Principles And Techniques of Modern Radar Systems
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Lecture – 12
FM-CW Radar

Key concepts: Concept of FMCW radar, Description of time-frequency curve in FMCW radar, Mathematical model of range measurement with FMCW radar, Mathematical expression for velocity measurement with FMCW radar

Welcome to this NPTEL course on Principles and Techniques of Modern Radar Systems. So, in previous lecture, we have seen CW Radar continuous wave radar. Now today we will see a variation of that CW radar called FMCW radar. Now the, we discussed that CW radar cannot measure range, because if you say send a signal continuously, then there is no timing mark on the signal that when it is starting and so, when the echo is received that it is with respect to which signal transmitted that is not known.

More specifically that I do not have a mark for time of transmission, I do not have a mark for time of reception. So, transit time we cannot measure and we know that, the radar range is measured by measuring these transit time. So, if we cannot accurately measure these transit time the measurement of range we cannot do.

Now, actually if you if we look at the frequency domain a CW radar; that means, it is operating at a continuous fixed frequency. So, the it is in the frequency domain, it will be a single function. So, there is no distinct mark there no sharp timing mark now. So, if I want to put that mark, I can broaden the spectrum. So, one way of broadening it, we know that modulation which communication people use for transmitting signal. Now modulation can broaden the spectrum of a signal. So, if we do that here though we are not sending it for communication purposes, but that can serve our purpose that type of radar is called FMCW radar, Frequency Modulated CW radar.

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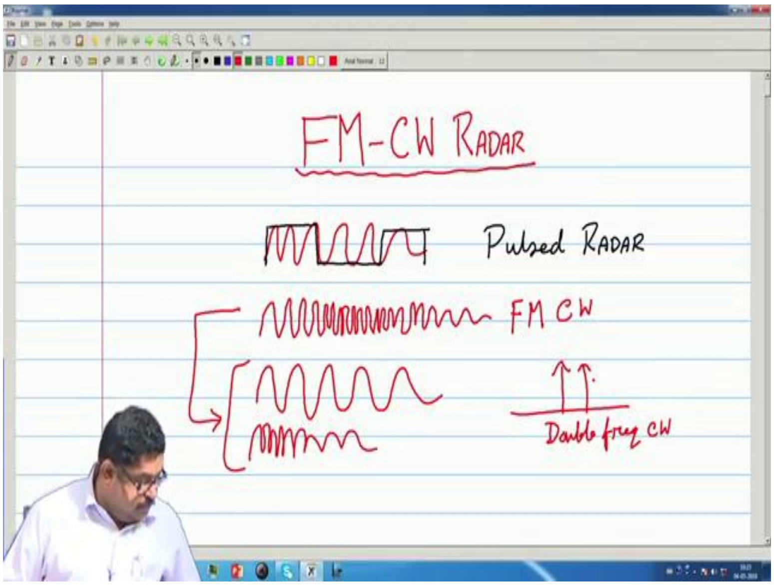

We know frequency modulation changes the spectrum, it gives a broad spectrum. So, you see this is one application of modulation; frequency modulation for measurement that we want to do range measurement for that modulation. Now if we do amplitude modulation actually that is the pulse radar, because I have a RF signal, now you see that if I put that this is my amplitude modulation. So, these on and off, this is basically nothing but pulsed radar.

We will see that later, we have understood various basic parameters of radar with the help of pulse radar; pulse radar wave form is something like these. Whereas, if I do the same thing in frequency, so, you see that I am changing the frequency. This is actually the FMCW radar. So, the timing mark here as you will see that is obtained, because I am changing the frequency of the signal in a predetermined fashion.

So, that will put the mark, also we will see that instead of sending one pulse if I send suppose this is one frequency, if I send another frequency signal. These also puts a mark, because in the frequency domain they will be given by two delta functions. So, that also is a broadening of the spectrum that also can give that mark. So, these are the two varieties of range modifying the CW radar for doing measurement. The first variety is called FMCW, the second one is called double frequency CW, both we will discuss.

First we will discuss the FMCW. So, which time I am changing the frequency in FMCW? Now more is my frequency deviation; that means, given an unit time the more I change can change the frequency, more will be the broadening of the spectrum and more will be my accuracy in the measurement. So, it is better that, if I can do more; if I can do more frequency deviation in the frequency modulation, but obviously, you know that there is a limitation because my electronics should support that. So, I cannot go on doing that indefinitely within the practical limit, I will have to do this.

Now, let us start the thing with a diagram. Actually in FMCW, the frequency is changed with time.

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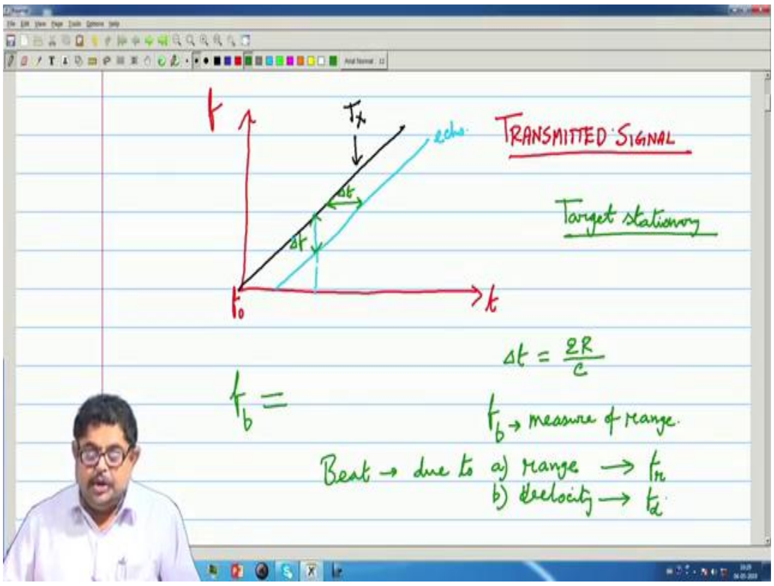

First we assume that it is being done in a linear fashion. So, later we will see that other fashions other things are possible. So, what I am doing? I am plotting the time verses frequency of the transmitted signals; transmitted signals. So, this is the plot of transmitted signal. So, let us say that at initial time the frequency is f naught and then I let us say change it in a linear fashion.

So, I can say that this is the transmitted signal. So, its frequencies changing linearly with time. Let me put so, obviously, after some time I will get the echo signal, because echo will take some time to come. So, the echo signal will be something like this. And these I am calling the echo signal and you see that suppose at a particular time; suppose this is the time I am interested doing the measurement that echo has come. So, that time already from the signal with which is the signal has been transmitted compared to that it has changed, because echo was sended before. So, maybe somewhere in here the transmitter signal was, but due to the transit time.

So, what is the transit time? At any frequency if I draw a curve, this is actually my transit time. So, I can say this is my delta t and this is echo signal. So, you see that at any time if I see, between the that time what is being transmitted and what has been received there is a frequency change. So that means, if I mix the transmitted signal at this time with the received signal at this time that this instant then there will be a beat frequency created.

Now here I am assuming that the target is stationary. So, this beat frequency is not due to the motion of the target. I am first doing that, later I will remove this restriction. So, even if the target is stationary; that means, it is not creating any Doppler, because there is no relative motion now but just because my modulation just because I am changing the frequency of the transmitted signal.

So, when I receive that these there is be a difference in frequency. Usually this so, this frequency if it is more, it will produce a beat, so, that we call beat frequency. So, this beat frequency will depend on what? Will depend on this delta t. And what is delta t? Delta t is proportional to the range, the more distance the target is the more delta t is there that we have seen that delta t is equal 2 R by c.

So; that means, if now this f b that is a function of delta t linear function of delta t and that it is also a function of range. So, basically by measuring this beat frequency I will be able to tell what is the range. So, this is the principle of FMCW radar. So, I can say that delta t is 2 R by C these we know and so, whatever received signal I am getting the echo, I am heterodyning that with a portion of the transmitted signal.

So, I am taking always the transmitted signal. So, these two I am mixing, I get a beat frequency f b. So, this beat frequency is a, I can say f b is a measure of range. Now if the target is moving, then also there will be a beat, because due to these there will be a difference also there will be a Doppler shift. So, these two will super impose. So, that is also beat frequency. So, to distinguish these two cases that due to range there is a beat. So, I will say that beat is due to two things, a due to the range of the target and b is due to the velocity of the target.

So, I will first distinguish that is why actually you see target I have initially assumed stationary. So, that I am getting the due to range what is the beat? So, this only for range that we call will be calling f r that frequency. So, beat frequency when only range is present, only due to range that is f r. Beat frequency only due to velocity that we will be calling f d f Doppler. So, in actual cases, it will be mixture of the two. Now I can first derive an expression for f r only due to range.

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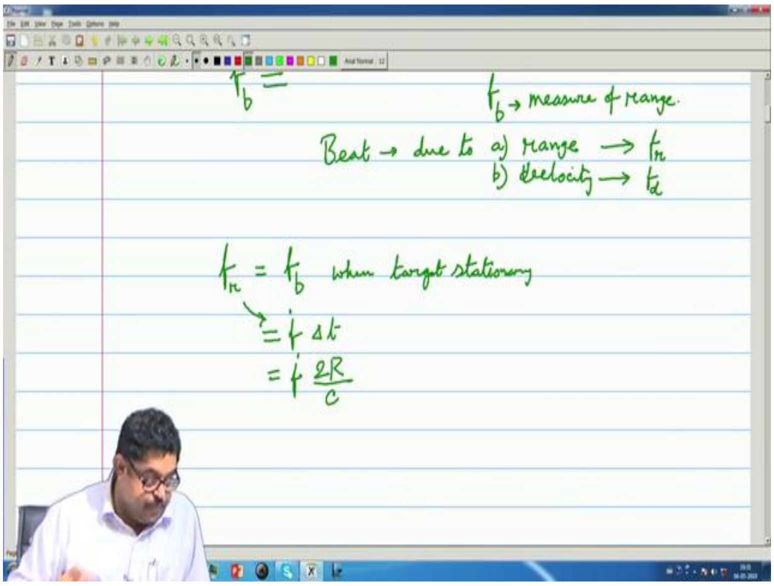
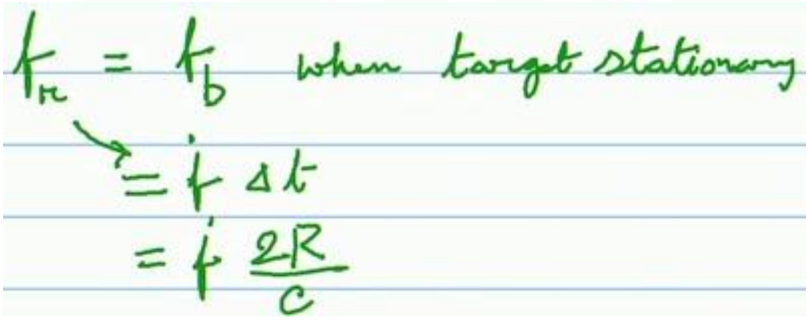


So, I can say that, f r is equal to f b when target stationary and can I say that the I know how I am changing the frequency, because if I look at that graph, that I know the sorry I know the fashion I am doing. So, I can say that in delta t time how much I have changed. So, there is a slope of this graph that multiplied by delta t will be the delta f.

So, I can write from these that, what is delta f? The change in frequency that is nothing, but d f d t into delta t or I can say f dot into delta t ok. So, these I know, because how what is the slope of this graph how my electronics is changing that I know that in what way I am varying. So, f dashed is known delta t I am measuring and so, I will be able to. So, now, we will write that



So, if we now we have to specify f dot. So, for that what I will do? I will do another graph.

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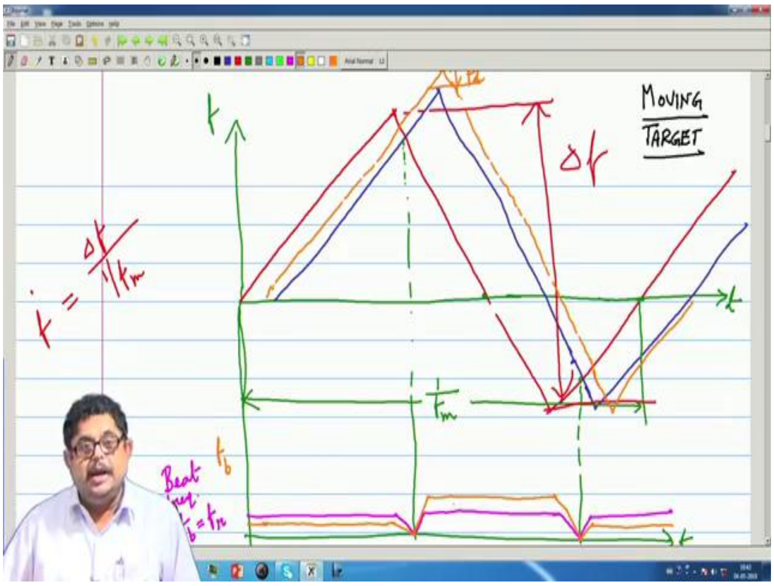
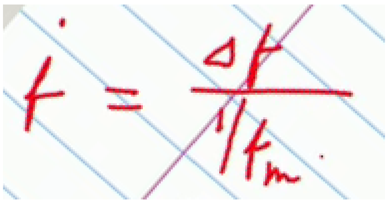


Now, so, again I am doing that, I have a transmitted frequency.

Now, in any practical circuitry you see, I cannot go on increasing the frequency. So, I will have to come down, because after the maximum frequency more cannot be an oscillator cannot go on changing its frequency indefinitely higher and higher. So, I will have to come down. So, I will have something like this oh sorry. So, this is my t this is my f. So, this is my transmitted signal and the echo signal I will get after some time. So, that will go then that will also come down. That will sorry, that will come down then that will go.

So, here I can say that, what is the rate or what is the time period of change of the frequency. So, I am starting from here. So, this is the full period. So, I can say that modulation period that I can call, because here it is going up. So, again another point, where it is going up, so, these I will. So, there is a modulation frequency. So, this is 1 by that. So, if modulation frequency is f m, then I can say that this time period is 1 by f m.

So, these and here I know that at what is delta f? So, maximum what I am changing you see that these and this is the minimum. So, I can say that this and this is the minimum. So, I can say that this is my total delta f. So, what is my f dashed, the d f d t that I can say these delta f the total divided by 1 by f m. So, I will write that what is my f dot? F dot is



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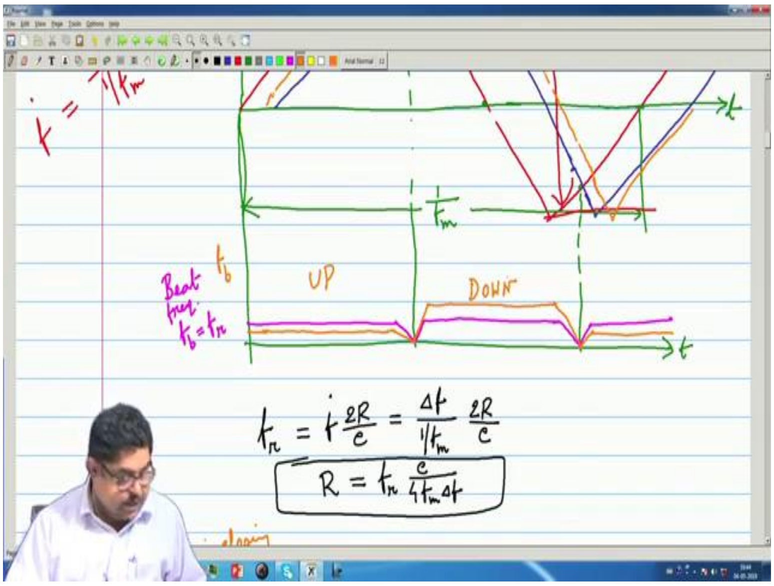

Now, you see that, if I plot the beat frequency in this same graph in time, what I will get this is an point you see that. I mark this one the transmitted signal frequency and received signal frequency they are meeting. Similarly, again they are meeting here; you see that this one then again that meeting is not here. So, if I plot this graph in some other color. So, I can plot the beat frequency; beat frequency due to range or you can say f b or that is at present that is f r.

So, how that will you see here everywhere that difference at any point these two are parallel graphs. So, here only where it is not parallel, when it has started coming down so, from that time, but; that means, I will get something like this then at from this point where the transmitted frequencies coming down. The difference between the 2 is coming down and at this point it will touch zero. So, it will be something like this is a beat frequency. Then again it will go up and then again it will go up to where again when this maximum minimum is there. So, that time again it will come down like that. So, this is a beat frequency curve.

So, you see that here only this is not constant, but here it is a constant. So, I can measure it and from that I will be able to find out. So, this is called turn around region, except turn around region that beat frequency is constant. So, and the beat frequency goes to zero no beat, when the transmitted signal frequency graph touches the received echo frequency graph. So, now, I can now I know f dot, because this is the property of the modulator which is in our hand. So, we can write from here, that what is the f r expression So, f r is,

$$f_r = \dot{f}\frac{2R}{c} = \frac{\Delta f}{1/f_m}\frac{2R}{c}$$

So, from this I can find out that what is the range R is

$$R = f_r \frac{c}{4 f_m \Delta f}$$

So, this is an very useful relation, that what is the range of the stationary target. So, the beat frequency observed f r C I know velocity of light. Then f m is the modulating frequency and the delta f is a total change in the frequency. So, now, with this I say that actually there is a let us say the sorry there is a Doppler. So, no more I am assuming stationary target, now I assume a moving target

So, what will happen there will be a Doppler? Now that Doppler may be positive may be negative, because depending on whether I have a closing target or receding target. If I have a closing target, there will be a positive Doppler. If I have a receding target there will be a negative Doppler. So, that means, this curve that means, the received curve this blue one that will shift up and down, but no shift in time axis. So, depending on the target closing or receding, the curve goes up signal frequency curve received signal frequency curve goes up or down with respect to the transmitted signal.

So, let us say that, if I have if I that will be a good color taken let me take this. So, what will be the this curve this curve will be something like this, this is the if I have a closing target. So, in time it will have to come same so, that means, this curve will cross these early will cross this curve and go somewhere in here and then come. So, you see that what is this difference.

So, from here to here, these difference is our Doppler f with small d. So, this and in the same graph down if I put this a thing. So, you see the if the curve goes up. So, the beat frequency will change. So, now, I will plot with these the I will say f b curve general beat frequency. So, it will be down. So, it will be constant here and it will in the time axis there is no change. So, it will go like this and in the other half you see, if you see this curve that he compared to the transmitted here the difference is less, but in the second half; that means, in the next half cycle the difference is much more.

So, the beat frequency will increase. So, this curve will go something like here. This again from here it will go down then again here this is it. So, you seen one half cycle the when the moving target is there so, the curve is below the constant target curve and in another half cycle it is above the constant half cycle curve. So, what is the effect of Doppler shift; Doppler shift has the increase the beat frequency in one half cycle in the other half cycle of the modulating period it has decreased. So, that now we will have to mathematically express.

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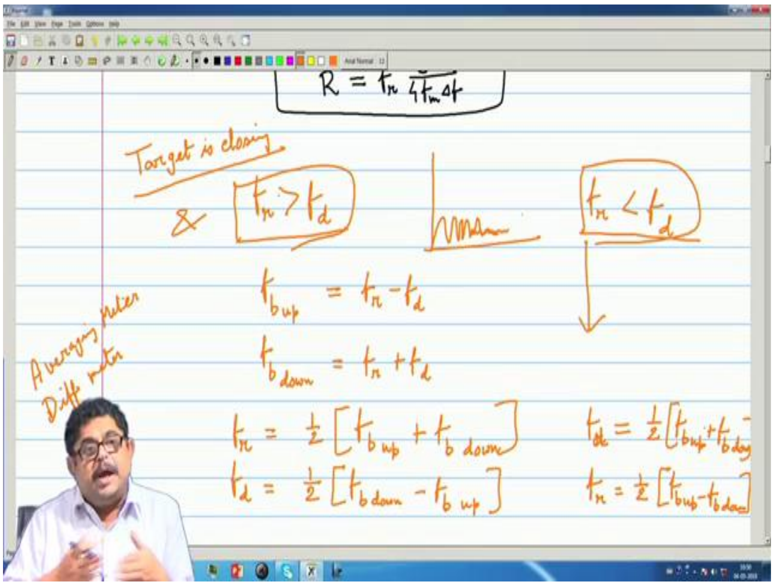

So, suppose the, for a moving target we know that it can be either closing or a. So, let us assume that target is closing. So, if the target is closing also to see that range is producing a frequency change and also the Doppler is producing the frequency change.

Now, range and velocity now how much frequency change they have produced, there can be two cases again. So, if after target is closing, we also have two cases whether f r is greater than f d or f r is less than f d. There are two cases, now usually in radars if the generally range is more and the velocity so, generally f r is greater than f d, but obviously, it comes very near then this condition also can come. So, first we are assuming target is closing as well as and f r is greater than f d. So, that graph, we have already drawn.

So, we can say that let us say that in the half cycle first sorry the first half cycle this is you see the total period is 1 by f m. So, this is 1 by 2 f m. So, this is we are calling up portion of the cycle and because here the curve is up than the stationary and this is the down cycle. With this terminology, I can say that

$$f_{b\,up} = f_r - f_d$$
$$f_{b\,down} = f_r + f_d$$

Now, can I do both the velocity and range measurement; that means, if I can find out f r and if I can find out f d separately then I can find both the velocity and the range. So, that can be easily done, you see that if I sum these two then I can then f d will cancel and f r is

$$f_r = \frac{1}{2}\left[f_{b\,up} + f_{b\,down}\right]$$

Now f b up and f b down can be measured, because I know the modulation cycles. So, in the first cycle I will put a counter on that measurement of f b that I will call f b up.

Similarly, I can measure f b down. I will some them I will get f r . If I take the difference between f b down and f b up then you see that

$$f_d = \frac{1}{2}\left[f_{b\,down} - f_{b\,up}\right]$$

So, on the other hand if I have this condition that this is a high speed target at short range in that case. So, this averaging meter, if you do this then the averaging meter so, that means, this is the averaging meter that will be giving you f d.

So, f d is

$$f_d = \frac{1}{2}\left[f_{b\,down} + f_{b\,up}\right]$$

This is the averaging meter and the difference meter that will give you that range. So, half f b up minus f b down

$$f_r = \frac{1}{2}\left[f_{b\,up} - f_{b\,down}\right]$$

you see the signs will change that is why I have taken that. So, to understand that, because I will have from these two measurement there will be an averaging meter; averaging meter there will be a difference meter. Now, what it is giving to know that we need to have an apriori idea that, what is this conditions?

So, this information is required. Then only we will be able to correctly interpret and generally that is known, because you know that already once you do the range measurement and if you follow you can find out whether it is the case because ranges generally measured before. So, this knowledge if it is has, it can correctly find out this. Now one question is that modulation of the frequency do we have to make it always linear? Because, that means, I will have to have a ramp then it should come down. So, that can we not have any other type of the particularly like our communication thing can we have a sinusoidal variation; that means, if my variation is frequency is varying like this then, obviously, the beat frequency won't be constant there over even for a half cycle.

But we will show in the next lecture that also is possible, only thing is if you have a multiple target detection problem by the FMCW radar. Then it is necessary to have linear modulation. Because each target should have a corresponding beat frequency, but if you were trying to detect a single target actually FMCW radars you see they are not used for mainly detection of the distant aircrafts or naval ships etcetera. They are used as I said in low range cases where suppose the altimeter; that means, height of the aircraft is measured by that etcetera radar altimeter where ground is your target. So, in that if you have to want to have that linear modulation like ramp type, then the cost increases complexity increases. So, sinusoidal modulation is better, but I know that the beat frequency would not be changing, because sinusoidal modulation means your transmitter frequency that is changing with what I have drawn that continuously it is going increasing I am not doing that in a hurry.

So, that beat frequency won't be constant over a half cycle, but we will show that by averaging you can find out that range that will be our discussion in the next class, because FMCW is mainly used for the short ranges. There if we can make the electronics or transmitter design simpler that will be better so, that we will take up next.

Thank you.