Electrical Measurement And Electronic Instruments
Prof. Avishek Chatterjee
Department Of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture- 24
Sensitivity, Accuracy, and Resolution of Wheatstone Bridge

Hello and welcome. We are discussing about measurement of resistance.

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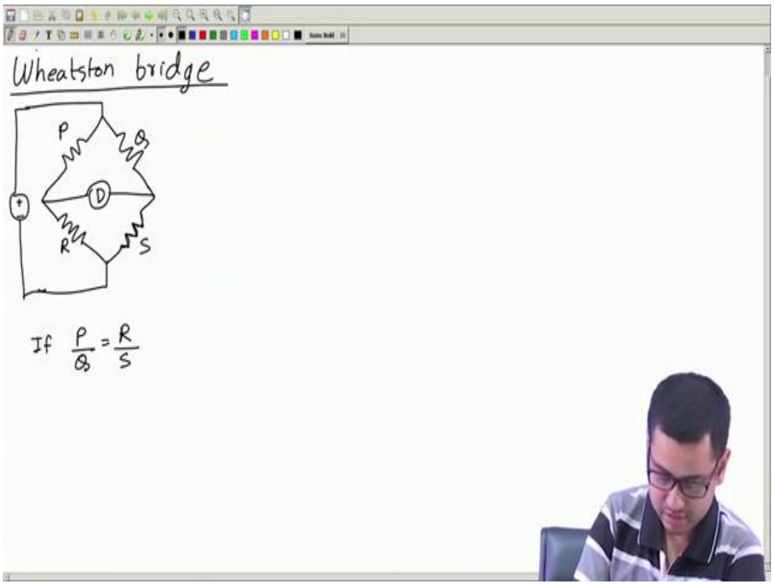

And so, today we will start our talk with Wheatstone Bridge which might be familiar to you. So, the bridge looks like a square and we have 4 resistances called them P Q R S, we connect power supply across to opposite terminals this is DC apply, because we are measuring resistance and we connect a galvanometer or any null detector D across the other two terminals and then you know that if the ratio of $\frac{P}{Q} = \frac{R}{S}$ then, the potential at this point and at this point will be same no current will flow through the detector. So, that is called the null condition.

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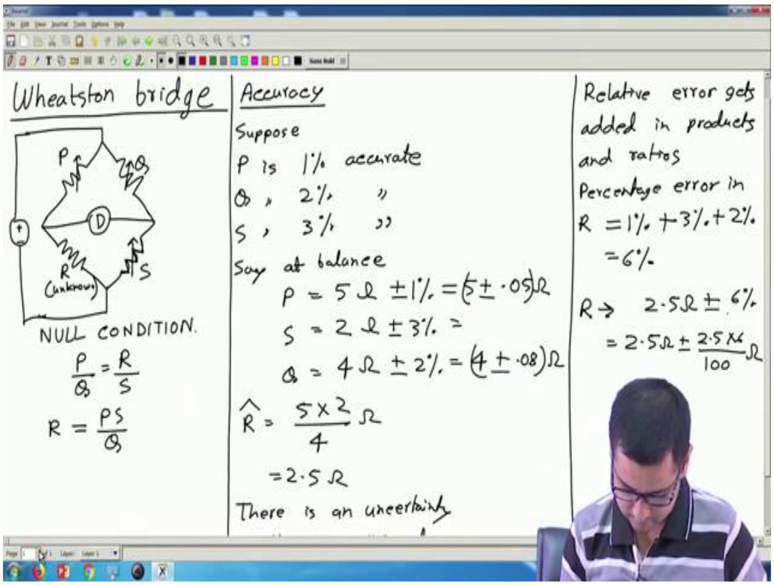

So, this is the null condition. Now, with this we can measure an unknown resistance suppose this R is unknown so and we want to measure the value of R and say this P Q R S these are known and variable resistances. So, we can set up a bridge like this, R is unknown and this 2 resistances we can change. So, we can change this 3 resistances. So, that null condition is achieved the galvanometer shows no deflection and then this condition is this balance condition is true from which we can write R= $\frac{PS}{Q}$, where P S and Q are known resistances. So, you know these values R can be found.

So, this is very simple. So, today we will talk about accuracy precision and sensitivity of this Wheatstone bridge. So, let us talk about the accuracy. If say suppose this resistances P Q and S which are known they are they have an accuracy or error of like say P is 1 percent accurate or inaccurate, Q is say 2 percent accurate and say S is 3 percent accurate ok. So, if this is the case and say at balance we see we observe that the value of P S and Q R say respectively like say 5 unit then maybe 2 unit and maybe 4 unit ok.

So, this means R = $\frac{5X2}{4} = 2$ but there is an uncertainty; there is an uncertainty in this measurement because the values of P S and Q those are not accurate we see that the value of P is 5 ohm say according to the marking or dial of this resistance box, but it has an accuracy or error of 1 percent so; that means, the value of P is actually 5 ohm plus minus

1 percent. So, which is like; so this is 5 plus minus. So, 1 percent means, 0.05 ohm ok. So, this is the range or of the of P within which P must lie.

So, P must lie between 4.95 ohm and 5.05 ohm. Similarly, we can. So, this is S is 3 percent accurate and similarly, Q is 2 percent accurate. So, they have arranged ok. So, like Q is 8 sorry 4 plus minus 0.08 ohm and so on ok. So, what is the limiting range of R roughly what is the. So, R is estimated to be 2.5 ohm, but it is not exactly or not necessarily 2.5 ohm, it has a range within which R should lie and what is that range? So, if you recall yesterday, we have sent that the relative error or percentage error gets added in products and ratios.

So, here R= $\frac{PS}{Q}$. So, therefore, the relative error or percentage error in measured value of R that will be given by percentage error of P, plus percentage error of S, plus percentage error of Q. So, which is for 1 percent for P, plus 3 percent for S, plus 2 percent for Q, which is 6 percent so;

Percentage error in R = 1%+3%+2%=6%

R= 2.5 ohm $\pm$ 6%

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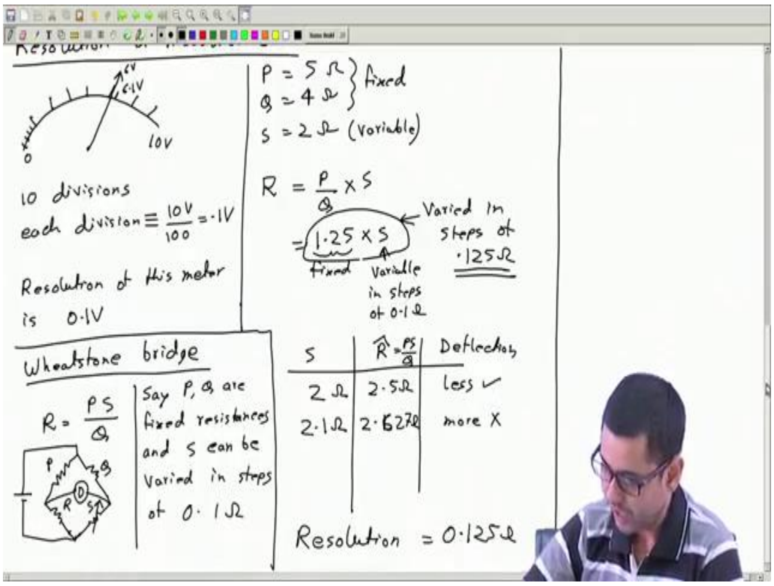

The resolution of measurement. So, what do you mean by resolution? So, possibly this is a new terminology we are introducing now. So, let us define resolution with an example.

Suppose I have a meter say voltmeter which has a range of 0 to 10 volt and say it has 100 durations. So, say it has in small divisions and a total of 100 divisions or 100 markings on the scale. So, 100 divisions; that means, each division of this voltmeter is equivalent to 10 volt divided by 100 ok so, which is 0.1 volt.

Now, if the pointer is say here maybe this indicates the value of 6 volt and then the immediate next marking this will indicate a value of 6.1 volt. So, the pointer can indicate values in multiple of 0.1 ohm and this is you can call as the list count or we call it the resolution of this meter. So, the resolution of this meter 0.1 volt. Now coming back to our Wheatstone bridge. So, we had unknown R is equal to P S times Q and so, this is the diagram let us draw it very quickly P Q R and S ok. So, I am not drawing the power supply and the detector now ok.

So, observe and say suppose that P and Q are constant resistances say P and Q are fixed resistances and S is a say decayed resistance box such that S can be varied in steps of say 0.01 ohm may be or may be say 0.1 ohm S can be varied in steps of 0.1 ohm, but P and Q they are fixed resistances if. So, then the value of P and P Q S they were 5 ohm, 4 ohm and 2 ohm.

Now, this 2 are fixed and this can be varied in steps of 0.1 ohm and then R is given by P by Q. So, from here multiplied by S. Now P is a fixed number Q is a fixed number. So, this is a fixed number. So, therefore, we can write this P by Q which is 5 by 4; that means, this is 1.25 multiplied by S and this S can be varied in steps of 0.1 ohm and this is constant this is constant fixed and this is variable in steps of 0.1 ohm.

Now, so, during the experiment what we do? So, we keep these two are fixed not variable we vary this S in steps of 0.1 ohm until and unless we get list deflection in this detector ok. So, in this detector we observe the deflection and vary the value of S until and unless we have minimum smallest amount of deflection in this detector. So, we are adding S; now S can be varied in steps of 0.1 ohm which means this quantity ok. So, this quantity together this is varied in steps of 1.25 multiplied by 0.1 which is 0.125 ohm 1.25 multiplied by 0.1. So, this is the minimum change that you can create in this quantity.

So, for example, when S is if S is let us make a table value of S and the estimated value of R which is this P Q by S. So, if we take S is equal to point S is equal to 2 ohm then this will be from this it will be come out to be 2.5 ohm next you can take S as and you can observe deflection then you take S as say 2.1 ohm this is the minimum change you can make in S.

And then the value of estimated value of R will be 2.5 plus this quantity. So, which will be 2.1 sorry 2.6 to 7 ohm and you observe the deflection. Suppose you see more deflection here it is less then this is a better estimation ok, but you can change your estimation only in steps of 0.125. So, this is the resolution. So, we call this as the resolution of the meter which is 0.125 ohm ok. So, now we have talked about resolution and accuracy another important quantity we will talk about is sensitivity.

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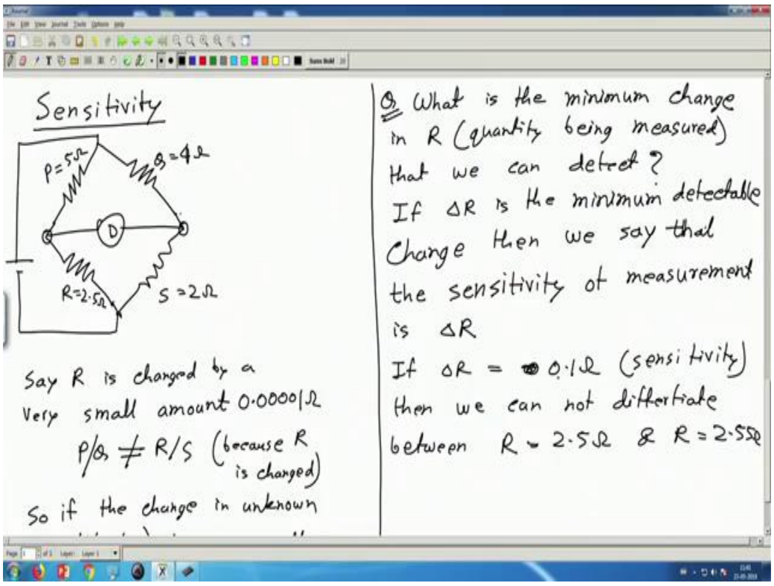

So, what do you mean by sensitivity. So, this is the bridge where we have say P Q R and S detector and the chosen value of Pi mean for this example is 5 4 and 2 5 ohm 4 ohm and 2 ohm. So, R came out to be 2.5 ohm.

Now, under this situation there is no deflection at all because the voltage here is same as the voltage here, but suppose this unknown R or this measurable quantity which you are measuring is changed it is changed by a small amount say R is changed by a very small amount is very small say like maybe 0.00001 ohm. So, small as small as you can think now of course, if you changed by even by a small amount now this $\frac{P}{Q} \neq \frac{R}{S}$ because R is changed. So, these two potentials at these two points they will no longer be same. So, some current should flow through the detector and we should observe on non-zero current.

But if this change is very small then the change in potential here between these two points will also be very small and the current through this detector will also be very small and we may not be able to observe that change ok. So, if the change in unknown quantity which is R is very small then we may not observe any deflection because the current through the detector will be very small.

So, we can ask the question what is the minimum change in R that is the unknown quantity being measured. So, what is the minimum change in R that we can detect ok. So, this question we may ask and say if delta R is the minimum change minimum of detectable change then we say that the sensitivity of measurement is delta R ok.

So, the minimum change in the unknown which we can detect which we can measure not a measure which you can detect or observe is called the sensitivity of the measurement if delta R is if say delta R is equal to 0.0 or say 0.1 ohm; that means, we can distinguish between 2.5 and 2.6 ohm which with the value of if the value of R is 2.6 ohm there will be some current observable current here and the bridge we will say it is not balanced, but if it is 2.5 ohm then no current flows. So, we can distinguish differentiate between the value of R equal to 2.5 and R equal to 2.6, but we will not be able to distinguish between R equal to 2.5 and say 2.55 because then. So, if delta R is 0.1 ohm.

So, this is the minimum detectable change or sensitivity then for both. So, then we cannot differentiate between then we cannot differentiate between R is equal to 2.5 ohm and say R is equal to 2.55 ohm because with R equal to 2.5 ohm no current flows through the detector and with R equal to 2.55 ohm such a small amount of current flows which we cannot detect that say that the pointer does not move at all we cannot observe the pointer movement at all. So, this is the meaning of the sensitivity now how to compute the sensitivity of this bridge or how to evaluate the sensitivity quantitatively that I think will we can do it with an example let us take an example.

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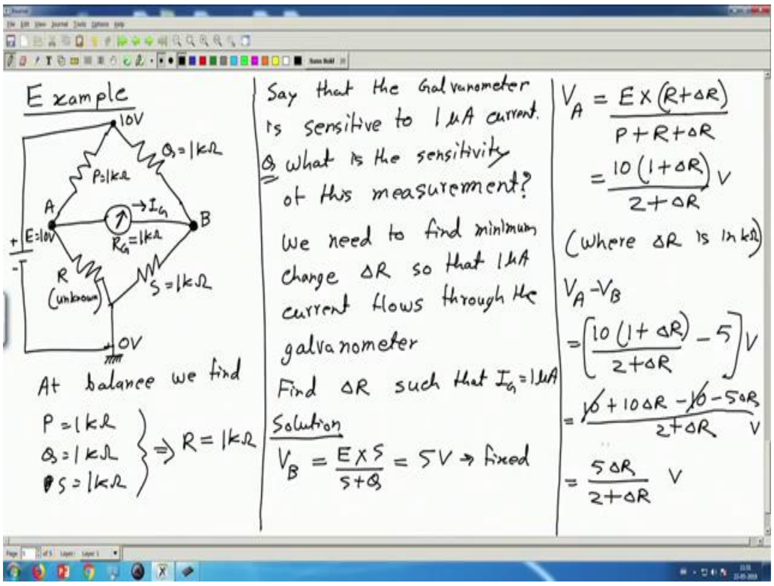

So, we will take a simple example say we have a bridge which is like this and this is the unknown this is unknown branch and say we have the detector here supply ok. So, for ease of calculation let us take a simple example where say this P is equal to 1 kilo ohm, Q is again 1 kilo ohm, R is unknown and S is once again 1 kilo ohm where we get the balance ok.

So, at balance we find P equals 1 k, Q equals 1 k, R sorry S is equal 1 k. So, this will imply that R which is unknown is once again 1 multiplied by 1 divided by 1 is 1 kilo ohm. So, this is very simple ok, but now say this voltage E is given to be 10 volt this is the supply voltage and say there is the internal resistance of this source is negligible ok. So, we are neglecting the internal resistance of the source or this conductor etcetera say this galvanometer has a resistance of R G call it once again may be 1 kilo ohm ok. So, we are just taking simple numbers.

So, R G is 1 kilo ohm and say that the galvanometer is sensitive to 1 micro ampere current so; that means, a minimum of 1 microampere current must flow through this galvanometer then only the pointer will move by an observable amount. If the current flowing through this galvanometer is less than 1 microampere then we cannot observe any pointer deflection the pointer will either not deflect at all due to deflection or even if it deflects it will be very difficult to identify that deflection. So, the minimum observable deflection is obtained with 1 microampere current ok. So, now, the question is what is the sensitivity of this bridge?

What is the sensitivity of this measurement ok? So, what is the sensitivity? So, for that so, we need to compute we need to find minimum change delta R in the value of R. So, that 1 micro ampere current flows through the galvanometer if I call this current as I G; G for galvanometer then we need to find out delta R such that I G is equal to 1 microampere. So, find delta R such that I G which is 0 under balance and now it will become 1 microampere. So, find out the value of the delta R ok. So, how do we solve it? Solution let us call this point as A and this point as B and let us take this as the reference point; that means, 0 volt point for calculation. So, this point is at therefore, 10 volt this is 0 this battery has an emf of 10. So, this point is at 10 volt.

Now; that means, the potential at B call that V B; V B potential at B it will be half of this 10 volt because this is 1 k this is 1 k. So, VB= $\frac{EXS}{S+Q}$ So, this is V B, now if you put the values it will come out to be. So, this is 10 volt 1 1 1, so, 10 by 2, so, this will be 5 volt and this is fixed ok, so, this is fixed.

Fixed means in the sense R becomes R plus delta R this potential is not going to change. Now what will be the potential at this point A V A; V A is equal to how much? Now V A that will depend on the value of R V A will depend on the value of R; if R changes V A will also change. So, VA= $\frac{EX(R+\Delta R)}{P+R+\Delta R}$.= $\frac{10\ X(1+\Delta R)}{2+\Delta R}$

VA-VB= $\frac{10\ X(1+\Delta R)}{2+\Delta R} - 5 = \frac{5\ X\Delta R}{2+\Delta R}$ V

that means, that will be the balanced condition and if delta R is non zero then this is the voltage which appears across the galvanometer here. Now how much current will flow through this galvanometer due to this voltage ok. So, for now let us find out what will be the value of I G? So, value of I G we have to find and let me copy this diagram.

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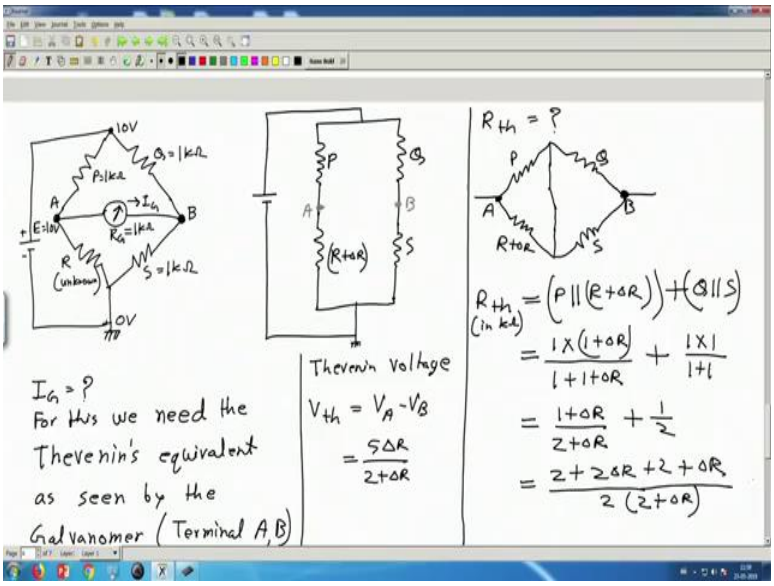

So, let us now we have to find I G; I G is equal to how much? For that we have to calculate the (Refer Time: 35:44) equivalent of the circuit as seen by this galvanometer ok. So, for this we need you can solve the same problem in different ways, but this is how I am going to solve for this we need the Thevenin equivalent as seen by the galvanometer; that means, from the terminal A and B terminal A B or port A B. So, between this two points we will find the Thevenin equivalent of the circuit. So, we will find the Thevenin equivalent as seen by the galvanometer let us draw the circuit first as seen by this galvanometer. So, let us remove the galvanometer and draw the circuit like this. So, this is I am going to draw it like this P Q R S between this we have the source and the detector was here this is point A and this is point B.

So the detector was here and now so, we need two things we need the Thevenin voltage and thee the Thevenin resistance. Now the Thevenin voltage the Thevenin voltage V th this will be nothing, but the open circuit potential difference between A and B. So, which is if I remove this then the potential difference between this two and here R has become R plus delta R.

So, this is nothing, but Vth = VA - VB as we have computed before because V B is always going to be 5 volt with respect to this point and this potential depends on the value of R plus delta R. So, we have computed V A using potential divided rule here and this is the difference between V and V B in the absence of this detector. So, this is same as computed before Vth = $\frac{5\ X\Delta R}{2+\Delta R}$ this is V the Thevenin. Now what is R the Thevenin? Thevenin resistance as seen by this ok

So, for that we have to remove this source. So, we have to short circuit this source and if we short circuit this source then starting from terminal a we have. So, let us write terminal A and terminal B here. So, from A we have P and Q. So, let us draw it like this P and then Q this goes to terminal B from terminal A. So, this is P this is Q this is like this P and Q and from the middle point between P and Q we have this source and there is another branch which is this R and S starting from A goes up to B.

So, let us draw it like this R and then S it goes to B. So, here we have R or actually R plus delta R and S here and the middle point between R and S from here we have, this source the source is here ok, but while computing the Thevenin resistance we should short circuit this source.

So, therefore, so, let us short circuit the source. So, this is the equivalent circuit as seen from terminal A and B. So, we have P Q R and S and the middle point is sorted ok. So, R th therefore, is the resistance between A and B will become P and R in parallel because this is in short this is sorted. So,

Rth (in k ohm)= (P ll $(R + \Delta R)$) + (Q ll S)

$$= \frac{1\ X\ (1+\Delta R)}{1+1+\Delta R} + \frac{1}{2}$$

$$= \frac{4+3\Delta R}{2(2+\Delta R)}$$

So, this is in kilo ohm in kilo ohm because everything here is in kilo ohm. So, I am not writing the unit, but I mentioning it here that R th is estimated in kilo ohm this comes out to be 1 plus delta R divided by 2 plus delta R plus this is half and then this will become 2 plus 2 delta R. So, check my calculations and please correct me if I make any mistake.

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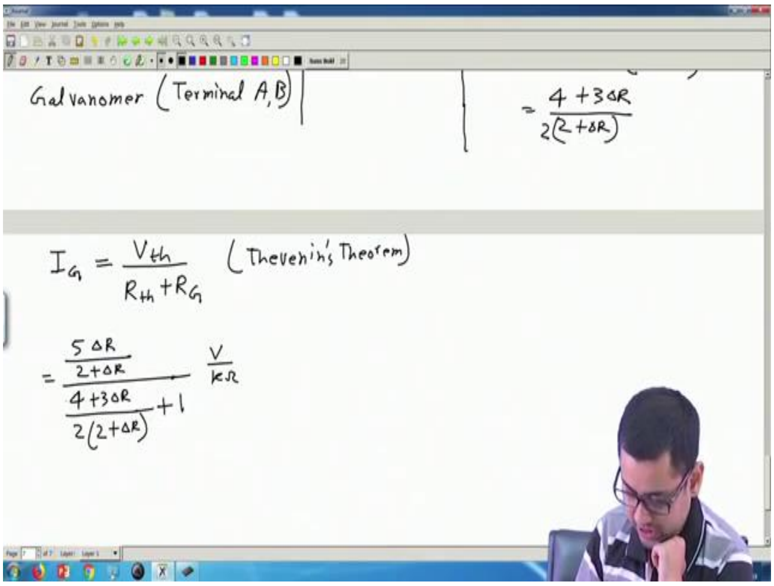

Now what will be the value of I G? I G will be nothing, but the current which will flow when we connect the galvanometer here. Now, if we connect the galvanometer here in this circuit the current that will flow through the galvanometer according to Thevenin's law

$$Ig = \frac{Vth}{Rth+Rg}$$

$$= \frac{\frac{5\Delta R}{2+\Delta R}}{\frac{4+3\Delta R}{2(2+\Delta R)}+1}$$

$$= \frac{5\Delta R\ X2}{8+\ \Delta R}$$

will be V th divided by. So, this will come out to be V the Thevenin divided by R the Thevenin plus R G according to Theremin's law, Thevenin's theorem ok.

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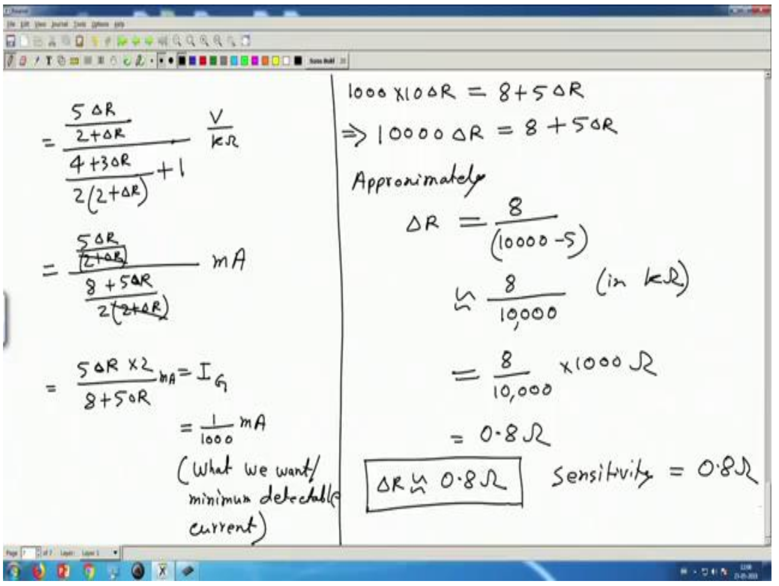

we want I G to be equal to 1 micro ampere we want this equal to 1 microampere which means ok. So, this is in milliampere 1 microampere means 1 by 1000 milliampere because. So, this is what we want or this is the minimum detectable current this is the minimum current that we can detect. So, the question we are asking that for what value of delta R this current will be same as the minimum detectable current 1 over 1000.

$1000 \text{ X } 10\, \Delta R = 8 + 5\, \Delta R$

$\Delta R = \frac{8}{10000-5}$ kohm

= 0.8 ohm

So, this is the minimum change in the unknown delta R which we can detect with due to which the pointer will move.

So, we say the sensitivity of the measurement is 0.8 ohm if R is changing by an amount less than this it will cause no pointer deflection we cannot detect it. So, in this video in this class we have discussed three concepts. Firstly, accuracy, then the resolution of Wheatstone bridge and finally, the sensitivity of the Wheatstone bridge and we have done a thorough calculation we have taken an example and have shown you how to do the calculation thoroughly.

Thank you.