Electrical Measurement And Electronic Instruments

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Lecture - 08
Derivation of Deflecting Torque in Electromagnetic, Electrostatic & Moving Iron Instrument

Hello and welcome friends. So, far we have studied a number of different types of instruments.

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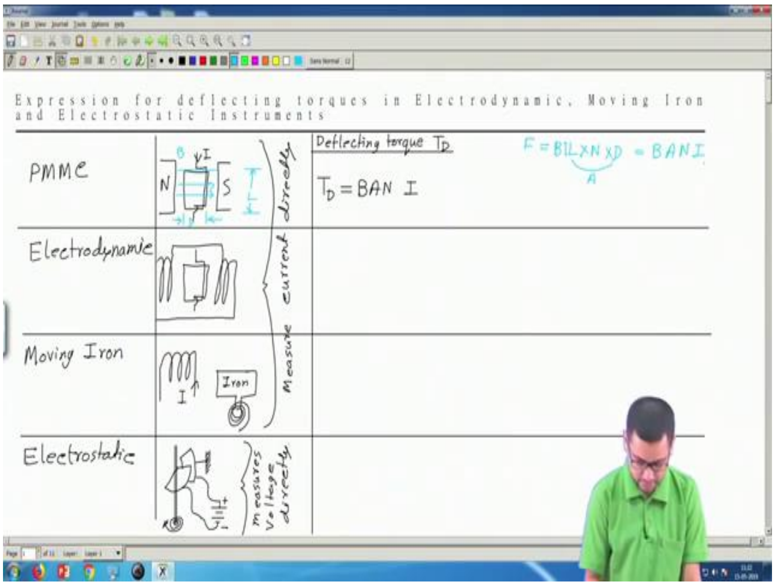

So, we can write them as we have studied PMMC instruments Permanent Magnet Moving Coil Instruments, then we have studied electrodynamic instruments or dynamometer type instruments, we have studied moving iron instruments and then we have studied electro static instruments.

So far we have studied these 4 types of instruments and so, we can recall them very briefly and very quickly, in a PMMC instrument we basically have a pair of magnets, permanent magnets. This is the speciality of PMMC instruments and between these two permanent magnets or the poles, we have a rotatable or moveable coil or moving coil. And, when any current flows through this moving coil the coil is turned due to electromagnetic torque acting on this current carrying conductors, and this is the working principle.

And then for electro dynamic instruments these permanent magnets are replaced by electromagnets. So, we have a pair of electromagnets, basically a pair of coils and we call these fixed coils and between this we have a moving coil just like the PMMC instrument. So, this is fixed coil, and this is the moving coil.

Now, we have talked about different types of moving iron instruments, attract attraction type, repulsion type and the simplest possible scheme is this the attraction type moving iron instrument, where we have a coil which carries the unknown current I and in front of this we have iron bar and this iron bar can rotate around this pivot. So, if any current flows through this coil, this coil will get magnetized and it will attract the iron bar. So, this is the working principle of an attraction type moving iron instrument.

And in case of an electrostatic instrument, what do we have? We have a metallic plate, conducting plate mounted around a spindle which can turn. So, this is the spindle attached with springs this can turn and then we will have another plate below it, this is fixed. So, this is attached to the frame this one cannot turn this one can turn. So, this is the construction and if we now apply any voltage between these two plates, if we apply a voltage, then this two will get charged with positive and negative charges and the electrostatic force between them will try to get them closer and that is how we can measure this voltage.

So, basically these 3 instruments therefore, they measure current directly. So, these 3 instruments measure current directly, because we need to pass some current through some coil this coil or this coil or this coil and the deflection of the pointer that will depend on this current. So, this instrument this instruments measure current directly and this instrument measure voltage directly.

Now, let us quickly recall the expression for the deflecting torque so, deflecting torque T_D, D for deflecting. So, for PMMC instrument we know this is given by,

$$T_D = BAN\, I$$

So, these formulas are so easy that you can derive it in a few seconds. So, for example, I want to find this torque expression. So, let us see how quickly we can we do it. So, we assume this flux density to be B so, this is the flux density B and we know this length of this coil is L the width or diameter of the coil is D.

You need not remember any formula in this subject you can derive it in few seconds so, that is the beauty of this subject. So, let me erase it. Now for electrodynamic instruments we know that T_D is equal to,

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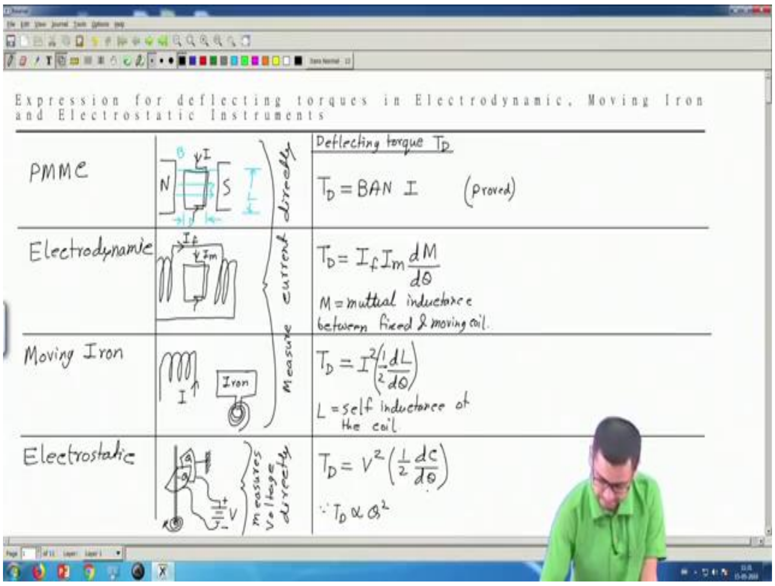

$$T_D = I_f I_m \frac{dM}{d\theta}$$

So, this is the torque expression here for moving iron how much is the torque? T_D will be proportional to I square. Why I square? Because this is proportional to the force of attraction between this coil and the iron bar and this is proportional to the strength of this electromagnet and the strength of magnetization of this iron bar both in turn is proportional to I.

$$T_D = I^2\left(\frac{1}{2}\frac{dM}{d\theta}\right)$$

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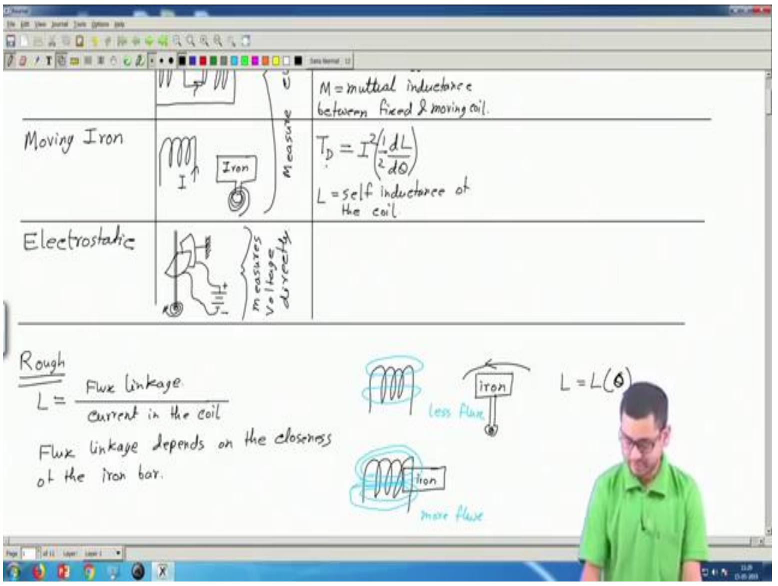

So, observe that what is self-inductance? Self-inductance L is nothing but the flux linkage of this coil when say unit amount of current flows through this coil. So, we can write the current in the coil in a denominator and in the numerator we write the flux linkage generated.

So, this is flux linkage per unit current flowing through the coil. Now, this flux linkage depends on the proximity or closeness of this iron bar on the closeness of the iron bar, because say when I have the coil here and the iron bar is at some distance little bit far from it then; obviously, the flux will be less, because the most of these flux has to flow through air only.

$$L = \frac{Flux\ linkage}{Curremt\ im\ the\ coil}$$

Now, if I bring this iron bar close to the coil and say I bring it even partially inside the coil then the reluctance of this area is reduced because this is now partly iron partly air and iron has lower reluctance, higher permeability. So, now we can have more flux we will have more flux than before so, more flux linkage so, less flux and here more flux. So, more flux means what? For same amount of current we have more flux linkage here for same amount of current we have lower flux linkage.

So, here the inductance self-inductance L will be higher than this case. So, self-inductance L is therefore, the function of the distance of the iron piece or we can say if this iron piece is a rotatable. So, this iron piece can rotate, and it can come closer or further.

$$L = L(\theta)$$

In this case we are talking about capacitance and in these two cases we are talking about inductance, here we have only one coil so, we are talking about self-inductance, here we have 2 coils we are talking about the mutual inductance between them.

$$T_D = V^2 \left(\frac{1}{2}\frac{dC}{d\theta}\right)$$

$$T_D \; \alpha \; Q^2$$

And one more thing which we often get confused is that, note that there is a term half here, half here and there is no half here. So, this is these are the torque expressions, expressions for deflecting torque for the 4 types of instruments we have studied so far. This we have proved, this is very simple, these 3 we have been proved in this video we are going to prove these 3 expressions.

So, before we begin so, this is going to be a long tutorial we will have we have to do 3 derivations, this will be a slightly longer video. However, if you are not interested or if you do not need the derivation you can skip the remaining part of the video from now.

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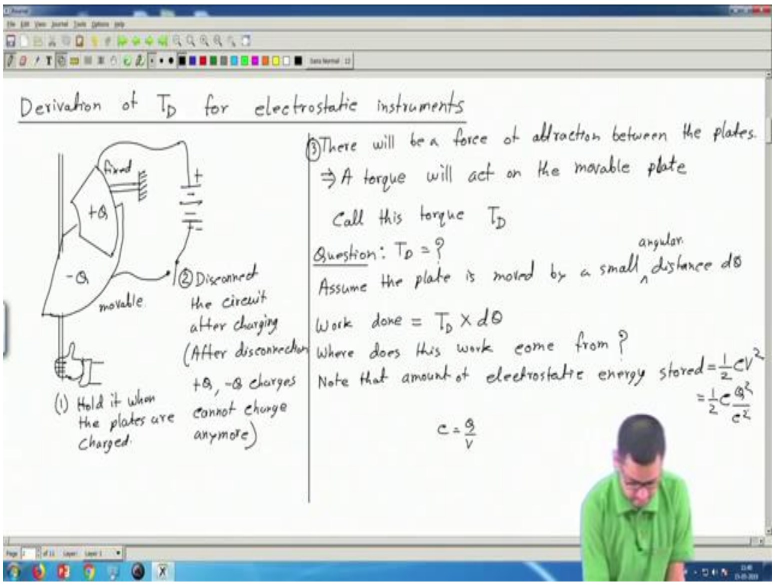

So, let us now start with the derivation of T_D; that means, deflecting torque for electrostatic instruments, I am starting with electrostatic instruments particularly because this derivation many of you might have already seen maybe in school. So, let me draw it quickly.

So, I have a spindled I have a pair of one plate here which is not connected to the spindle, another plate, may be below it and this one is connected to the spindle. This can rotate, this cannot rotate, this is fixed and their size is normally same, although my diagram is not that good, their size looks different and we have this one connected rigidly to the frame of the instrument this is fixed, this is movable.

Now, if we have as we have already said, if we apply a voltage between these two plus minus and then we will have some charge positive charge Q here, then negative charge Q here. Now once this and the moment when this charging is happening I will hold this so, I will hold this maybe by hand, imagine that I am holding it by hand, my drawing is not that good, but imagine I am holding this. So, that this plate cannot move, hold it when the plates are charged, this is you have to do you have to hold it when the plates are being charged.

Now once this charging is over it will not take much time it, then disconnect this. So, I have a switch so, I can disconnect this. So, step 2: disconnect the circuit after charging. Now what will happen, the charge which is already accumulated, plus and minus Q that cannot change, because the circuit is now broken. So, this charge cannot change any more so, this plus Q charge will remain here and minus Q charge will remain here.

So, after disconnection plus Q and minus Q charges cannot change anymore, because there is no path for this charge to flow. Now after this, so now this is disconnected, but now observe that there will be a force of attraction between the 2 plates, which means a torque will act on the moveable coil. So, a torque will act on the movable moveable plate.

So, now let us call this torque call this torque as T_D. So, the question is T_D is equal to how much how much will be the value of T_D, to find the value of T_D we will use the principle of conservation of energy very simple. Now, assume that there is a torque which is acting on this plate which is trying to rotate this plate. Now, what I have to do? I have to hold this plate at it is position. I have to apply equal and opposite amount of torque then only it can stay here without moving.

So, when it is not moving the torque that I am applying is also T_D. Now assume that I loosen my hand a bit slightly. So, that my torque is slightly less and then this plate will move towards the other plate, say this plate is moved by a small distance d theta assume the plate is moved by a small distance d theta, angular distance of course. So, how much is the work done in this process.

$$Work\ Done = T_D \times d\theta$$

$$Elecrostatic\ energy\ stored = \frac{1}{2}CV^2 = \frac{1}{2}C\frac{Q^2}{C^2} = \frac{1}{2}\frac{Q^2}{C}$$

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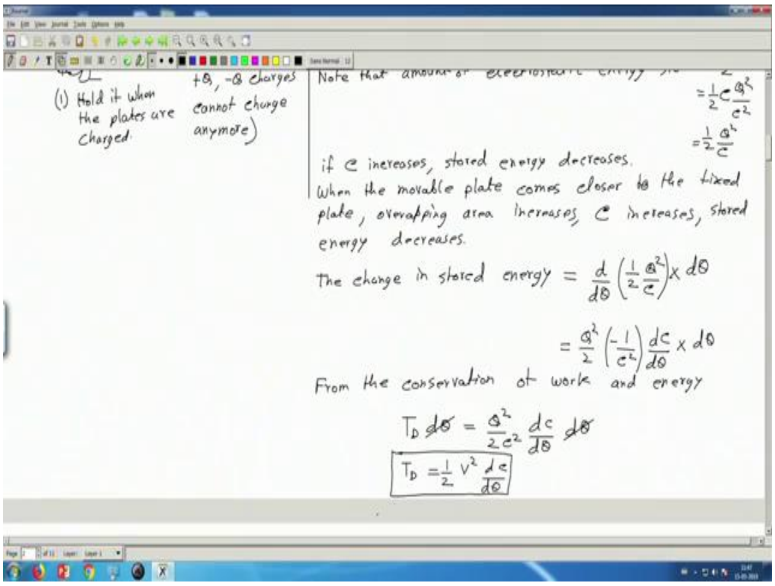

Now see when this plate is moved towards the other plate, the overlapping area between them increases; that means capacitance C increases and if capacitance is increasing then the stored energy will decrease.

That means, this is the energy which is pained to do this work so, this work done is done by spending this stored energy. Now, when the movable coil comes closer to the fixed plate; please forgive me if I am doing this mistake again and again. When the moveable plate comes closer to the fixed plate overlapping area increases; that means, C increases that means, stored energy decreases.

Now, how much is the change in stored energy when this plate is moved by an angle of theta? So, this is the total change of stored energy and this we can write if we differentiate.

$$Change\ in\ stored\ energy = \frac{d}{d\theta}\left(\frac{1}{2}\frac{Q^2}{C}\right) d\theta$$

$$= \frac{Q^2}{2}\left(-\frac{1}{C^2}\right)\frac{dC}{d\theta} d\theta$$

So, this is the change in stored energy and this is with a negative sign indicating that the energy is decreasing, because capacitance is increasing and we know that it is in the denominator if capacitance increases store energy decreases. So, this is the amount of decrease of stored energy and this must be equal to the work done from the conservation of work and energy. Now from the conservation of work and energy, we can write that this work done, which is this, should be equal to this change in store energy.

From conservation of energy,

$$T_D d\theta = \frac{Q^2}{2}\frac{1}{C^2}\frac{dC}{d\theta}d\theta$$

So, this is the expression for torque which we have written before and now we are proving it. So, this is the derivation for the expression of deflecting torque in an electrostatic instrument.

$$T_D = \frac{1}{2}V^2\frac{dC}{d\theta}$$

Next we will do 2 more derivations, derivation for deflecting torque in electrodynamic of it will do later before that we will do moving iron and then electrodynamic.

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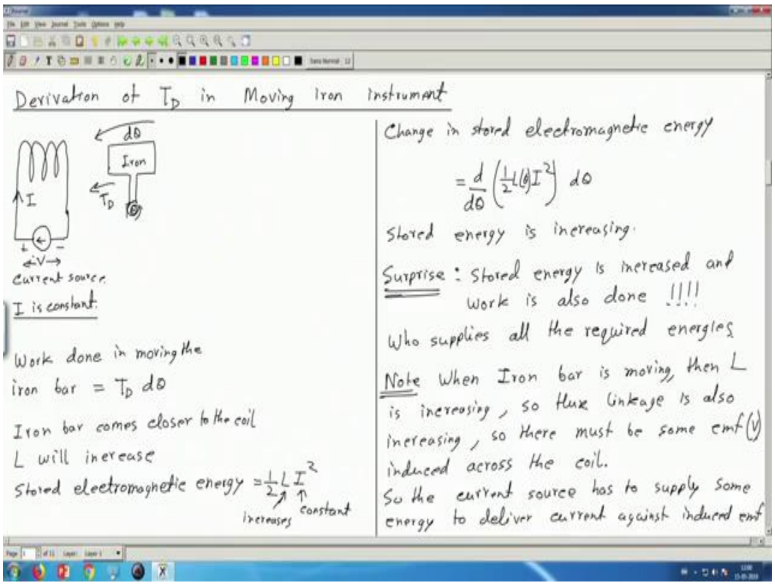

So, two more derivations we will do in this video. Derivation of T_D deflecting torque in first moving iron instrument. So, we will take this simplest one which is an attraction type moving iron instrument which has a coil, which carries an unknown current I that we want to measure, in front of it we have an iron piece which can rotate around some pivot.

And so, this is iron or any magnetic material in principle it can be and this can rotate it can come closer or further, depending on whether the spring is stronger or this electromagnetic force is stronger. So, this is the scheme.

So, let us take or consider this scheme similar derivation can also be done assuming the person type instrument or other variants of this moving iron instrument, but this is simplest to do this, that is why I have chosen this. So now, when some current is flowing so, for this current to flow let me attach a current source.

So, this is a current source, what is the function of a current source, the function of the current source is to keep this current I constant, no matter what happens an ideal current source is supposed to keep this current always constant and for that it can apply any amount of voltage, any amount of potential difference it can generate which is required at any moment to keep this current at the same value of I. So, that is the function of this current source.

So, I is constant, like in the previous derivation we had stored in a charge constant in this in this derivation we will use the idea that the current I is constant, but the voltage here. So, this is a current source and the voltage between these two points I am calling it as V between these two points across the current source. This side I am taking as my positive reference, this side as my negative reference, this can vary this can vary with the time, but current I cannot vary.

Now, what happens when current flow, this will apply some force or a torque which will try to rotate this iron bar towards the left. So, this is the torque T_D this is the direction of the torque and let us forget this spring for now. Once again let us assume that this iron piece moves by an angle of d theta. If so, then work done in moving the iron piece is given by the torque multiplied by the angle T_D d theta. So, this is the work done so, some work is being done.

Once again the question is, who does this work or where does this the required energy come from? Now if this is moved by an angle of d theta so, iron bar comes closer to the coil now what will happen, self-inductance L will increase why? because the iron is coming closer to the coil. So, the reluctance of the flux path is being reduced more flux can be produced with the same amount of current. So, L will now increase.

And if L increases, then the stored electromagnetic energy also has to change. So, we can write the change in stored electromagnetic energy is given by this expression for energy.

$$Stored\ electomagnetic\ energy = \frac{1}{2}LI^2$$

$$Change\ in\ stored\ electomagnetic\ energy = \frac{d}{d\theta}\left(\frac{1}{2}L(\theta)I^2\right)d\theta$$

So, this is the total change in a stored electromagnetic energy. And this energy increasing or decreasing, this is increasing because this is the expression for stored energy L is increasing because the iron bar is coming closer. So, L is increasing so stored energy is increasing. So, stored energy is increasing.

Now, we are in a surprise the stored energy is increasing and yet some work is done in moving this iron bar. So, where is this energy coming from, because stored energy is not decreasing so, we cannot say this work done is done at the expense of stored energy, because stored energy itself is also increasing. So, we have an increase of energy stored energy as well as some work done. So, there must be some other source of energy transfer which is supplying both this energy, can we recognize what is that source, that source is this.

So, let me first write a surprising thing, stored energy is increased and work is also done. So, who supplies all the required energies? Now, note that when the iron piece is moving, then L is increasing so, flux linkage is also increasing. So, there must be some emf induced across the coil, because we know according to Faraday's law if flux linkage is changing then there must be some emf induced across this coil and this emf we are calling it V the value of this emf you are calling it V.

And then this current source has to deliver current against this induced emf, because this induced emf according to Lenz's law will always try to oppose this current. Maybe Lenz's law is not that easy to see here. But we can always show that this voltage will act in a direction such that it is trying to oppose this current. Therefore, this current source has to deliver some energy in the process of supplying this current against the induced emf. So, the current source has to supply some energy to deliver current against induced emf.

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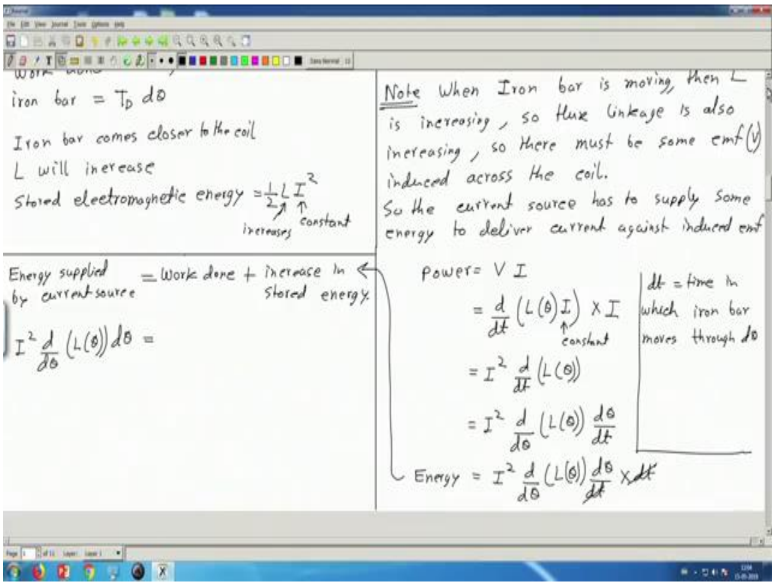

$$Power = V\,I = \frac{d}{dt}(L(\theta)I)I$$

$$= I^2\frac{d}{dt}\big(L(\theta)\big) = I^2\frac{d}{d\theta}\big(L(\theta)\big)\frac{d\theta}{dt}$$

$$Energy = I^2\frac{d}{d\theta}\big(L(\theta)\big)\frac{d\theta}{dt}\times dt$$

So, this is the energy supplied by the current source. So, this should be equal to the work done in moving the iron bar plus the increase in stored electromagnetic energy.

$$I^2\frac{d}{d\theta}\big(L(\theta)\big)d\theta = T_D d\theta + \frac{1}{2}I^2\frac{d}{d\theta}\big(L(\theta)\big)d\theta$$

$$\frac{1}{2}I^2\frac{d}{d\theta}\big(L(\theta)\big) = T_D$$

This is the expression for the torque which we have stated before.

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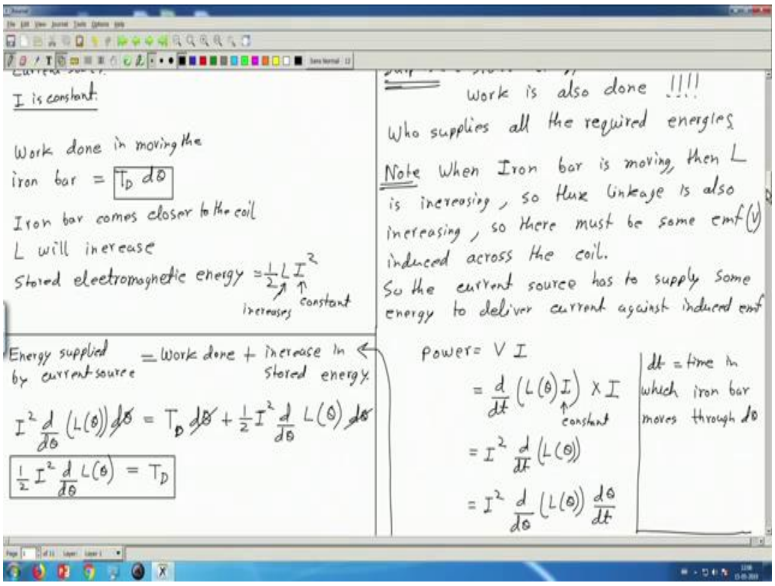

We have also stated the meaning of this expression, but in this video we are doing the derivation so, this is also done. Now the last thing that remains is the derivation for torque for electrodynamic instruments and once again you can expect you can guess that we are going to use the same strategy of energy conservation. So, you may try to yourself before looking at the remaining part of this video or you may look at the remaining part of the video.

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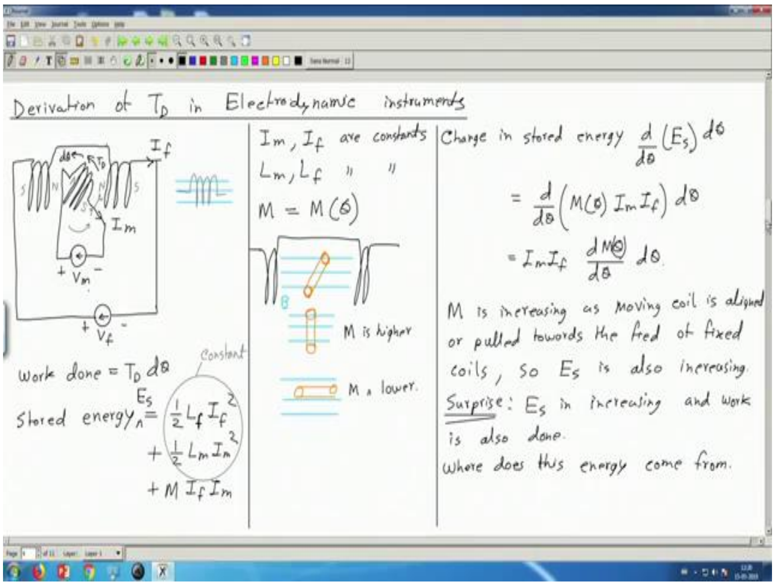

So, the next thing is derivation of T_D in electro dynamic instrument. So, what do you have in an electrodynamic instrument, we have 2 coils 2 sets or 2 sets of coils, one fixed coil set of fixed coils and between this we have moving coil, see I draw it at some arbitrary angle, the current in the fixed coil we call it If, the current in the moving coil we call it I_m.

Now, let me connect both these to 2 current sources. So, that these currents remain constant. So, I have 2 current sources delivering 2 currents through moving and the fixed coils. Now, let us recap a bit and then see that the torque which is acting on this coil is T_D and

$$Work\ done = T_D d\theta$$

$$Stored\ energy = \frac{1}{2} L_f I_f^2 + \frac{1}{2} L_m I_m^2 + M I_f I_m$$

Now, in this expression which are the terms that are constant I_m I_f are constants, why because they are being supplied by constant current sources so, these are constants. (Refer Time: 55:46), L_f L_m these self-inductances they depend on the coils themselves, they had a proximity with respect to some other a magnetic material etcetera. These terms are not changing, because although this moving coil is moving, but it is self-inductance is not going to change at least if some other nearby magnetic material is not there. Why this is a function of theta? Because so, let us see one second let me recap say if I have the fixed coils like this it is generating some flux and let this flux density be B in any left to right or right or the left direction not so important now. And let me draw the moving coil, like I used to do it earlier like this and this is one side of the coil and this is another side of the coil. So, this side is perpendicular to this plane of paper and this side is also perpendicular to this plane of paper.

Now, what is the flux linkage, the flux linkage is the amount of flux that goes through the coil, which crosses the coil. Now, if I have this coil at say, this orientation like this, then a lot of flux can go through it, but if I have this coil in perpendicular orientation, then all flux lines slide or glide across coil, because they do not go through they do not in go through the coil ok. So, here we have less flux linkage.

So, here we can say the flux linkage of the moving coil due to some amount of current in the fixed coil is lower compared to the flux linkage in the moving coil due to the same amount of current in the fixed coil here it is higher. So, here M is higher, here M is lower. So, M is therefore the function of the angle or angular position of this coil.

$$Change\ in\ stored\ energy = \frac{d}{d\theta}(E_s)d\theta = \frac{d}{d\theta}\left(M(\theta)I_f I_m\right)d\theta$$

$$= I_f I_m \frac{dM(\theta)}{d\theta} d\theta$$

Now, is the stored energy going to increase or decrease, it is going to increase because we know that when currents flow through these coils. So, they will generate some polarities like maybe say this is north, this is south, and if here I have south and this north for the moving coil, then we know the this 2 will attract each other and therefore, they will try to get aligned, basically all magnets try to get aligned due to the forces between them.

So, this coil will come to so, it will rotate like this and this will become aligned. So, after the rotation the moving coil will come to this position, that means so, this is same as this position because now the flux lines can all pass through the turns of the moving coil. So, mutual inductance M is increasing. So, M increases if this coil is pulled towards the magnetic field of fixed coil M increases. So, M is increasing as moving coil is aligned or pulled towards the field of fixed coil, M is increasing. So, Es will also increase because this is proportional to M is also increasing. So, once again we have the same surprise here Es is increasing and work some additional work is also done. So, where does this energy come from? Where does this energy come from? This must come from against again through this current sources why, because when this moving coil is getting moved M is changed. If M is changed then flux linkage will also change and if flux linkage is changed some emf will be generated across this.

So, let us call this some emf which can generate here as V_m and this emf as if any emf is generated here. Of course, there will be some emf generated here call that V_f because M is changing, if M is changing then flux linkage of both these coils will change which will generate this voltage.

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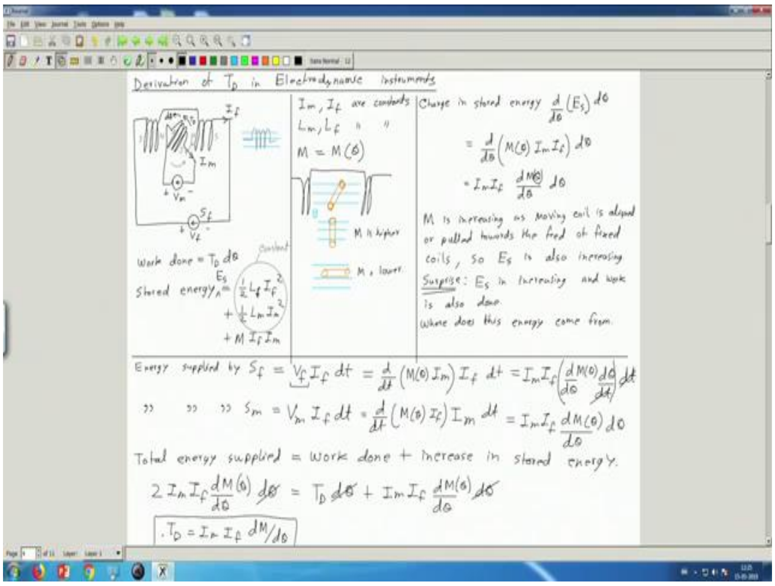

$$Energy\ supplied\ by\ S_f = V_f I_f dt = \frac{d}{dt}(M(\theta)I_m)I_f dt = I_m I_f \left(\frac{d}{d\theta}M(\theta)\frac{d\theta}{dt}\right)dt$$

$$Energy\ supplied\ by\ S_m = V_m I_f dt = \frac{d}{dt}\big(M(\theta)I_f\big)I_m dt = I_m I_f \frac{dM(\theta)}{d\theta}d\theta$$

$$Total\ energy\ supplied = V_m I_f dt = work\ done + increase\ in\ stored\ energy$$

$$2I_m I_f \frac{dM(\theta)}{d\theta}d\theta = T_D d\theta + I_m I_f \frac{dM(\theta)}{d\theta}d\theta$$

$$T_D = I_m I_f \frac{dM}{d\theta}$$

That was the derivation for torque in electrodynamic instruments. So, let us conclude this video by saying that all these derivations are very interesting, particularly for those who loves physics a bit for them this will be very interesting that is why we have done this. But, for solving problems for exams etcetera, maybe this for our exam which will have at the end of this course, these long derivations may not be that important.

What will be more important is this end results, because this is what an engineer may need to apply again and again in his regular practice, but the beauty of understanding this derivation is that once you know the underlying physics you can apply the same idea, the same knowledge similar topics and you can expand your domain of knowledge in this way.

So, you can apply this idea and solve more complicated problems in similar or related topics thank you, this was a long video.

Thanks for staying with us..