Principles And Techniques Of Modern Radar Systems
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Lecture - 22
Tutorial Problems on CW and Pulsed Radar (Part II)

Key Concepts: Tutorial 4

Welcome to this NPTEL lecture on Principles and Techniques of Modern Radar Systems we were seeing the we are doing tutorial. So, Problems on CW and Pulse Radar this is the Part II.

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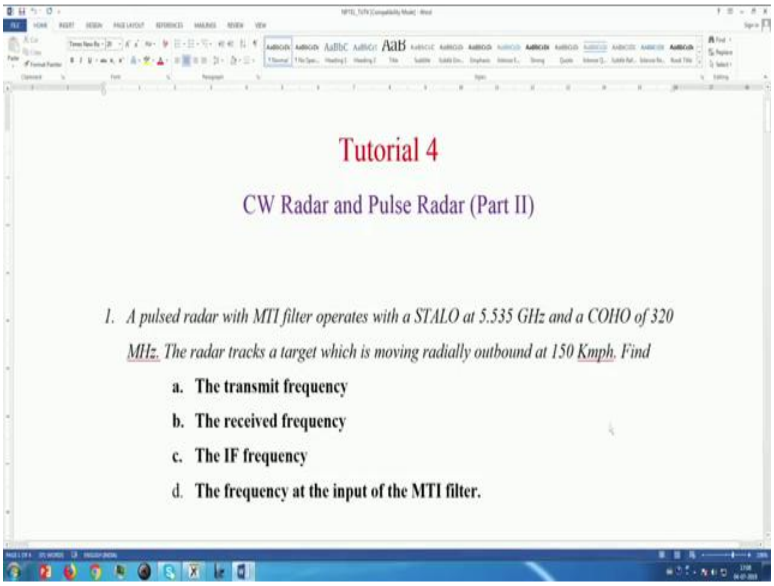

So, this is the question today, a pulse radar with MTI filter; that means, the classical pulse radar thing operates with a STALO at so and so, giga Hertz and a COHO of so, and so, mega Hertz. The radar tracks a target which is moving radially outbound at 150 kilometer per hour find the transmit frequency the received frequency the IF frequency the frequency at the input of the MTI filter. So, you see its a classical thing we if you just remember the block diagram we can do this let us try these.

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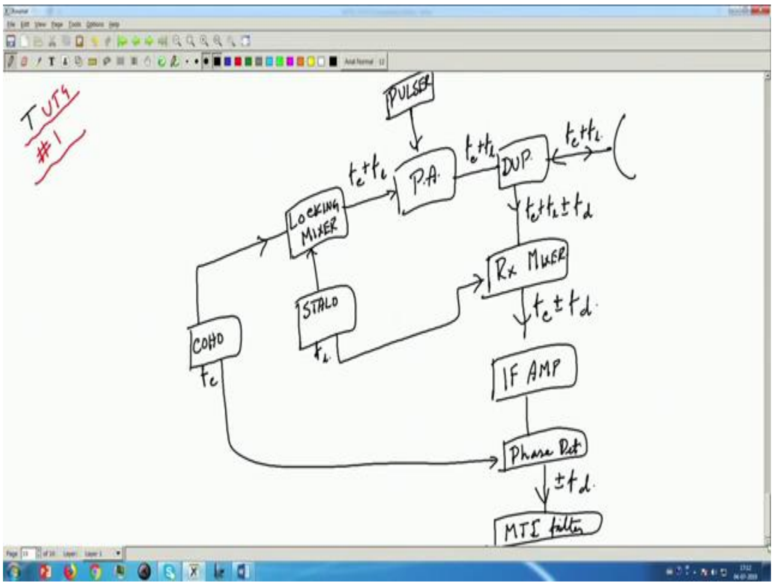

So, this is tutorial 4 problem 1 let us see. So, first I will have to if you want to draw this we will need to draw the block diagram. So, COHO and STALO if you remember this was COHO f c and STALO f l. So, they are put to a locking mixer. So, this output is f c plus f l and then that here is a power amplifier; power amplifier to this power amplifier if you remember that modulator pulse are that thing also come, but that does not change the frequency. So, this is f c plus f l then it is put to the duplexer and from the duplexer it goes to the antenna.

So, that means, the transmit frequencies f c plus f l and through this antenna it also comes here this is the receiving part. So, first there is the R x mixer. So, this R x mixer is comes here; that means, this frequencies actually f c plus f l plus or minus f d depending on the Doppler of the thing. So, R f mixer. So, how R f mixer steeps? So, this is f l; that means, you will live with f c plus minus f d then that is given to the IF amplifier then from there it goes to the phase detector and to this phase detector this COHO is fed so; that means, basically phase detector that also you now it’s a mixer. So, here you are getting f d that is put to the MTI filter this was the block diagram. So, basically the transmit frequency. So, let us now calculate the transmit frequency.

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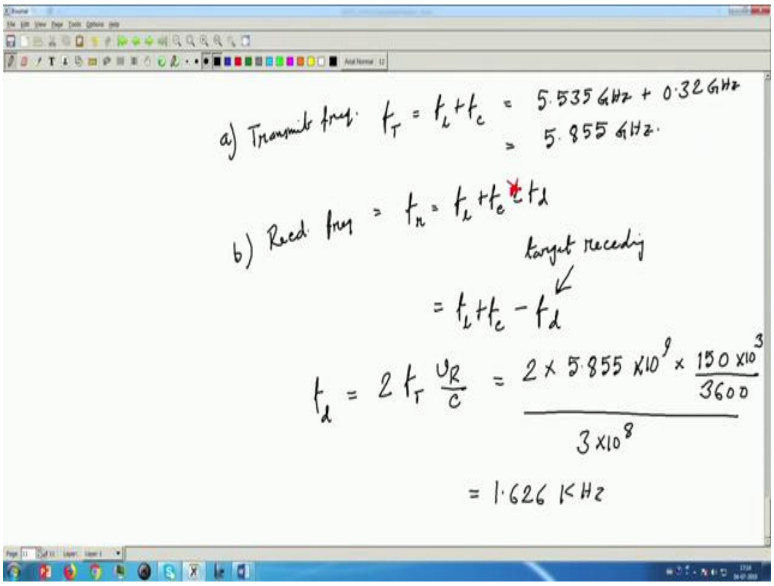

So, that is part a transmit frequency is f T if we call that is f l plus f c. So, that is 5.535 gigahertz plus 0.32 giga Hertz. So, that will be 5.855 giga Hertz as simple as that then what is the received frequency? Now received frequency is if we call it f r that is nothing, but f l plus f c plus minus f d. Now out of these which is said that the target is receding target receding target receding means, I will get this will be negative. So, this will go out; that means, this will be minus f d.

So, I can say that it is f l plus f c minus f d because target is receding and what is f d? F d is 2 f T v R by C. So, 2 into f T is just now calculated 5.855 remember this is f T actually this is a transmitted frequency. So, 2 into is giga Hertz then v R. So, v R is 150 kilometer per hour. So, 150 into 10 to the power 3 by hour; hour means 3600 and then divided by C. So, that will give us 1.626 kilo Hertz. So, Doppler is 1.626 kilo Hertz.

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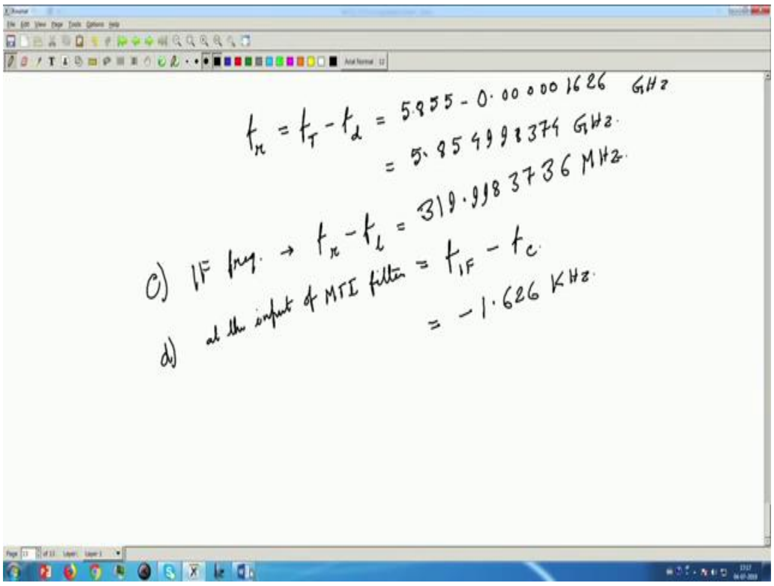

So, we can calculate what is f r. f r will be f T minus f d. So, that is 5.855 minus 0.000001626 everything in giga Hertz. So, it is 5.854998374 giga Hertz ok. Then part d question is, what is part d question d? Question d is the IF frequency. So, next we will have to do c that is f r we got. So, this we have not done IF frequency.

So, IF frequency is f r minus f l. So, that if you do f r is this f l you already know 5.535. So, that if we do it will come to be 319.9983736 mega Hertz and last is what is the frequency at the input of at the input of MTI filter that is f IF minus f c f IF is this f c is known. So, it is minus 1.626 kilo Hertz. So, this is the Doppler shift. So, it is in the negative side there is a Doppler of these that is all. So, very conceptual problem.

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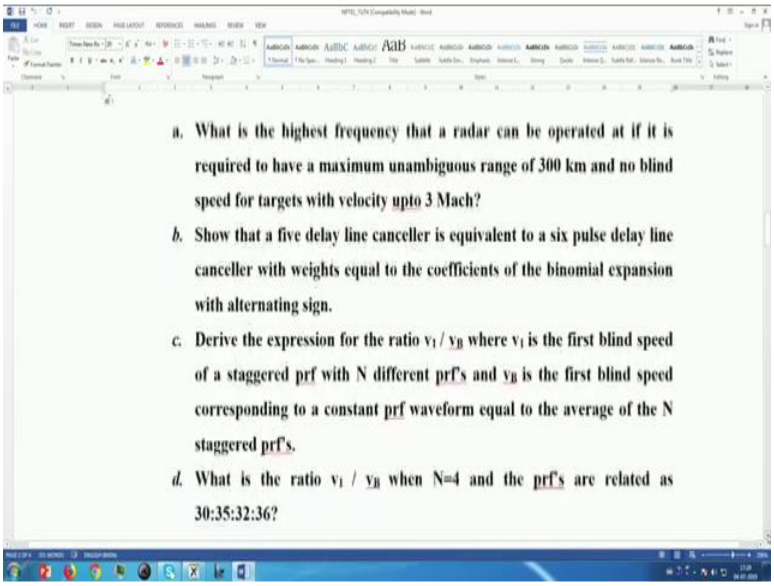
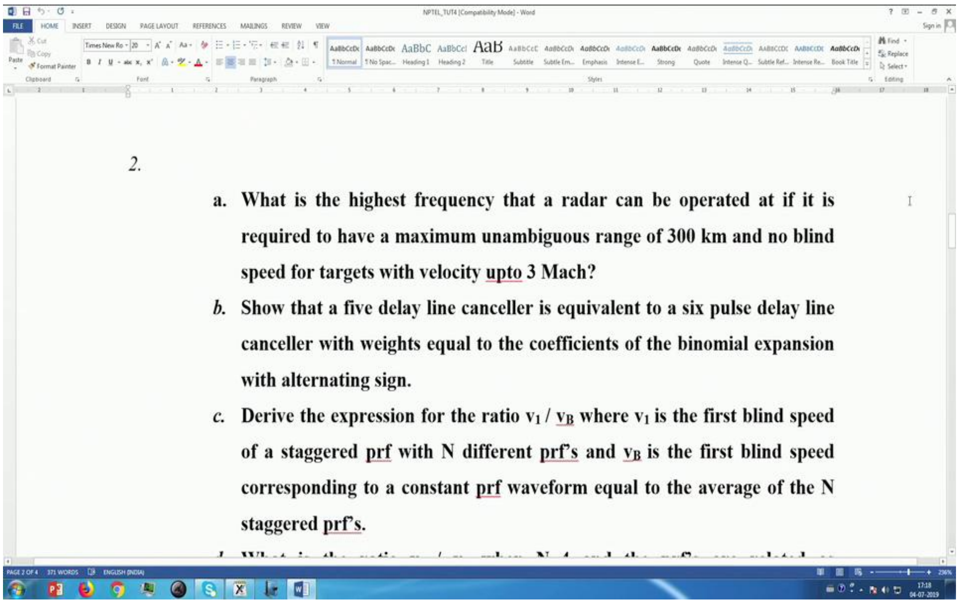


a. What is the highest frequency that a radar can be operated at if it is required to have a maximum unambiguous range of 300 km and no blind speed for targets with velocity upto 3 Mach?

b. Show that a five delay line canceller is equivalent to a six pulse delay line canceller with weights equal to the coefficients of the binomial expansion with alternating sign.

c. Derive the expression for the ratio v_1 / v_B where v_1 is the first blind speed of a staggered prf with N different prf's and v_B is the first blind speed corresponding to a constant prf waveform equal to the average of the N staggered prf's.

d. What is the ratio v_1 / v_B when N=4 and the prf's are related as 30:35:32:36?



2.

a. What is the highest frequency that a radar can be operated at if it is required to have a maximum unambiguous range of 300 km and no blind speed for targets with velocity upto 3 Mach?

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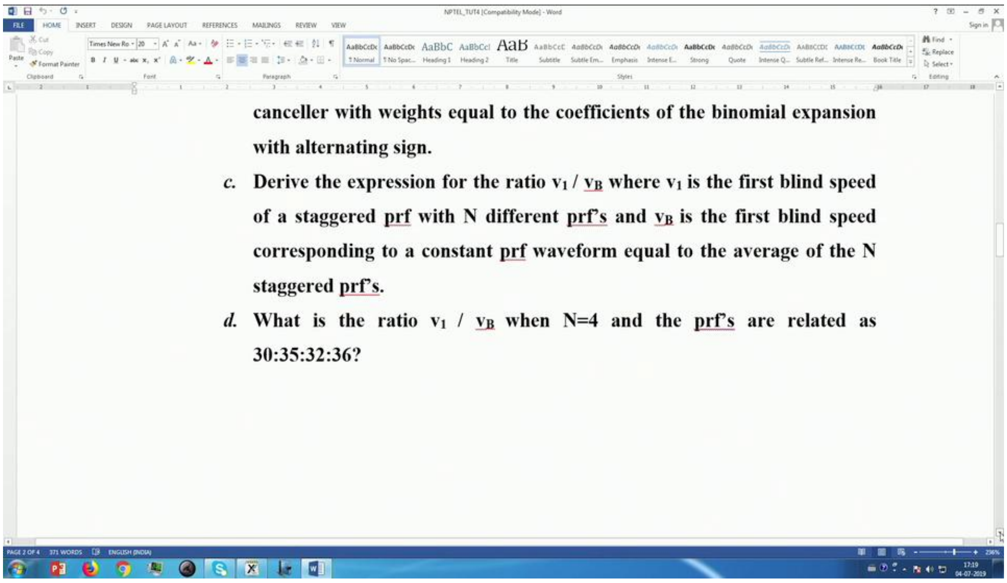
Let us go to the next problem what is the highest frequency that a radar can be operated at if it is required to have a maximum unambiguous range of so, and so, kilometer and no blind speed for targets with velocity up to 3 Mach. Mach I think you know one Mach means the speed of sound that is 330 meter per second. So, aircrafts etcetera missiles (Refer Time: 10:51) they have this unit 3 Mach means 3 into velocity of sound 8 Mach means 8 into velocity of sound etcetera.

Generally up to 8 9 Machs we the aircraft fighter aircraft or this space crafts we can go. So, this is the first part the second part is show that a five delay can canceller is equivalent to a six pulse delay line canceller with weights equal to the coefficients of the binomial expansion with alternating sign. Derive the expression for the ratio v 1 by v B where v 1 is the first blind speed of a staggered prf with n different prfs and v B is the first blind speed corresponding to a constant prf waveform equal to the average of the n staggered prf. What is the ratio this is a particular value is given and so, this is a let us see this problem first that let us solve the first part what is the highest frequency.



canceller with weights equal to the coefficients of the binomial expansion with alternating sign.

c. **Derive the expression for the ratio v_1 / v_B where v_1 is the first blind speed of a staggered prf with N different prf's and v_B is the first blind speed corresponding to a constant prf waveform equal to the average of the N staggered prf's.**

d. **What is the ratio v_1 / v_B when N=4 and the prf's are related as 30:35:32:36?**

So, that we can have that targets with up to 3 Machs because most of the time we know that what are the enemy targets. So, in this case let us say that 3 Mach is their maximum target. So, we can easily find out that this should be within our below or blind speed. So, by that you can do.

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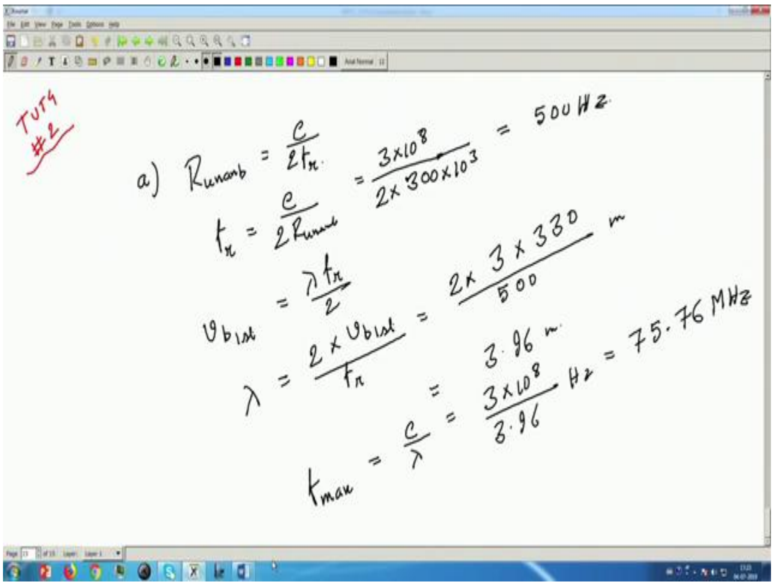

So, I can say this will be tutorial 4 problem 2. So, first part is we know again that r unambiguous related to the prf. So, c by 2 f r and so, from that f r we can calculate f r is c by 2 R unambiguous and that is 3 into 10 to the power 8 by 2 into this 300 into 10 to the power 3 that has been said that. So, that turns out to be 500 Hertz is the prf.

So, v b first is lambda f r by 2. So, that is actually we have to find this lambda because that will give us the frequency. So, if you solve for lambda, lambda is 2 into v b first by f r. So, 2 into what is 2 into v b first. So, 3 into Mach. So, 330 I am assuming speed by speed of sound 500 which is metre. So, lambda is 3.96 metre. So, you can say that f max that will be c by lambda. So, 3 into 10 to the power 8 by 3.96 Hertz so, that is 75.76 mega Hertz ok.

So, up to 75.76 mega Hertz you can safely operate the targets will be detected ok. So, what is the next problem? Show that a five delay line canceller is equivalent to a six pulse delay line canceller with weights equal to the coefficients of the binomial expansion with alternating sign this we have shown also in the theory, but let us show.

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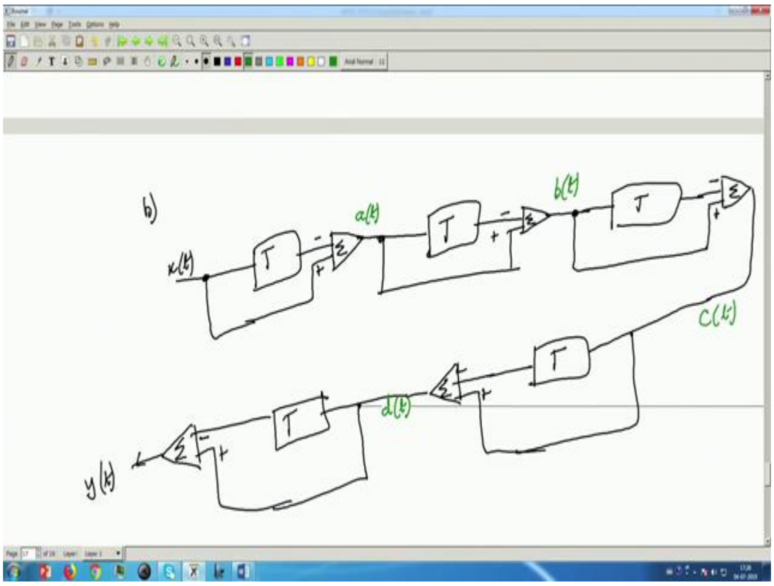

So, the tutorial. So, we have five delay line so; that means, this is a T, this is a T, this is a T, this is a T this is a T five delay line consecutively. So, this one and we know there will be a summer. So, from here it is directly going and readily. So, if this is x t coming and then this is that loop, this is minus, this is plus this again will go and one will come from here, again there is a summer so, this one is negative this is positive. So, this will be finally, our y t. So, five delay line. So, we can now write. So, for ease of things we can give some names that this one we are calling a t, then let us say this one we are calling b t, then this output we are calling c t, this one we are calling d t rest is ok.

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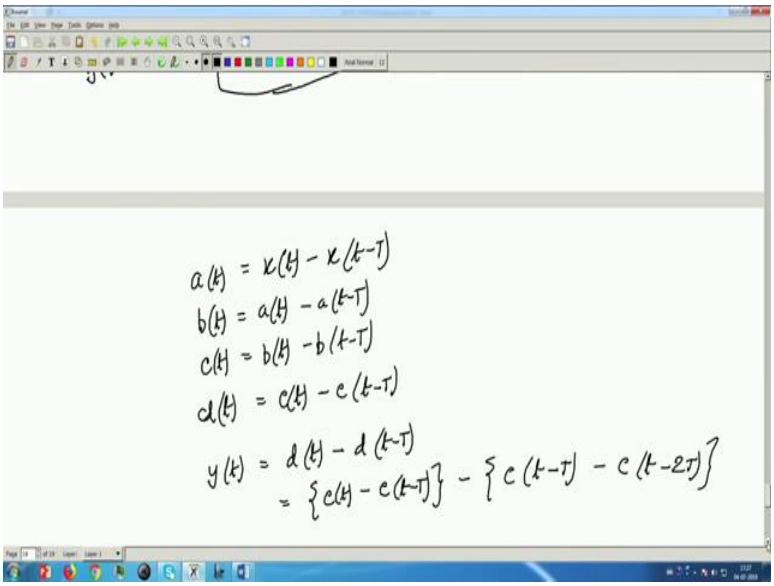

So, I can write what is a t; a t is x t minus x t minus T b t is a t minus a t minus T c t is b t minus b t minus T, d t is c t minus c t minus T. So, what is y t is d t minus d t minus T. So, d t is nothing, but c t minus c t minus T minus c t minus T minus c t minus 2 T.

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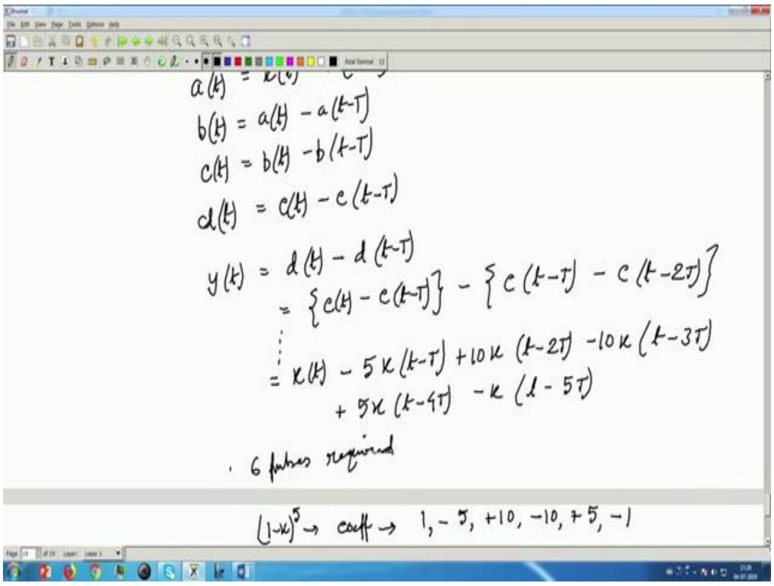

Then you can go on putting these values and finally, I can show if you put all those values gradually. So, you will come up with this x t minus 5 x t minus T. So, I am here there are steps easily you can fill up those steps 10 x t minus 2 T minus 10 x t minus 3 T plus 5 x t minus 4 T minus x t minus 5 T. So, that shows that you require 6 pulses one is this 0 1 2 3 4 5…… 6 pulses.

So, that is why 6 pulses are required that as shown to get this output required and also you can find out that what is 1 minus x whole to the power 5. Its coefficients are binomial. If we do binomial expansion of 1 minus x whole to the power n. So, coefficients are 1 minus 5 plus 10 minus 10 plus 5 minus 1 same here 1 minus 5 10 minus 10 plus 5 minus. So, we have shown whatever has been asked for.

Now, come to the next part of the problem this part c derive the expression for the ratio v 1 by v B where v 1 is the first blind speed of a staggered prf with n different prf this is an important thing. So, let us do this that part c.

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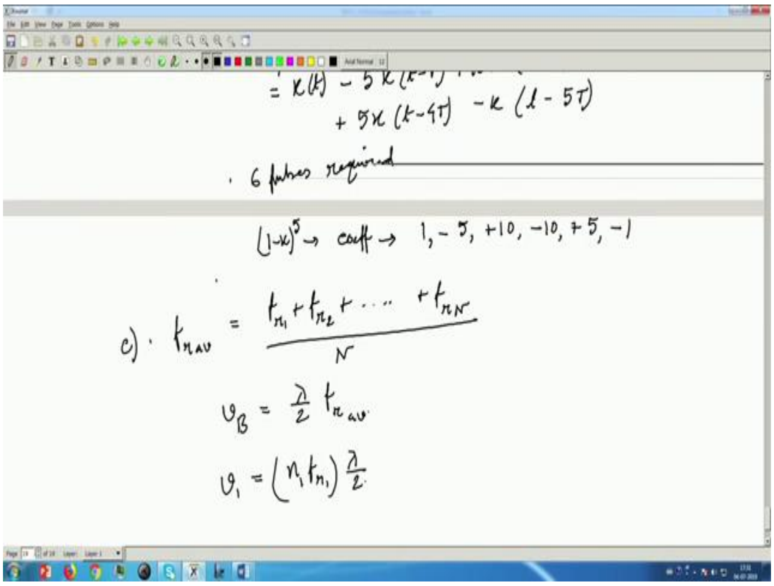

So, suppose we are doing f r average is f r 1 plus f r 2 plus f r n….. n different prf s staggered. So, f r average will be this. So, v B will be lambda by 2 f r average also what is a first blind speed due to say f r 1? I am calling that v 1. So, that is nothing, but n 1 f r 1 into lambda by 2.

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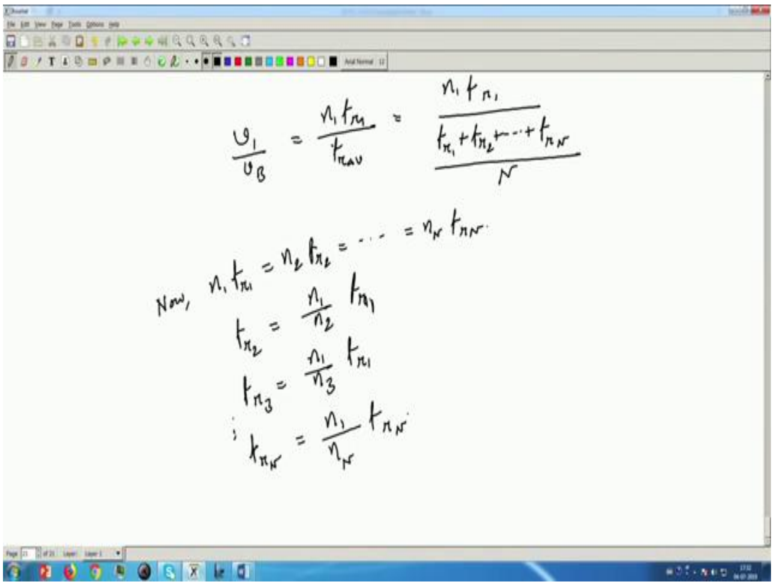

So, I can take this ratio as v 1 by v B is n 1 f r by f r Av. So, that is n 1 f r 1 by f r average is nothing, but f r 1 plus f r 2 plus f r N by N. Now we also know that the relation between all these are like this. So, one by one we can all express every f r 2 f r 3 etcetera in terms of f r 1. So, f r 2 is nothing, but n 1 by n 2 f r 1 f r 3 is n 1 by n 3 f r 1; f r N is n 1 by n N f r 1.

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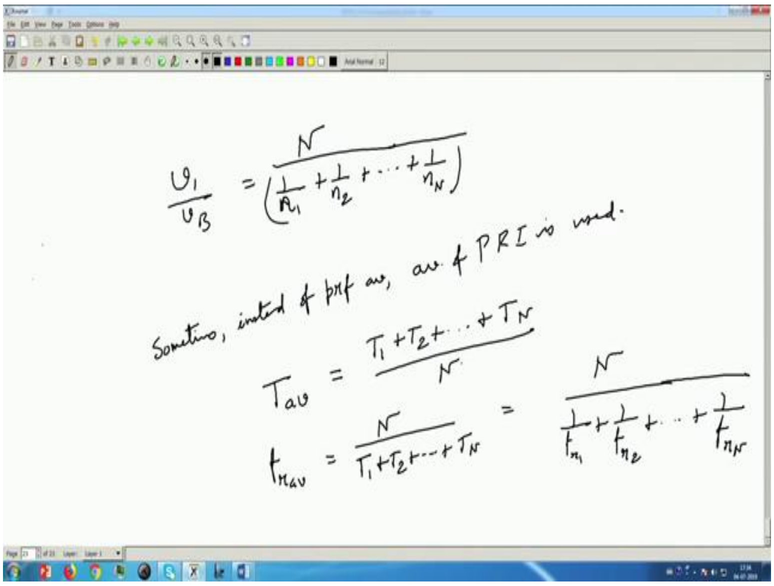

So, putting all these we can get v 1 by v B will be N by 1 by n 1 plus 1 by n 2 plus 1 by n N ok. Now, sometimes this average instead of average of prf. So, I will write sometimes instead of prf average of PRIs is used in that case the formula is slightly different. So, let us do that. So, in that case we can write T average will be T 1 plus T 2 plus T N by N. So, f r average in that case is N by T 1 plus T 2 plus T N. So, that is N by 1 by f r 1 plus 1 by f r 2 plus 1 by f r N.

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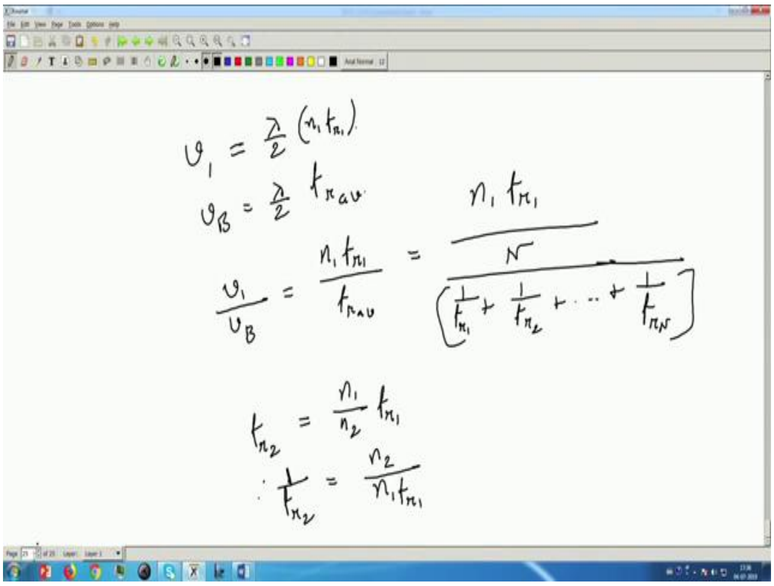

Now, v 1 we know lambda by 2 n 1 f r 1 v B is lambda by 2 f r average. So, v 1 by v B is n 1 f r 1 by f r average is n 1 f r 1 by n by 1 by f r 1 plus 1 by f r 2 plus 1 by f r N etcetera ok. Now, as before we can express all these f r 2 f r 4 from that formula as. So, this earlier we use now we require this 1 by f r 2 that will be n 2 by n 1 f r 1.

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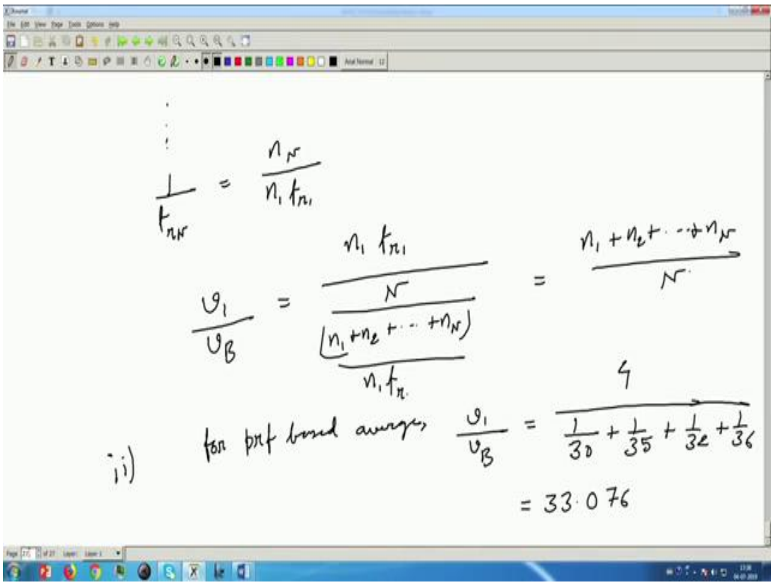

Then 1 by f r N is n N by n 1 f r 1 etcetera. So, putting this we can get v 1 by v B is n 1 f r 1 by N by n 1 plus n 2 plus n N by n 1 f r 1. So, that will finally, give you n 1 plus n 2 plus n N by capital N. So, this two formulas are a bit different, but let us see their effect is more or less same. So, for the given values for prf based average let us calculate what is this ratio of v 1 by v B. So, it is 4 by 1 by 30 plus 1 by 35 plus 1 by 32 plus 1 by 36. So, that is 33.076 ok.

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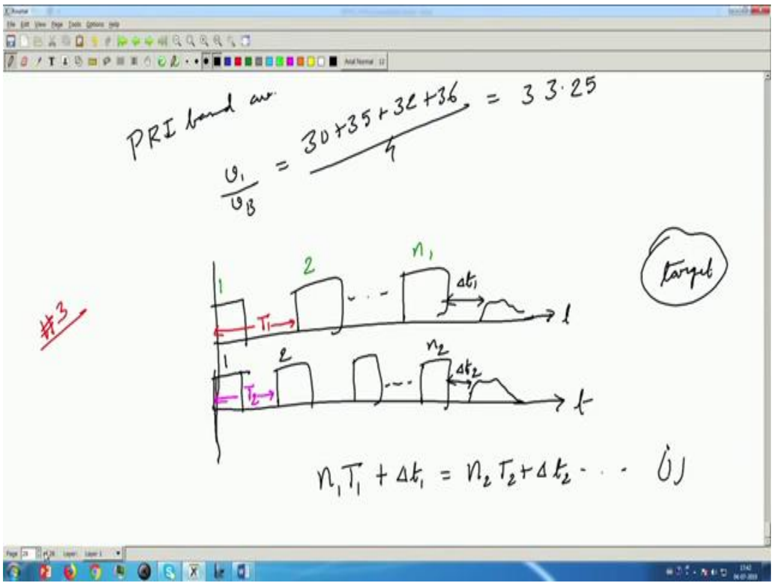

Whereas PRI based average that will give you that v 1 by v B is 30 plus 35 plus 32 plus 36 by 4. So, that will be 33.25. So, in one case 33.25 in other case 33.076. So, these two formulas give slightly different values, but the order of improvement with staggered PRF is always present you see always you are getting almost a 30 times improvement on there a thing.

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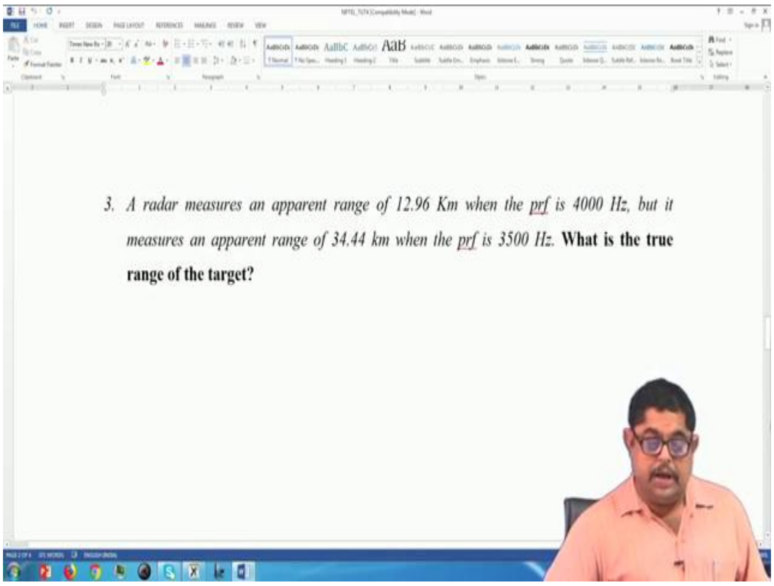

So, I think this problem we have seen there was another problem a radar measures an apparent range of 12.96 kilometer, when the prf is 4 kilo Hertz, but it measures an apparent range of 34.44 kilometer when the prf is 3500 Hertz, what is the true range of the target? So, this is again an interesting problem. So, I will say that problem 3. So, basically the scenario is like this your radar is sending. So, I can name them that let us say this is one this is the second pulse this is the n 1 pulse and I have the target here.

So, the echo is coming suppose this actual echo is come here. So, let me call this as delta t 1. So, after n 1 pulse transmission I have got the echo now in this case the prf is different. So, first one is this then. So, the echo is coming in the same time, but this one is delta t 2 and this numbers are different 1, 2. So, last one is here let us say n 2. So, these are all with time.

So, what I can always say that in reality echo is coming same in the first case let us say after n 1 pulses I am getting after transmission of n 1 pulses I am getting this time is important. So, this is my let us call T 1 and this is my T 2. So, I can always equate these two times you think that I can always write n 1 T 1 because this time is same. So, n 1 T 1 plus delta t 1 is equal to n 2 T 2 plus delta t 2 this is my equation number 1.

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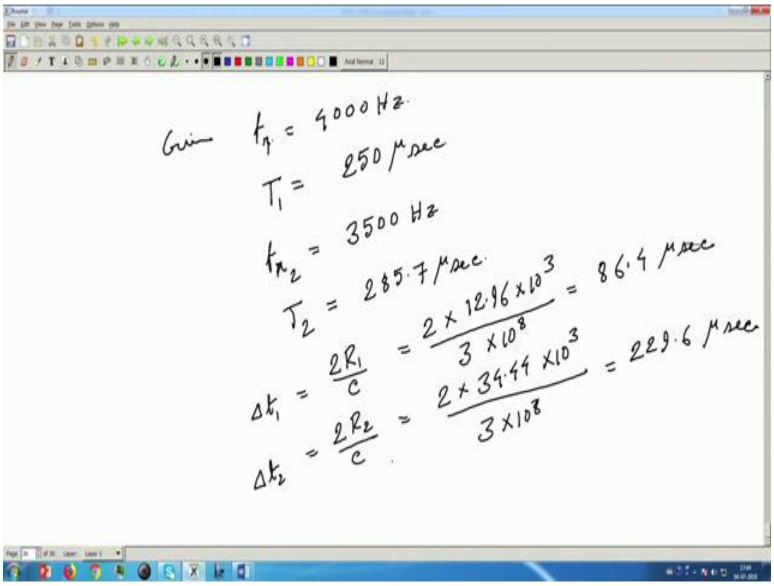

Now, what are given? Given things are f r 1 is 400 Hertz. So, T 1 I am getting T 1 is 1 by this. So, that will be 250 microsecond then f r 2 given that is 3500 Hertz immediately it says that T 2 capital T 2 is 285.7 microsecond delta T 1 is 2 R 1 by C. So, that is 2 into 12.96 into 10 to the power 3 by 3 into 10 to the power 8. So, this comes to 86.4 microsecond and delta t 2 is 2 R 2 by C that is 2 into 34.44 into 10 to the power 3 by 3 into 10 to the power 8. So, it is 229.6 microsecond. So, now we will have to trial and error solve these.

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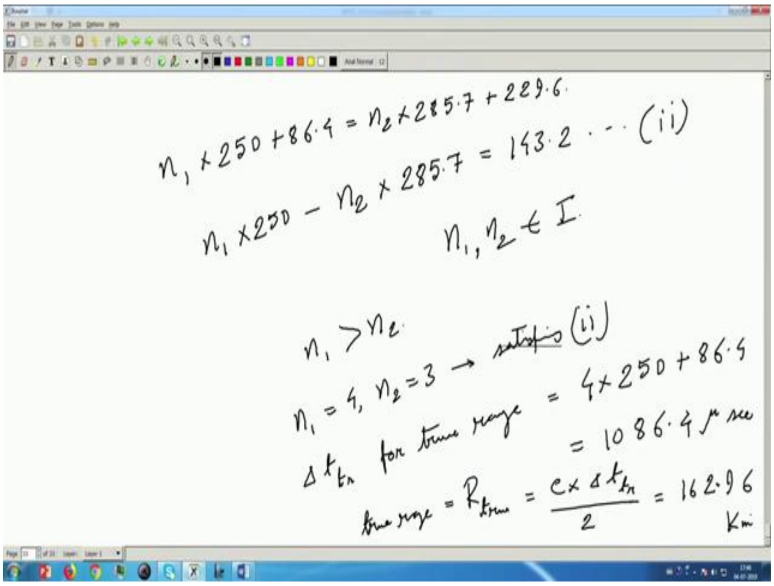

So, n 1 into 250 plus 86.4 is n 2 into 285.7 plus 229.6. So, the equation is n 1 into 250 minus n 2 into 285.7 is 143.2 this is my equation 2 and I know n 1 n 2 belongs to integer. So, it is apparent from this equation that n 1 is greater than n 2 by trial and error you can find that n 1 is equal to 4, n 2 is equal to 3 satisfies equation 2. So, delta t u t r for true range will be 4 into 250 plus 86.4 that is 1086.4 microsecond. So, true range is R true is C into delta t tr by 2 that is 162.96 kilometer.

Please go through this is an important problem and I conceptually clears many thing that how in the cases actually the radar by just thus true readings apparent readings, apparent range readings when you are varying 2 prfs you can always find what is the true range this is the way radar finds true ranges.

Thank you.