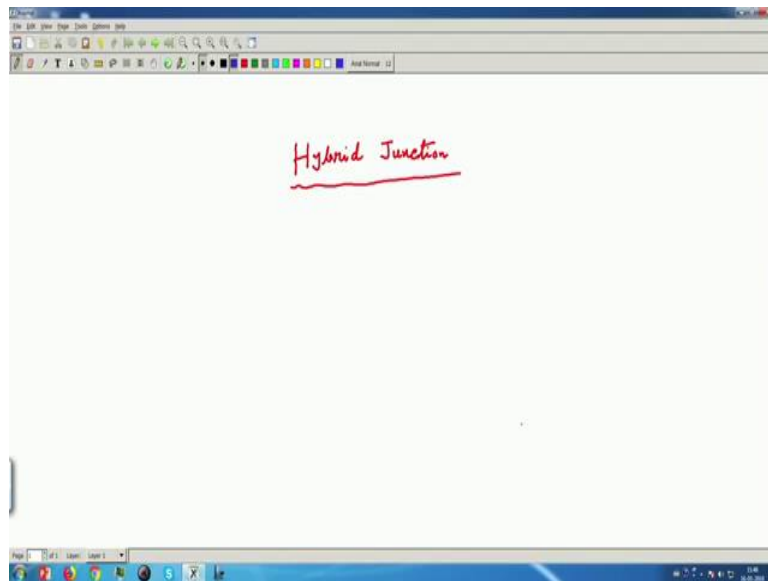


Principles and Techniques of Modern Radar Systems
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Lecture – 30
Tracking Radar (Contd.)

Key Concepts: Magic T, rat race, directional coupler

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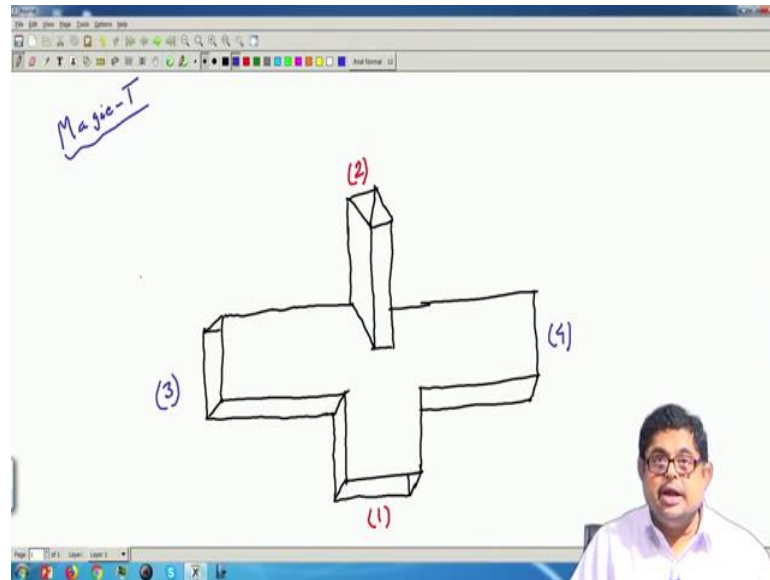


Welcome to this NPTEL lecture on Principles and Techniques of Modern Radar Systems. So, in the last class we have started seeing the hardware components those are necessary to have the mono pulse ratio. So, in that we were we have seen feeds, then we have seen started seeing the hybrid junction.

Now, hybrid junction we have discussed that it is a 4 port device there are two input ports there are two output ports and two properties are required from this hybrid junction one is that if we give the signal to port 1, it should not come to port 2 and it should power should get equally divided between the 3 and 4 output ports, they may be in phase they may be in quadrature, but power should be equally divided. And also we have seen that if two signals with proper phase are applied to the input ports of the hybrid junction two input ports of the hybrid junction, then in one output port we should get sum in another we should get the difference, output will be proportional to this.

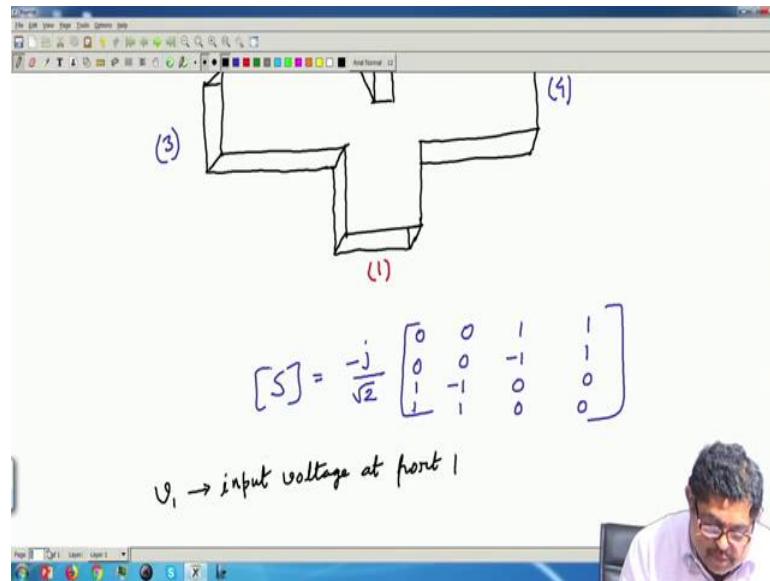
So, now the relative phase between the two inputs that depends on the particular type of hybrid it is either 0 degree or 90 degree. So, now, we will see that some of the examples of this thing.

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So, that first that we will see is called a magic T you know we in undergraduate courses you have seen this also you can refer to the NPTEL lecture on basic components of microwave engineering, there we have discussed the principles of this in detail magic T. There is nothing magic in it; it is basically combination of one E plane junction and one H plane junction and here this is a magic T. So, this 1 and 2 are the input ports, 3 and 4 are the output ports.

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And we know its S matrix, the scattering matrix that is given by minus j by root 2, please remember scattering matrix is the voltage wave ratios reflected by incident. So, that turns out to be 0 0 1 1, then 0 0 minus 1 1 and 1 minus 1 0 0 and 1 1 0 0.

$$[S] = \frac{-j}{\sqrt{2}} \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 0 & -1 & 1 \\ 1 & -1 & 0 & 0 \\ 1 & 1 & 0 & 0 \end{bmatrix}$$

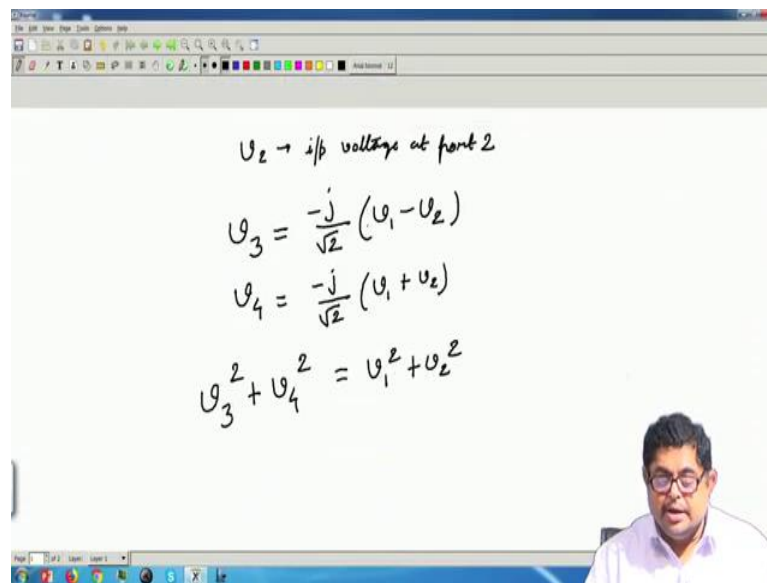
So, this is the S matrix. So, from this S matrix we can say few things that if when the input is given at port 1, we can see that equally that gets divided to port 3 and 4 you see the first column. So, that is if the input is given to port 1 then equally it gets divided between the ports 2 and 4 and nothing comes out of port 2 because of this 0.

Similarly, when input is given to port 2 then also equally divided to port 3 and 4 nothing comes out at port 2, but you see that the in port 3 signal and port 4 signal is out of phase in case of if the signal is given to port 2, if the signal is given to port 1 the signal at port 3 and signal at port 4 are in phase. So, we can say that if simultaneously two signals are given at port 1 and port 2. So, then their sum will be appearing at port 4 and their difference will be appearing at port 3.

So, 1 and 2 inputs if they are in phase then s and d output also will be in phase and input and signal and output signal differs by propagation phase, but that is immaterial because

relatively that does not matter. So, theoretically this is the behaviour, but actually it is a frequency sensitive behaviour because this length of this Ts. So, they are designed for a particular frequency. So, they are not broadband, so if your signal has a bandwidth somewhat larger then there will be problems, but that is a thing. So, let us first discuss the ideal things; so let us say that v_1 is the input voltage at port 1. Remember yesterday we have discussed in detail that this v_1 is actually a complex voltage because this is an RF signal.

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And let v_2 is the input voltage at port 2. So, from the scattering matrix easily we can write what will be v_3 ; v_3 will be minus j by root 2 v_1 minus v_2 and v_4 will be minus j by root 2 v_1 plus v_2 ok.

$$U_3 = \frac{-j}{\sqrt{2}} (U_1 - U_2)$$

$$U_4 = \frac{-j}{\sqrt{2}} (U_1 + U_2)$$

So, depending on polarity convention of v_1 and v_2 , v_3 , v_4 may get interchanged, but we should always have that one output port we have the signal is proportional to sum and in one output port it is difference.

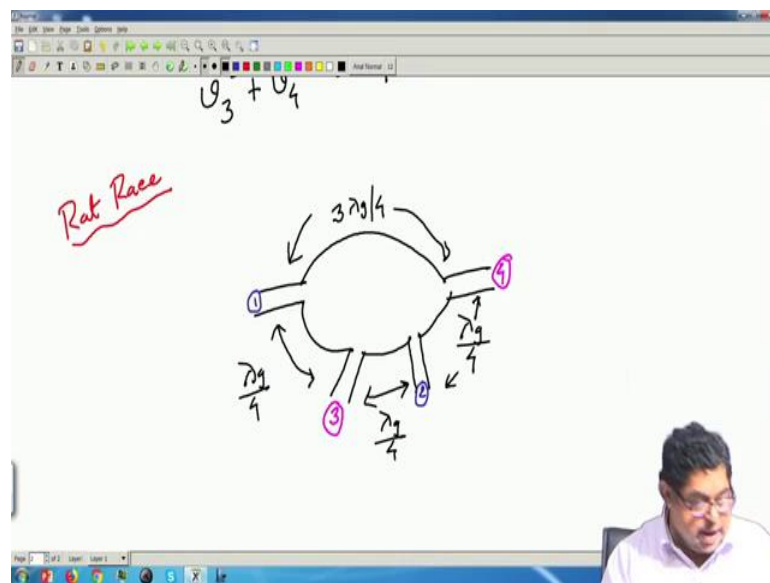
Now, the factor of this root 2 is coming to make the power balance. So, we can check that if we find what is v_3 square plus v_4 square that becomes v_1 square plus v_2 square.

$$V_3^2 + V_4^2 = V_1^2 + V_2^2$$

That means, power balance is there between the input and output that is required because we are assuming a lossless T junction. So, and also the input and output are interchangeable; that means, we can make three four input ports and one two output ports ok.

So, this is one example this you are familiar with the magic T, but magic T has one problem that construction wise this port 4; that means, the plane this port 2 that is in a different plane. So, it becomes a bit more bulky; so in cases where we have planar type of thing it may not be suitable.

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So, the second example we will see that another example of hybrid is rat race. So, this is also example of a hybrid here the in a rat race you know it is a transmission line based structure. So, there are 4 ports let me call this as 1 and this is 2 these are the input ports and let me take some other colour. So, this is 3 and this is 4, now construction wise this distance is generally taken $3\lambda_g$ by $4\lambda_g$ is the guided wavelength. So, and these are all λ_g by 4.

So, if we give signal at port 1 you can see that it reaches, the signal reaches port 4 by 2 different paths 1 is having length $3\lambda_g$ by 4 another is this $3\lambda_g$ by 4. So,

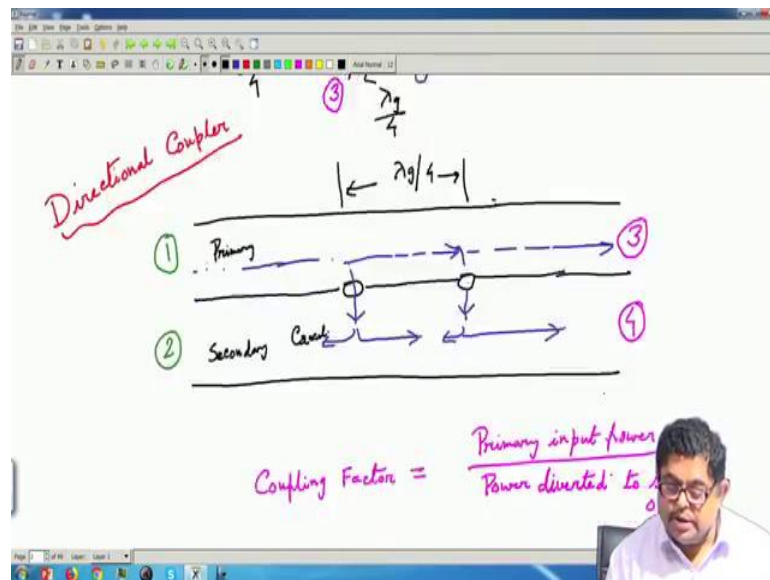
path difference between the two signals that are travelling to port 4 they are 0; so they gets reinforced in port 4.

Similarly, the same signal reaches port 3 by two different paths one path is $5\lambda/4$ by 4; that means, basically $\lambda/4$ and another is this. So, again the path difference is $\lambda/4$. So, that gets reinforced, then it reaches port 2 by two paths the path difference between the two that is $\lambda/2$. So, they cancel out; so nothing comes from this.

So, they are satisfying the property the first property of rat race that if signal is given to 1, it gets equally divided between 3 and 4 output ports and in port 2 it does not come. Similarly, if we give the signal at port 2 then you can check from this that equally they get divided into 3 and 4 and nothing comes out at port 1. And so if same phase signals are given at port 1 and port 2, then you can see the sum will be at port 3 and difference at port 4, so s and d are in phase.

So, phase relationship of rat race you can see explicitly the construction is done at $3\lambda/4$ I mean depending on $\lambda/4$. So, depends on lengths the phase relationship, so it also is a narrow band device. So, this is another problem because actually we will be dealing with pulse, so we require a bit more broader things.

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So, we will see the third example of hybrid that is a directional coupler this also you all know from your study of microwave engineering also you can see that this couplers we have discussed in detail in our NPTEL course on basic building blocks of microwave engineering. Now, what is a directional coupler? A directional coupler is actually; so there are two holes, two openings here and actually this is the primary guide and this is the secondary guide.

And so we can say that the waves they are coming here, so they will come here also one will go on and again here there will be a path here and this wave will go and similarly these waves they come here they also go here and this wave also comes here and goes here and now let us give the names. Let us say that port 1 is here and port 2 is here the two ports and let us say this is port 3 this is port 4.

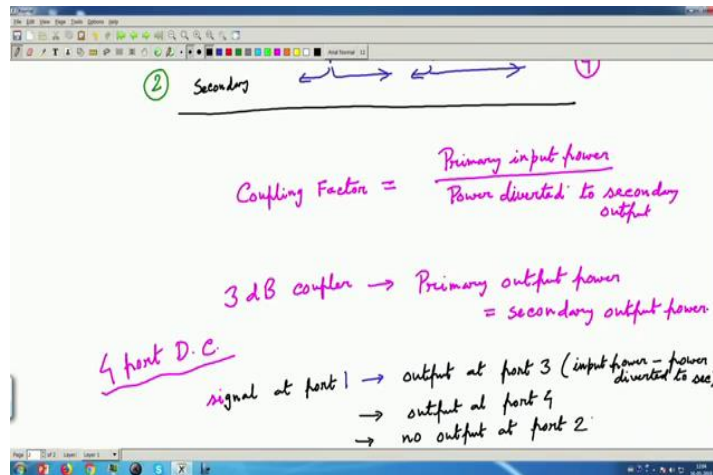
So, what is a coupler? Just I am refreshing your memory that a coupler is a device that taps off some fraction of the power flowing through a primary wave guide or transmission line into a secondary wave guide or transmission line. The coupling factor you all know the coupling this is the characteristic parameter of any directional coupler. So, this is primary input power by the power directed to secondary output; that means; that means output in the secondary guide.

$$\text{Coupling Factor} = \frac{\text{Primary input power}}{\text{Power diverted to secondary output}}$$

So, if I give power here whatever power I am giving input power divided by the power in port 4 because this is the secondary guide.

If I give power in the port 2, then this will be primary this will be secondary. So, that time it will be power given here divided by power in port 3. So, this you know now mono pulse will require 3 dB couplers.

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So, 3 dB coupler means that the primary output power and secondary output power they are you can say diverted this is to be more precise it will be power diverted to this.

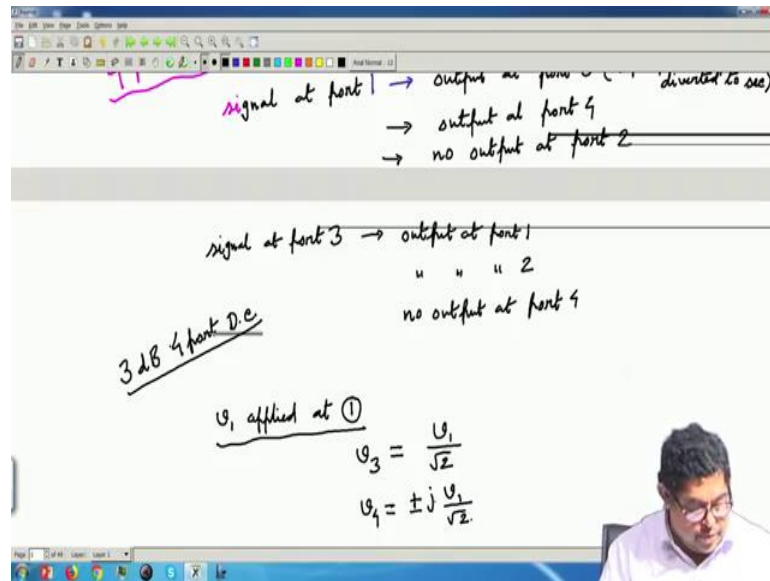
So, 3 dB coupler means the coupling factor is 2. So, primary output power, it shows that primary output power is equal to secondary output power. So, now, a directional coupler, so previously I have defined coupler. Now, what is a directional coupler? A directional coupler is one that produces an output at a given secondary port when power flows through the primary port in one direction, but not in the other direction.

Again I am repeating a directional coupler is one that produces an output at a given secondary port when power flows through the primary port in one direction, but not in the other direction. So, in mono pulse we will be requiring a 4 port directional coupler, in labs I think you have used 3 port directional couplers there also 4 ports are there, but one of the port generally the second port is isolated.

So, only in one you keep the power, but in mono pulse operation the hybrid that we require if directional coupler goes there that will be the four port. So, now, if we give signal say the; so let me write 4 port directional coupler. So, produce output at one primary port and at one or other of two secondary ports depending on direction of power flow in primary path.

Now, if I give signal or let me use this one. So, signal at port 1 suppose I have given the signal at port 1. So, what I will get? I will get output at port 3, how much power I will get that input power minus power diverted to secondary. Also I will get output at the secondary port that is port 4 and since it is directional coupler no output at port 2 because it is directional.

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Similarly, if I give right; similarly, if we give signal at port 3; port 3 then we will get output at port 1, output at port 2 you can easily check from the diagram because I do not know whether I have indicated the construction signal at port 3. So, output at port 1, output at port 2 no output at port 4. I am not able to locate. Yes.

So, here one details I have not given that this is lambda g by 4, then only you will see that in this direction the signal path difference is such that they are getting cancelled. So, I can say if I give power here it is cancelling here and here they are getting added in port 3 it is going. So, this thing I think you all know.

So, now, we will see the for the 3 dB coupler we will write quantitatively this thing that, I said that we require a 3 dB directional coupler, 3 dB 4 port directional coupler. And now when all the ports are match terminated; that means, there is no reflected power from the ports after going to the output it is getting extracted. So, in that scenario let v_1 is applied at port 1; v_1 applied at port 1.

So, we know if we give see port 1, there will be output at port 3, there will be output at port 4 they will be equal because of 3 dB. So, 3 and 4 will get equal power, but what will be the phase relation now you should see that. So, if we see that v_3 that will be v_1 by root 2 again 3 dB power. So, in voltage terms that will be v_1 by root 2 and what will be v_4 ? V_4 will be either plus or minus $j v_1$ by root 2.

$$V_3 = \frac{V_1}{\sqrt{2}}$$

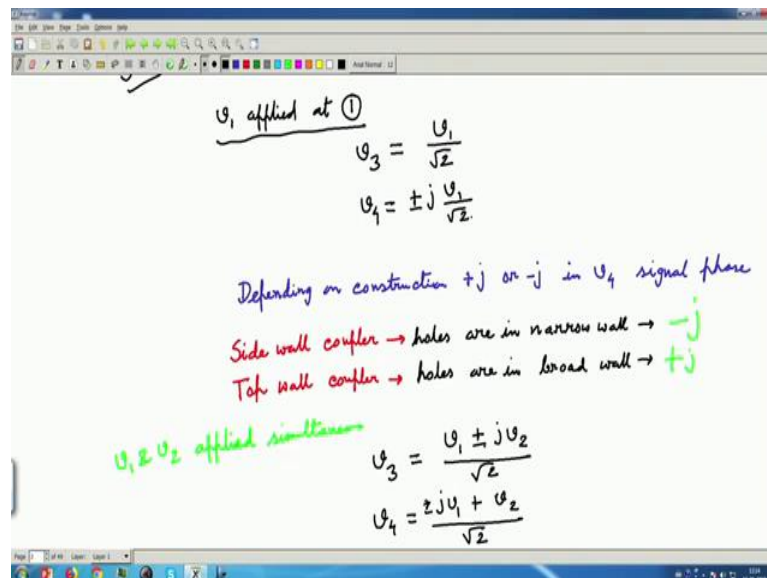
$$V_4 = \pm j \frac{V_1}{\sqrt{2}}$$

Now, who would why I am saying plus j because this is depending on the construction of the coupler as we earlier discussed that in case of hybrids you can have a 0 degree or 90 degree or plus j or minus j type of couplers.

So, depending on construction of coupler this whether you will have plus j or minus j now let us see this drawing. So, you see this holes they are the coupling paths, now the holes if they are in the broad wall or broad wall of a coupler then; that means, holes are in broad wall then we get a plus j; that means, if the in a rectangular wave guide now this is a rectangular wave guide. So, this is the broad wall. So, this or the bottom one.

So, if the holes are there that is a broad wall holes are in broad wall that time plus j will be the phase change and if we the walls are in the narrow walls this blue colour is the narrow wall this side or the back side. So, if the holes are there then also you can have the coupler. So, then you have the minus j phase gets added.

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V_1 applied at ①

$$V_3 = \frac{V_1}{\sqrt{2}}$$

$$V_4 = \pm j \frac{V_1}{\sqrt{2}}$$

Depending on construction +j or -j in V_4 signal phase

Side wall coupler → holes are in narrow wall → -j

Top wall coupler → holes are in broad wall → +j

V_1, V_2 applied simultaneously

$$V_3 = \frac{V_1 \pm jV_2}{\sqrt{2}}$$

$$V_4 = \frac{2jV_1 + V_2}{\sqrt{2}}$$

So, I will write that which sign to take that depends on the construction, depending on construction. So, depending on construction plus j or minus j in V_4 signal phase. Now,

the side wall coupler. What is a side wall coupler? That holes are in narrow wall I have already indicated which is narrow wall of an waveguide, then we get minus j.

On the other hand if we have a top wall coupler, then holes are in broad wall, then we have plus j. So, this choice is open to the designer based on compactness required he can choose either of the drawing. So, now, similar thing we can write for v 2. So, if v 1 and v 2 are applied simultaneously; so v 1 and v 2 applied simultaneously; that means, v 1 is the voltage applied at port 1, v 2 is the voltage applied at port 2. So, what will happen to port 3?

So, v 3 will be v 1 plus minus j v 2 by root 2 and v 4 will be plus minus j v 2 plus j v 1; j v 1 plus v 2 by root 2.

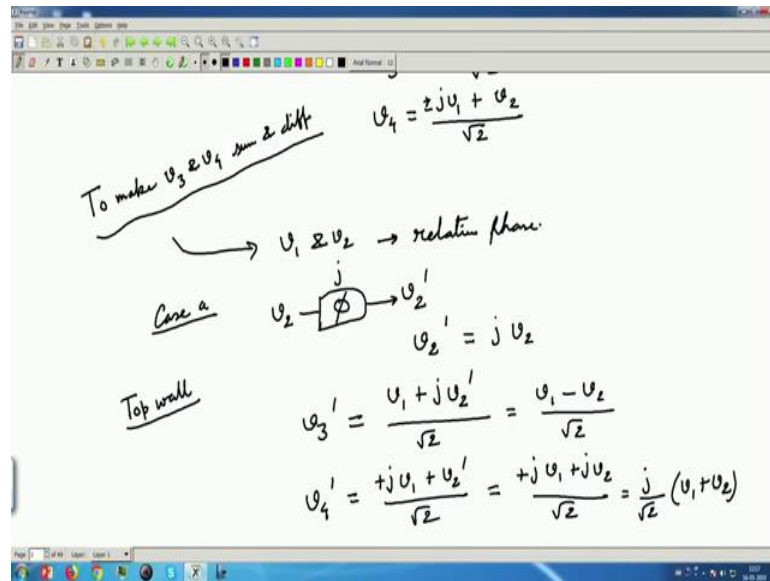
$$V_3 = \frac{V_1 \pm jV_2}{\sqrt{2}}$$

$$V_4 = \frac{\pm jV_1 + V_2}{\sqrt{2}}$$

So, in both the equations depending on the coupler suppose you have a side wall coupler, then in v 3 expression you take this minus sign and in v 4 expression you also should take minus sign here. If you have a top wall coupler then in this expression you have plus sign here and here also plus sign. So, that should be a thing that you cannot interchange that. So, the same sign of the quadrature term applies to both equations.

So, now, you see that v 3, v 4 if we want to make sum and difference, then v 1 and v 2 should have a phase difference between them. This is clear because this is not directly giving you the sum and a... because there is a phase here.

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So, there are various possibilities. So, to make v_3 and v_4 sum and difference type of thing what we need we need that v_1 and v_2 should have a relative phase between them; relative phase. We will see one or two choices suppose case a that let we put a phase shifter of phase shift 90 degree so; that means, v_2 signal that we put to a phase shifter of. So, that will be let us say 90 degree phase and we call that v_2' that we apply to the directional coupler; that means, v_2 is delayed by 90 degree by a phase shifter preceding the input. So, what will be output to port 2 of coupler? So; that means, what is the relation between v_2' and v_2 I can say jv_2 .

$$v_2' = jv_2$$

And let us say that we are making a top wall coupler. So, now, we are freezing the design. So, top wall let us say; that means, plus sign only I will have to take. So, now, I can write in this case what will be v_3' you see I am writing v_3' to differentiate it between this case, that where v_1 and v_2 were in phase here there is a phase difference that v_2 is delayed by a 90 degree.

So, if you do v_3' it will be v_1 plus same as this plus j instead of v_2 I should write this divided by root 2. So, this is nothing, but v_1 plus, but what is v_2' ? j and j ; so that will give me a v_1 minus v_2 by root 2 and what will be v_4' ? So, v_4' dashed

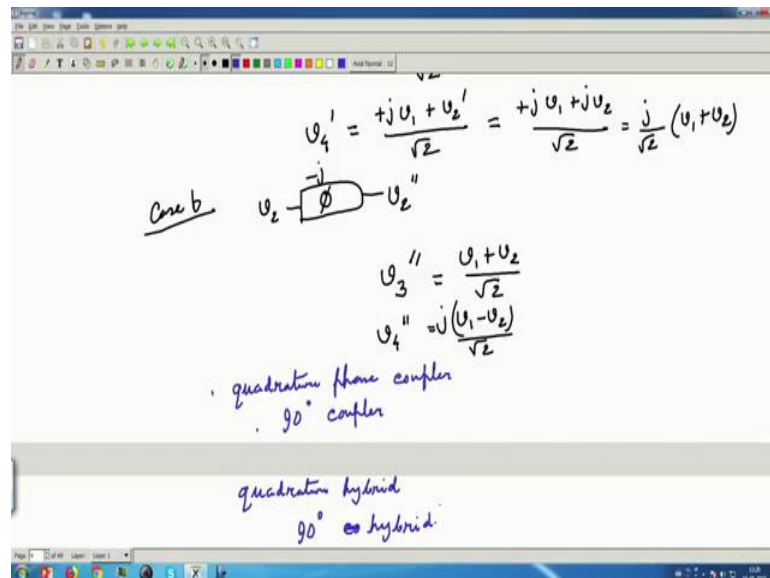
will be that plus $j v_1$ plus v_2 dashed by root 2, now put the value of v_2 dashed. So, plus $j v_1$ plus $j v_2$ by root 2, so it is j by root 2 v_1 plus v_2 .

$$V_3' = \frac{V_1 + jV_2'}{\sqrt{2}} = \frac{V_1 - V_2}{\sqrt{2}}$$

$$V_4' = \frac{+jV_1 + V_2'}{\sqrt{2}} = \frac{+jV_1 + jV_2}{\sqrt{2}} = \frac{j}{\sqrt{2}} (V_1 + V_2)$$

You see now if I have this relative phase between v_1 and v_2 by 90 degree, then the 3rd port is giving me difference and 4th port is giving me sum.

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Now, there can be other choices also you can see that if I have case b, then let us say that the picture is v_2 then we put a phase shifter, but here it is minus j and let us call that v_2 double dashed. So, in this case if you do you will see that v_3 double dashed that will be v_1 plus v_2 by root 2 and v_4 double dashed will be $j v_1$ minus v_2 by root 2.

$$V_3'' = \frac{V_1 + V_2}{\sqrt{2}}$$

$$V_4'' = \frac{j(V_1 - V_2)}{\sqrt{2}}$$

So, whether you want to make the 3rd port as a sum port or difference port these are the two choices you can make also if you instead of a top wall coupler if you take a side wall

coupler the whole thing will change. So, think, but the important point is one of the output ports is giving me a signal proportional to sum another is giving me a signal proportional to difference of the two input signal.

So, that what we are looking for earlier. So, in the directional coupler we can have that and directional coupler does not have like magic T any as any different horizon, sorry, vertical planes. So, it is easier to construct, you can have planar structures of this also. So, this coupler; that means, is called quadrature phase coupler various names of this coupler.

So, quadrature phase coupler or it sometimes is called 90 degree coupler. So, all these are different names of the same coupler. Sometime called quadrature hybrid, sometimes called 90 degree hybrid; 90 degree hybrid etcetera. Now, spacing of the holes that you have seen that since it is $\lambda/4$. So, that λ is calculated for a particular frequency, so it is narrow band.

So, how to make it broad band? People have researched that and found out that if you use more number of holes; that means, instead of two holes you have more holes or if you instead of holes you can have slots cut that frequency sensitivity is reduced and that makes it broad band and so then the device can be used over a useful frequency range you know that the radar pulse is a narrow band pulse, but at least over that pulse the thing should work. So, that people have achieved.

So, we have seen this examples of hybrids. So, we have discussed magic T, we have discussed the rat race and we have discussed the directional coupler of this 90 degree hybrid. So, we saw okay that they can give the sum and difference, but our ultimate thing is that mono pulse ratio that how to make, that we have not till got. So, in the next class we will discuss that how to make using these hybrids phase shifters and other circuitry, how we can design a hardware circuit which can give us a mono pulse ratio d by s or that is a complex ratio or d by $s \cos \alpha$ sort of thing if d and s are real. So, that thing we will discuss in the next class.

Thank you.