic of this noise, that whether it is white whether it is Gaussian all those things, it’s a general input noise PSD that can be always found because you take the various samples of noise either you can take ensemble things; that means, spatial samples or time samples and there are various properties of noise by which we can say that we can find the correlation between them, then we can find the Fourier transform of that and that is the noise power spectral density.

So, N i f if it is there then I can write these. Now again name this equation that this relation between the output noise power and input noise power spectral density that will then come as my equation 7. So, now, I am in a position to find the output; output means matched filter output. Output SNR at t is equal to t 0. So, I will call it SNR t minus t 0 that will be the signal power, the signal power was equation 6 by equation 7. So, I can write that minus infinity to infinity X f H f e to the power minus j 2 pi f t 0 df then I will take the square divided by this that is minus infinity to infinity N i f H f square df then this. So, this is the SNR.

$$SNR(t-t_0) = \frac{\left| \int_{-\infty}^{\infty} X(f)\, H(f)\, e^{-j2\pi f t_0}\, df \right|^2}{\int_{-\infty}^{\infty} N_i(f)\, |H(f)|^2\, df}$$

So, this equation let me call equation number 8 ok. So, now, I recall that what is the maximum value of that etcetera that actually this is this the you can see that we know Fourier transform etcetera those are complex quantities. So, H f is complex, X f is complex, e to the power minus j that is also complex, N i f is complex, H f square that is real. So, now, the question this is real, but this N i f is complex. So, it is a one complex thing by one complex thing. So, that thing also there form is square. So, that is mathematicians have solved this problem that what is the maximum upper bound or what is the upper bound for these thing. So, that is call Schwarz's inequality Schwarz's inequality.

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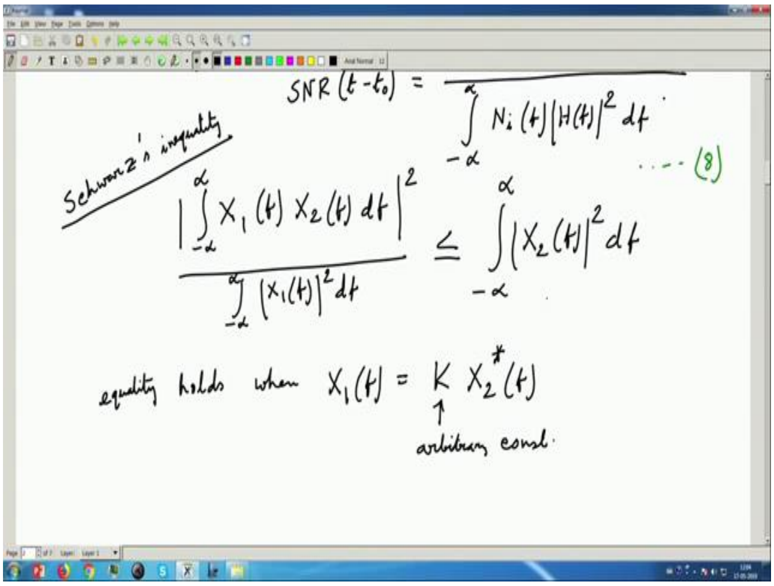

So, we will take the help of that what is the Schwarz's inequality? Schwarz's says that minus infinity to infinity. If you have two complex function one is X 1 f and another is X 2 f and if you have something like this that their product integral of their product square divided by one of that square df; that means, product of two complex functions they are integral then these over the whole range of f and then that integral square divided by the some another integral infinite integral. So, if you have that then you have the upper bound given by these that upper bound is minus. So, this ratio always is has an upper bound that is X 2 f square df.

$$\frac{\left|\int_{-\infty}^{\infty} X_1(f)\, X_2(f)\, df\right|^2}{\int_{-\infty}^{\infty} |X_1(f)|^2\, df} \leq \int_{-\infty}^{\infty} |X_2(f)|^2\, df$$

This is called Schwarz's inequality and he also said that this upper bound gets achieved. So, the equality or upper bound because always this ratio is less than the right hand side. So, equality holds when the X 1 and X 2 they are related by a constant. So, equality holds when X 1 f is some constant K into X 2 f with the conjugate.

$$X_1(f) = K\, X_2^*(f)$$

So, this is a thing. So, here I will say that this is a arbitrary constant so; that means, when these two complex functions. So, they are related like this, then you get that this ratio is equal to this and that is a maximum value. So, we will have to apply this Schwarz's inequality in our case.

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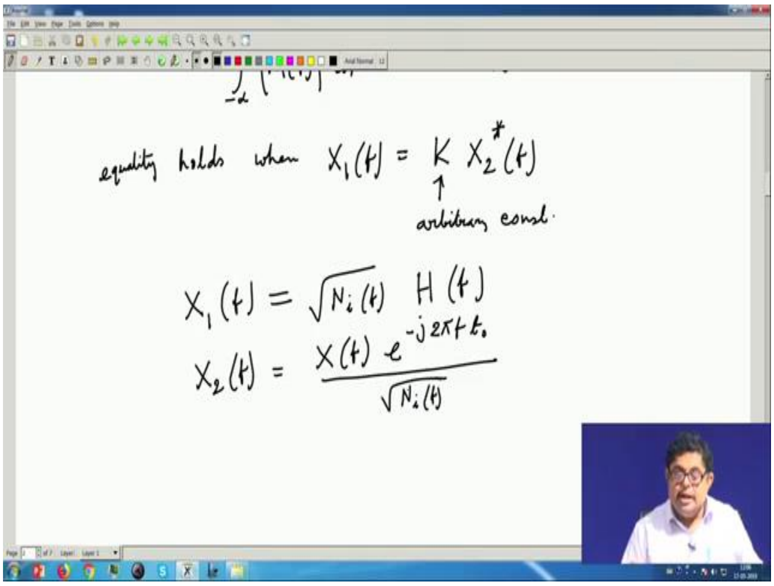

So, what we do? We now make that we will have to from our that relation 8 SNR relation we can choose judiciously this X 1 f and X 2 f. So, X 1 f if I choose like this you see that N i f root and outside H f and X 2 f if I choose as X f e to the power minus j 2 pi f t naught by root over N i f.

$$X_1(f) = \sqrt{N_i(f)}\; H(f)$$

$$X_2(f) = \frac{X(f)\, e^{-j2\pi f t_0}}{\sqrt{N_i(f)}}$$

So, if we have this choice then we know that maximum value of these. So, then you can see that if we look at equation 8 you see that the actually integral. So, this SNR this equation if you see that these expression is nothing, but same as this LHS of the Schwarz's inequality if we choose X 1 2 and X 2 f in that manner. So, now we know that maximum value of this SNR will be what.

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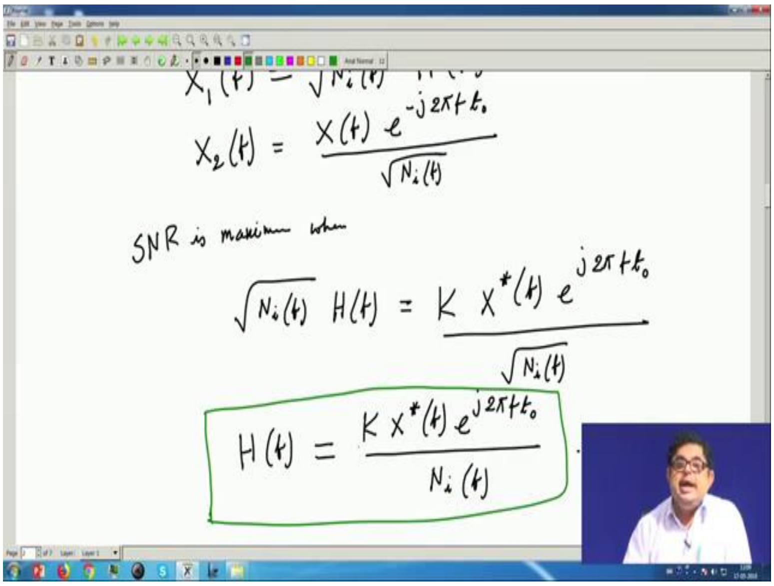

So, we can say that SNR is maximum when root over N i f H f is equal to sum constant K into X star f e to the power j 2 pi f t naught divided by root over N i f.

$$\sqrt{N_i(f)}\; H(f) = K\, \frac{X^*(f)\, e^{j2\pi f t_0}}{\sqrt{N_i(f)}}$$

You see this is a real number that is why we can take this it is ok. And this is the maximum and what from this you see we are getting the famous relation that H f gets determined. So, what is H f it is our matched filter transfer function, but it is H f is equal to K X star f e to the power j 2 pi f t 0; that means, some phase then divided by sorry that will not be I think square root that will be N i f.

$$H(f) = \frac{K X^*(f) e^{j 2\pi f t_0}}{N_i(f)}$$

So, this is a very very important thing and I can now give an equation number to 8, it is equation number 9 also I am making this in highlighted that this will be use. So, we were looking for what is the filter that can maximize SNR. So, this is the filter that can maximize SNR on because I know what is the what signal I transmitted I know. So, I can find out what is the X star f, also I know if I know what is the delay then I can find these and the if I know the noise input noise to the match filter, I know the power spectral density for that N i f then I can find only K need to be chosen.

So, this is an important relation and we have this is actually basis of match filter and once I have H f I can find h t etcetera. So, this actually was our goal. So, we have got that goal that matched filter, we could find and an expression for match filter in terms of the noise input noise coming to the filter and also in terms of the signal. So, that is a remarkable achievement and we will discuss more on this match filter in the next class.

Thank you.