Principles And Techniques of Modern Radar Systems
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Lecture – 20
MTI Improvement Factor

Key Concepts: Mathematical model of MTI improvement factor, Description of subclutter visibility, Implementation of an N-pulse MTI filtered with tapped delay line.

Welcome to this NPTEL lecture on Techniques and Principles of Modern Radar System. So, we were discussing MTI filter performance metrics. So, in the previous lecture we have seen the clutter attenuation the first metric, but we have told that time that actually we need to see that attenuation with respect to the level of the signal. So, something like signal to clutter ratio at the input and output should be compared that is the MTI improvement factor.

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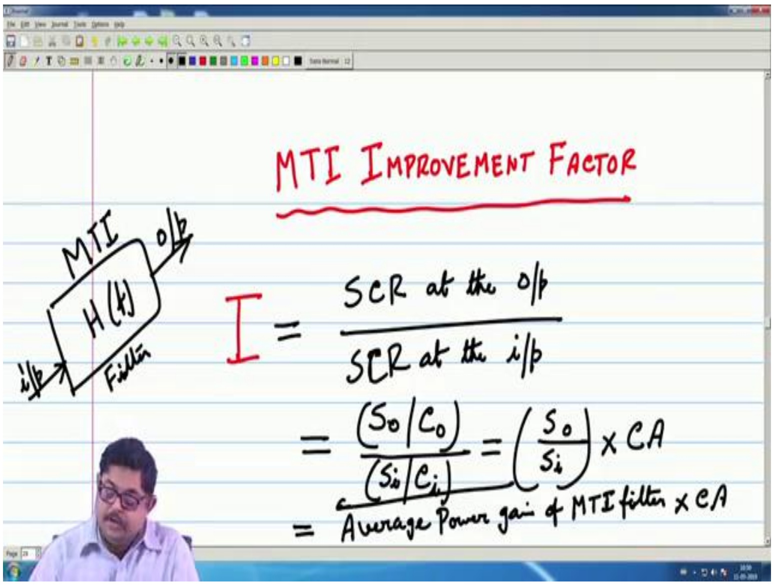

So, I is I is the symbol of MTI improvement factor, it is SCR at the output of whom? Obviously, this is the MTI filter.

So, the SCR here this is the input and this is the output. So, SCR at the output by SCR at the input; so that means, I can write S naught by C naught where S is the signal power at the output, C we have already said clutter power divided by Si by Ci. So, this I can write that the signal power ratio at the output by input into the clutter attenuation what we have already seen. So, then what is S o by S i? So, this I can say that average from here I can write that average power gain of MTI filter and this is the clutter attenuation.

$$I = \frac{\text{SCR at the o/p}}{\text{SCR at the i/p}} = \frac{(S_o/C_o)}{(S_i/C_i)} = \left(\frac{S_o}{S_i}\right) \times CA$$

$$= \text{Average Power gain of MTI filter} \times CA$$

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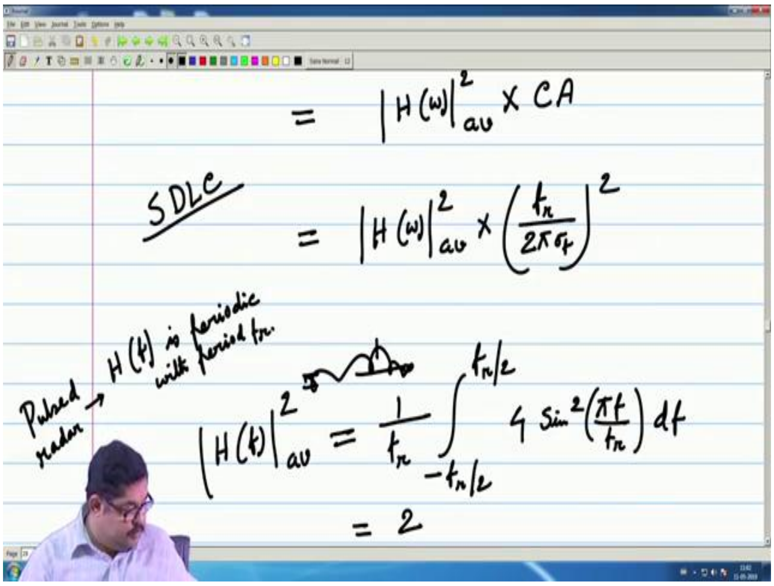

So, we know if the transfer function is H f, so, this can be written as that H omega square, but there should be average why because at all the frequencies it may not be same. So, we will have to take some averaging and give that.

$$I = |H(\omega)|^2_{av} \times CA$$

Now for if I want to calculate this for a single DLC again we specialized as SDLC. So, we know this CA we have already found. So, I can put that value f r by 2 pi sigma f square.

$$I = |H(\omega)|^2_{av} \times \left(\frac{f_r}{2\pi\sigma_f}\right)^2$$

Now, we will have to evaluate this one. So, we know that in a pulsed MTI radar, the H omega or H f it is periodic with period fr. So, this thing will turn out to be that 1 by. So, we know that I can write that H f is periodic with period fr this is for a pulse radar we have seen that pulsed radar. So, this I can what will be the this average thing that I can find out. So, let me separately do it that H f square average that will be 1 by f r minus f r by 2 to plus f r by 2 and then for a single DLC I know that expression sin square pi f by f r d f. So, this is the average, if you do that this whole thing it can be easily done sine square integration you can do, the answer will be 2.

$$|H(f)|^2_{av} = \frac{1}{f_r}\int_{-f_r/2}^{f_r/2} 4\sin^2\left(\frac{\pi f}{f_r}\right) df$$
$$= 2$$

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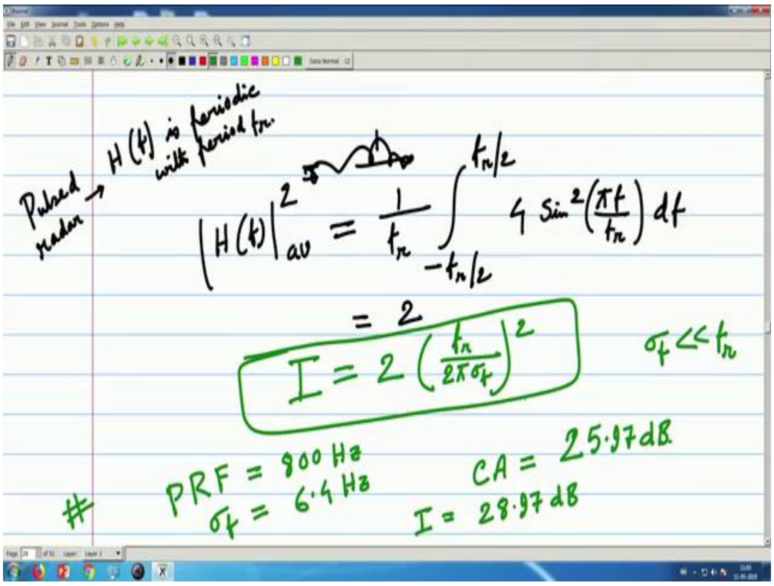

So, we can find the for a MTI improvement factor for a single DLC implementation it will be 2 into f r by 2 pi sigma f whole square.

$$I = 2\left(\frac{f_r}{2\pi\sigma_f}\right)^2$$

This is a very useful relation that it is this something like clutter attenuation, but multiplied by 2 for single DLC and also I am again reminding the region of validity that sigma f should be less than fr then only this is valid because we have assumed that in evaluating that.

So, let us take an example that suppose a certain pulse radar is has a PRF 800 Hertz and suppose the clutter RMS spread that sigma f is 6.4 Hertz a very typical value. So, what will be the MTI improvement factor for this we can calculate what is for CA Clutter Attenuation that fr value we know 800 Hertz. So, this expression basically that if we do that put it sigma f is given, this is given, this will turn out to be in db scale 25.97 dB and this factor of 2 will give a 3 dB. So, MTI improvement factor for this will be almost 29 dB.

So, you see that the MTI filter is giving a 29 dB thing very good gain even a single DLC and actually you can now calculate that instead of single DLC if I give a double DLC etcetera what will.

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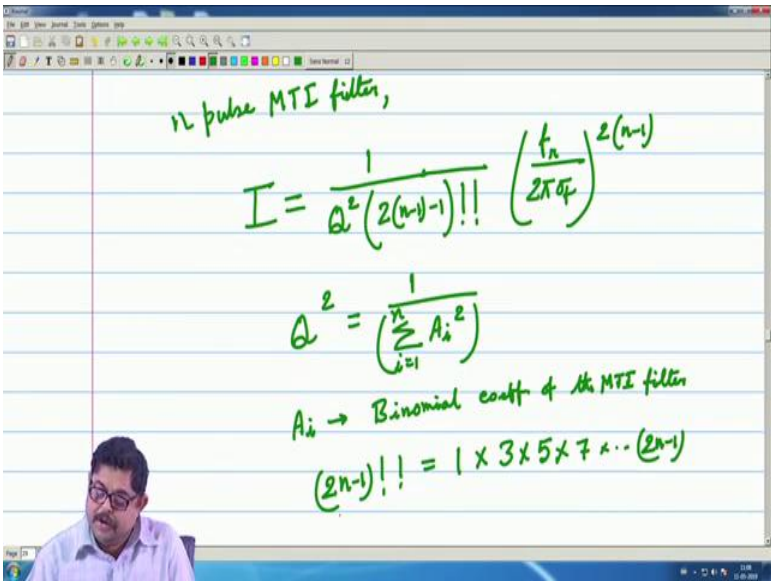

So, for I can write that for n pulse 2 pulse 3 pulse that n pulse MTI filter if you do that evaluation, the improvement factor is given by a general formula 1 by people have derived it I have not gone into that, but you if you know these you can find what is the improvement factor.

$$I = \frac{1}{Q^2 (2(n-1)-1)!!} \left(\frac{f_r}{2\pi \sigma_f} \right)^{2(n-1)}$$

Well you see it is a double factorial notation I have given, now I will explain that, but what is Q square? So, Q square is 1 by sigma i is equal to 1 to n Ai square

$$Q^2 = \frac{1}{\left(\sum_{i=1}^{n} A_i^2 \right)}$$

and Ai is nothing, but the binomial coefficient of the MTI filter. Now this thing see what are these actually MTI filter we have seen that, higher order things we can always implement them by a transversal filter and there are weights those weights are the coefficients and it can be shown that they are nothing but binomial coefficient.

If we choose it that way then that gives you the optimal thing we will discuss that. So, Ai is that and what is a these double factorial things? That a odd double factorial is 1 into 3 into 5 into 7 up to 2 n minus 1 and even double factorial is 2 into 4 into 6 into up to 2 n.

$$(2n-1)!! = 1 \times 3 \times 5 \times 7 \times \cdot \times (2n-1)$$
$$(2n)!! = 2 \times 4 \times 6 \times \cdots \times 2n$$

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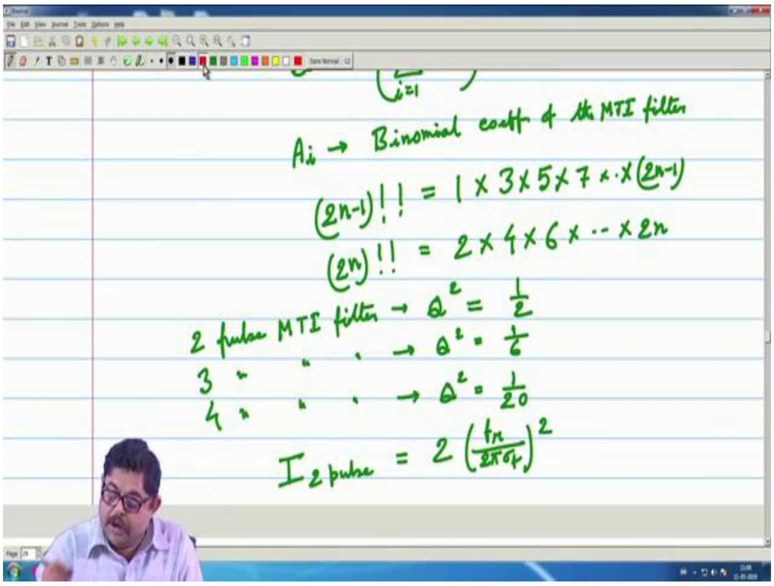

So, we can now find out from that formula that for a 2 pulse MTI, 2 pulse MTI filter what will be Q square? Q square will be you can find out because you have those coefficients. So, it will come out to be half for a 3 pulse MTI filter Q square will be 1 by 6, this you can easily check for a 4 pulse MTI filter Q square is 1 by 20 and so, you can easily write that what is I; that means, MTI factor for 2 pulse that we have already seen this, but from this formula again you can check this will be 2 f r by 2 pi sigma f whole square.

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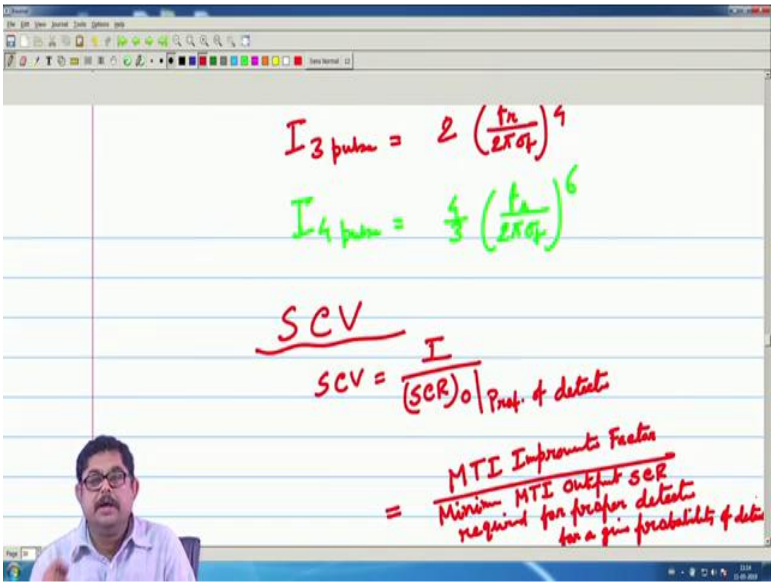
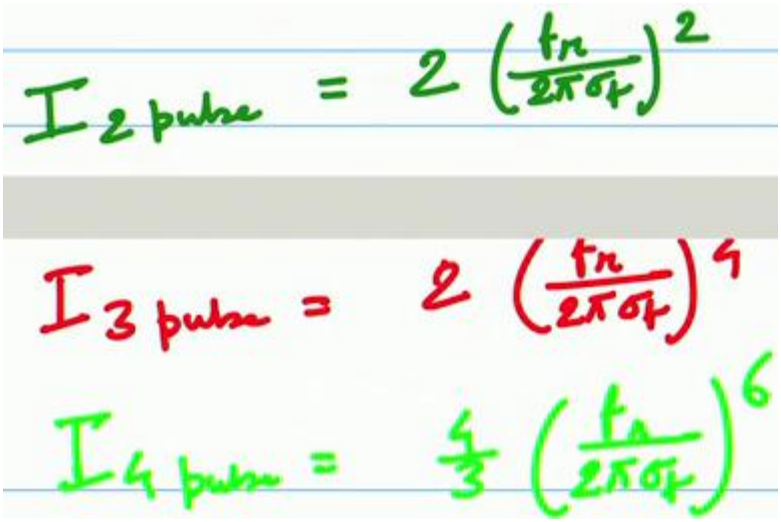


Now, what is the corresponding thing for 3 pulse? I 3 pulse will be 2 fr by 2 pi sigma f whole to the power 4 and I 4 pulse that will be 4 by 3 fr by 2 pi sigma f whole to the power 6.



So, you see that if we have more pulses we process then the improvement factor goes with a square ratio; that means, for a 2 pulse it is fr square, for a 3 pulse it is fr whole to the power 4 and remember that fr divided by sigma that is a very huge number. So, you are getting a huge improvement in a thing.

Now, actually sub clutter visibility you just I will qualitatively say we because we have not seen certain thing. It is actually when a radar makes a processing it is with a given constraints, those are called probability of detection and false alarm etcetera that we have not discussed yet, but sub clutter visibility is this thing, but how much under those given parameters what is the performance metrics. So, I will describe it that, sub clutter visibility it is describes the radars ability to detect non stationary targets embedded in a strong clutter background for a given probability of detection and false alarm.

So, for a given constraint how much it can see targets embedded in a strong clutter background. So, for example, a radar with 10 dB SCV will be able to detect moving targets whose echoes are 10 times smaller than those of clutter. So, we have seen that sometimes the RCS of the clutter may be 60 70 dB higher than the RCS of the target. So, sub clutter visibility is an indication that whether the thing will be able to detect or not. So, I am just writing SCV is defined as the moving target indicator by the signal to clutter ratio of the output for a given probability of detection.

$$SCV = \frac{I}{(SCR)_o}\Bigg|_{\text{Prob. of detection}}$$

So, you see it is I, but under that minimum MTI output signal to clutter ratio required for proper detection for a given thing. So, for your later you will understand this meaning actually it is the numerator is MTI improvement factor, but denominator is the minimum MTI output SCR required for proper detection for a given probability of detection. As I said that it is scaled by that what is the minimum SCR that requires you are scaling by that. So, that gives you the sub clutter how much clutter thing you can process.

Now, a few things about the implementation. So, we have seen the performance matrices how to calculate that. We have got a formula also by that you can find for various stages of MTI filter what will be the thing. Now I will say that if we recall the if I the implementation of delay line canceller DLCs.

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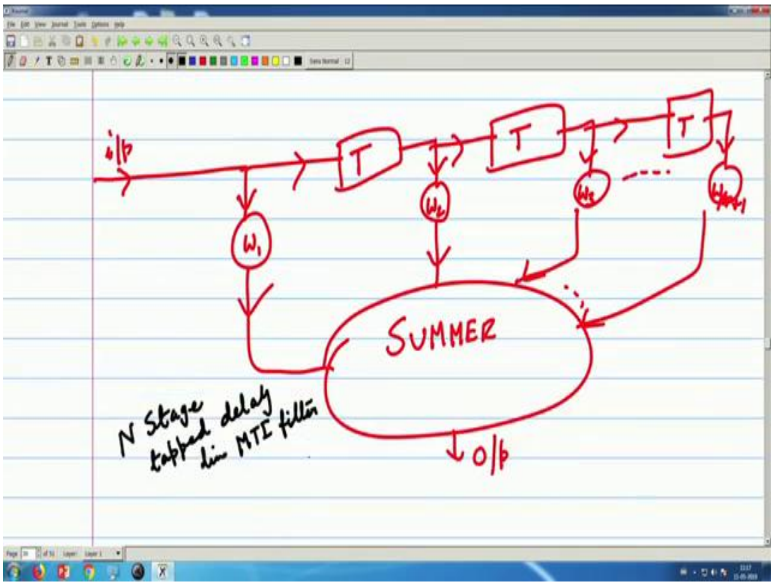
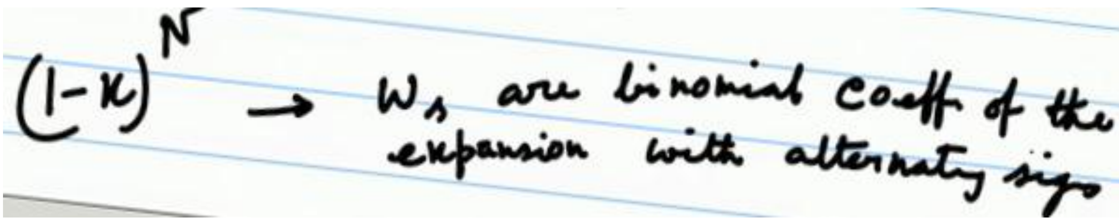


So, we can say that basically if the input is coming, what the a is doing that it is first given giving some weights, then there is a delay I will write T, then there is another weight then further delay there is another weight then delay. So, I will give this further weight like this. So, if this is the last one I can call that if there are a n stage tap delay thing, then the final thing will be n plus 1. So, I can say there are some a thing and all these are summed. So, there is a summer with which all these are connected. So, finally, all these are put here.

So, this is a summer summing network and you get the output from here. So, this is the block diagram of a n stage n stage tapped delay line MTI filter. This we already have seen now what people have shown that if this weights how to choose these weights W 1 W 2 W 3 W 4? Now you have 2-3 cases we have seen and that time we have not recognized, but actually you will see that the way we have got those values they are nothing, but the their binomial coefficients of 1 minus x whole to the power N.



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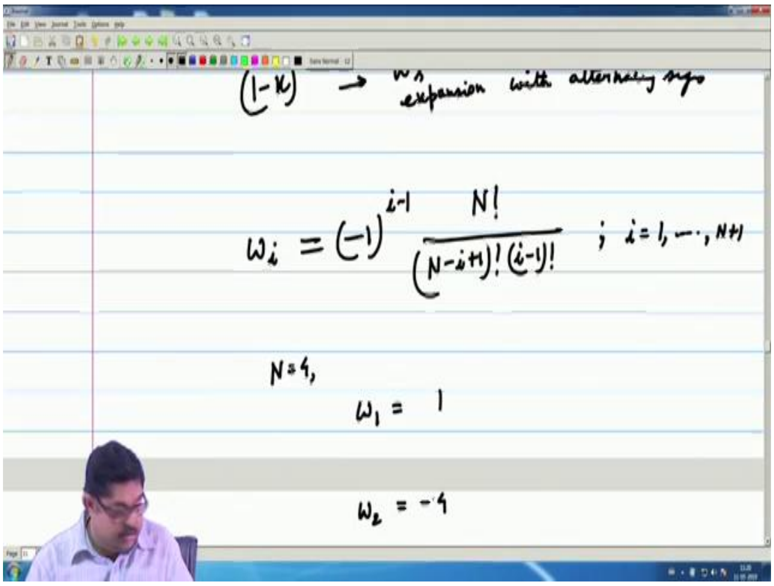

In our case actually these W 1 W 2 W 3 etcetera they are nothing, but the binomial coefficients; that means, the coefficient of the expansion 1 minus x whole to the power sorry this will be capital N with alternating signs. So, I can say that the Ws are binomial coefficients of the expansion with alternating signs that turned out and this is nothing, but our this what we have said that n stage cascaded single DLCs so; that means, we know the binomial coefficients. So, I can write that what is says that W i is minus 1 whole to the power i minus 1 this will make this alternating and N factorial by N minus i plus 1 factorial into i minus 1 factorial where i is equal to 1 to N plus 1.

$$\omega_i = (-1)^{i-1} \frac{N!}{(N-i+1)!\,(i-1)!} \quad ; \quad i = 1, \ldots, N+1$$

So; that means, you can always do that. So, in you can check suppose for if you take capital N is 4. So, basically N stage that thing. So, what will be W 1 what will be W 2 and you can see that in your actual thing you can also do that. So, let us say that because up to 3 we have done. So, let us do for 4 also that N is equal to 4 then if you find from here what will be W 1?

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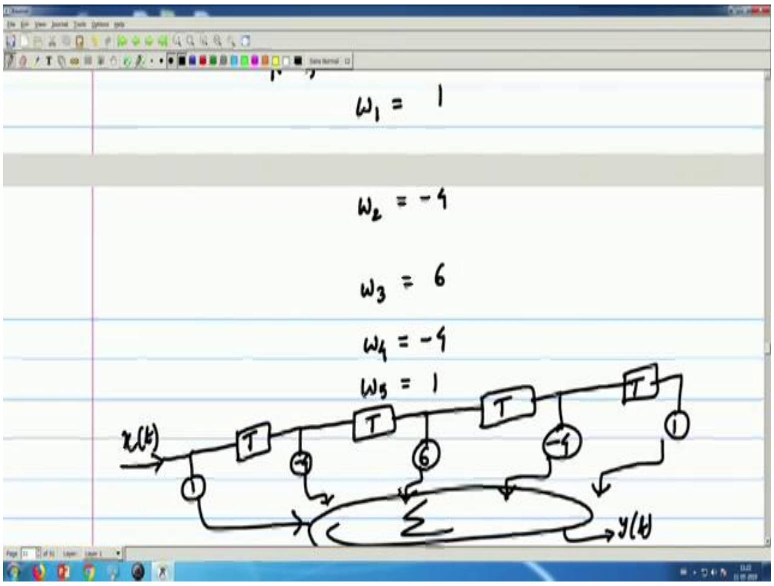

If you put it you will see that W 1 will be 1 W 2 that will be minus 4 W 3 that will be 6 W 4 that will be minus 4 and W 5 that will be 1 and so; that means, the thing will be you are giving an input then you are giving a weight of 1 then you are putting a delay, you are giving a weight of minus 4 then you are giving another delay, you are giving a weight of 6, then you are giving another delay you are giving a weight of minus 4, then you are giving the another delay and giving a weight of 1 all these you are summing. So, there is a summer and you are getting the output. So, if this is x t you are getting y t.

Now, this is from binomial thing and from your 4 pulse, I can see that if you have x t you are giving.

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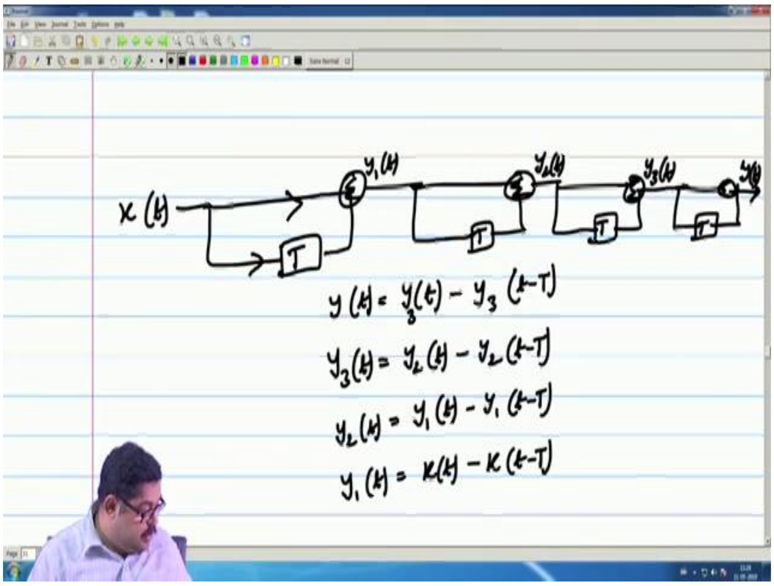
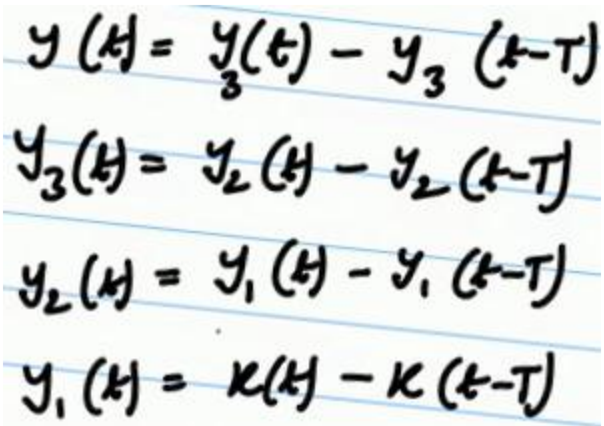


So, a delay you are getting here you are putting sum. So, this is one stage, then there will be another stage, then there will be another stage and then there will be the fourth stage. So, this output is y t. So, 4 cascaded single DLC.

So, you can write the expression if I call this as y 1 t, this one y 2 t this is y 3 t final one is y t. So, you can write it that what will be the final one; that means, y t will be y 3 t sorry y 3 t minus y 3 t minus T, then y 3 t will be y 2 t minus y 2 t minus T and y 2 t will be y 1 t minus y 1 t minus T and y 1 t is x t minus x t minus T.



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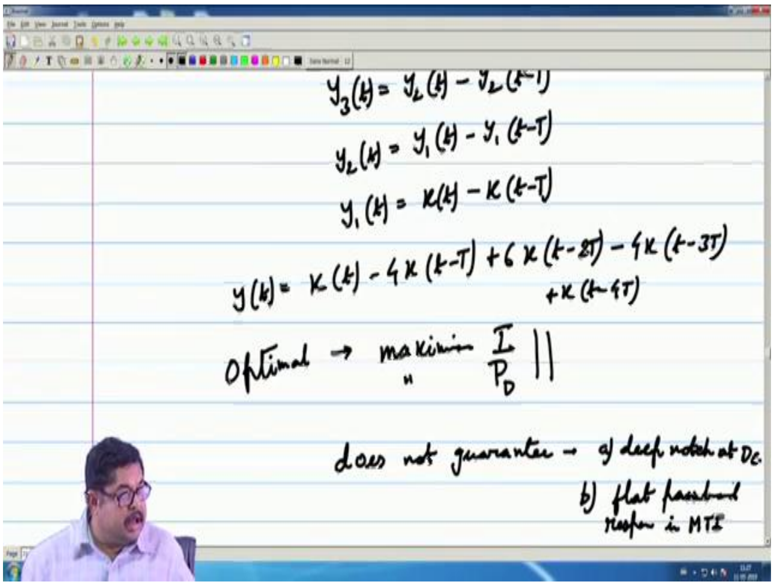

So, you can put this one by one here and finally, we will see that you can write y t as x t minus 4 x t minus T plus 6 x t minus 2 T minus 4 x t minus 3 T plus x t minus 4 T.

$$y(t) = x(t) - 4x(t-T) + 6x(t-2T) - 4x(t-3T) + x(t-4T)$$

So, you can check these weights are nothing, but the binomial filter.

Now, this binomial coefficient choice; that means, this implementation this is optimal this is optimal in the sense that it maximizes. So, this is optimal in the sense that it maximizes the MTI improvement factor I also it maximizes the probability of detection. So, if we choose it in these way people have proved that because you can have other implementations also 4 stage etcetera, but this is the bast in the sense of this it maximizes the MTI improvement factor and also it maximizes the probability of detection.

So, maximizes I and maximizes P D probability of detection these we will see later in the detection theory what does it mean now we write it, but, but it does not guarantee. So, these two things is very useful, but it does not guarantee I will say that these does not guarantee, 2 things a deep notch at DC; that means, that notch at DC does not get guaranteed by these for that you will have to find various things and also the flat pass band response in MTI.

So, this is a open problem that determination of W i so that apart from improving I and PD these two a and b these 2 also is at it. One answer to that we have seen is the recursive filter, but still there are other things that how to choose those values so, that we can get a good a. So, this is a still evolving research area that how to choose these and ok.

So, I think we close this lecture here some more things about this MTI filtering because this is a very important research area and with the technology this is much focus is there on radar research researchers, they concentrate here that how to improve these MTI detection etcetera. So, we will discuss something more and that how to determine that which stage how many stages of DLC will take that thing we will discuss in the next class.

Thank you.