Electrical Measurement And Electronic Instruments

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Lecture – 38
Impedance measurement

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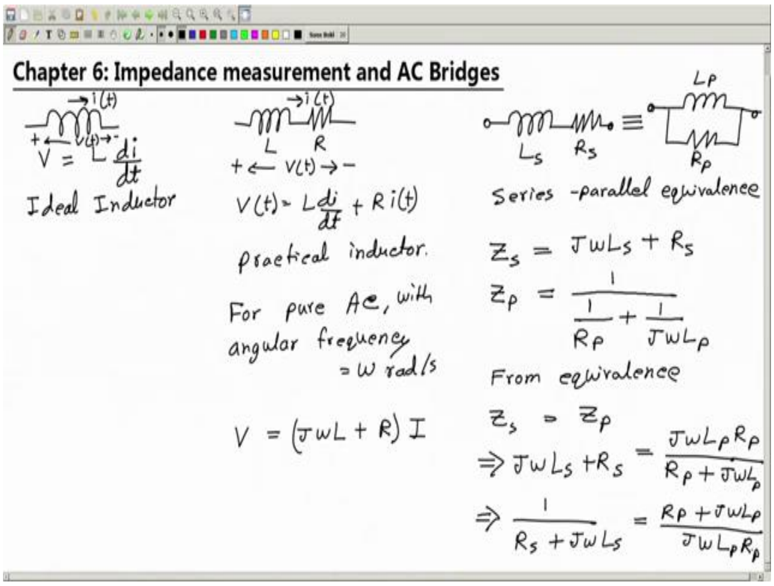

Welcome back. Today, we are going to start a new chapter. But possibly this is the most boring chapter, we are going to a deal with. We are going to talk about AC bridges and measurement of impedance that means, not just resistance, but could be capacitance, inductance or some combination of them ok. So, let us begin. So, before we go to AC bridges, we will talk about series and parallel equivalent of a practical inductance or a practical capacitor ok.

So, when we draw the symbol, this is an ideal inductor. So, this is the symbol of an ideal inductor where, this rule that EMF or voltage = L $\frac{di}{dt}$. This rule is true with the proper sign ok. So, depending on which direction and in which direction, you are considering the current and in which direction you are measuring the voltage. So, if I am measuring the current from left to right, this is i t and if I am measuring the voltage with left side as the positive and right side as the negative side ok. Then, we have this relationship V = L $\frac{di}{dt}$. and this rule for an ideal inductor should be perfectly valid.

So, this is an ideal inductor ok. But there is no ideal inductor in reality. Why? Because any inductor is after all coin made up of some conductor and any conductor is any conductor must have some resistance ok; any coil, any conductor must have some ohmic resistance. I mean nothing is like a superconductor in reality. So, and practical in inductor will always have some resistance which we can think that this resistance is in series with the inductance ok.

So, this call this R, call this L and then, the relationship between the current and voltage, we can write it as ok. Once again, if I take this side as my positive reference, this side as my negative reference. Then, one we can write that V (t) == L $\frac{di}{dt}$.+ Ri (t) ok. So, this is a practical inductor and we also know that for AC, where i is sinusoidally evading ok. So, for special case AC, perfect pure sinusoidal AC ok; so that means, is voltages and currents and voltages are all like sin omega t cos omega t, pure AC with angular frequency equal to omega.

Omega the unit should be radian per second. Then, this relationship we can write using phases like V = (JωL+ R) I . So, this is what we all know and here, this resistance which is in series is quite intuitive. It is easy to understand, why do we have this resistance. This is because any conductor, any coil must have some resistance, some ohmic resistance which is intuitively should be series with the inductance ok.

But now, what I am going to show you is that is this an inductor pure inductor in series with the pure resistance ok, call this L s R s; s stands for series because this two were in series, is always equivalent to another circuit where we have this inductor in parallel with a resistance call this L p and R p; p because they are in parallel. So, for so given a series combination of a particular value of L s and R s, we can always find a value of L p and R p such that these two circuits are equivalent. Equivalent in the sense the impedance between these two terminals and impedance between these two terminals is same ok.

So, this is series parallel equivalence for an practical inductor. Now, how can we find the value of L p and R p, if L s and R s are given or vice versa. That is very easy. So, consider this side, this L s in series with R s. So, we can write the total impedance

Zs = JωLs+ Rs

$$Zp = \frac{1}{\frac{1}{Rp}+\frac{1}{\mathrm{J}\omega L\mathrm{p}}}$$

From equivalence

Zs = Zp

$$J\omega Ls + Rs = \frac{1}{\frac{1}{Rp}+\frac{1}{\mathrm{J}\omega L\mathrm{p}}}$$

$$\frac{1}{\mathrm{J}\omega Ls + \mathrm{Rs}} = \frac{\mathrm{J}\omega L\mathrm{p\ Rp}}{Rp+\mathrm{J}\omega L\mathrm{p}}$$

$$\frac{Rs-\mathrm{J}\omega Ls}{Rs^2+\omega^2\ Ls^2} = \frac{1}{Rp} + \frac{1}{\mathrm{J}\omega L\mathrm{p}}$$

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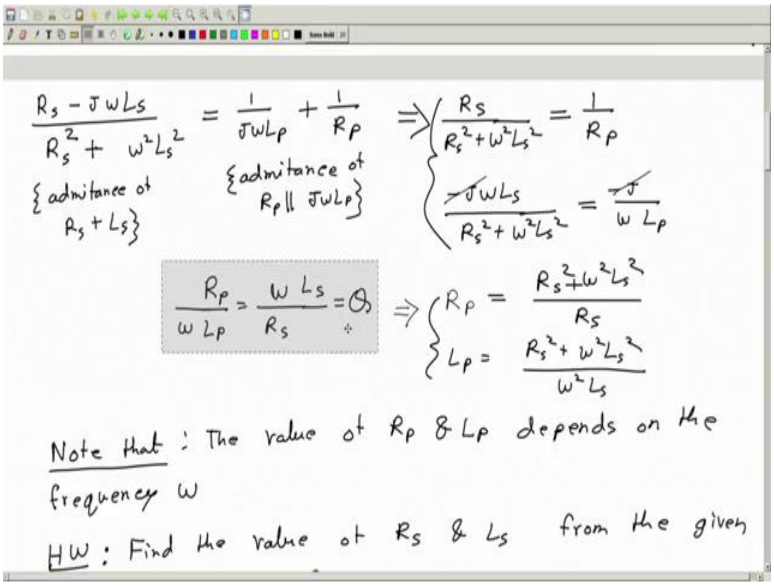

$$\frac{Rs}{Rs^2+\omega^2\ Ls^2} = \frac{1}{Rp}$$

$$\mathrm{Rp} = \frac{Rs^2+\omega^2\ Ls^2}{Rs}$$

$$\mathrm{Lp} = \frac{Rs^2+\omega^2\ Ls^2}{\omega^2\ Ls}$$

So, given R s and L s, we can thus find R p and L p such that the parallel combination of R p and L p will be same or equivalent to the series combination of R s and L s. Note that the value of R p and L p depends on the frequency omega ok. So, this equivalence will be true for a particular frequency; whatever frequency you choose, its not true for all frequencies ok. So, for say 50 hertz if you get sum value of R p and L p; for 100 hertz, the value of R p and L p will be different.

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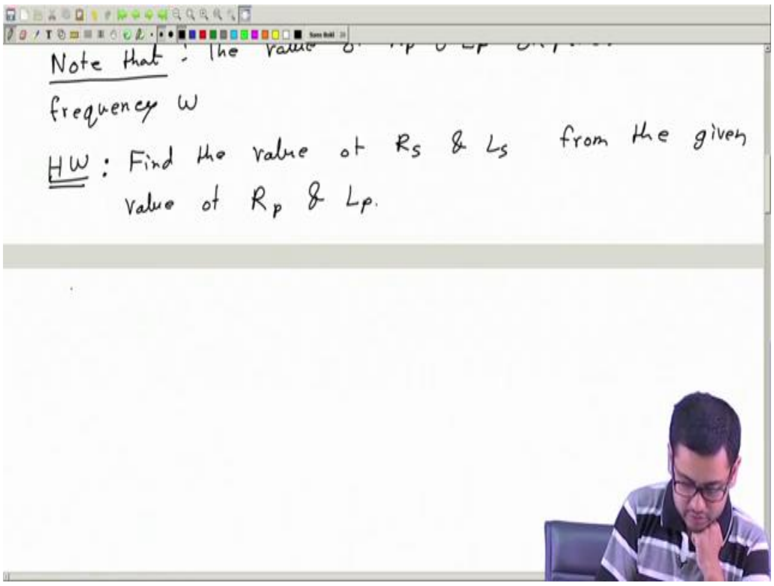

Now, let me just give you a small homework, which is very easy to do. So, in here we have found the value of R p and L p, given value of R s and L s. Your homework will be find the value of say R s and L s from the value of or from if the give if the value of R p and L p is given from the given value of R p and L p ok. So, this is a small exercise I would like you to practice. So, now next thing we are going to talk about is capacitance.

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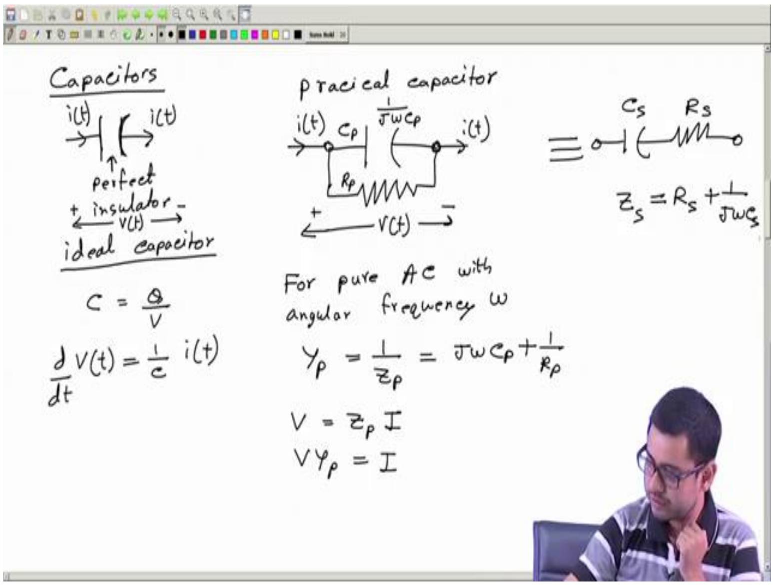

Now, an ideal capacitor which we denote with just the symbol or maybe sometimes like this ok. For this, no current can actually flow through one plate to the other through this dielectric because this is a perfect insulator ok. So, this is an ideal capacitor ok. And for this, the relationships like C= Q / V from which you get

$$\frac{d}{dt} V(t) = \frac{1}{c} i(t)$$

This is perfectly true for a for an ideal capacitor and where, the current that flows is this i t or this i t; no current goes through the insulator. It flows only from outside to one plate and from the other plate to outside and the voltage that we are referring here is this voltage lefts let me draw it here. V (t) with this side positive and this side as the negative difference.

So, this is an ideal capacitor. Now, a practical capacitor we will always have some leakage from one plate to the other through the dielectric. So, a practical capacitor will always have some leakage current flowing from say this side to this side and how do you model that leakage current? By giving a path for the leakage current path or a conductance in parallel to this capacitor. So, this is a conductor resistance and conductance are equivalent one over resistance is conductance. So, I have giving some conductance, nonzero conductance ok. So, non infinite resistance such that some current can flow from here to here.

So, this is the model of a practical capacitor which is a parallel combination of a ideal capacitor with a resistor ok. So, and here the what is the relationship between V and i ok.

So, for that let us first find the impedance of this circuit ok. So, and let us take for simplicity, let us take for pure AC which angular frequency omega ok. So, this part will have an impedance of call ok, if this capacitance is C p, if this resistance is R p; p because they are in parallel. Then, the impedance of this capacitor is 1 over J omega C p ok. So, this is the impedance and R p same as R p, impedance of resistance is R p.

$$Yp = \frac{1}{Zp} = J\omega\, Cp + \frac{1}{Rp}$$

V = Zp I

V Yp =I

Now, I am claiming that any such practical parallel combination of capacitor and resistor is always equivalent to a capacitor in series with R register; where, I will denote the value as R s and C s respectively; s for series. Now, so my claim is that any such parallel combination is always equivalent to a series combination. Now, how can we find the equivalent series combination? Once again, the same trick. So, we have to equate the impedance of this with the impedance of this circuit. So, we can write. So, for this we can write Z s which is the series combination of this 2. So,

$$Zs = Rs + \frac{1}{J\omega Cs}$$

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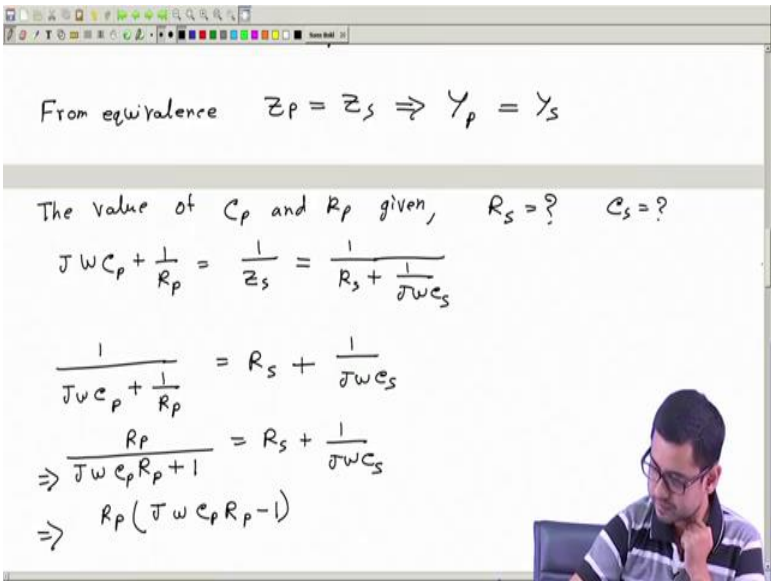

And now, for this two circuits to be equivalent, we can write that Z p = Z s ok. Now, Z p =1/ R p or we write we can also write it this way Y p=Y s. Now, given the values of Y p C p and R p, we can always find the value of R s and C s. Similarly, we can do it in the other way given the value of R s and C s, we can find the value of C p and R p. Now, let us do it this way. Say the value of C; C p and R p given and we want to find R s equal to how much; C s equal to how much ok? So, you have to just manipulate this equation, we have to do with in a way such that the it becomes easy ok.

So, let us first write it in terms of admittance or maybe in terms of the yes, I think it will be easier if we do it in terms of admittance ok. So, we can write

$$J\omega\, Cp + \frac{1}{Rp} = \frac{1}{Zs} = \frac{1}{\mathrm{Rs} + \frac{1}{J\omega Cs}}$$

$$\frac{1}{J\omega\, Cp + \frac{1}{Rp}} = \mathrm{Rs} + \frac{1}{J\omega Cs}$$

$$\mathrm{Rs} = \frac{Rp}{1 + \omega^2\, Cp^2\, Rp^2}$$

$$\frac{1}{J\omega Cs} = \frac{-J\omega Cp\, Rp^2}{1 + \omega^2\, Cp^2\, Rp^2}$$

$$\frac{1}{Cs} = \frac{\omega^2 Cp\, Rp^2}{1 + \omega^2\, Cp^2\, Rp^2}$$

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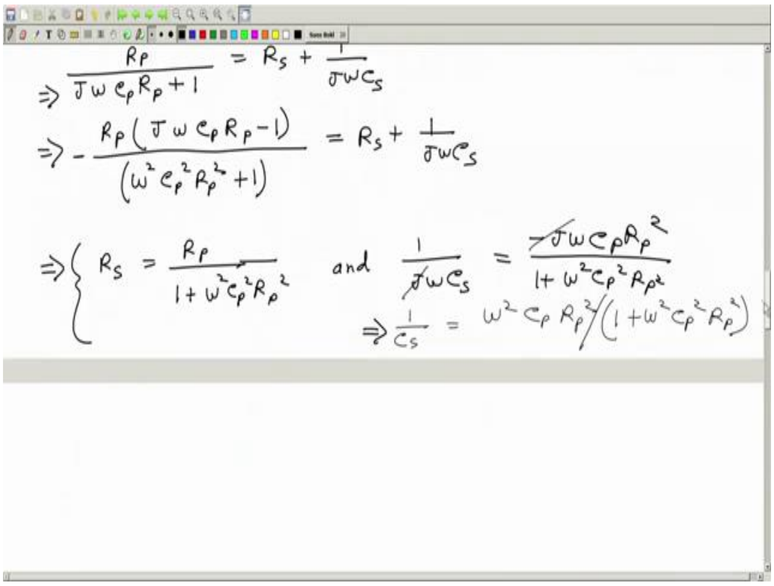

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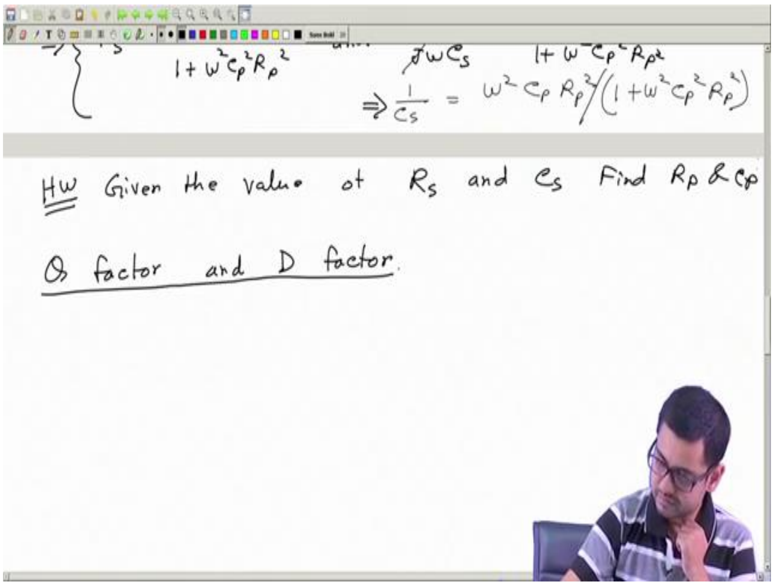

Now, for you the homework will be given the value of R s and C s, find R p and C p? Here, we have found value of R s and C s from R p and C p. Here your task is opposite ok. Now, in this class, we will define two more terminologies. Q factor and D factor. This will be the last topic for this class ok.

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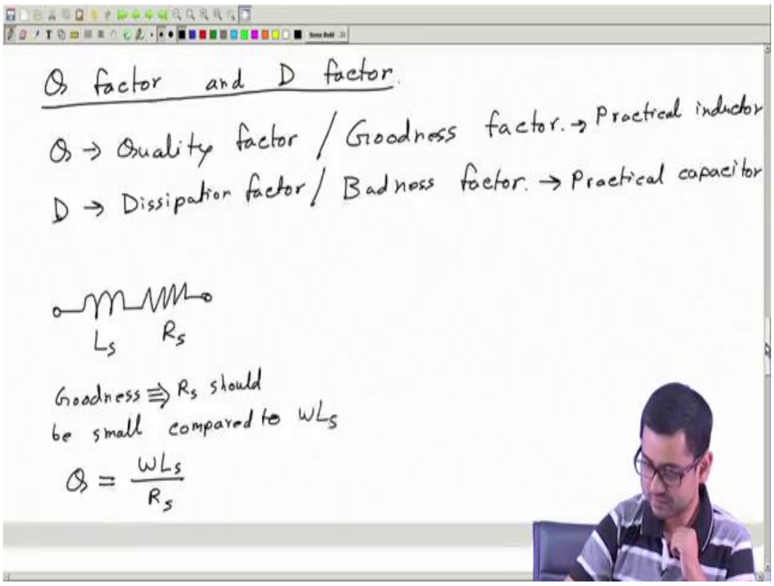

So, generally the word Q stands for quality ok. So, this is like quality factor or you can call it, I often call it this is a goodness factor. It is a measure of measurement. It is a measure of goodness; how good something is and D stands for dissipation factor. I often call it, this is the degradation factor or a badness factor.

Now, a small note is that the term Q factor in electrical engineering is often used in multiple context. It has several definitions in several different context like in the context of filters, in the context of resonant circuits and also in the current context which you are discussing. So, you should not get confused because unfortunately, the same name is used in different ah cases different scenarios ok.

Now, this quality factor is normally used for a practical inductor. Although, you can extend this thought for capacitance as well, but this is normally used for stating how good an inductor is and this is D factor is normally used for a practical capacitor to state how bad a practical capacitor is. Now, if you think of a practical inductor which is always can be modeled as a series combination of some pure inductance and the pure resistance, then the measure of goodness is equivalent to the measure how ideal this inductor is.

So, this goodness factor or Q factor will tell us that how close this practical inductor is to an ideal inductor. When it will be good then? It will be good only if this R s is small compared to the value of the inductance ok. So, goodness implies R s should be small compared to the impedance of this part J omega L s or if you consider the magnitude only then you forget the J.

So, this is the magnitude of the impedance of this pure inductive part and this is due to the coil resistance. There could be other factors too which we will talk about later. So, this impedance actually not only the coil resistance, it also reflect other losses suggest like core loss etcetera, we will talk about that later, right now ignore that ok. So, therefore, we can write that Q.

$$Q= \frac{\omega\ Ls}{Rs}$$

And alternatively, if L s is large, then also the quality will be better. So, this is how you can remember this. Now, so this is in terms of; so, this is quality factor in terms of L s and R s. Now, if you go back, we have seen that any L s R s series combination can always be written as can always be model as an L p R p parallel combination such that the value of L p and R p is like this.

$$\frac{Rp}{\omega Lp} = \frac{\omega\ Ls}{Rs} = Q$$

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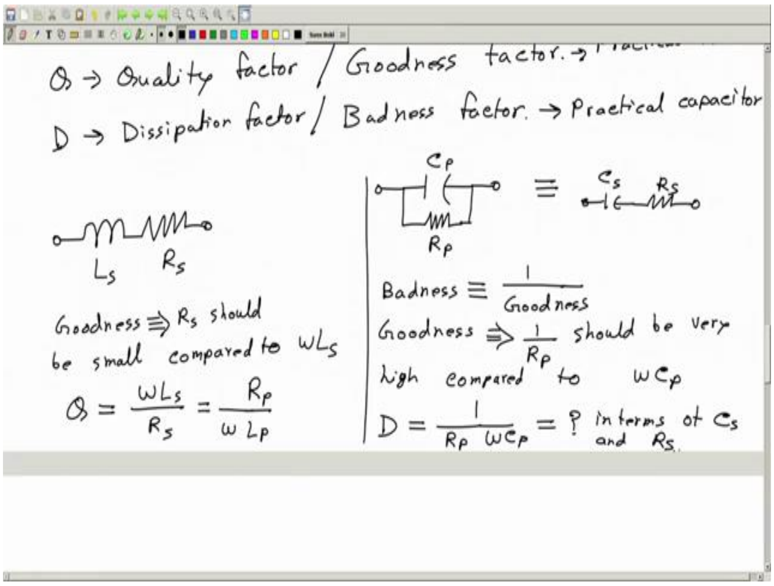

Therefore so, we can write it like this as well ok. So, in the parallel combination Q factor will be better, if parallel resistance is higher compared to the inductance. Parallel resistance higher means no current will flow through the parallel path of this inductor ok. So, this is the definition of Q factor quality factor and the way to remember it. You remember that quality is a measure of goodness and goodness means how close this practical inductor is towards an ideal inductor; an ideal inductor should have nodes resistance in series. So, R s should be small compared to omega L s. So, R s here should be small, omega L s should be high.

Similarly, D factor this as I said is a badness factor or degradation factor you can call is normally used for practical capacitor which is always lossy or leaky, some current always flow through the dielectric which we model using this parallel resistance ok. So, now, the badness is equivalent to how far this C p R p combination is from a ideal capacitor. So, badness= 1/goodness and now, goodness will imply how; how ideal this combination is which means this R p should be very high. So, that no current can leak through this resistance. The conductance should be very small or this resistance should be very high.

So, goodness means the conductance that means, 1/R p should be very high compared to this conductance and this conductance is J omega C p ok. So, therefore, the D or the badness factor will be given by. So, it will be bad, if this R p is small; if R p is small, more current flows. So, therefore, we have it in the denominator and it will be also bad if this conductance; conductance of or I call that susceptance of this part should is low ok. If this is also low, then we write it as like this ok, J you can ignore because we are talking in terms of the magnitude only. So, this is the dissipation factor ok.

$$D=\frac{1}{Rp\ Cp\ \omega}$$

Now, this dissipation factor is in terms of R p and C p. Now, we know that this is always equivalent to some C s, R s in series. What will be the value in terms of C s and R s that is your homework? Ok. So, in this video, in this class, we have learned two important concepts; number one series and parallel equivalence of a practical inductor or a practical capacitor and how to compute the series parameters if the parallel parameters are given or how to compute the parallel parameters if the series parameters are given. This is one thing we have learned.

And the other thing which we have learned is these two terms new terms Q factor and D factor. The way to remember it is remember that Q factor means quality factor which is a measure of goodness. Normally, it is used for inductors and D factor dissipation factor is a badness factor and therefore, Q factor defines how close up and practical inductor is to a to an ideal inductor and D factor signifies how far a practical capacitor is from an ideal capacitor.

Thank you.