Principles And Techniques Of Modern Radar System
Prof. Amitabha Bhattacharya
Department of E and ECE
Indian Institute of Technology, Kharagpur

Lecture – 13
Double Frequency CW Radar

Key Concepts: Analytical expression of FMCW radar with sinusoidal modulation, Phase measurement based CW radar, Two-frequency based CW radar.

Welcome to this NPTEL lecture on Principles and Techniques of Modern Radar System. We were discussing FMCW radar in previous class, we have seen an FMCW radar with triangular modulation that up ramp and down ramp both are there now, we say that can we have a sinusoidal frequency modulated CW radar? Yes the answer is yes if you are detecting a single target and if the target is at short range then you can use. So, that analytic derivation we will do today.

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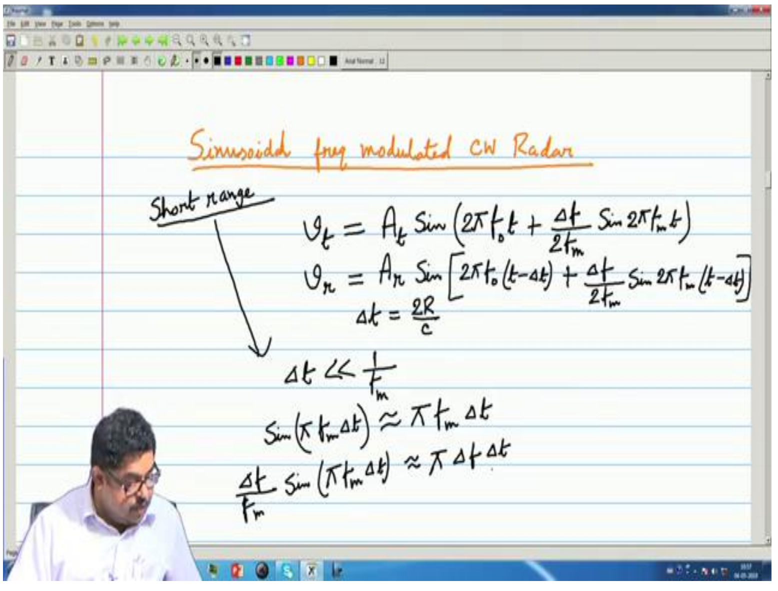

So, let the transmitted frequency a transmitted signal voltage that is something you give A t amplitude the amplitude does not matter here because everything will depend on the frequency. So, sin I can write I have a sinusoidal modulator. So, sin 2 pi f naught t this expression I think you know that because in your analog modulation classes the frequency modulated wave is like this

$$v_t = A_t \sin\left(2\pi f_0 t + \frac{\Delta f}{2 f_m} \sin 2\pi f_m t\right)$$

So, it is modulating the frequency is getting modulated with a modulating frequency f m this expression is known. So, let the transmitter is using that transmitter; that means, simply an FM transmitter you are using and what is the received signal. So, and now received signal will be delayed by a time delta t. So, I will do that the amplitude I am writing

$$v_r = A_r \sin\left[2\pi f_0 (t-\Delta t) + \frac{\Delta f}{2 f_m} \sin 2\pi f_m (t-\Delta t)\right]$$

$$\Delta t = \frac{2R}{c}$$

Now, I have assumed because we have seen that generally these FMCW radars are used for detecting short range targets. So, short range; so, what is the meaning of short range; that means, delta t. So, short range means delta t is much smaller than 1 by f m; that means, my the modulating frequency compared to that it is much much smaller. So, sin of pi f m delta t when I am writing this actually; that means, the can I write that f m delta t if I multiply with pi also that also is much smaller than one. So, we know that sin theta is equal to theta for small arguments. So, can I write this equal to pi f m delta t and so, this expression delta f by 2 f m this what will happen to this expression can I write now that delta f by f m then sin pi f m delta t that will be then pi delta f delta t.

$$\Delta t \ll \frac{1}{f_m}$$

$$\sin(\pi f_m \Delta t) \approx \pi f_m \Delta t$$

$$\frac{\Delta f}{f_m} \sin(\pi f_m \Delta t) \approx \pi \Delta f \Delta t$$

So, what will be now I know V t I know V r in V r I will have to make this simplification and then I know that the receiver of this radar will heterodyne that and produce a beat voltage.

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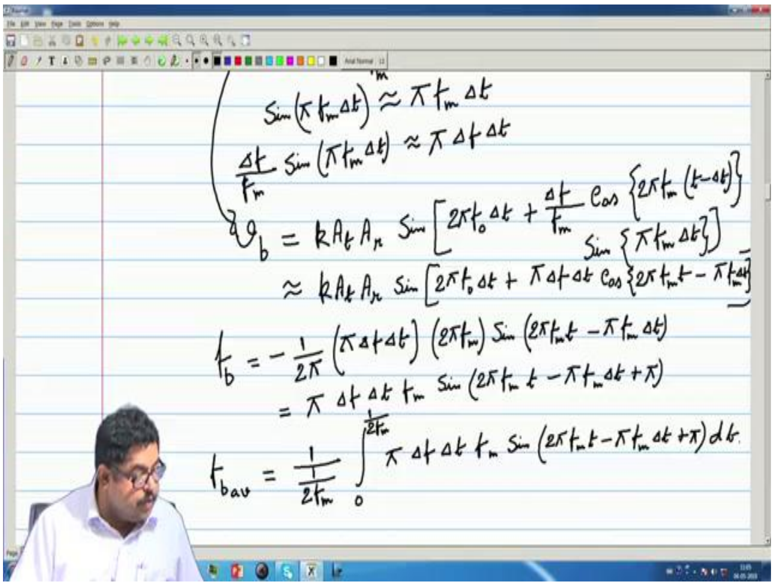
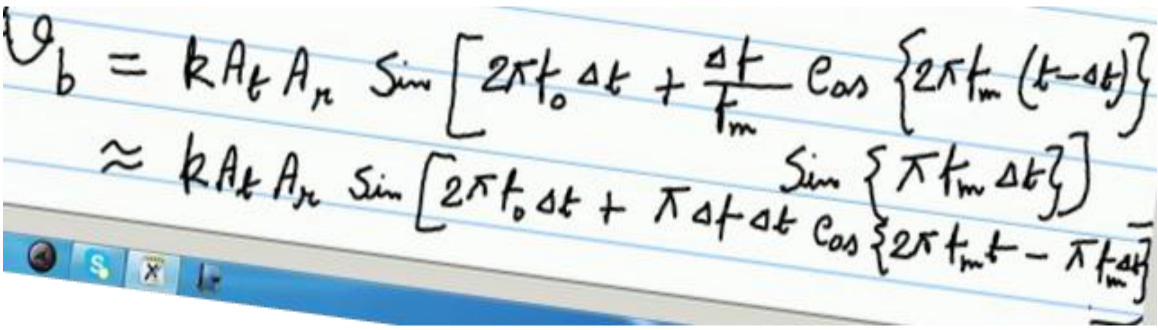


So, what is v b expression I can say some multiplier A t A r then you see it is basically sin. So, I know that when it mixes the basically the argument of this both of the sin function. So, they will be difference. So, can I write 2 pi f naught delta t plus pi delta f delta t this you can do just you do on your own I am just helping you with the major step.

So, it is sin minus sin. So, cos terms will come. So, let me give another step. So, that you later do not have problem plus delta f by f m cos that 2 is going because there are cos 2 sin term. So, there is a 2 there so, cos 2 pi f m t minus delta t into sin pi f m delta t.

This is the expression that these two from here it comes then you put these simplifications and so, with that simplification you can now write that



Now, this is the beat signal. So, what is the beat frequency? So, you see beat frequency is this argument that should be differentiated with time and that will give us the beat frequency or that will actually give us the angular beat frequency will have to divide that by a. So, I can from here I can write that what is f b? f b is you do it this is a you see that only this term is time dependent these are all invariant to time. So, they are going to the phase of the thing only these terms. So, cos if I differentiate there will be a minus sin due to that from a angular frequency to this and then I have

$$f_b = -\frac{1}{2\pi}\left(\pi \Delta f \Delta t\right)\left(2\pi f_m\right) \sin\left(2\pi f_m t - \pi f_m \Delta t\right)$$
$$= \pi \Delta f \Delta t f_m \sin\left(2\pi f_m t - \pi f_m \Delta t + \pi\right)$$

So, it you see that sinusoidally this frequency is changing with time. So, beat frequency is not constant of time, but I can measure the average I can put an averaging meter. So, let me see what will happen if I average that I am calling f b average.

So, that will be we know how to take average. So, over the 1 half cycle I will make that average because we know that up frequency and down frequency are always change. So, over a half modulation period we will do that;

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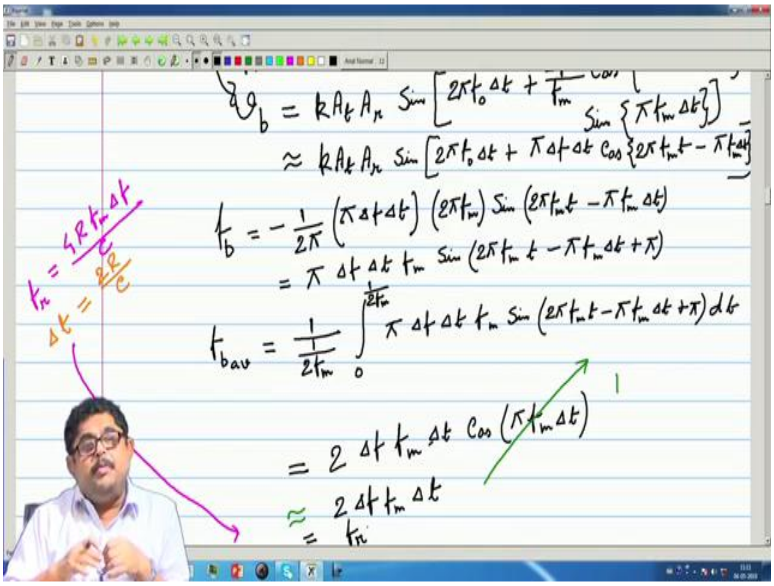

$$f_{b\,av} = \frac{1}{\frac{1}{2f_m}} \int_0^{\frac{1}{2f_m}} \pi \, \Delta f \, \Delta t \, f_m \sin\left(2\pi f_m t - \pi f_m \Delta t + \pi\right) dt$$

$$= 2 \, \Delta f \, f_m \, \Delta t \cos\left(\pi f_m \Delta t\right)$$

Now we already have seen that f m delta t this their product that is much less than 1. So, we can say that cos pi f m delta t this thing will under that assumption that short range. So, this is becoming 1 ok. So, this then approximately becomes that 2 delta f f m into delta t;

$$= 2 \, \Delta f \, f_m \, \Delta t \cos\left(\pi f_m \Delta t\right) \quad \cos\left(\pi f_m \Delta t\right) \rightarrow 1$$

$$\approx 2 \, \Delta f \, f_m \, \Delta t$$

now already you see these terms delta f f m their product we have seen because f dashed that was that delta f by 1 by f m. So, delta f f m and from there we have related to r.

So, we can immediately bring r here. So, this or let me write here that remember this that already we have found that f r is 4 R f m delta f by c. So, if I put that here then I can write that and also we know the value of delta t or that also let me write by some other color we know these two things that delta t is always we know 2 R by c. So, if we put them here then we get this f b average that will be become you will see that it. So, what will happen f r you put this two it will be coming f r. So, it also is directly giving you f r. So, if you make beat average that will give you a f r.

$$= 2 \, \Delta f \, f_m \, \Delta t \cos\left(\pi f_m \Delta t\right) \quad \cos\left(\pi f_m \Delta t\right) \rightarrow 1$$

$$\approx 2 \, \Delta f \, f_m \, \Delta t$$

$$= f_r$$

Now, here we have assumed a stationary target case, but if you do the derivation for Doppler also it will be; that means, even if you use a sinusoidal thing only thing you need to put an extra averaging meter actually it can be generally shown that any reasonable shape modulation wave form can be used to measure range if average beat frequency is measured only thing you need in that a positive cycle and a symmetric negative cycle.

So, various other types of modulators are also used. So, also Doppler shift if it is present it will be there. So, up shift and down shift time there should be equal; that means, that symmetry that should be there now major application of this type of FMCW radar is on board all the radar all the aircrafts they measure the they have an altimeter by which they measure their height from the ground when you fly etcetera you will see that always the on the go nowadays modern aircrafts they show that now it is at so and so height. So, that time actually that ground is the target it is sending a continuous wave signal it is frequency modulating and receiving the thing then it is doing this business and going there.

So, also you see that r c s of the ground is a good scatterer thing at microwave frequency; obviously, at various frequencies the r c s of ground is different, but at microwave frequencies these radars operate. So, there it has a good r c s. So, also the range is short. So, with a low transmission power also you can get reasonably good return for having a simple radio detector the antenna also need not have high gain a low gain antenna will do usually nowadays even patch antenna or a simple dipole antenna or monopole antenna that can be done.

Now, also the Doppler is small because radial velocity of the target with respect to that means, the change of the velocity of the radar that is not much. So, Doppler is neglected. So, it is assumed moving stationary target also here there are no clutters because ground here generally ground is a good source of clutter, but here ground is the target, so, no question of much clutter. Now in industry this c w radar is heavily used suppose in a blast furnace the molten iron in blast furnace what is the distance. So, distance measurement of molten iron in blast furnace I said that if that distance is not known then actually at what time you will have to take it out otherwise the molten iron will blow the blast furnace.

So, by sending these you find out that how much molten in there what is the distance has it come to the point where we will have to take it out. So, all this measurement is done; now in the aircraft the detection etcetera. So, in the Hawk air defense; that means, that time in the detecting and aircraft also in 50s this FMCW radar was used because that time the pulse radar was not good, but nowadays pulse radar was not used much and another problem with this for detection of pulse radar for detection of aircraft the that time the transmitter is high powered. And so, the receiver and transmitter then needs to have large amount of separation for FMCW in pulse radar that problem is not there that is why pulse radar has become more compact and nowadays it can give you much more range compared to FMCW so, that is used. So, we will see the block diagram of this FMCW radar next because.

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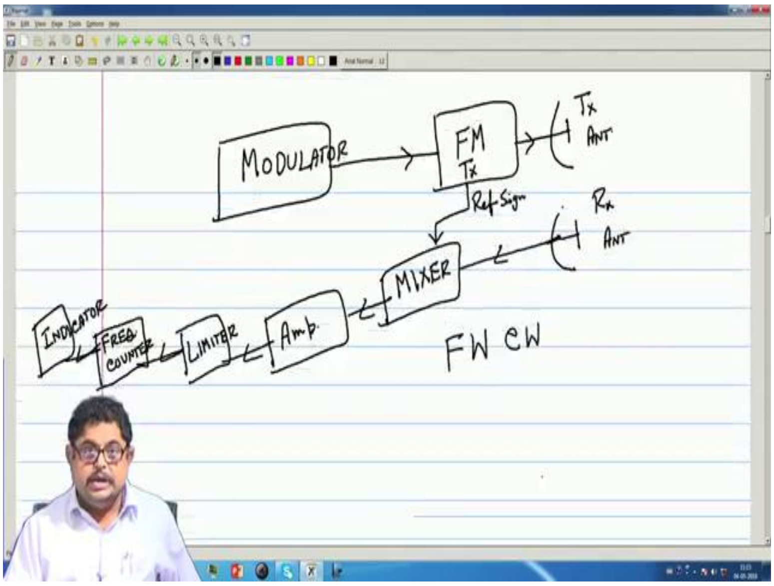

So, block diagram is you have a modulator then that modulator is feeding a FM transmitter and then that is put to an antenna. So, then so, I could say that transmit antenna this is your received antenna. So, the received signal is coming here then that is put to an mixer r f mixer and in that r f mixer from this one also a reference signal is brought. So, mixer is fed with the received echo and this reference signal.

Then you know the beat frequency is already produced then you need an amplifier then there will be a limiter and then there is a because amplitude you are not interested. So, you put a limiter put to a frequency counter it will make its job easier frequency counter and that will go to an indicator. So, it will tell you what is the range because indicator will have that mapping and. So, I will write this. So, this is the block diagram of a FMCW radar ok.

Now, now we will see that a as I said this is the FMCW now range measurement also can be done with CW radar by not doing frequency modulation, but we doing something in frequency that is called double frequency radar that is an alternate principle. So, I will discuss that why because in simpler versions that will also do instead of having a modulator because modulator is a bit more complex.

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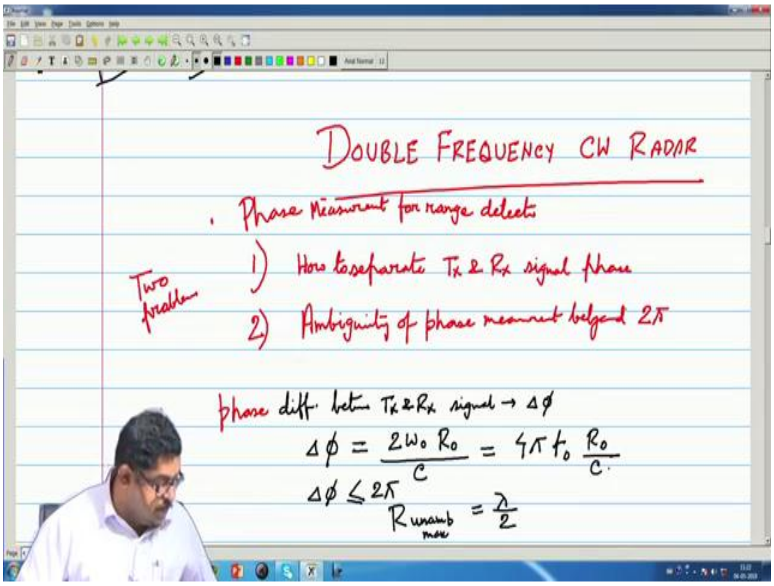

So, you can have now here actually this. So, I will start that sorry double frequency CW radar. You see that till now we are discussing that radar is measuring the range by measuring the time; now here we will not measure time we will use another measuring thing that is phase. You know suppose I have a signal. So, that signal is coming back. So, it is covered the extra distance. So, that you know that any wave if it goes. So, that there is a space change because we say that phase constant into the distance traversed. So, if it traverse 2 hour distance there will be correspondingly that beta into that 2 hour that type of phase change.

So, between the transmitter and received signal if I can measure that phase change then also I have the range information because that phase phase is also proportional to the range. So, these can be done, but there are two problems one is that how we will separate the transmitted signal and received signal in a CW radar because you do not know which one you transmitted and which because you are continuously sending. So, you are also after the initial delay you are also continuously receiving.

So, how will you separate agreed, but if you can do that let us say somehow I can put some mark on the signal. So, that I know there that what is the that corresponding to that mark. So, what is the transmitted signal what is the received signals what is the transmitted signal's phase what is the received signal's phase then I can do another thing is that phase measurement. So, two problems 1 I said that how you. So, first thing I will say that this is a phase measurement we are doing for detecting range phase measurement for range detection number 1 in this principle two problems 1 I say that how to separate transmitted and received signal phase which one is transmitted.

Now, that problem we will solve by putting two frequencies that we will show, but there is another problem that phase measurement now has an ambiguity because you know that phase is the periodic function time is not a periodic function. So, I can measure any time, but phase after 2 pi actually if I measure 3 pi that is basically 1 pi and all the values there that thing. So, phase measurement should be restricted from a for a 2 pi duration. So, if phase if range is proportional to phase range will also get restricted and that is true we will see that there is only very few length actually it is almost some lambda by 2 or something. So, that is not useful, but we will see that how people have overcome these two problems.

So, I will say that ambiguity of phase; ambiguity of phase measurement beyond 2 pi. So, these two we will see, but let us say that if I can measure phase then how I can measure range you see what is the phase difference; phase difference between transmitted and received signal let that be delta phi if that is there and we know the from our knowledge of electromagnetic theory that how much this will be it is nothing, but

phase diff. betn Tx & Rx signal → $\Delta\phi$

$$\Delta\phi = \frac{2\omega_0 R_0}{c} = 4\pi f_0 \frac{R_0}{c}.$$

So, I can have a phase detector I can do, but then this portion I need to restrict or this whole thing I need to restrict delta phi should not be more than 2 pi. So, my condition is delta phi should be less than equal to 2 pi to have this ambiguity problem and that means, you see that if you put it here what is R unambiguous range measurement maximum that turns out to be lambda by 2 as I saying.

$$\Delta\phi = \frac{2\omega_0 R_0}{c} = 4\pi f_0 \frac{R_0}{c}.$$

$$\Delta\phi \leq 2\pi$$

$$R_{unamb_{max}} = \frac{\lambda}{2}.$$

Now in microwave region then it is not useful, but these two problems that how to separate transmit or how to put marks on a CW signal and this marks that the though phase can be measured, but this is a thing these two can be circumvented by transmitting 2 signals 2 signals differing only in frequency very slightly. Let us say f 1 and f 2. So, that is the double frequency signals beauty.

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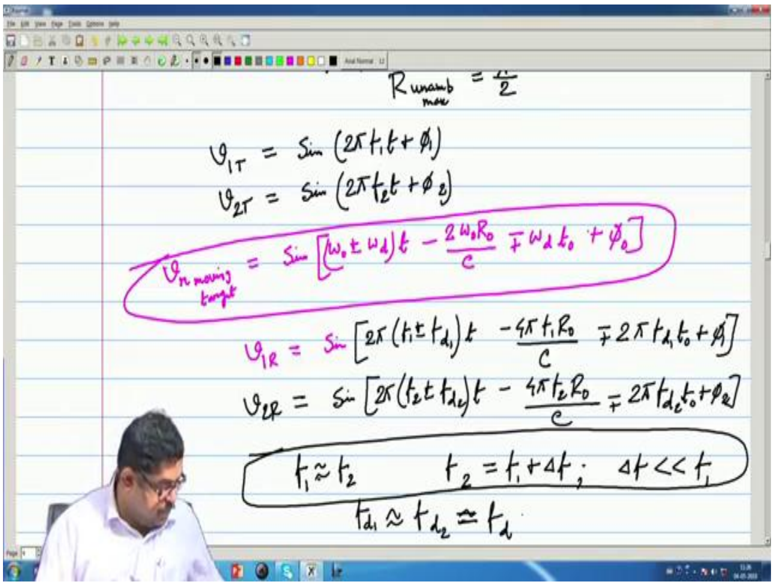

So, let us say that we are transmitting v 1 T those are transmitted signal sin 2 pi f 1 t plus phi 1 and v 2 T we are transmitting deliberately I am not writing amplitudes because ultimately I will be mixing or detecting phases that also is a mixer. So, not a problem plus phi 2 let us say phi 1 and phi 2 are the initial phase.

$$v_{1T} = \sin(2\pi f_1 t + \phi_1)$$

$$v_{2T} = \sin(2\pi f_2 t + \phi_2)$$

Now, please remember that at the basic principles class we have derived that or for a CW radar we have derived that for a moving target this I will write by this pink one we have already derived an expression for the moving target received signal from a moving target that I am writing that received signal v r from a moving target moving target case we have derived that was sin omega naught plus minus omega d t.

$$v_{r\,\text{moving target}} = \sin\left[(\omega_0 \pm \omega_d)t - \frac{2\omega_0 R_0}{c} \mp \omega_d t_0 + \phi_0\right]$$

Please this you refer we have already derived that deliberately that time I said that keep it later we will use it now we will use it this was the expression that we have derived earlier.

So, we are sending the two the transmitter is sending two such signals f 1 and f 2. So, we can easily write what will be v 1 r that sin sorry that we use this black ink sin 2 pi f 1 plus minus f d 1 t minus 4 pi f 1 R naught by C minus plus 2 pi f d 1 t naught plus phi 1 t naught is nothing, but the initial time and v 2 R similarly I can write sin 2 pi f 2 plus minus f t 2 t minus 4 pi f 2 R naught by C minus plus 2 pi f d 2 t naught plus phi 2 okay;

$$v_{1R} = \sin\left[2\pi(f_1 \pm f_{d_1})t - \frac{4\pi f_1 R_0}{c} \mp 2\pi f_{d_1} t_0 + \phi_1\right]$$

$$v_{2R} = \sin\left[2\pi(f_2 \pm f_{d_2})t - \frac{4\pi f_2 R_0}{c} \mp 2\pi f_{d_2} t_0 + \phi_2\right]$$

t naught is nothing, but the initial time and r naught that time we defined that it is the range at time t is equal to t naught.

Now, we are saying that transmitter is sending f 1 f 2 f 1 and f 2 are close by f 1 is almost equal to f 2, but definitely different and let us assume that f 2 is f 1 plus delta f where this delta f is much much smaller than f 1.

$$f_1 \approx f_2 \qquad f_2 = f_1 + \Delta f; \quad \Delta f << f_1$$

$$f_{d_1} \approx f_{d_2} \simeq f_d$$

So, with this is a very important thing that basically to nearby frequencies. So, this will make f d 1 almost equal to f d 2 and let us call that value f d.

$$f_{d_1} \approx f_{d_2} \simeq f_d$$

Now, the receiver will get this signal that heterodynes to the corresponding transmitted signal separately.

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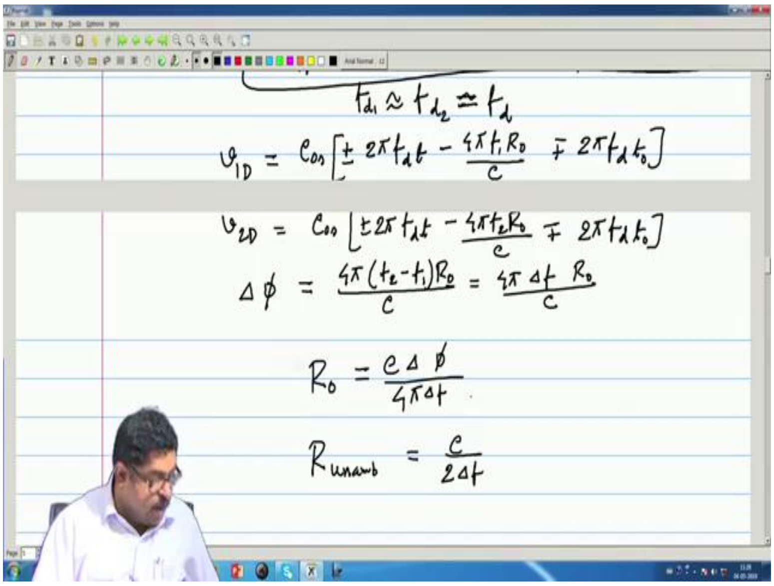

So, Doppler signal it extracts. So, it is v 1 or beat frequency extracts. So, v 1 be that voltage signal will be cos if you do that sin minus sin. So, we will get cos plus minus 2 pi f d t minus 4 pi f 1 R naught by C minus plus 2 pi f d t naught and v 2 D is cos n plus minus 2 pi f d t minus 4 pi f 2 r naught by c minus plus 2 pi f d t naught.

$$v_{1D} = \cos\left[\pm 2\pi f_d t - \frac{4\pi f_1 R_0}{c} \mp 2\pi f_d t_0\right]$$

$$v_{2D} = \cos\left[\pm 2\pi f_d t - \frac{4\pi f_2 R_0}{c} \mp 2\pi f_d t_0\right]$$

So, these two signals we find the phase. So, phase is what that 4 pi f 2 minus f 1 R naught by C. So, it is 4 pi delta f R naught by c.

$$\Delta\phi = \frac{4\pi (f_2 - f_1) R_0}{c} = \frac{4\pi \Delta f \, R_0}{c}$$

So, from here I can you see easily find out what is my range R naught is C delta phi by 4 pi delta f;

$$R_0 = \frac{c\,\Delta\phi}{4\pi \Delta f}$$

delta f I know because I have transmitted. So, you see that now what is my unambiguous range R again if I put the restriction it is less than 2 pi R unambiguous that is C by 2 delta f.

$$R_{unamb} = \frac{c}{2\Delta f}$$

So, you see instead of lambda by 2 just by making two different frequency I can do that I can do the measurement also. So, this is a very simple technique and by what factor actually I have changed the unambiguous thing this is a practical thing.

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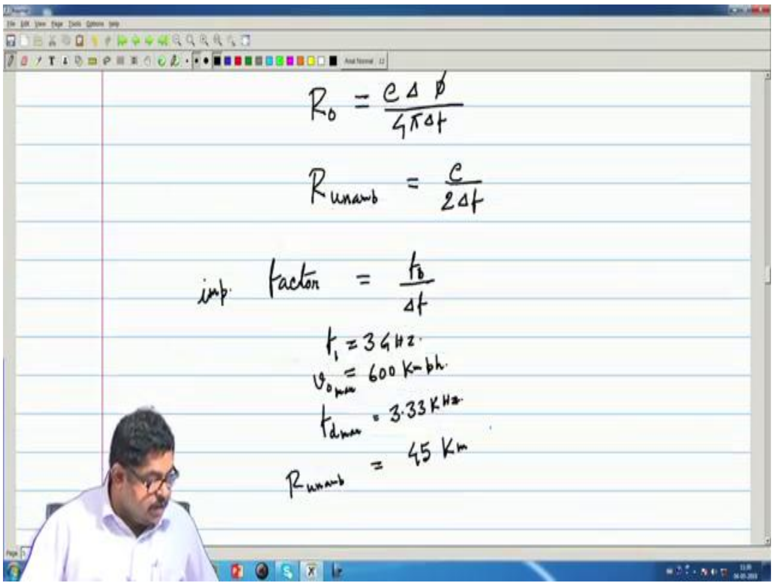

So, the factor of improvement or improvement factor of range is f naught by delta f now f naught means actually f 1.

So, delta f is very small. So, by a large factor we can do that. So, let us say that this is f 1 or f 2 whatever.

So, what is my suppose I have a f 1 3 giga Hertz let us say that target velocity v naught is 600 kilometer per hour now immediately I can find out what is the maximum this is the maximum velocity v naught max the target's maximum velocity. So, f d max that you have formulas by that you can find out that it will come out to be 3.33 kilo Hertz and. So, if you put that the R unambiguous that will become you will see 45 kilometer. So, this is usable. So, actually you see that 2 frequency means you are putting the mark in the spectrum. So, that two problems I said the first problem is solved here and second problem you see these technique has made this thing because it has changed the whole spectrum and by which the unambiguous range also changed. So, this phase detector can be used.

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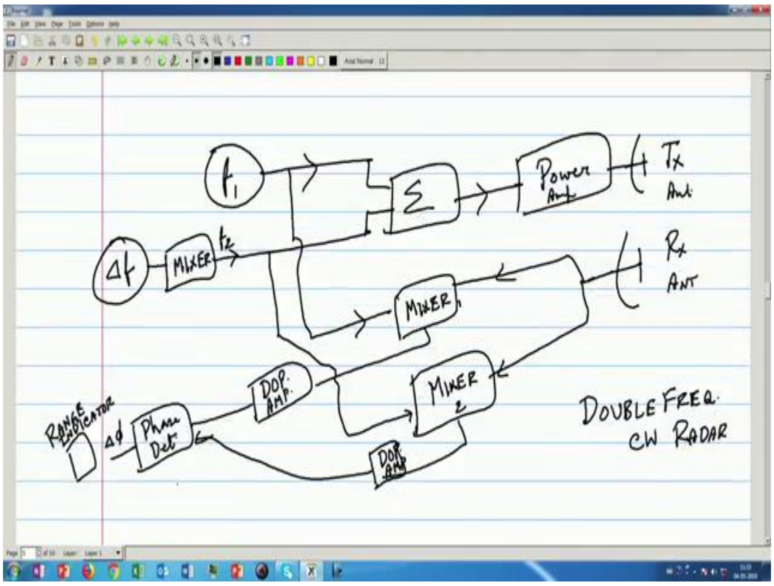

Let me just briefly give its block diagram that it will have a suppose a f 1 oscillator from that also there will be a delta f oscillator and that delta f oscillator will be put to a mixer and. So, this output will be f 2 and both of these are put to a summer and then there is a power amplifier and then it is transmitted to the antenna. So, this is the transmitter side then in the receiver side you will have the receiving antenna that will take it then they will put the signal to two mixers mixer at that time we are saying. So, you can say mixer 1 mixer 2 ok.

So, then who will give suppose this mixer one. So, from this you will take a path you are giving. So, these two the output will come. So, this you are giving. So, mixer 1 that will put to a Doppler amplifier and that is put to a phase detector. similarly this mixer 2 these signal is put received signal and from this f 2 you take this. So, this output again put a Doppler amplifier and put to this phase detector this is giving you the delta phi and then you have a range indicator this is a simple circuit only thing is these two oscillators are only thing. So, this is your double frequency CW radar and only thing is I think all of you know phase detector in phase modulation many times you have done basically phase detector is what analog multiplier circuit.

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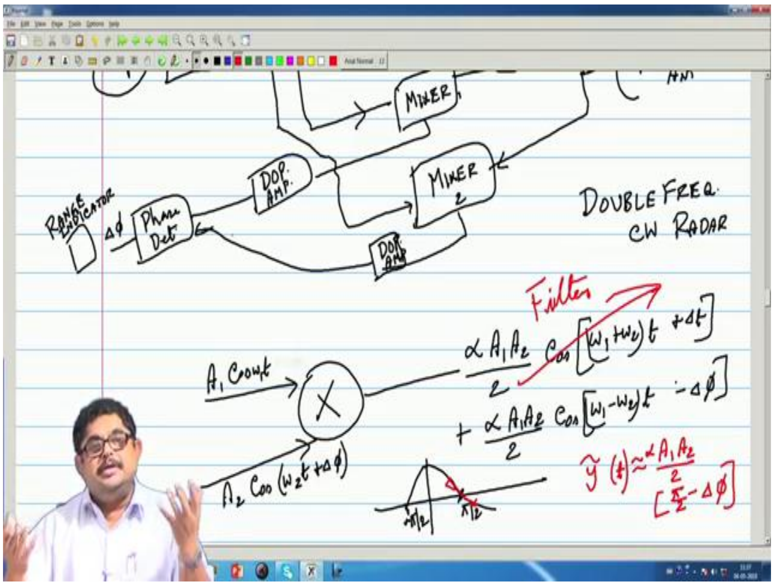

So, you have a multiplier you give. Let us say A 1 cos omega t cos omega 1 t you give A 2 cos omega 2 t plus delta phi. So, it will give you alpha some constant I am assuming A 1 A 2 by 2 cos omega 1 plus omega 2 t plus delta phi plus alpha A 1 A 2 by 2 cos omega 1 minus omega 2 t minus delta phi. So, you can after this you put a filter. So, that will remove the high frequency also if omega and omega 2 are nearby. So, basically so; that means, this term is going out by a filter by a filtering you can do that and then this term is very nearby.

So, this is almost one and so; that means, you can put an phase locked loop in the circuit and that phase locked loop. So, if we call this signal as y t. So, phase lock loop you know that this is the cosine function. So, this is your pi by 2 this is your minus pi by 2. So, near this you have a linear region and you know that this y t that can be approximately written in the linear region as alpha A 1 A 2 by 2 then instead of this cos I can write pi near that region if I can lock it in this linear region pi by 2 minus delta phi.

So, you see that this is a bias, but this. So, y t is proportional to delta phi. So, you can easily in this region if you can lock after this filter you put in phase lock and then your with delta phi the signal. So, you can easily find range the more precise you are in this measurement more precise will be your range indicator. So, this circuit is simple this is the phase detector circuit just for your refreshing I have given you this. So, this is a simple circuit you can make it and that makes a very simple CW radar.

So, with this we are finishing CW radar any of you I encourage please buy these components and make the radar these are simple components nowadays available very cheap from your pocket you can buy that and you can make your own radars only do not transmit with much power then you may caught by that spectrum regulation agency, but with a low power you can no problem 1 milli watt etcetera that is not a problem ok. So, do that these are simple radar enjoy this radar in the next class we will go to the radar which you cannot make from your own pocket those are pulse radars we will start seeing that.

Thank you.