Electrical Measurement And Electronic Instruments
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Lecture – 37
Kelvin – Varley Potential Divider

Welcome we are studying potentiometers. So, we have studied DC and AC potentiometers and in this context another equipment often becomes very useful which is a Potential divider. In this class, we will see what a potential divider is we will take a small example, in the context of potentiometer to understand why are Potential Divider can be useful. And, then finally, we will study Kelvin-Varley Potential divider which is the focus of this class ok.

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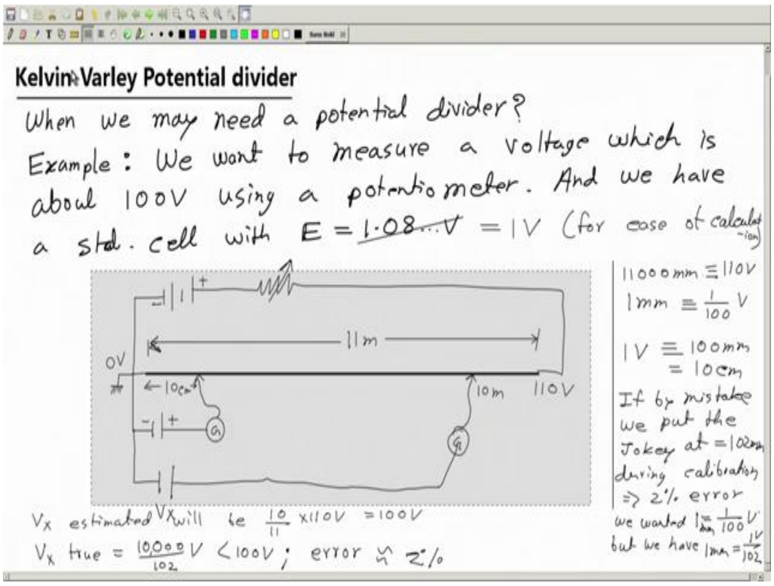

So, let us start this class with the question, when we may need a potential divider? So, the answer we can, see with an example and the example is as follows, we want to measured voltage which is around; which is about say 100 volt.. So, we want to measure it using a potential divider, using a potentiometer and we have a standard cell with EMF, E= 1.08 something ok.

So, let us take it for ease of calculation let us take it to be 1 volt only ok. So, let us take it 1 volt for ease of calculation. Now,–_suppose the length of the potentiometer is 11 meter, say this length is 11 meter ok. Now, so we want to measure 100 volt and I mean something around 100 volt, it is only possible if the voltage between the two points of this wire is greater than the unknown voltage which is 100 volt. So, therefore, we definitely need a supply, DC supply which will provide more than 100 volt here.

So, if I take this side as the reference 0 volt, this side should be around maybe 110 volt ok, that is a easy number. So, if I choose to have 110 volt here this will imply. So, 11 meter is equal to 110 volt

11000 mm = 110 V

1 mm = 1/100 V

1 V = 100 mm = 10 cm

Now, so, therefore, for calibration we have to set the Jokey of these standard cells, this is the standard cell minus side plus side this has to be consistent minus and plus. So, let these be the galvanometer. So, you have to set the Jokey somewhere around in centimeter, exactly at 10 centimeter and then adjust this current to get the null.

Now, if we make a mistake instead of setting it to say 10 centimeter, say if we set it to 10 centimeter plus minus 1 millimeter. So, that is a mistake. So, if by mistake we put this Jokey during calibration Jokey at. So, it should be at 100 millimeters say we put it at by mistake say 102 millimeter ok. So, we have a 2 millimeter mistake during calibration, which means 2 percent error ok. So, why 2 percent; basically now so, we wanted that 100 mm= 1 volt, but we have so, let me write it this way. So, he wanted 1 mm=1 /100 volt and we actually have 1 mm = 1/102 volt.

So, we have. So, now, this is a 2 percent error approximately, if you remember the rule for error in product and division, then you know that this is approximately a 2 percent error in the voltage per millimeter. Therefore, during the measurement ok; so, if we get the balance so, this is the V_X unknown, if we get the balance somewhere here at 10 meter ok, then we will estimate V_X as so,

$$V_X = \frac{10}{11} x\, 110 = 100\, volt$$

So, this is V_X estimated, but

V_X true = $\frac{10000\ V}{102}$ < 100 V error $\approx$ 2%

This is because we are setting I mean we have the full length equal to 100 and 10 volt and so, 1 volt is within a very small distance and we can make a mistake ok. In setting this Jokey instead at 100 millimeter we may set it at 100 and 2 millimeters so, that is the mistake. And, the mistake will be more severe say if we want to measure a voltage which is around 1000 volt ok. So, if that was the case.

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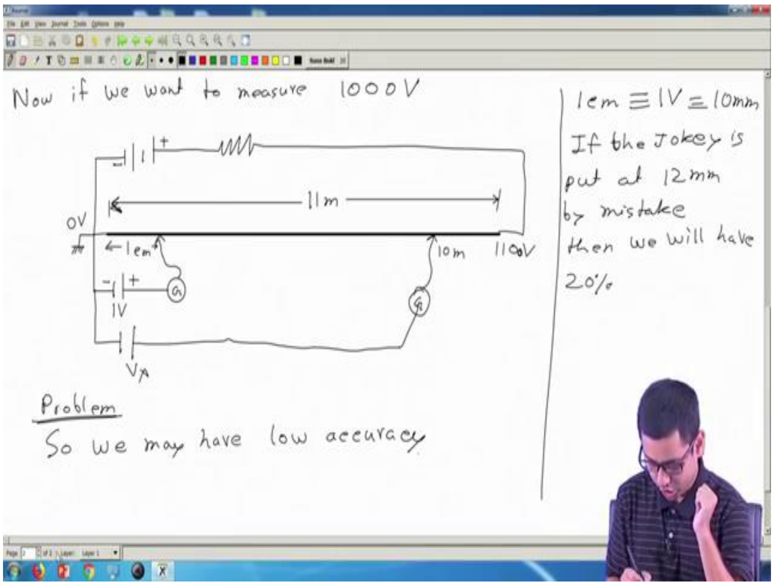

So, now, if we want to measure that just let us do the calculation very quickly. Now, if we want to measure say something around 1000 volt ok. So, I am increasing the value of the voltage that I want to measure, but the standard cell is still 1, it has the same EMF 1 volt because standard cell voltage is not going to change ok. So, then we have to set this point somewhere around 1100 volt and then we should do the calibration by putting the Jokey at 1 centimeter ok. So, this is like now 1 cm = 10 mm = 1 volt. while setting the Jokey, if we make say 2 millimeter mistake, if the Jokey is put at say 12 millimeter by mistake, by manual error or due to the error in this scale whatever the reason is, then we will have 20 % error.

So, we will finally, have 20 % error in the measured voltage so, that is the problem. Now, there is a solution ok. So, the problem is we may have low accuracy and now the solution to this problem is as follows.

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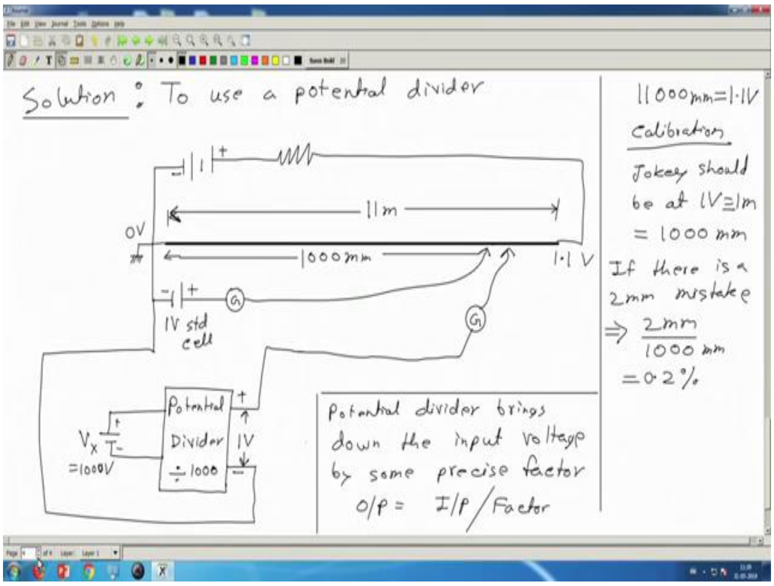

If this is 1000 volt instead of measuring this voltage directly with a potentiometer what we can do? We will use something called a potential divider ok and, say will give this input V_X which is 1000 volt. So, will give V_X as the input to this potential divider and the function of the potential divider is to divide the input by some factor, say this divides by a factor of 1000 ok.

So, then we will have output which is around 1 volt if this is around 1000 volt, output will be around 1 volt and, this voltage we will apply to this potential divide potentiometer. So, that is one side, another side goes like this. And so, the voltage that we are actually measuring is 1 volt we are not measuring 1000 volt measuring a much smaller voltage. Therefore, we can choose to set this side to have something around 1.1 volt that is enough ok.

And, if we choose so, if 11 meter or 11, 000 millimeter, if we choose it to be 1.1 volt then for calibration Jokey should be at 1 volt, because this is 1 volt standard cell. So, Jokey should be at 1 volt which means 1 volt is equivalent to from this you can find this will be 1 meter or 1000 millimeter ok. So, we have to put the Jokey somewhere down here at a distance of 1 meter or 1000 millimeter from the left side.

Now, once again if we make some small mistake say a 2 millimeter mistake so, if there is a 2 millimeter mistakes. Now, this will imply how much error 2 mm/ 1000 mm. So, this is equivalent to 0.2 percent, which is much lower than the previous two cases, where we had either 2 percent error or 20 percent error ok. So, we can have much smaller error, if we use a potential divider.

So, the solution is to use a potential divider and let me recap what a potential divider is a potential divider is lower I mean it brings down these input value, input voltage to a lower value. So, potential divider brings down the input voltage by some precise factor it has to be precise ok.

$$O/P = \frac{I/P}{factor}$$

If, we want to divide it by 1000 we should divide exactly by 1000 it should not be 1001 then of course, there will be some error due to that which we have not considered in this example. So, this may also have some error which can increase this estimated error ok. So, that we have not discussed we have assume this potential divider to be ideal ok.

But, in any I mean just a small note even say if this is 1000 and I mean 1 by an error so; that means, it will cause 0.1 percent error and so, that is not as large as 2 percent or 20 percent in the previous example. So, this error can still be small depending on how good this potential divider is plus, minus signs are important ok yes. So, now, let us see how a potential divider can be made.

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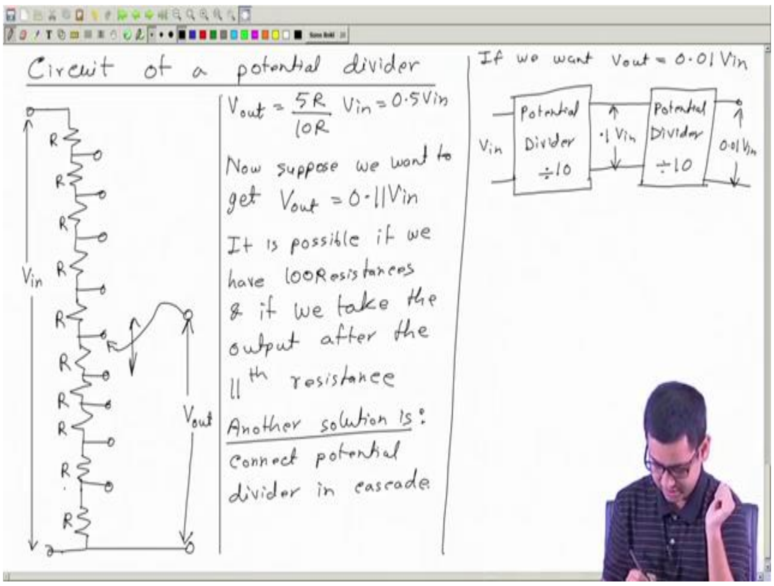

So let us now see circuit of potential divide, we have already seen why is it useful. So, this is basically a resistive divider it is very simple. So, it is a set of resistances connected in series, so, 1 2 3 4 let me take say 10, 5 6 7 8; 1 2 3 4 5 6 7 8 9 and so, 1 2 3 4 5 6 7 8 9 10. And, say if all these resistances are,R equal, we need this to be equal. And, if we now apply a voltage here say call it V in this is the input voltage and if we take the output or if we measure the output between say these two terminals, then we have actually V out is equal to in this case V out = $\frac{5R}{10R}$ Vin = 0.5 Vin

Now, we can move this Jokey up or down we can connect it here, here, here, here or anywhere. So, accordingly we can get V out which is a fraction or V in. So, with this circuit we can get any fraction of the input which is like 0.1 times, 0.2 times, 0.3 times, or 0.7, 8, 0.9. So, any fractions are like that of this V in we can get with this circuit.

Now, suppose we want to get V out = 0.11 Vin. So, how is that possible? That is possible; if we have say 100 such resistances R and then if we take the output from the 11th position ok. So, it is possible, if we have 100 resistances and if we take the output after the 11th resistance; so, count 1 2 3 4 so, on 11 resistances total 100 resistances are there, count 11 and then put the Jokey there, then you can get the exactly 0.11 of V in ok.

So, output after 11th resistance, but then you need 100 resistances and 100 equal resistances there should be as equal as possible this chain is becoming longer. Now, if I need V in to be say 0.1, 2, 3 up to three decimal places then; that means, I need 1000 resistances, for 4 decimal place division we need 10,000 resistances, it is very clumsy and not a good idea.

So, another solution is connect potential dividers in cascade ok, like this. So, let this be one potential divider and say if this is the input V in and say if we want V out to be equal to say 0.01 of V in ok, then we can take two potential dividers. So, this will divide by a factor 10, this will divide again by a factor 10, the output of this potential divider will fit to the input of the second one. So, this voltage will be 0.1 times V in and then here we will get 0.1 times V in; 0.01 ok so, this is a solution.

Now, how to make this cascade using this potential divider; so, let us try that.

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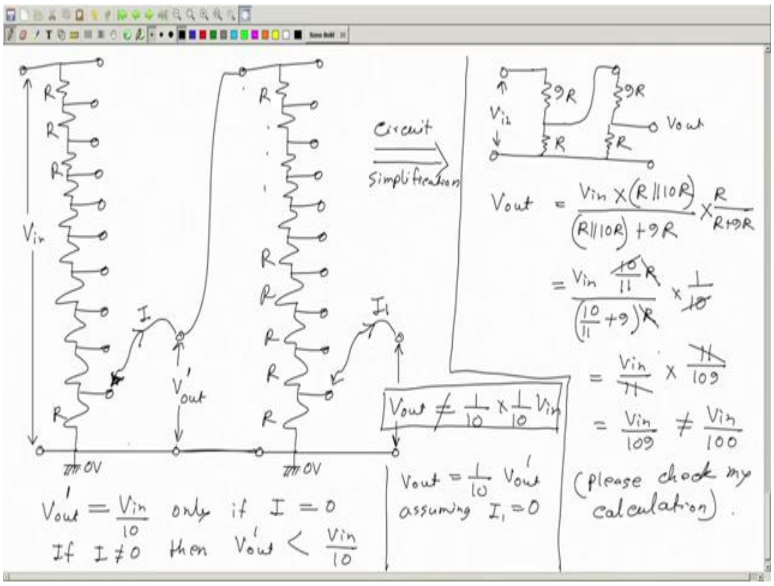

So, let me draw two potential dividers with 10 resistances, 1 2 3 4 5 6 7 8 9 10 ok; 3 4 5 6 7 8 9 10 ok. And so, this is one potential divider this side is the input V in and V out is between this terminal you can take this as the reference for all measurement. So, V out will be between this and wherever you connected ok.

So, if you want to divide by a factor of 10, then you have to connect it here after the first resistance; then let us make another one ok. Now, if I want to divide by a factor of 100 so, this is dividing by a factor of 10, now this is also dividing by a factor of 10 so, what I can do, I can do it like this.

So, I will connect this is here, I will bring this output here, I will connect this and I will take the final V out from here. So, this is the V out; so, this is like 1 over 10 and this is like another 1 over 10 which means 1 over 100. So,

$$V\ out = \frac{1}{10}\ x\ \frac{1}{10}\ Vin$$

so, all these resistances are equal here also all these resistances are equal ok, but the question is will it really will so, think of it for a moment and you can pause the video at this point think of it and then come back and see is it really so, or not.

And, I am saying this is really not equal to, why? Because so, this circuit so, let us do some circuit simplification ok. So, circuit; so, some circuit simplification will give it like so, I have 9 R here and then 1 R this is V in, this is this part and from here I take the output to the input of the second stage again I have 9 R here and 1 R here. So, I write them as like this 9 R and R. So, this is V out with respect to this point ok, this is V in and V out is between these two terminals.

Now, let us calculate how much is V out? So, let us calculate it directly. So, if we do a direct calculation then this R and this two, 9 R and R. So, this 2 R in series 9 R and R, these 2 R in series and this R is in parallel with that. So, we can write

$$V\ out = \frac{Vin\ x\ (R||10R)}{(R||10R)+9R}\ \text{x}\ \frac{R}{R+9R} = \frac{Vin}{109} \neq \frac{Vin}{100}$$

Please check my calculation, if I have done any mistake please point it out correct it yourself it is not difficult, but the conclusion is this V out is not 1 over 100 equal to V in. So, this is the observation and why is it so, actually this is because – so, if we think that this V out call this V out prime intermediate. If, you think this is equal to 1 over 10 as V in that is only true if there is no current flowing through this or through this. So,

V out' = V in /10 only if this current I =0

If there is a current, we call that a loading effect, then this relationship is not true ok. Once again this potential divider rule is only true if no current is flowing out that is very basic ok. And, if some current is flowing out, then basically this resistance will be in parallel with something else. So, it is true value will be smaller than just R, because there is a parallel resistance which is this. So, the potential that we will have here actually is less than this expected value.

If, $I \neq 0$ then V out' < V in /10

And, then there is if we assume there is no loading effect here, if we just have an ideal voltmeter or potentiometer connected here. So, no current is flowing so, this part will give 1 by 100 that is for sure V out is equal sorry 1/ 10, V out prime assuming this current called $I_1 = 0$, that is possible if we have a ideal voltmeter here or a potentiometer here then. So, of course, then V out < V in /100, that is what we have seen it is less than V in / 100 it is according to my calculation V in/109; check my calculation please. So, what is the solution?

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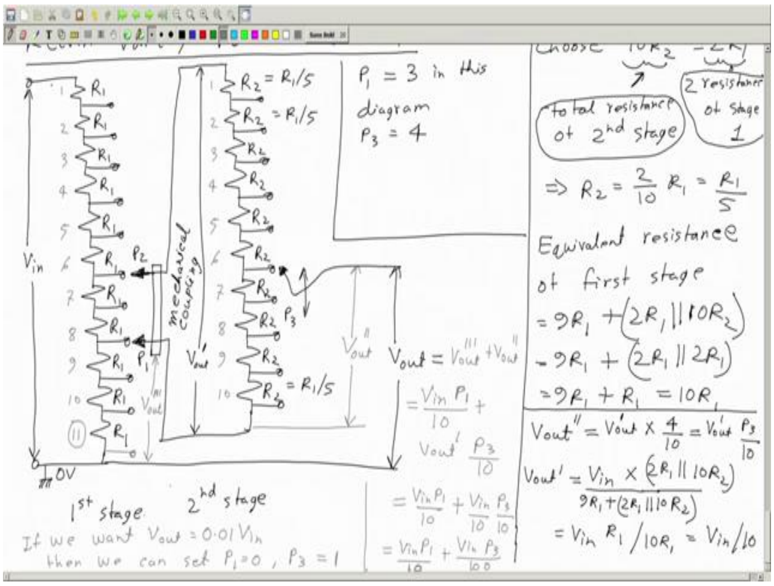

So, now we are going to see a new circuit which is called Kelvin-Varley potential divider, where the previous problem will not exist. What was the problem, the problem was due to the loading effect of the first stage, by the second stage. The second stage of this potential divider is drawing some current I and therefore, the output from the first stage is not as expected, it is less than the expected value ok.

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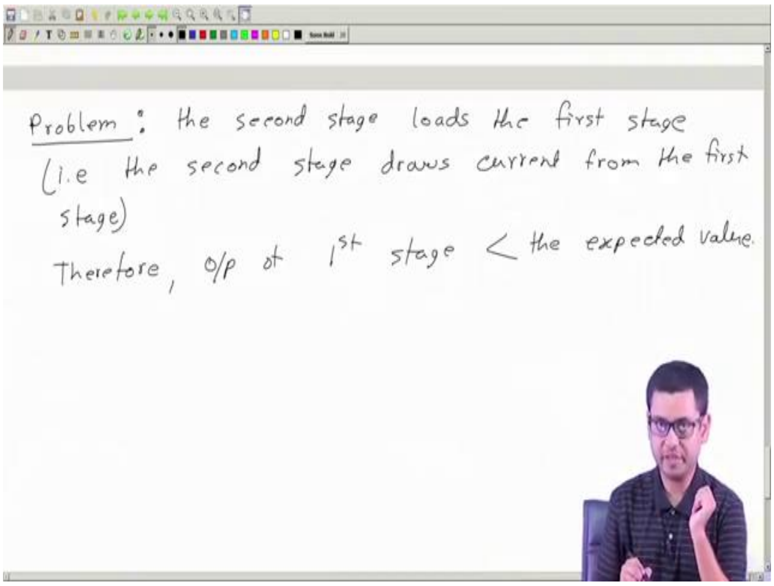

So, let me first also write the problem. The problem of in the above circuit is that, the second stage loads the first stage which means; when which means the second stage draws current from the first stage. Therefore, output of 1st stage is less than the expected value ok; expected value I think you understand in the previous case expected value was 1/10, but the actual value was less than that ok.

Now, the solution to this problem is a small modification, but very nice and well thought of modification of the previous circuit. What is the modification? The modification is like this so, I will first take say 10 resistances, 1 2 3 4 5 6 7 8 9 10, I will take extra 1 why, I will tell later 11 ok. So, this is let me count let me put this counts 1 2 3 4 5 6 7 8 9 10 11, there are 11 resistances ok.

In the first stage so, this is first stage there is an extra one. So, note this extra one' extra resistance and all these resistances are equal of course, so, their value you can take as R R R R R and R all equal. Now, in the second stage I will take only ten resistances, this extra one is not required. Now, what I will do is so, you know the input will of course, be at the first stage so, this is the input V in.

And here there will be a nice trick; the trick is like this. So, we will have two Jokey's instead of one and these two Jokey's will be mechanically coupled. So, they this 2 move together ok. So, this is mechanical coupling so, this two pointers move together. If this pointer is connected here the next pointer will be connected to the switch after leaving 1 switch ok. So, this two will be always across 2 resistances ok.

So, you cannot move them individually you have to move it together only and they will always connect in parallel to 2 resistances always. And, now the next thing I what I will do is; so, call this resistances as if this is R 1 1 1, 1 for stage 1 call these R 2 all these are equal of course, but not necessarily equal to R 1. So, I will choose this total resistance of stage 2 which is $10 R_2 = 2 R_1$

So, this is total resistance of 2nd stage and this is resistance of I mean 2 resistances, 2 resistances of stage 1 ok. So, I will choose it in this way so; that means, I will choose $R_2 = \frac{2}{10\,R1}$. If, I do this and then the output I will take from this point as the reference call this reference point. So, output will be taken from this reference point and so, between this reference point and one point so, this you can move you can connect it anywhere to this 2nd stage. So, between these two points I will take V out ok.

So, the trick is this, the main trick is here that this mechanical coupling this moves together and the 2nd trick is the choice of this resistances ok. So, I have chosen 2 R 1 same as this 10 R 2; now, how does this help? Now, the total resistance in the 1st stage or total equivalent resistance is how much R 1 times 1 2 3 4 5 6 ok, 1 2 3 4 5 6 then 7 8. So, 8 and 9; 9 R 1 plus this and this in parallel ok.

So, the equivalent resistance of first stage will be $= 9R_1 + (2R_1 \| 10 R_2)$

$$= 9R_1 + (2R_1 \| 2 R_1)$$

$$= 9R_1 + R_1 = 10 R_1$$

So, equivalent resistance of this part is 10 R 1 only ok. Now, if the lower pointer call this call this P 1 P 2 and this P 3 ok, P 1 is the position of the lower pointer which is in this case 3 ok. So, P 1 is the; P 1 = 3 in this diagram. And, P 3 is equal to 1 2 3 again three. Ok. Let us make it let us change it 1 2 3 4 in this diagram ok.

Now, what will be the value of V out that is the question? Now,

$$V_{out}'' = V_{out}' \times 4/10 = V_{out}' \times \frac{P3}{10}$$

$$V_{out}' = \frac{Vin\ x\ (2R1\ ||10R2)}{9R1+(2R1\ ||10R2)} = \frac{Vin}{10}$$

So, now, how much is V out; V out will be ok, let us have another notation call this potential here to here to here, V out triple prime 1 2 3. So, this is V out will be then this put. So, let us start from here 0 2 this point V out 3 V out triple prime and, then so, this is same as now this point is same as this point and from we have another voltage V out double prime ok.

$$V_{out} = V_{out}''' + V_{out}'' = \frac{Vin}{10}\, P1 + V_{out}' \times \frac{P3}{10} = \frac{Vin}{10}\, P1 + Vin \times \frac{P3}{100} =$$

So, now, if we want say 0.01 volt so, if we want if we want V out = 0.01 V in then we can say it P 1 = 0 and P 3 = 1. If we set P 1= 0 this term goes away P 3 = 1 is means it will be V in/100. So, we will have the desired output. So, we will bring this pointer down here at 0 level down so, this will automatically come together with it here we will bring this pointer after the first resistance here ok. So, and now if you want say any other arbitrary value.

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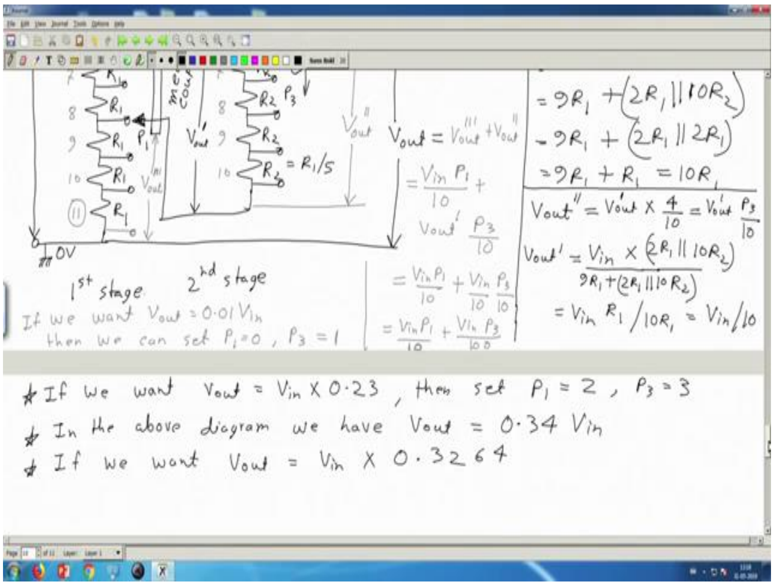

So, if we now want V out = V in x 0.23

then set P 1 = 2 and P 3 = 3. So; that means, bring this pointer down here after 2 resistances and this pointer after 3 resistances; 1 2 3 bring it here ok. So, this way you can get any decimal fraction of V in as the V out. And, similarly now if we see in this diagram, in this particular diagram how much is V out P 1 is 1 2 3, P 3 is 1 2 3 4.

So, this is 3 this is 4; so, therefore, in the above diagram we have V out will be equal to 3 and 4 so, 0.34; 0.34 V in. So, you can get any decimal fraction of V in this way ok. So, and now suppose you need to get a fraction a decimal after say you up to 4 decimal places ok.

So, if we want V out = V in x 0.3264, then what do we do, can we get that from here? No, with two stages we can get up to 2 decimal places, if we want to get up to 4 decimal places, we need 4 such stages. So, let me draw the circuit and this will be the end of today's class after I draw this circuit ok. And, I request you instead of seeing my drawing you try it to yourself you stop the video here this will be the last thing I will do so, but I will request you do it yourself try it yourself if you need any help continue with the video.

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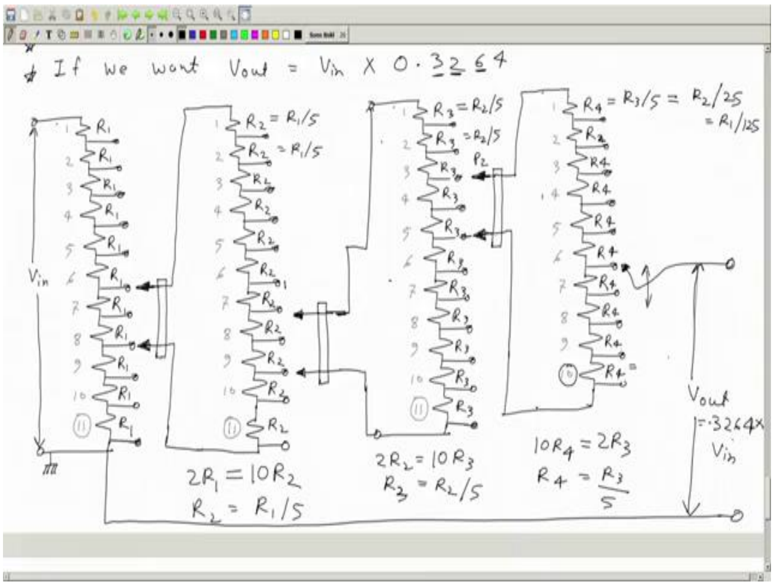

So, I copy this; so, I already have two stages. So, I will copy it once again. So, that I have four stages and now I will do the editing here on top of it. So, this is one stage, this is another stage, this is the third stage and then I will have the fourth stage. So, what I need? I need 11 resistances in the first stage, but since now there is a third stage I also need 11 resistance in the second stage as well ok. So, we have 11 resistances both in first and second stage and we have to ensure that, this total resistance is same as these 2 resistances.

And I am telling you that this total resistance is going to be same as 10 times R 2, not 11 times R 2 10 times R 2 why hold on for a few minutes it this total resistance is going to be 10 R 2 not 11 R 2. So, therefore, we want here–_10 R 2 = 2 R 1, which means R 2 = R 1/ 5 like this fine. So, let me write it directly R 2 is R 1 by 5 and these two are mechanically coupled together. So, they move together, here also in the 3rd stage we will have a mechanically coupled arrangement. So, this two pointers move together ok; so, you can move this two together you cannot move them alone.

Now, once again while choosing these resistances I have to make sure call this R 3. So, these are R 3 we have 11 of them and then this last stage you call R 4, here we have 10 of them ok, call this R 4 ok. Now, here we have 10 of them and here we have 11 right, but so, what we will want? We will want that this 10 is same as this 2 because this 2 and this 10 R parallel and I want that this 2 and this 10 become equal so, that the effective value is same as not 2 R 3, but only 1 R 3 ok.

So, here we will want that 10 R 4 same as 2 R 3; that means, R 4 will be R 3/ 5. So, you can write here R3/ 5, so, on always. So, therefore, now how much is the total resistance of 3rd stage, total equivalent resistance we have 11 of them, but these 2 are in parallel with another equivalent 2. So, this 2 and this 2 becomes only 1. So, actually this is equivalent to 10 resistances; this is equivalently 10 resistances ok, not 11 only 10 and then this 10 resistances; that means, 10 R 3 is coming in parallel to 2 R 2. So, we will again want that 2 R 2 = 10 R 3 meaning R 2 equal to or R 3 equal to R 2 / 5.

So, here you can then write this as R 2/ 5 so on. So, therefore, this will become so, R 3 is R 2 by 5; so, this will become R 2 by 25 and so on ok. And, then the total equivalent resistance of this is 10 R 3, this 10 R 3 is coming in parallel with 2 R 2. So, then we want 2 R 2 same as 10 R 3 so, R 3 same as R 2 by 5 ok.

So, oh this I already have written. So, here how much resistance do you have this 2 and the parallel thing is equivalent to only 1 resistance. So, although there are 11 actually it is equivalent to only 10, that is the key of this circuit, there is only 10 resistances; equivalent 10 resistances this 2 and this everything together becomes only 1 ok.

So, there; that means, we have 10 R 2 in total here and this 10 R 2 must come in parallel with 2 R 1. So, 2 R 1 we want it to be 10 R 2 so, R 2 therefore, R 1 by 5, which is already written here ok. So, then this is R 1/ 5, this is R 2 / 5 so; that means, it will be actually R1/ 25 and then therefore, this will be equal to actually R1/ 125 divided by 5, divided by 5, divided by 5 ok. So, this is how the resistances should be and note we should have 11 resistance, 11 resistances, 11 resistance, in first second third in all stages except the last stage the last stage should have only 10 resistances ok.

This is another important fact. And, now last thing where is the output taken from the output will be taken from this reference point so, this as the reference. So, the output will be taken between this reference and this final Jokey, this has 2 Jokey’s mechanically coupled these has 2 Jokey’s mechanically coupled similarly this, the last one has only 1 Jokey. And, the output is between this reference point very important; between this reference point and here, this is the output ok. Now, if you ok; now in this configuration according to what we have drawn how much is V out so, to calculate V out so, we can do it like this we have 1 2 3 resistances.

So, V out will be therefore, point thee 1 2 3 then here we have 1 2 3 4, 4 up to the lower resistance ok. So, 0.04; another 0.04 means 0.34, then here we have 1 2 3 again. So, 0.003; that means, totally 0.343 let me make it smaller 0.343 and then finally, here we have 1 2 3 4 from the lowest 2.1 2 3 4 resistances so, another four this times V in this diagram ok.

Now, what we actually wanted? We wanted 0.3264. If this is what we want then we have to adjust this Jokey's ok, this is this should be after 3 1 2 3 fine, this should be after 2. So, this point should come here, after 2 resistances. So, let me bring this down exactly after 2 resistances here like this. So, we have 1 2 3; 3, 1 2, 2 done, then next is points 006. So, this should go therefore, after 1 2 3 4 5 6 it should come here, 1 2 3 4 5 6. So, we do it 1 2 3 4 5 6 fine and ok. So, we got 0.6 done then 4; that means, this would come after 4, 1 2 3 4 it is already there. So, that is great. So, then we actually now have what we wanted 0.3264 times V in. So, we are done.

Thank you.