Principles and Techniques of Modern Radar Systems
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Lecture – 18
Multiple PRF MTI Radar

Key concepts: Multiple PRF MTI radar, PRF staggering, Analytic expression of transfer function a three pulse canceller with different PRI, Improvement on the first blind speed with staggered PRF

Welcome to this NPTEL lecture on Principles and Techniques of Modern Radar System. Now we were discussing MTI filter, out of that we have seen that there is a good MTI filter that is the one recursive with feedback loop. So, that one gives you a good pass band response and also deep notch. Now we will see that how we can extend the first blind speed with the radar without disturbing the lambda or fr. So, that we will discuss that is called multiple PRF MTI.

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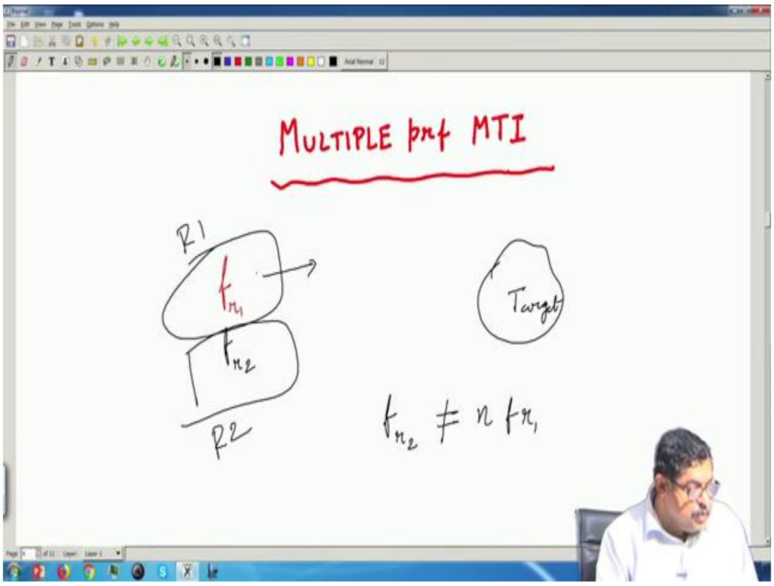

So, now, to understand the concept, first think that there are two pulse radars with different PRFs suppose, PRF fr 1 and another PRF is fr 2 and they are trying to detect a target. So, obviously, since so, this is one radar; radar 1 and this is another radar; radar 2 they are trying to detect the same target. So; obviously, the first blind speed of this radar is different from the first blind speed of this radar, because their PRFs are different.

Now, even if they are operating at same rf frequency, their first blind speeds are different. So, target undetected by one radar maybe visible to the second radar or vice versa; provided this fr 1 and fr 2 they are not integral multiples of each other. That means, fr 2 should not be, if this relation holds then if one misses that the other will do that so, that we can imply. But this is a costly affair having to detect one target; one radar itself is costly then putting two is costly, but this concept can be used that the same radar, let us multiplex the PRF in time domain; multiplex the PRF in time domain that is called multiple PRF MTI radar.

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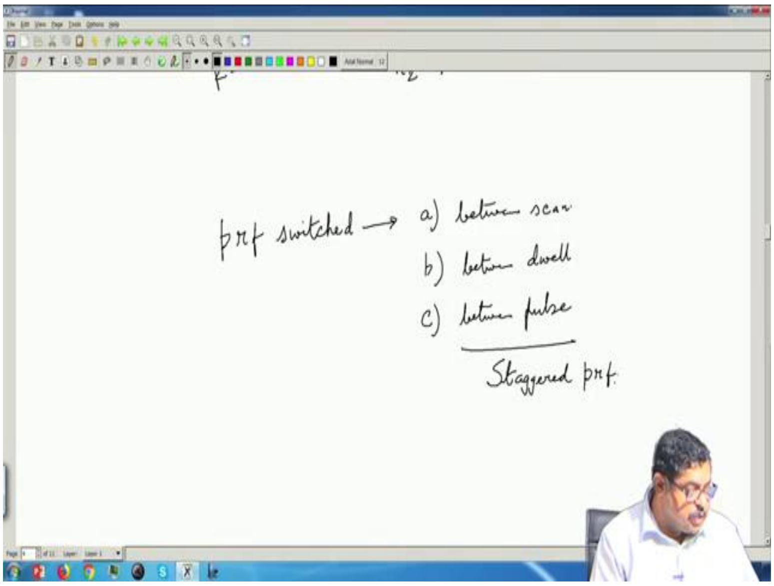

Now so, we can say that instead of two radars, 1 radar can multiplex the 2 PRFs now there are various ways by which this can be done. So, we can say PRF switched; PRF can be switched one scheme is between scan; that mean, in one scan you know antenna scans so, in one scans the PRF is something and the next scan it is a different PRF, then some radars they do it between dwells. Dwells means when you are at a particular beamwidth; that means, when you are scanning 1 beamwidth scan is called dwell.

So, the every time the antenna scan half beamwidth the PRF is switched so, next PRF; another PRF is there or the another scheme is between pulse. So, at after sending 1 pulse with a particular PRF the next pulse is a; so, this one we will discuss the others are similar the concept. So, this is called switching PRF pulse to pulse, this is called staggered PRF ok.

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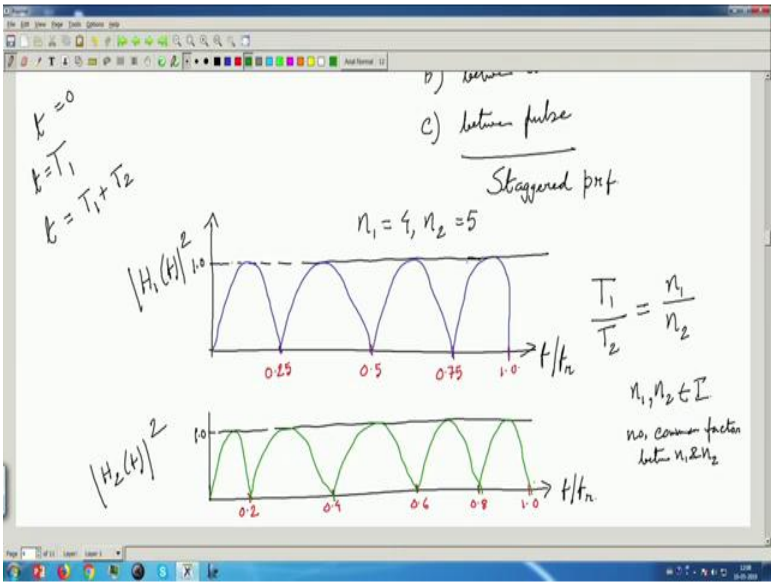

So, we will discuss this; so, one thing is that we will choose the PRFs in such a way that; the that second pulse will be sent at suppose 1 pulse is sent after an interval t is equal to T 1. So, if first pulse is sent at t is equal to 0 then the second pulse is sent at t is equal to T 1, then the third pulse is sent at t is equal to T 1 plus T 2. If we did not stagger every pulse we will send t is equal to T 1 we will send 1 pulse, t is equal to 2 T 1 we will send a pulse, t is equal to 3 T 1 we will send a pulse.

But here you see that the first pulse we are sending at t is equal to 0, t is equal to T 1 we are sending the second pulse, t is equal T 1 plus T 2. We are sending the third pulse and we are choosing, so that that T 1 and T 2 they are not integrally related; that means, T 1 by T 2 will be in n 1 by n 2

$$\frac{T_1}{T_2} = \frac{n_1}{n_2}$$

where, n 1 and n 2 belongs to an integer and also no common factor between n 1 and n 2, obviously, except 1; 1 is common factor of every number. So, no common factor between n 1 and n 2.

So, I will now to draw that graph I need some numbers. So, let us take one example is n 1 is 4 and n 2 is 5. So, what will be the magnitude response of the first one. So, I know that; one second, so, this will be, if I draw this is 1.0 and this side is f by fr. So, I can draw these let me fix it.

So, n 1 is 4, so, we are considering a single DLC, so I know that it will be something like this that there will be a notch here. So, this will be the thing. Now, what happens for the second one who has these n 2 is equal to 5 what will be the response? So, let me, so there we know that, so the response of the two a things; that means, had we been the first one that will give this, the second one with n 2 is equal to 5 will give this response.

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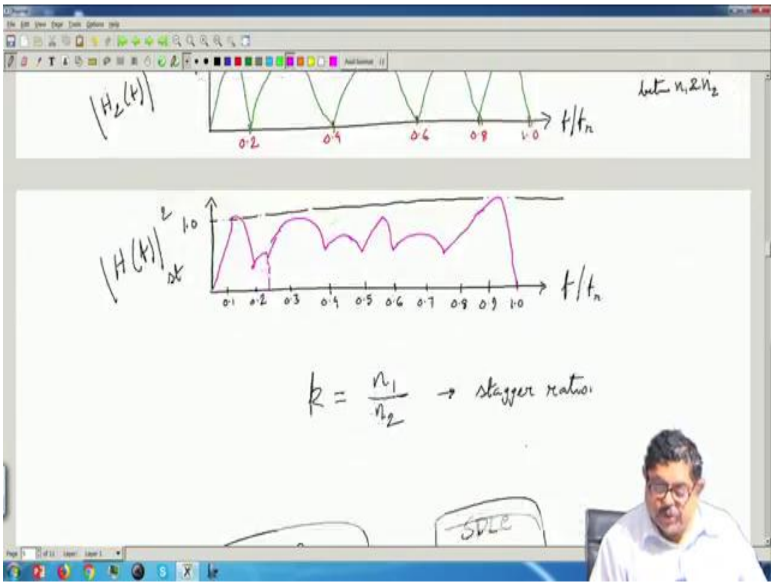

So, if we now sum these two, you will find; actually this is a good MATLAB exercise, you yourself can do it and see that in this case the, when we will see the staggered one, basically we will show that these will be, all these notches; that means, these nulls they will go, but before that we need to have some analysis.

So, I will later complete this part that how it will look with a staggered PRF, but we need to understand. And this ratio k is equal to n 1 by n 2 that is called a stagger ratio.

$$k = \frac{n_1}{n_2} \rightarrow \text{stagger ratio}$$

Now this choice of the stagger ratio is very important, lot of theoretical work goes on to find out what is the stagger ratio. I will indicate that still there are some open research problems in these, how to choose these stagger ratios etcetera. So, what I did consider a three-pulse canceller with PRI's. You know these T 1 T 2 I have already introduced that radar those are called PRI's. So, with PRI's the first one I have shown that is having a PRI of T 1 and the second one with a PRI T 2.

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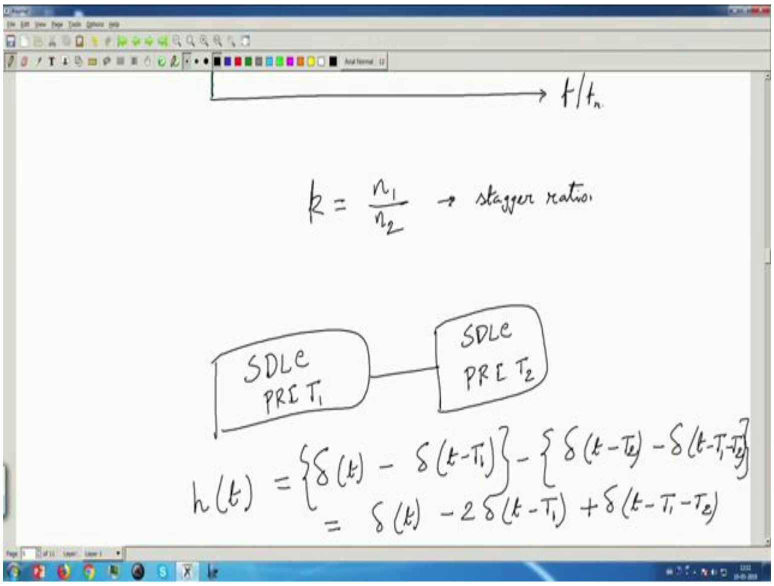

So, if we have that so that means, I have a I can say one SDLC with PRI T 1 and then another SDLC with PRI is T 2. So, this will make a three-pulse canceller. So, we can easily write what is the impulse response; that it is delta t minus delta t minus T 1, then delta t minus T 2 minus delta t minus T 1 minus T 2. So, we know it is delta t minus 2 delta t minus T 1 plus delta t minus T 1 minus T 2.

$$h(t) = \{\delta(t) - \delta(t-T_1)\} - \{\delta(t-T_2) - \delta(t-T_1-T_2)\}$$
$$= \delta(t) - 2\delta(t-T_1) + \delta(t-T_1-T_2)$$

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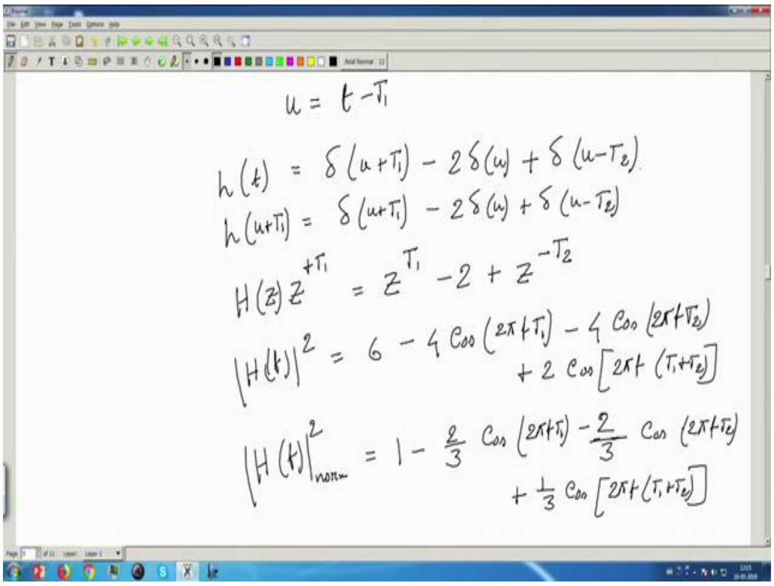

So, here you can put a change of variable that u is equal to t minus T 1 and that will make that your h t is delta u plus T 1, sorry minus 2 delta u plus delta u minus T 2. So, this one also we need to change h u plus T 1 is equal to this delta u plus T 1 minus 2 delta u plus delta u minus T 2.

$$u = t - T_1$$

$$h(t) = \delta(u+T_1) - 2\delta(u) + \delta(u-T_2)$$
$$h(u+T_1) = \delta(u+T_1) - 2\delta(u) + \delta(u-T_2)$$

So, then we can go to the z transform that what will be the z transform for these that H z z to the power plus T 1 is equal to z to the power T 1. This will be minus 2, this will be z to the power minus T 2.

$$H(z)\, z^{+T_1} = z^{T_1} - 2 + z^{-T_2}$$

So, this we can manipulate and easily find what is H z; H z will be 6 minus 4 cos or I can write directly in terms of omega also, because we know how to come from z domain to this. 2 pi fT 1, instead of omega I am writing. So, this also I can write in terms of f. 4 cos 2 pi fT 1 minus 4 cos 2 pi f T 2 plus 2 cos 2 pi f T 1 plus T 2. So, we can normalize these with these DC value 6. So, I can write H f square normalized is 1 minus two-third cos 2 pi f T 1 minus two-third cos 2 pi f T 2 plus one-third cos 2 pi f T 1 plus T 2.

$$|H(f)|^2 = 6 - 4\cos(2\pi f T_1) - 4\cos(2\pi f T_2) + 2\cos[2\pi f(T_1+T_2)]$$

$$|H(f)|^2_{norm} = 1 - \frac{2}{3}\cos(2\pi f T_1) - \frac{2}{3}\cos(2\pi f T_2) + \frac{1}{3}\cos[2\pi f(T_1+T_2)]$$

So, this thing now I am in a position to plot it. So, this plot now I can complete that this is my level 1. So, this is my I can say that H f square of the 3 DLC staggered. So, I will write staggered one. So let me take this color and put that let me take 0.1, 0.2, 0.3, 0.4, 0. Actually in MATLAB you can do these well, but am just giving you roughly an idea that how it looks like, that starting from here, actually it has something here, then again you can now here, then it goes up and then 0.4 it has a null and all, then 0.5 again it has a null, then it will go up and again at 0.6 it will have a null, then 0.75 it will have a null, then finally, it goes and comes to 1.

While you see what I have seen these, these are the points of null. So, when they are added, actually since nulls are there, so, when you are cascading them so, at 0.2 there is a null, at 0.25 there is a null, local nulls, but you see the whole fun thing is making it more uniform. And by that it is the null instead of having at 0.25 at 0.5 etcetera you are putting it much ahead. So, this is the first blind speed coming and we will find out that how much improvement we have got compared to the original ones.

So, at least there you can see that if whatever was the PRF of any your original one at least by 4 or 5 times or their LCM times it has been shifted. So, this shows that a; obviously, one thing is it is not as uniform as the original one, but it has some local nulls here, but we know the technique that if we put the proper recursive filter then this thing can be easily (Refer Time: 20:50), because if we could have make that sine square type of thing uniform. So, this is making uniform is not a problem with the recursive filter. So, with the recursive filter these thing is the. Now the question is how far we have advanced; that means, how far we are successful in pushing the blind speed.

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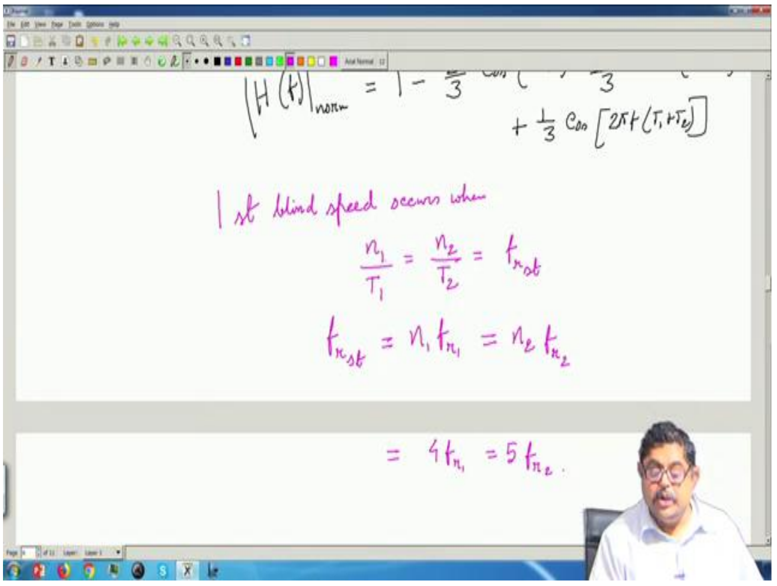
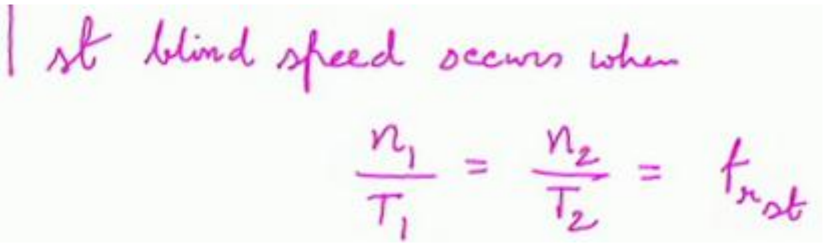


So, that will now calculate that first blind speed will occur when I can say from that graph that n 1 by T 1 is equal to n 2 by T 2 is equal to fr staggered.



That means, basically when the nulls are coming that some integral multiple of 1 by T 1 and some other integral multiple of 1 by T 2 when they are equal then the null is coming, as in this case with 4 by 5, so it will come at that 4 into some number and 5 into some number we know that is the 20. So, at 20 it will come, so here it is, this is the general thing.

So, we can say that the what is fr staggered? You see very simple relation n 1 into what is 1 by T 1, it is the PRF of the first one; that means, the PRF with which the radar operated at the first between the, on the second pulse that is these or it is also same as n 2 into 1 by T 2 is nothing, but fr 2. So, how far it is pushed either n 1 times it has pushed or n 2 times it has pushed.

$$f_{r_{st}} = n_1 f_{r_1} = n_2 f_{r_2}$$

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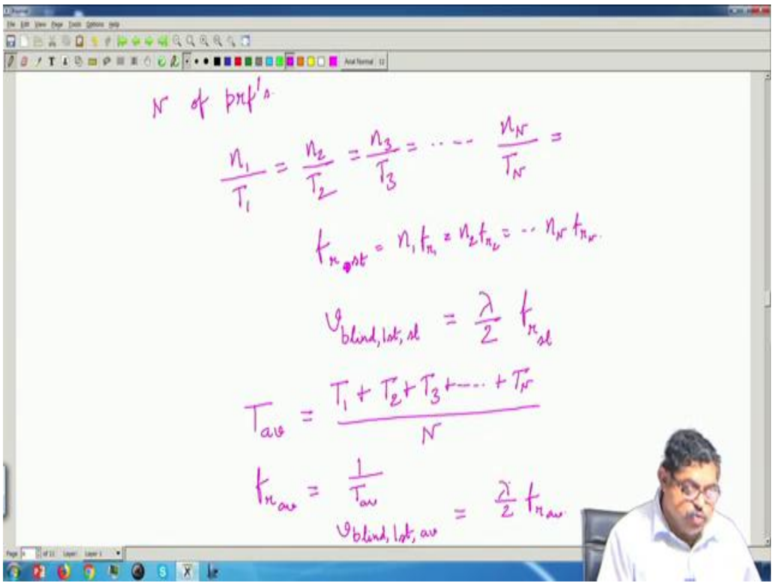

And in our case if we now specialize for that, in our case we can say that fr staggered is nothing, but 4 f r 1 or 5 f r 2 and so, 4 f r 1 is 5 f r 2, but in general. Now what is the general thing for these that in general, suppose here we have assumed in the previous example only two PRFs we are multiplexing. Now in many cases you need to multiplex more to push it further.

So, let us say that there are capital N number, N number of PRFs are there. So, we know the equation will be n 1 by T 1 is equal to n 2 by T 2 is equal to n 3 by T 3 is equal to n N by T N.

N of prf's

$$\frac{n_1}{T_1} = \frac{n_2}{T_2} = \frac{n_3}{T_3} = \cdots \frac{n_N}{T_N}$$

So, this relation will hold, so we can find what is the first blind speed. So, from here we can find what is the staggered PRF.

So, any one of these is the staggered PRF become the first blind speed and from there we can find out what is a fr staggered. As before you can see the same equation we are writing f staggered is any of them. So, fr staggered will be n 1 fr 1 or n 2 fr 2 etcetera up to n N frN.

$$f_{r_{st}} = n_1 f_{r_1} = n_2 f_{r_2} = \cdots n_N f_{r_N}.$$

Now how much we are successful to compare that we can say that compared to a constant PRF radar, how much these multiplexing of PRF has pushed the change. So, from staggered what is the relation to the blind speed.

So, v b or v blind first staggered. So, we know what is that relationship that is lambda by 2 into fr stag.

$$v_{blind,1st,st} = \frac{\lambda}{2} f_{r_{st}}$$

This lambda is the RF carrier frequencies lambda, so fr stag. So; that means, if we can make fr stag more blind speed will be also more. Now to compare it with a constant PRF, we need to find the average time, because you see different pulses have been sent on different time. So, we will have to compare it with a average time. Now this T average is nothing, but can be written T 1 plus T 2 plus T 3 plus T N by N

$$T_{av} = \frac{T_1 + T_2 + T_3 + \cdots + T_N}{N}$$

and we can say that a radar operating with a PRF which is average of these that will be nothing, but 1 by T average

$$f_{r_{av}} = \frac{1}{T_{av}}$$

and so, its v blind first average that will be this lambda by 2 f r average.

$$v_{blind,1st,av} = \frac{\lambda}{2} f_{r_{av}}$$

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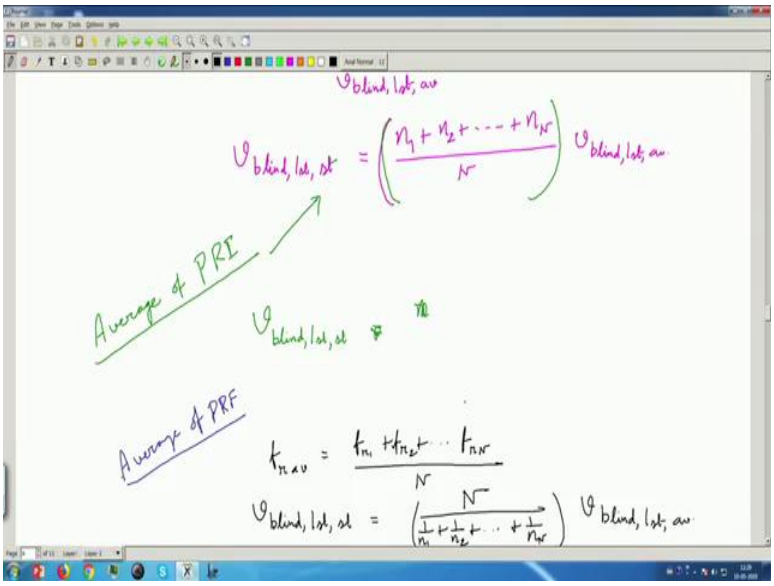

So, if we do that, so we can make the relation that v b or blind v blind first stagger is nothing, but n 1 plus n 2 plus n N by N into v blind first average.

$$v_{blind,1st,st} = \frac{n_1 + n_2 + \dots + n_N}{N} v_{blind,1st,av}$$

So, this number by which we are getting the improvement. So, this number is our improvement factor, that staggered, this staggered one compared to a constant PRF is how much.

Now we will derive this relation now, it is very simple, you how I am coming from here to here so, I will say also this average taking; actually there are two ways the average can be taken, they give slightly different result. These either you can take the average of PRI's or you can take the average of PRF's.

Now, they give slightly different values, but more or less things are same. So, I will now or I would leave it, that you can do it. I am just giving you the result in the example we will see or assignment you will have to derive these. So, I am writing that if you take the average, for the constant PRF of radar, average of the T average, of average of, sorry I write, average of PRI then your improvement that v blind first staggered becomes n 1, I whatever I have written, This is we have these you can show that, but if I have average of PRF; that means, if we have fr average is fr 1 plus fr 2 plus fr capital N by N then if you do you will see that v blind first staggered is equal to the formula is N by 1 by n 1 plus 1 by n 2 plus 1 by n N into this v blind first average.

$$f_{r\,av} = \frac{f_{r_1} + f_{r_2} + \cdots f_{r_N}}{N}$$

$$\vartheta_{blind,\,1st,\,st} = \left(\frac{N}{\frac{1}{n_1} + \frac{1}{n_2} + \cdots + \frac{1}{n_N}}\right) \vartheta_{blind,\,1st,\,av}$$

So, see these two formulas are slightly different, but anyone can be used as in the practical assignments in the examples we will see, that anyone PRF you can use, so for making the comparison and there you will see that it is an order of improvement higher; that we can push the blind speed further. So, that completes our discussion on the putting the blind speed etcetera. Now we will have to find what is the composite a thing, because I have not shown you that how I have done this graph; that how to, in general how to find the H f.

Actually, you see that these graph I have, this expression is derived for multiplexing two PRFs, but in general if there are more number of PRFs then how to do that. Also one thing I am not going into that how to choose the numbers n 1 n 2, because choosing those numbers that they do not have common factors, but you see choosing them gives you the actual multiplication factor.

So, how to choose that there are various theories, we are not going into that, but in the next class will continue this discussion and we will find for staggered PRFs, how to find the a thing, some properties of PRF. And then we will also define various figure of merits for the this MTI filters, particularly MTI improvement factor that we will find out and we will then compare all these things that how much clutter they are attenuating and also how much signal they are attenuating.

Basically, some sort of a SNR type of thing that will that type of concept we will see and we will make that performance evaluation and from that we will see, so that we will complete. So, MTI filter we will be considering, but what we have seen in this lecture and the previous lecture that what is an MTI filter made of, how to make a good DLC with feedback circuit so that your pass band thing is uniform. Now we have seen how to extend your blind speed and then we will see that if we go on multiplexing etcetera, how is; how that improves the blind speed extension and then we will see some parameters which will evaluate quantitatively that how good our MTI filter is.

Thank you.