Principles and Techniques of Modern Radar Systems
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Lecture – 19
Multiple PRF MTI Radar (Contd.) & Clutter Attenuation

Key concepts: Quantitative analysis of MTI filter, Analytical expression of clutter attenuation of an SDLC

Welcome to this NPTEL lecture on Principles and Techniques of Modern Radar System. Now, we were discussing MTI filter in that we have also seen staggered PRF MTI radar. So, what I will now take an example and see that what we have discussed, based on that how we can calculate the first blind speed, what is the effect of pushing the first blind speed further away by staggering the PRF.

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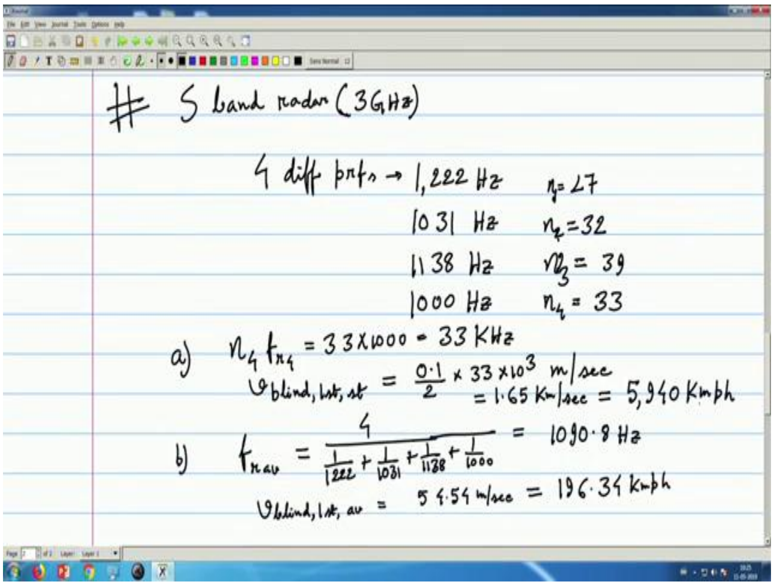

So, let us consider a sorry, consider a problem that a S band radar, let us say at 3 Giga Hertz utilizes staggered waveform with 4 different PRFs, 4 different PRFs. Let us say the PRFs are 1222 Hertz then 1031 Hertz then 1138 Hertz and 1000 Hertz. So, let us calculate, the first part is calculate the first blind speed of this radar. So, this staggered radar, what will be the first blind speed? So, here we want to find out that what is the common or what is the common multiplier of this.

So, actually that is a there are various theories for finding these numbers, but here let us say that we can take it as for this one I can take n. So, we are calling it n 1 27, n 2 let us say 32, then n 3 is 39 and n 4 is 33. If you see that if you multiply 33 with this, you will and 27 with this you will get the same thing. So, this has been obtained and you see that there are no these numbers are given. So, we can say that first blind speed that will occur at anyone, let us say that let us say the last one because there is 1000 here. So, first blind speed occur at the Doppler which is n 4 f r4. So, that will be 33 into 1000. So, 33 kilo Hertz is the Doppler. So, what is the v blind first for staggered? We know this formula is simple that lambda by 2 into this.

So, we can make this is 3 Giga Hertz. So, lambda will be 0.1 in meters by 2 into this 33 kilo Hertz. So, this blind speed will come in meter per second. So, if you do this, this will be 1.65 kilo meter per second or if you convert it to kilo meter per hour, it will be 5940 kilo meter per hour. So, you see as I said that typically the aircrafts (Refer Time: 05:03) is 1000 etcetera kilo meter per hour. So, if the first blind speeds come like here it is good, but let us compare that had we not done this staggering and suppose we have operated with a constant PRF.

Now, which is the how to get the constant PRF that let us take some average. So, if that is taken as the PRI average then we know that in that case our f r average that formula we have given. So, that will be 4 by 1 by this 1222 plus 1 by 1031 plus 1 by 1138 plus 1 by 1000 in Hertz. So, that will be 1090.8 Hertz. So, what will be the blind speed due to that? Blind first average so, that if you calculate that again those things. So, it will come to 54.54 meter per second.

So, here it was kilometer, but in kilometer per hour if we convert that it is 196.34 kilometer per hour. So, you see compared to a 200 kilometer per hour by staggering, we are pushing it almost 30 times I can say. So, you see thus there is tremendous improvement in these, but what is the price we are paying definitely in nature we cannot get anything free.

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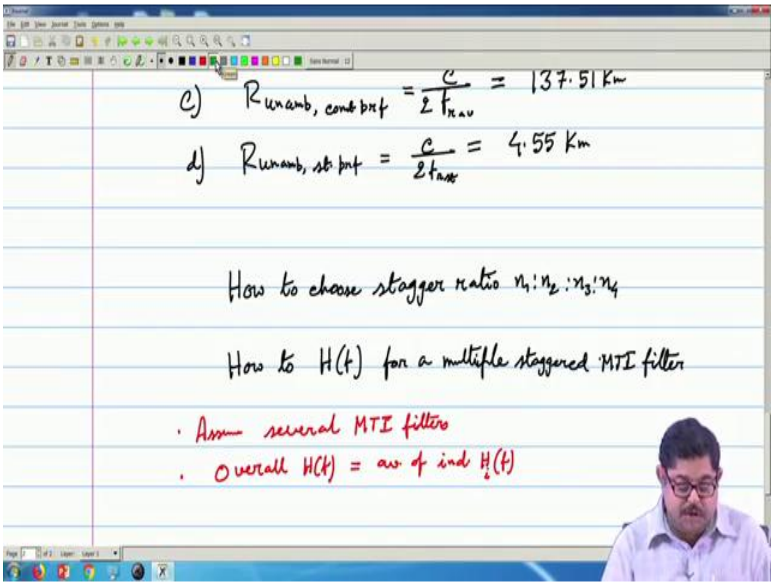

So, the price we are paying in the unambiguous range. So, for this average a thing, what will be the unambiguous range? So, unambiguous range, for this constant PRF radar so, that will be c by this two f r av c by 2 into. So, if you do that, that will give you 137.51 kilometer whereas, what is that scales for the staggered PRF radar? So, R unambiguous the staggered PRF in that case c by 2 f r staggered. So, f r staggered we have already found out it is 33 kilo Hertz. So, we can put that, that c and those things that will give you 4.55 kilometer.

So, you see that drastically this is getting reduced. So, if you are more interested in the velocity, you go for this. So, we have seen the effect of staggering. Now, I will come back to the point where we have left last time that you see the four PRF staggering is a common thing, but we do not know how to make the transfer function of the MTI filter, if there are four staggering because we have solved the problem for two staggering frequencies.

So, also there is another point that how to choose; how to choose the stagger ratio; that means, the point I was saying that how to choose that n 1 is to n 2 is to n 3 is to n 4. So, there are various algorithms those are mathematical things I am not going into that details. But, generally these staggered ratio is chosen closer to unity and that gives you good results.

Now, to come back that how to find the H f for a multiple staggered MTI filter. So, the two one we have seen that we can always do which two. So, to determine in this case the algorithm for more than two staggering PRFs first, we will say that assume there are assume several MTI filters one for each combination of two staggered PRFs.

So, you determine now for two you know how to do. So, you can find out the individual PRFs for them then you overall response overall H f is nothing, but average of individual H f or I will say H i f. So, what does that mean? Suppose I have that case of four.

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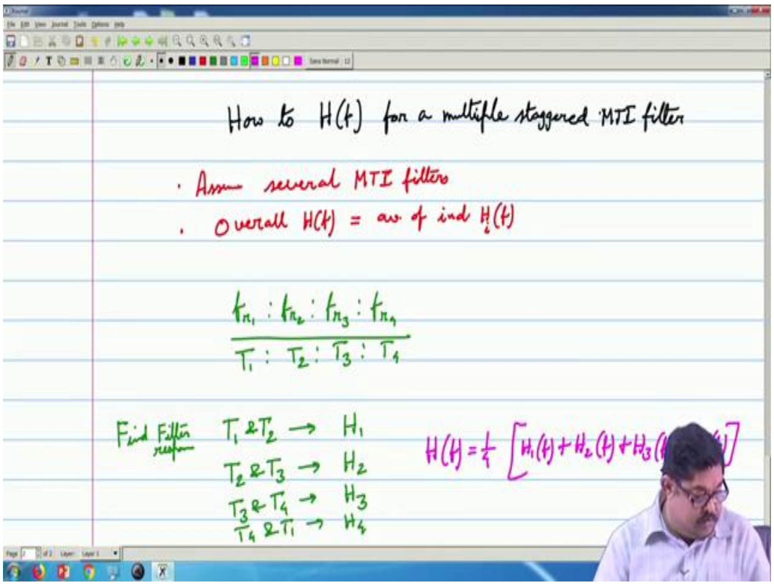

So, if I have the four staggering PRFs let they be f r1, f r2, f r3 and f r4 four staggering PRFs. The corresponding PRIs will be T 1 T 2 T 3 and T 4. So, what we will do first we will find the filter response with let us say first find filter response with T 1 and T 2 that we have already seen and let us call that filter response.

So, find filter response by staggering f r1 f r2; that means, So, let us call it T 1 then you do it with T 2 and T 3 let us call that filter response H 2 then T 3 and T 4 that is H 3 let us call and finally, that T 4 and T 1 that we will be calling H4. Now, by that overall H f, what will be the overall H f? Overall H f is 1 by 4 into H 1 f plus H 2 f plus H3 f plus H 4 f ok.

$$H(f) = \frac{1}{4}\left[H_1(f) + H_2(f) + H_3(f) + H_4(f)\right]$$

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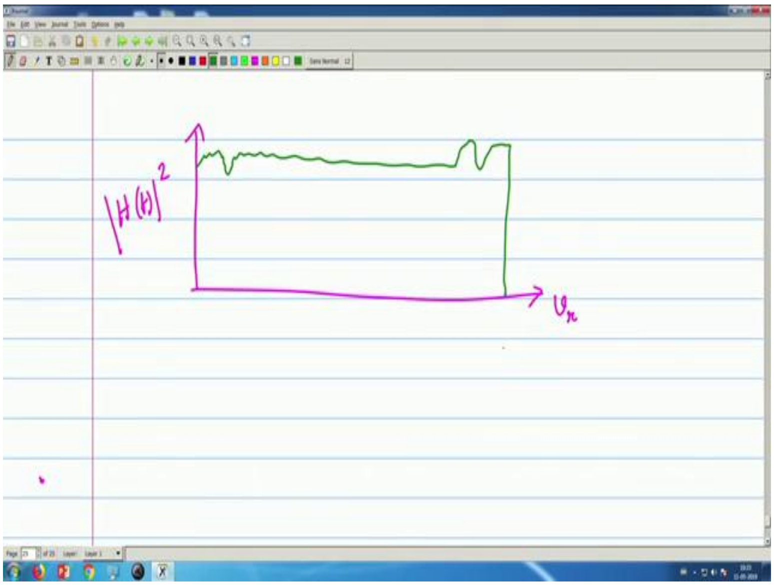

So, we can draw the gain that Hf square for this overall response versus the relative velocity v r. So, the graph will be there will be some initial ripples, but otherwise that will be a flat response up to the first blind speed. So, clutter is removed, you see first blind speed pushed away and more or less uniform response over MTI pass band.

So, this is a good way now we will see that how to characterize this filters that each MTI filter we require some performance metrics to measure that how much it would remove clutter. So, there are three performance metrics.

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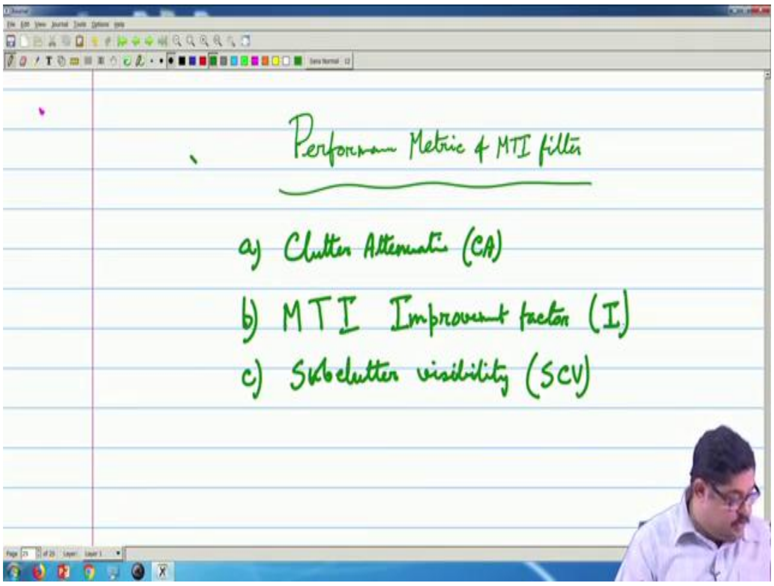

So, I will say that performance metric of MTI filter. There are a three such, one is called clutter attenuation or its symbol is CA, the second one is called moving or MTI improvement factor and the third one is called sub clutter visibility.

So, we will discuss each one of them. So, MTI improvement factor this one is called sub clutter visibility and MTI improvement factor is generally given the symbol I ok. So, we will now discuss one by one.

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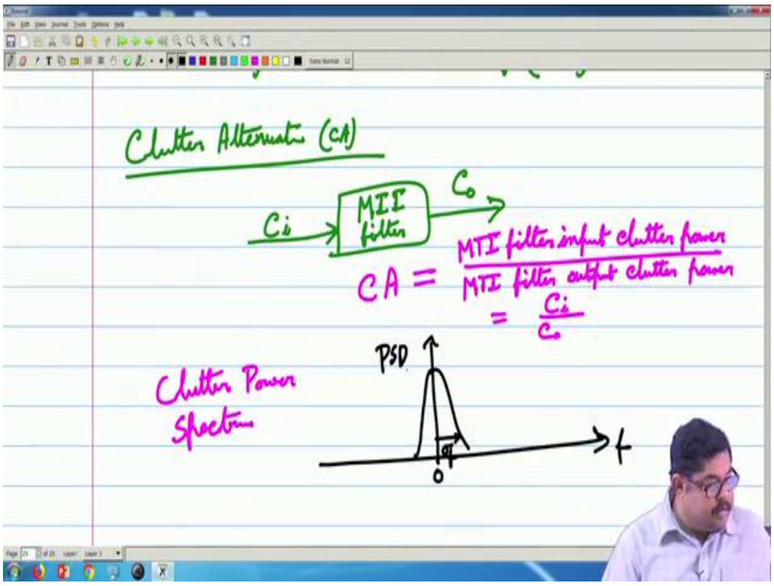

So, first is the clutter attenuation C A. So, consider that I have the MTI filter. So, the input to this is the signal as well as corrupted with clutter. So, if the clutter power input clutter power, clutter is a random process, but we know that we can through the PSD power spectral density we can find its power. So, let us say that that is input is C i and output clutter power is C o; in that case the C A clutter attenuation will be the ratio of these. So, I can say that MTI filter input clutter power divided by MTI filter output clutter power; that means, basically it is a ratio of C i by C o ok.

$$CA = \frac{\text{MTI filter input clutter power}}{\text{MTI filter output clutter power}} = \frac{C_i}{C_o}$$

Now, if we can model the clutter power then or clutter we can model. So, if we can model the power spectral density then we can find out the power let us do that. So, let us assume that the clutter is modeled as a Gaussian distributed power, you know that generally we take Gaussian because it is mathematically tractable also there is a central limit theorem which says that, if there are no dominant scatterers and there are any were dominant scatterers of this clutter and no one is dominating. But, there are many usually four five is there that is sufficient then whatever may be the individual clutter power spectrum the overall will be Gaussian.

So, with that knowledge we can say that if we model the clutter power spectrum power spectrum. Now, also I recall that we have seen that actually clutters' power spectrum is something like this, if this is your f then frequency. So, there will be center at 0 it has a spread of sigma f and that we have seen at the beginning. Also, we know that due to the pulse radar, that spectrum will get repeated at the integral multiple of f r.

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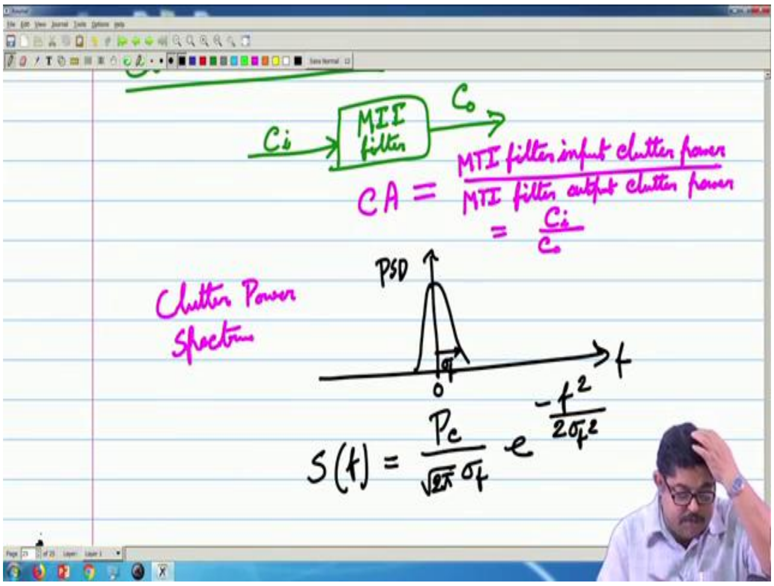

Now, first this is the PSD so, let us analytically express it that if clutters power spectrum is S f and; obviously, it is centered at 0. So, we can write it as P c by Gaussian PDF. So, 2 pi sigma f e to the power minus f square by 2 sigma f square.

$$S(f) = \frac{P_c}{\sqrt{2\pi}\,\sigma_f} e^{-\frac{f^2}{2\sigma_f^2}}$$

Here we have assumed that it is a 0 mean that is why mean is not coming here and what is Pc? P c is the I should.

I should write, what is P c? P c is the constant clutter mean power or clutter mean power that is P c. So now, I can calculate from that, what is the input power? So, input power will be from minus infinity to infinity I will have to have that S f df this is power spectral density S f. So, overall frequency if I integrate I will get so, this is nothing, but the integration of that P c by root over 2 pi sigma f e to the power minus f square by 2 sigma f square df.

$$C_i = \int_{-\infty}^{\infty} S(f)\, df$$
$$= \int_{-\infty}^{\infty} \frac{P_c}{\sqrt{2\pi}\,\sigma_f} e^{-\frac{f^2}{2\sigma_f^2}}\, df$$

Now, you know that if I take out P c out then what is this? 1 by root over 2 pi sigma f e to the power minus f square by 2 sigma f square df and this is nothing, but area under the whole Gaussian curve minus infinity to infinity, we know that value is 1. So, the answer will be P c.

$$= P_c \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\,\sigma_f} e^{-\frac{f^2}{2\sigma_f^2}} df$$

$$= P_c$$

Now, next we will have to determine the output power. So, I have the MTI filter now, I am know the input I am giving that I am giving that Sf. So, what will be the output that I can write from the filter theory?

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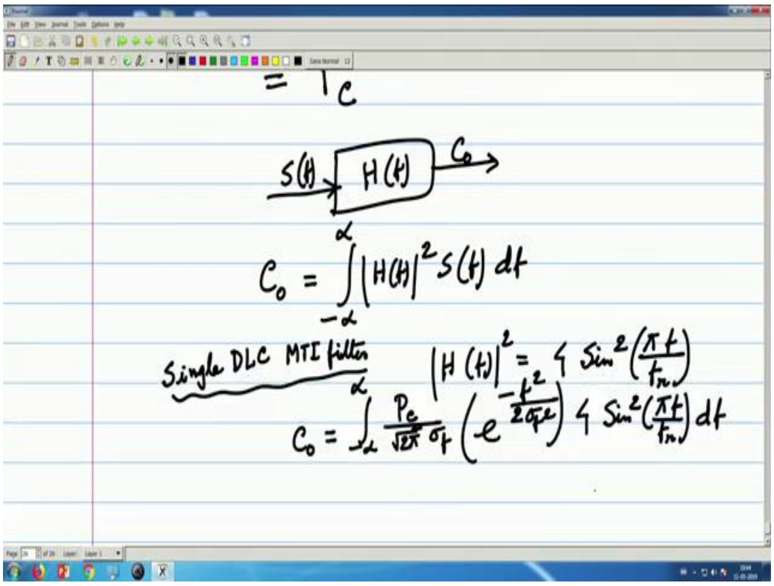

Remember, this S f is powered spectral density and output I will get C o. So, C o is minus infinity to infinity H f square S f df so, this integration we need to do.

$$C_o = \int_{-\infty} |H(f)|^2 S(f)\, df$$

Now, H f will have to now take some value. So, if you have the single DLC suppose single DLC MTI filter, if I have that then I know that what will be H f, H f square we have already derived 4 sin square pi f by fr.

$$|H(f)|^2 = 4\sin^2\left(\frac{\pi f}{f_r}\right)$$

So, we will put that.

So, C naught will be minus infinity to infinity P c by root over 2 pi sigma f then e to the power minus f square by 2 sigma f square into this 4 sin square pi f by f r df.

$$C_0 = \int_{-\infty}^{\infty} \frac{P_c}{\sqrt{2\pi}\,\sigma_f}\left(e^{-\frac{f^2}{2\sigma_f^2}}\right) 4\sin^2\left(\frac{\pi f}{f_r}\right) df$$

Now, as we have seen that if you look at the spectrum that this spread is very small.

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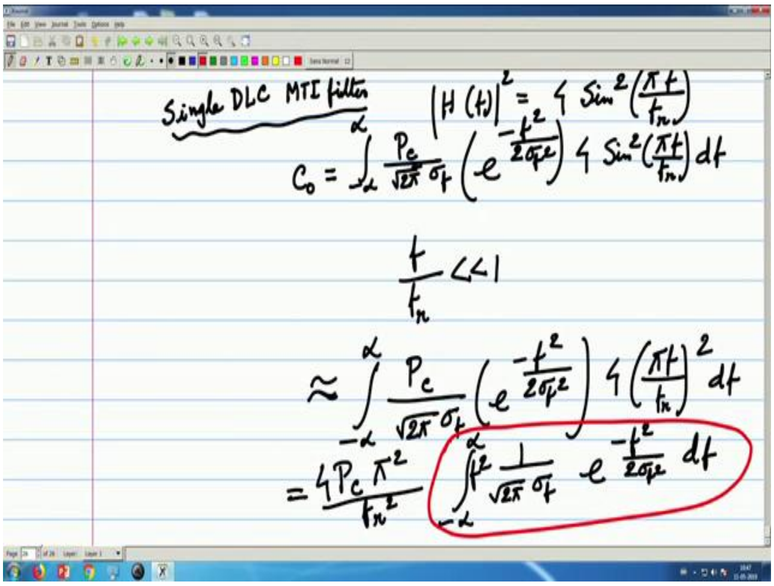
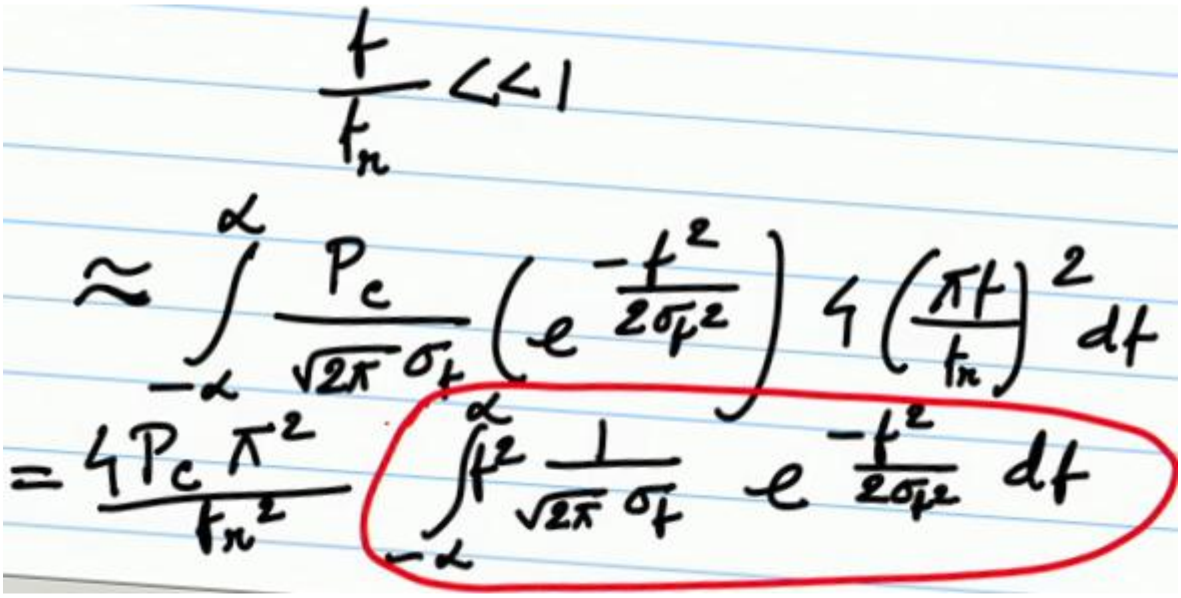


So, I can utilize that and we can say that this power spectrum, the clutter spectrum that is significant only at small f. So, I can assume that f by f r that is much much less than 1 because only over a very small spread, the power spectrum is significant, but f r is quite larger than sigma f. So, f by f r is very small if I do that then you know this sin square term can be approximated by the square of the argument sin argument. So, I can with that I can write this that minus infinity to infinity P c by root over 2 pi sigma f e to the power minus f square by 2 sigma f square into 4 into pi f by f r whole square df ok.

So, I think you have recognized, what is this because I can take all these things out I am deliberately taking out those things which I do not need 4 P c will go out pi square will go out and here I will take that f r square, it is also a constant with respect to f. So, I am left with minus infinity to infinity 1 by root over 2 pi sigma f then e to the power minus f square by 2 sigma f square and actually I should say that f square into df.



So, can you see what is this? Can I say that this part is nothing, but the second moment of the Gaussian pdf.

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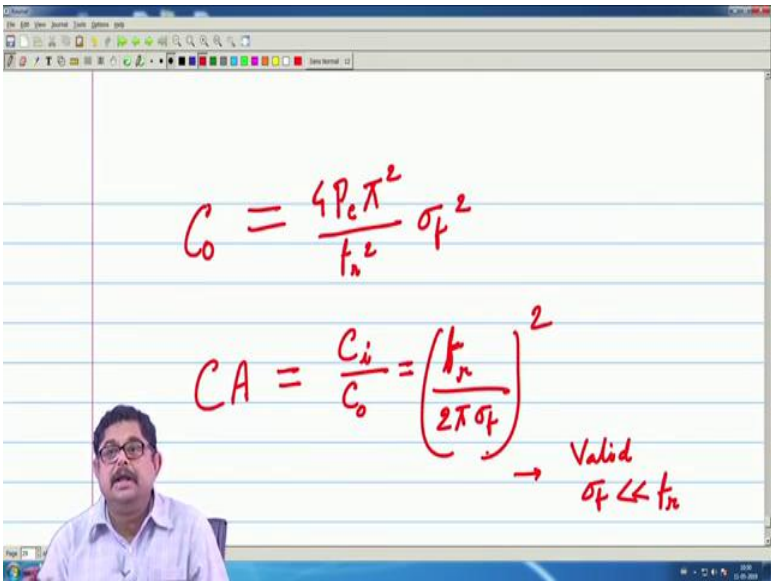

So, I can easily write that it was C o, C o will be the 4 P c pi square by f r square into sigma f square

$$C_0 = \frac{4 P_c \pi^2}{f_r^2} \sigma_f^2$$

because second moment is the sigma f square for a zero mean case. So, this so, now I can find out, what is clutter attenuation? Clutter attenuation is C i by C naught and I have seen that both the things I have evaluated. So, if I put that it will be f r by 2 pi sigma f whole square and remember that this is valid provided I have sigma f is much much less than f r.

$$CA = \frac{C_i}{C_0} = \left(\frac{f_r}{2\pi \sigma_f}\right)^2 \rightarrow \text{Valid } \sigma_f \ll f_r$$

Usually it is true that f r the PRF that is there that is much larger than sigma f, but if you have that in a very the platform motion and other things sometimes, may make that sigma f is not this is valid then you will have to go to the basic equation and try to find out evaluate that integral, but otherwise this is a good one. So, you see that for a single DLC this is a thing for clutter attenuation. So, it is proportional to the f r square the PRF square and since sigma f is very small.

So, you have a good number so, lot of clutter gets attenuated ok. But as thus in noise that we are more interested in because this is a detection problem. So, we are more interested here also like communication that, what is the how much clutter is suppressed with respect to the received echo signal power level; that means, signal to noise ratio sort of thing. So, that we call signal to clutter ratio.

So, that is more important because clutter may be reduced, but if the signal to clutter ratio is not improved due to the MTI filter then there is no point. So, that is the second metric will come, the MTI improvement factor that takes care of that something like your noise figure in case of noise. So, the corresponding thing in this MTI filter domain is called MTI improvement factor we will take up that in the next class.

Thank you.