Principles and Techniques of Modern Radar Systems

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Lecture - 03
Radar Equation

Key concepts: Mathematical model of radar equation

Welcome to this lecture on NPTEL course on Radar Systems. As we discussed that radar is a complete system. So, we engineers always try to understand any system by doing a model that model we will discuss today, that model is called radar equation.

So, it is a basic model of the whole radar system. Now, it is a very simplified model there are various simplified assumptions in this model. And in the practical situation the situation is somewhat different and various assumptions that we make some of them do not hold there, some of them hold there. So, but the beauty of this model is that it actually ultimately reaches that final real system by various modifying this model.

So, we will see that today we were discussing this model at the starting of the course. At the end of the course we will again revisit this model and we will make various changes in the assumptions of this model and we will be more realistic model will come up. So, actually that is the way by which all electronic systems are studied or basically any engineering system is studied. So, we will start with that radar equation.

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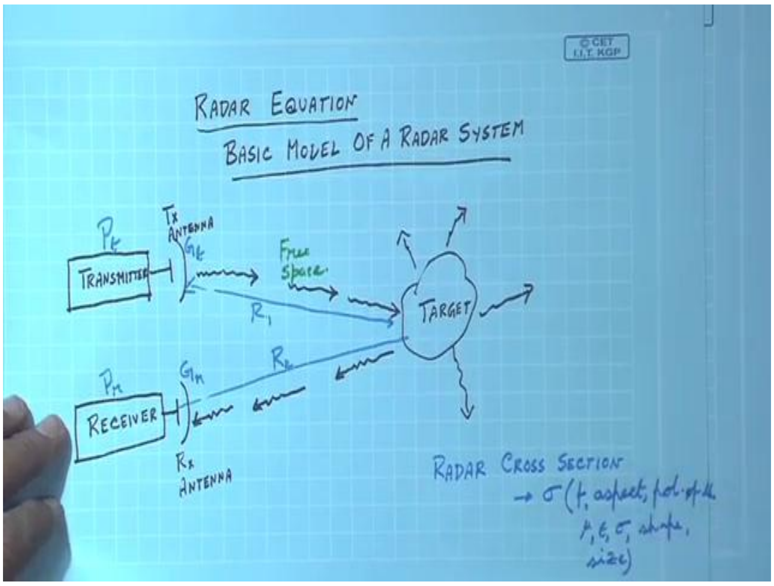

So, what radar equation says that there is a transmitter. That transmitter is giving the power the RF power and that is being radiated to the free space medium. Here it is an assumption that the whole medium is free space, there are no other scatterer etcetera any nearby and we are this transmitter is through this transmitting antenna, a microwave antenna, it is pumping that power to the free space, you see the electromagnetic wave are going it is also going in other direction. But you know the transmitting antenna generally all antennas has a spatial gain.

So, it will try to point to the expected zone of the target, so more power is coming here compared to other directions. So, now it is there is an target, so it is hitting that target. Then as you know that if any object any target if it gets illuminated by the electromagnetic wave, then if it is a metallic target we know it will reflect.

But some portion will still go inside up to the skin depth etcetera, but, but it is even in real metallic things not an ideal metallic things some portion goes inside maybe some portion gets transmitted etcetera. But also what happens the target absorbs the energy and some of that energy it again emits as a source that is called scattering.

So, it is different from reflection, refraction etcetera. So, scattering actually in radar we are interested in the scattering because this scattering gives the inner information of the body, because here the target is behaving as a secondary source. So, some of the information about the characteristic of the target we get from that. So, that scattered energy is now coming back it is also going in other direction, but depending on the target it the receiver.

So, this target also will have some gain type of thing, so it is not that it is an isotropically radiating thing depending on it is shape, size, material, parameters some of the energies in, some of the direction is special direction is getting more sorry angular direction is getting more energy compared to other directions. So, whatever energy is coming that is falling on this radar receiving antenna. There is a receiver.

So, that antenna is taking that power and then putting it to the receiver and receiver is measuring that received signal that is called echo by radar people. And then various measurements that we have discussed various functions of radar are taken care of by processing that echo, so this is the model now.

So, we will say that mathematically if we want to describe this model let us say that transmitter that has a peak power P t. Here I am writing P t, but remember this is the peak power because also there is an this model I am saying though this model is valid for all types of radar.

So, but we will try to understand it later by a pulse radar, because that is easier to understand. Though this model is valid for c w radar also and other type of radar also. Now, so the power radiated power given by transmitter is P t now that is given to the antenna we are assuming here no loss of the antenna. So, generally microwave antennas are lossless.

So, actually loss is very few, so we generally neglect it. Now, that transmitting antenna let us assume that it has a gain of G t. t subscript stands for transmitting side. Similarly, the power received by the receiver let us call P r and this receiving antenna let us say it has a gain of G r. And this distance radial distance from the transmitter to receiver that distance let us call this R. And so, transmitter to target and target to receiver it can be different as we have discussed earlier that if it is a by static radar it will be different.

But let us or let us say R 1 and R 2 are the two distances. And now target when I say scattering actually there is a parameter that we will discuss next that is called radar cross section. Actually radar cross section is some equivalent electrical area of the target, please remember that this is not same as the geometrical area that we will see also it is depend on which direction you are seeing from.

So, the because scatter energy in this direction is different from scatter energy in some other direction also at which frequency you are seeing. It actually is a function of what is the shape of the object, what it is made off, what is the polarisation of the waves that is transmitted and polarization of this scattered wave etcetera. So, on various parameters it depends, but the beauty of this radar cross section it is a parameter for mathematical modelling of the system we introduced this radar cross section. This is a characterization of the target it is generally called sigma it is unit is metre square and in dB scale it is generally called dBSm .

So, it is not the again I am saying it is not the geometrical area, it is the electrical area that the receiver is saying in this direction etcetera. So, now we can start having a mathematical model equivalent to this radar equation that will be called equation. So, you see that transmitter is giving P t power in this direction of target. This target is far away from the transmitter.

So, it is in the far field of the transmitting antenna, so basically plane waves type of thing is illuminating the target. Similarly, the receiver is far away from the target these are all assumptions of this model that it is receiver is far away and in radar that is mostly valid because radars is it at kilometre sort of distances etcetera several 100 kilometres.

So, we can say that R 1 R 2 both are very large compared to the, this dimension of this antenna or the far field which is 2D square by lambda typically you call it. So, this, so what are the then assumptions of this model?

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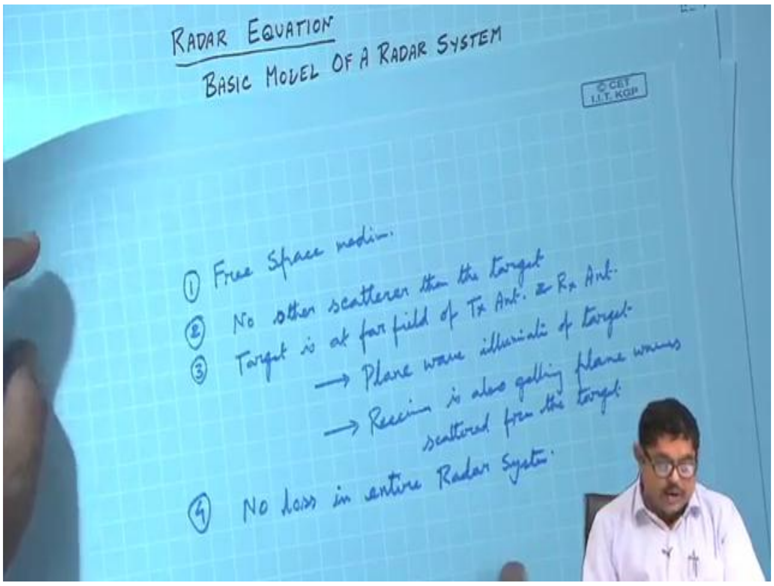

Let me write that one thing is there are medium is free space; free space medium, no other scatterer than the target. The target is at far field of transmitting antenna and also I can say receiving antenna. Now, if I, so basically plane wave sort of illumination this free face, this target is far field that means I can say that basically plane wave illumination.

The first part that the target is at far field of the transmitting antenna that gives plane wave illumination of target and the second part that the receiver is at the far away distance from target. That means, R 2 is very large that gives that the receiver is also getting plane waves scattered from the target.

So, with this assumptions and this G t and G r they shows the spatial gain of the target in these direction; that means, this transmitter and target there is an angular direction. So, in that direction the gain is G t and gain function is G t and similarly the target and receiver they are at a particular orientation from the reference direction.

So, in that direction this G r, if we do this then we can write the, we can relate the knowing this from P t we can find an expression for P r that will help us to connect this whole model mathematically that we will do now. So, if I have a transmitting power P t and that is being radiated to free space by an G t. So, what will be the power intensity received by the target?

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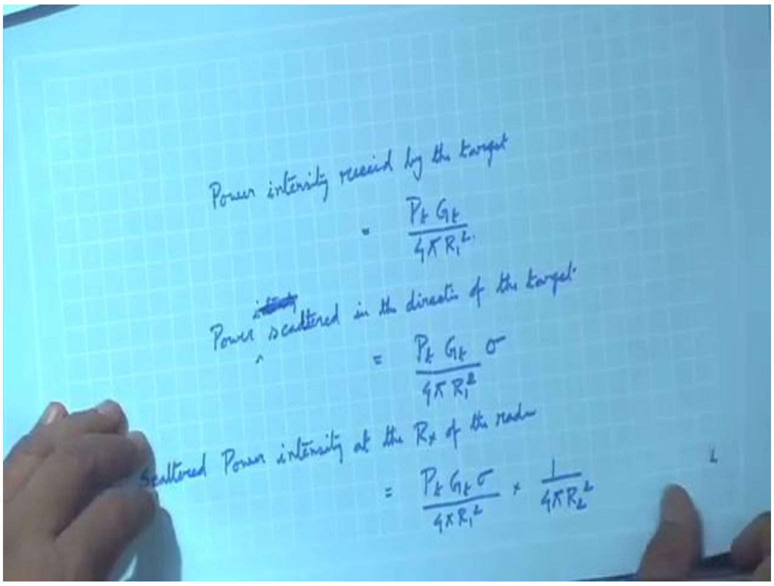

Can I say that power intensity received by the target will be

$$\frac{P_t G_t}{4\pi R_1^2}$$

Now, as I said that target intercepts this power and reradiates or scatters the power in the direction of the receiver and let us say that radar cross section of this target is sigma.

Please remember that this sigma is dependent on the frequency of this transmitting source it is the dependent on the angle that it is making; that means, we call it aspect angle then polarisation of the wave. Then also it is dependent on the material parameters of the body; that means, mu, epsilon and sigma of the target it depends on shape of the target then it is a function of size of the target etcetera. It is function of many things suppose I am I know that value is sigma.

So, I can say that power scattered or power intensity to be specific, power intensity scattered in the direction of the target that will be

$$\frac{P_t G_t \sigma}{4\pi R_1^2}$$

So, this is the power that is sorry this is the it is not intensity. So, at this point I am writing that again in this direction powers that is scattered, so power scatterd not power intensity here. So, what will be the power intensity at the receiver of the radar, so since this distance R 2 is very large.

So, it will be the plane wave, we know plane wave is a spherical wave sort of; it is a approximation of the spherical wave at a large distance. So, we can write that this intensity will be

$$\frac{P_t G_t \sigma}{4\pi R_1^2}\frac{1}{4\pi R_2^2}$$

because that variation we know from our basic electromagnetic understanding that it is a plane wave falling, so it will be like this.

Now, this is the intensity this is intensity this is the scattered power, so, you can also say that this is the scattered power intensity at the received radar. So, we come to the next that how much power now the receiver receiving antenna will take. So; obviously, we know that it has a spatial gain, so into this, but these again actually will be, so the receiving antenna any antenna that also has an effective area that is also an electrical effective area.

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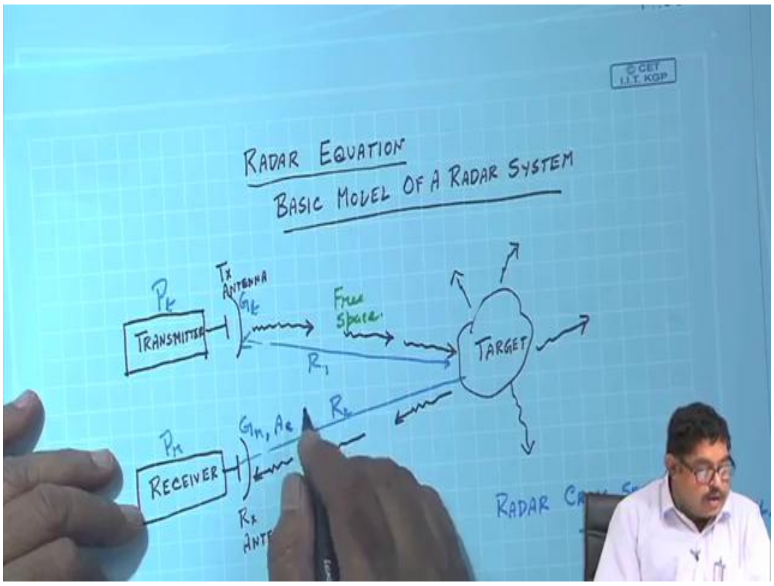

So, we can say that power let that so; that means, here any antennas it is receiving characteristic is called. So, let I am calling it A e, so it is the effective area of receiving antenna.

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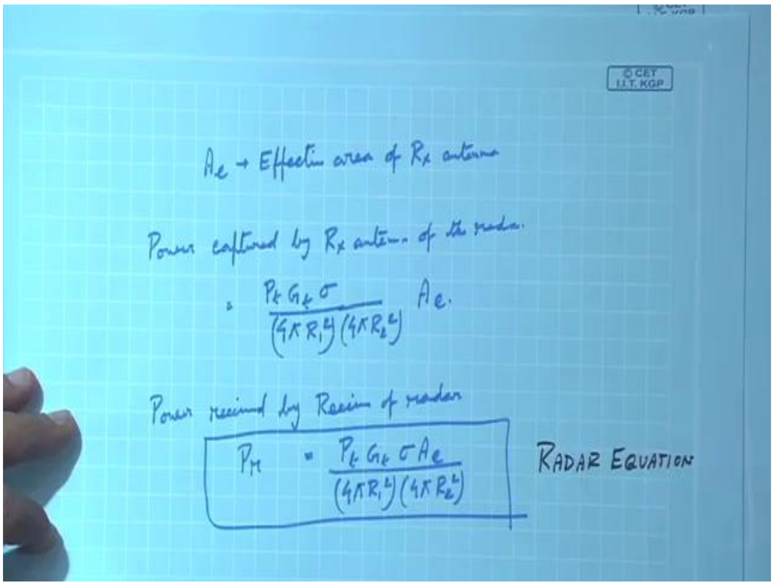

So, let me say now that A e is effective area of receiving antenna. Actually this A e parameter is required for any receiving antenna for transmitting antenna I am not specified it, because it is transmitting, but for receiving antenna we need effective area.

So, already the power intensity at the receiver or I can say at the Rx antenna, because the antenna and the receiver they may be at different places. So, it is basically at the receiving antenna this is the intensity. So, what will be the power captured by the antenna? So I can say the power captured by receiving antenna of the radar will be

$$\frac{P_t G_t \sigma}{4\pi R_1^2} \frac{1}{4\pi R_2^2} A_e$$

we assume that this again is a microwave antenna lossless antenna.

So, this whole power will be given to the receiver, so I can say that power received by receiver of radar is

$$P_r = \frac{P_t G_t \sigma A_e}{4\pi R_1^2 4\pi R_2^2}$$

So, this we know that from our if we look at this diagram we call that power received by the receiver P r, so now we can say that P r is this. So, we see now mathematically everything is related, so this is the radar range equation.

Now, here people you can further manipulate this, but this is the basically radar equation. So, because this expresses what these whole block diagram express the same thing is expressed here. So, this is the physical model this is the mathematical model of the radar, this is called radar equation radar equation.

Now, we see we will just go on making some more assumptions, but those are specific to some radar. But, this is the fundamental one because the fundamental assumptions of this thing is that the whole medium is free space though we know that in reality that may not be the case. No other scatterer than the target; target is at far field of transmitting antenna and receiving antenna.

You can also say that actually there is another assumption that no loss in entire radar system; that means, there are no loss in the transmitter receiver transmitting antenna, receiving antenna. So, this is also a lossless system, but you see that if I write like this there is a spreading loss etcetera because that is why you are getting all this mathematical things.

So, spreading loss is assumed in free space because whenever a spherical wave goes it has a spreading loss that loss is incorporated, that is why this 1 by 4 pi R 1 squared, 1 by 4 pi R squared that variation actually gives a loss, because as I go more and more I am the power intensity is reduced, so that is in the form of a loss.

But that is not loss in the system, so the lossless system is assumed for the radar. So, under this 4 assumptions this radar range equation that is this radar range equation or radar someone calls radar range why I will come just now, but it is a more correctly should be called radar equation ok.

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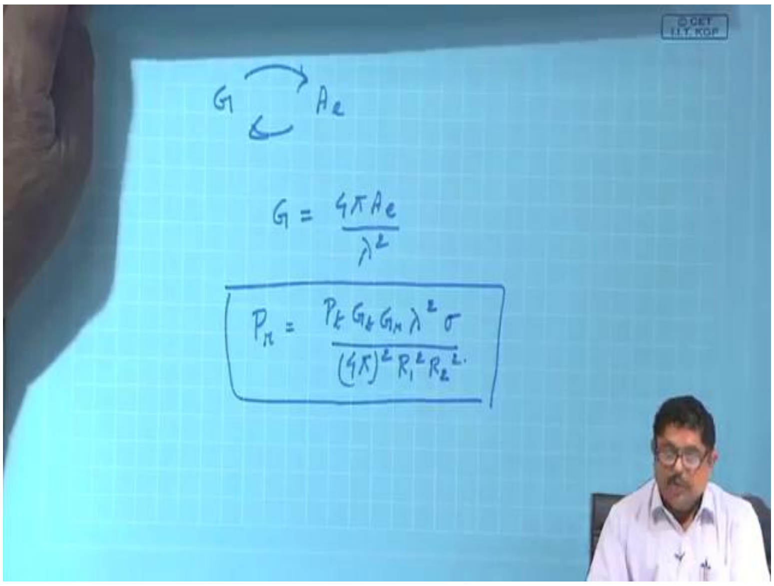

So, this model we have seen. Now you see that for any antenna the it is transmitting gain G and it is effective area A there is a relation. So, G and A they have a relation between them, that relation is I think in antenna classes you have seen that relation is

$$G = \frac{4\pi}{\lambda^2} A_e$$

Where, lambda is the electromagnetic wave's wavelength A e is the effective area. So, we can express this receiving antennas A e in terms of it is own transmitting gain G r. So, if you do that just put this.

So, you will get the expression for P r is.

$$P_r = \frac{P_t G_t G_r \sigma \lambda^2}{(4\pi)^3 R_1^2 R_2^2}$$

This is one; that means, this equation actually gives you this equation if you take this help of this that is one thing we can do. So, this is another radar equation.

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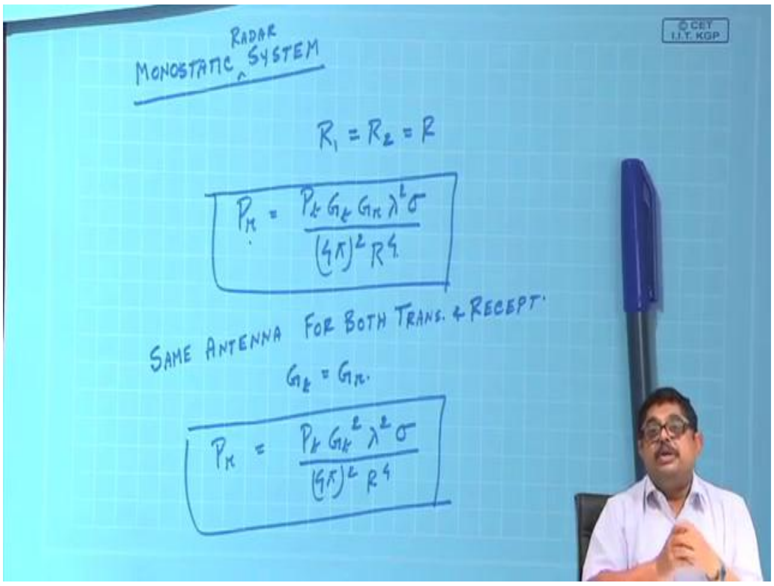

Now we can make a another assumption that if we have a mono static system, we have discussed this thing in previous classes mono static sys radar system mono static you can say radar system. That means, what that receiver and transmitter are co located, if receiver and transmitter are co located then R 1 and R 2 what was R 1 and R 2? R 1 is the distance of the target from transmitter R 2 is distance of receiver from the target.

So, they becomes equal if you do that the radar equation becomes P t G t where is the last equation. Last equation was this

$$P_r = \frac{P_t G_t G_r \sigma \lambda^2}{(4\pi)^3 R^4}$$

So, this shows us that you see power received, so this is again a radar equation power received is proportional to 4th power inversely proportional to 4th power of the distance. Now this is a very severe it has a lot implication, because now we are saying you see in a communication system the when the receiver receives the power it is just 1 by R square if receiver is far away from the transmitter in a communication system.

But in the radar actually it is a two way journey, because the signal is going to the target the target is taking that and scattering back and then again it is travelling the same path that is giving a R to the power 4. That means, a power received by the receiver of the radar is very very small compared to a communication receiver whatever power it receives.

So, the challenges here will be extract the information from that weak signal and you know that actually we have assumed that there are no noise etcetera in the system, there are no losses etcetera, but when noise will be present then the noise will be comparable to this receive system. So, eliminating that noise understanding that there is a signal and then extracting that information there are huge challenges for that. That we will see later, that is why the radar needs a much more sophisticated techniques than the usual communication systems that is one.

Another assumption we will see that generally in a radar; radar is a costly system and if you have a mono static system. Then why you and you see the actually that will see in your assignments etcetera that the costliest part of the radar is the antenna. Though antenna is a passive device compared to transmitter it seems that what I am saying, but actually you will see that the transmitter is a high power device transmitting.

But the electronics is well developed there, where as antenna that has various challenges that is why making an antenna that is the one of the it will say it is the most costly equipment. Though it is nothing it some metals etcetera, but it is actually the main engineering knowledge require is required to design this radar antennas.

So, that is why people say that why we will have two different antennas, because that will add to the cost instead we will use the same antenna and we will try to use the same antenna and in the some extra circuitry, so that we can multiplex between transmitter and receiver. If we do that then; that means, I am saying that same antenna, because when I am sending the signal I am not listening unnecessarily because that time I cannot listen also.

Because it is a huge power it is getting transmitted and there if I want to hear for that look signal it is no meaning that is why radar keeps its eye, keeps its ear closed that time. And when it stops sending that is way you it starts sensing or listens to the echos etcetera.

So, the same antenna for both transmission and reception, if you do that then I can say that G t and G r both are the transmitting gain of the antennas, G t was for transmitting antenna, G r is for the receiving antennas transmitting gain. So, they becomes equal if it is there, then P r

$$P_r = \frac{P_t G_t^2 \sigma \lambda^2}{(4\pi)^3 R^4}$$

So, this is another radar equation. Now from here I can say I will be deriving various parameters of radar basic parameters and by that we will start gaining an understanding of these. But you see that unless and until we have this model we could not say that what will be the those parameters.

Because now, one of the parameter is radar is for ranging, so what is the maximum range a radar can make. Now, that unless and until I have modelled this in a mathematical form I could not answer, but now from this model I will be able to answer that. Actually like that I can say that not only that what is the range also how much power is required to have certain range how much gain of the antenna is required given the power all these questions now I can answer.

Also we will see that when I am doing the measurement I need to say with confidence that whatever I am measuring is correct, so there are some ambiguity in measurements any measurement has some ambiguity. So, there will be some ranges which are called ambiguous range some ranges are called unambiguous range etcetera. What is the minimum range what is the maximum range?

So, all these questions now I will be able to answer, because I have a model of the mathematical model of the whole thing. First a physical model and engineering physical model, then a mathematical model and now I will be answering that that we will be see from the next class.

Thank you.