Electrical Measurement And Electronic Instruments
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Lecture – 23
Error Calculation

Welcome back ok. So, in this video we will take up a lighter topic easier topic. So, we will talk about errors and accuracies; so, this is going to be a simple topic ok.

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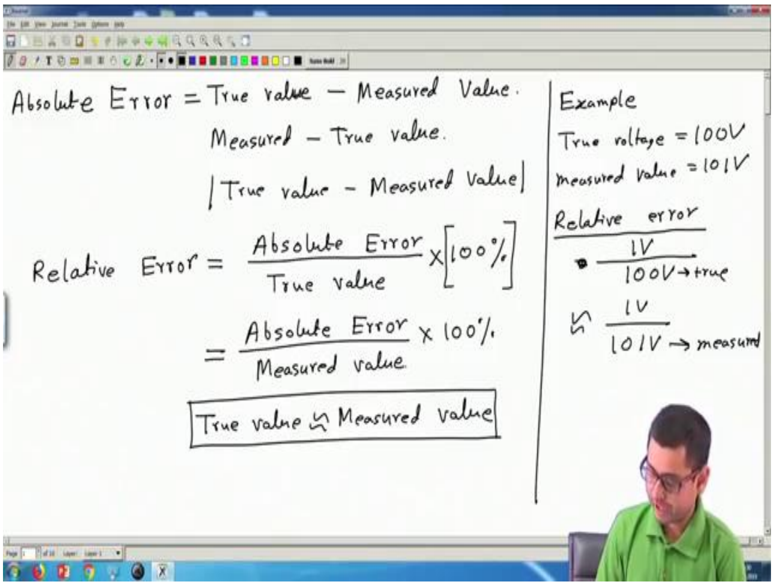

So, Error, what is an error and how do we measure it. So, if any quantity whose true value say I have a measuring voltage whose true value is 100 Volt, but my meter shows it is to be 101 Volt, then we can say that this 101 minus 100 is 1 is the error; 1 Volt is the error, if this is true value and if this is what the meter shows ok.

Absolute Error= measured value – true value = true value - measured value

So, both definitions are used in practice, I mean depending on the situation. So, you should not be stuck with in the fact that no say this is the right definition this is wrong definition or this is right, this is wrong it is not like that in practice we use both the definitions depending on the situations. So, either is ok, and if you do not bother about this sign you can just put a mod, even a mods you can just also use a mod of true value minus measured value.

So, all these definitions are used depending on the situation ok, but this error we will call as an absolute error and we can also define what is called the relative error, what is the relative error?

Relative error = $\frac{absolute\ error}{true\ value} = \frac{absolute\ error}{measured\ value}$

if you want to express it in percentage, then you can multiply it with a 100. So, this is optional ok; so, this is optional you can either express it in percentage or in per unit. So, this 100 multiplication with 100 is not required. So, depending on in the way you want to express it you have to multiply it with 100 or not.

In practice if the meter or the method is not that bad, I mean true value should be close to the measured value ok. They generally they should not be very different they are generally close therefore, whether you are dividing by the true value or measured value these two should come out to the same ok.

So, depending on the situation you can either use this definition or this definition. So, that depends on the situation. So, for example, as I was saying if true voltage of in any circuit it is say 10 Volt; say 100 volt, and measured value is 101 Volt, then relative error

Relative error = $\frac{absolute\ error}{true\ value} = \frac{absolute\ error}{measured\ value}$

$$= \frac{1}{100} = \frac{1}{101}$$

So, these two numbers are same. So, this is absolute divided by the true value. So, this is true value and this is the measured value both are quite similar quite close.

So, you can use either definition if nothing is specified exactly what you should do, then you can use either definition and your result should be similar.

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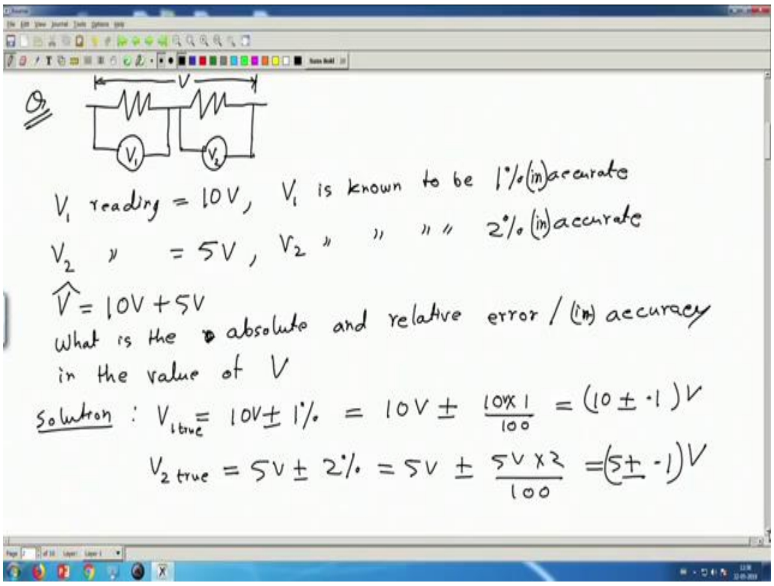

Now let us take some problems; so, question. The question is suppose I have a circuit like this, where I have 2 resistances in series and I measure the voltage across this 2 resistances with 2 Volt meters, this is call it voltmeter 1 and voltmeter 2 ok. Now

V1 reading = 10V, V1 is known to be 1% accurate

V2 reading = 5 V, V2 is known to be 2% accurate

V 1 is known to be which means that the value of the true value of the voltage can be 10 Volt plus minus 1 percent ok; that means, plus minus 0.1 Volt. And V 2 reading is say 5 Volt and V 2 is known to be say 2 percent accurate, I should rather say inaccurate because when I say 2 percent accurate we actually mean its 2 percent inaccurate or the error can be 2 percent. Although we generally say 2 percent accurate I mean in colloquial language, we often say 2 percent accurate or 2 percent or 1 percent accurate, but we actually mean it is 1 percent inaccurate or 2 percent inaccurate.

Now the question is; so, what is this voltage across the 2 resistance? V so, we will we can estimate this as V 1 plus V 2 two voltages in series so, that is 10 Volt plus 5 Volt ok. So, this is the estimated value of the total voltage V. And now we will ask what is the error or say relative error or say we will compute both absolute and relative error in the value of V. So, we can ask the same question often in different forms we can also say so, what is the absolute and relative say accuracy maybe we can say actually we mean in accuracy not accuracy inaccuracy.

Although we normally say accuracy we actually mean inaccuracy when talking about errors. So, this terms you should be a bit careful, I mean not careful I mean should be open minded about ok. So, you should not memorize these terms that error is this minus this by this or accuracy is this formula it is not like that. So, depending on this situation from the wording you have to understand what is meant.

And the best practice which I will suggest is if you have any confusion always ask, if somebody is saying, if somebody is selling you ammeter saying this is 10 percent accurate, best thing you ask that seller that, do you actually mean it is 10 percent inaccurate or what is the definition of your inaccuracy? Do you mean that the true value minus the meter reading is always going to be within the 10 percent of the true value?

So, it is best if you ask whenever you have confusion and in general you have to apply your common sense so, that is why I was saying in my introductory video that this subject requires a lot of common sense. So, from using common sense we have to understand what is actually meant. And once again if you have any doubt ask, in our course as well, in our quizzes if you have any doubt in any question ask ok. So, that is the best practice ask question ok. So, what is the; so, the now the question is, what is the absolute or relative error or in accuracy in the measured value of V.

$V_{1true} = 10V \pm 1\%$

$V_{2true} = 5V \pm 2\%$

$V_{true} = V_{1true} + V_{2true}$

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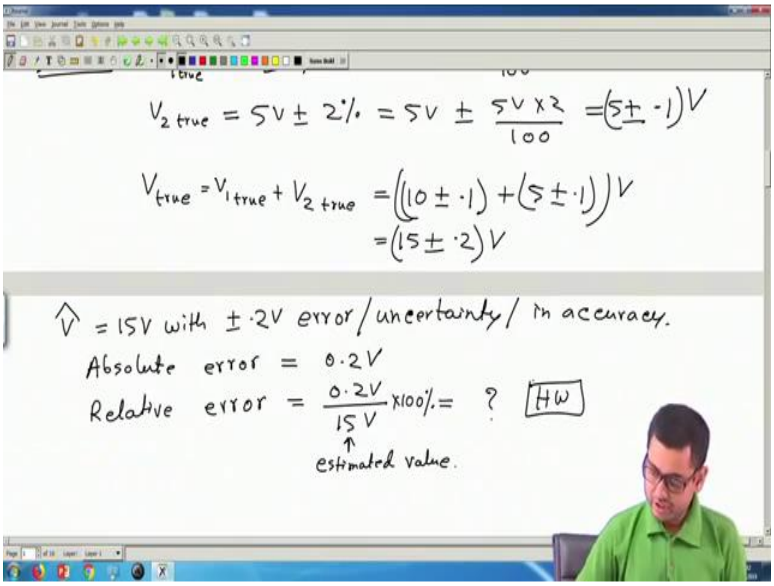

And therefore, V true which is V 1 plus V 2, V is this total voltage V 1 is here V 2 is here. So, V true will be V 1 true plus V 2 true, this will be 10 plus minus 1 plus 5 plus minus 1. So, then this will be 15 ok. So, 15 and then I can have this as plus or minus independently I can have this as plus or minus. It may happen when this is plus this is also plus, it may happen that when this is plus this is minus, it may happen that when this is minus this is plus and similarly this is minus this is minus. So, all possibilities are there and what is the worst case extreme possibility? Extreme possibility is that this will be 10 plus and this will be also 5 plus. So, that will be 15 plus 0.1 0.1 0.2.

And the other extreme possibility is 10 minus and 5 minus. So, we can have both of them minus together. So, this is another extreme possibility 15 minus 0.2. So, the worst case this we also call the limiting case is that V true is 15 plus minus 0.2 therefore, the we can estimate V as 15 Volt with plus minus 2 Volt uncertainty or error or inaccuracy whatever you call plus minus 2 Volt error or uncertainty or we call it inaccuracy ok.

So, absolute error is therefore, 0.2 Volt ignoring this sign, this is 0.2 Volt and relative error this will be 0.2 Volt. Now; so, I am dividing it with 15; here 15 means the estimated value or measured value not the true value ok. So, in this we are using the definition that relative error is absolute error divided by the estimated value, why; why are we not using true value here? Because we do not know the true value, we cannot use that definition. Because the true value can be anywhere between 15 plus 0.2 and 15 minus 0.2; that means, the true value can be anywhere between 14.8 and 15.2 we do not know. So, we cannot use that definition.

So, in this situation we definitely have to use this definition where the denominator is estimated value. So, then this will be let us multiply it with a 100 and you can compute what this value is ok. This you can find the value with a calculator, I do not have a calculator and right now so, you can do it homework ok.

So, this is how to calculate the absolute and relative error in this situation.

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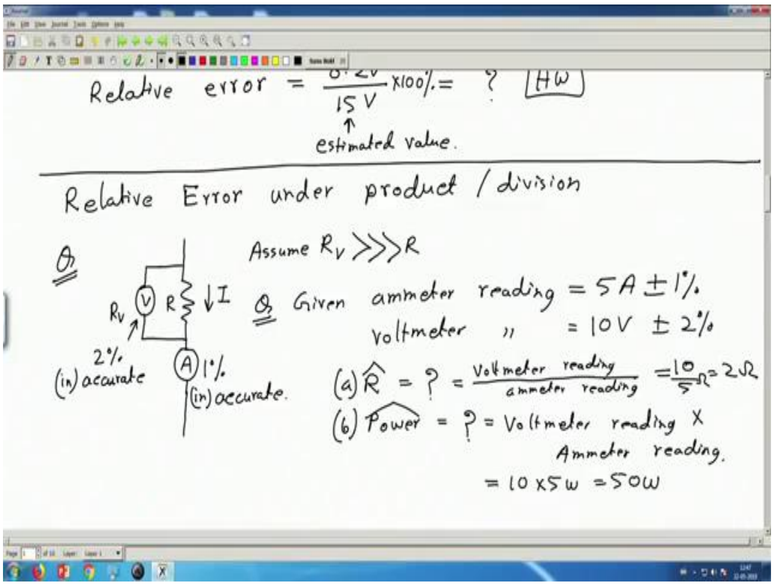

Now, let us consider how relative error is aggregated if when we take products of quantities. The topic title is relative error under product or division ok. So, consider a situation a question, say we have resistance which is carrying some current I, we are measuring this current with an ammeter A and this is I and the voltage drop across it is V we are measuring the voltage drop with a voltmeter V. For now you can ignore whether the ammeter is measuring the current through this resistance alone or it is actually measuring this current plus this current.

Now for simplicity, assume R v voltmeter resistance is very very high compared to this resistance R so, this current is very small, you can ignore that for now for this calculation. Now say the question is given ammeter reading equals say 5 ampere and the ammeter is known to be say 1 percent accurate; that means, 1 percent inaccurate actually. So, I will not write this in always. So, and say this voltmeter is 2 percent accurate. I actually mean inaccurate I am not writing it again and again I think the meaning is now clear to you. 2 percent accurate I mean, it is not a meter which is only 2 percent accurate who will buy that, but if we say it is only 2 percent inaccurate its meaningful. So, this is actually 2 percent inaccurate although we are; we say that this is accurate ok.

So, given the ammeter reading 5 ampere and plus minus 1 percent and voltmeter reading is equal to say what 10 Volt plus minus 2 percent, what is the value of the resistance? R is equal to what? So, this is part a, and part b is power consumption is in this resistance is how much? So, R we know this is like a voltmeter reading by ammeter reading.

So, this is voltmeter reading by ammeter reading ok and the power is voltmeter reading multiplied by ammeter reading ok. So, this will be; this resistance will be voltmeter reading is 10, this is 5. So, 10 by 5 is 10 by 5 is equal to 2 ohm and in this case this will be how much? This is thin into 5 Watt. So, this is 50 Watt ok. So, this is the estimated value of R and this is the estimated value of power ok.

This is not the true value of R or this is not the true value of power because the both the voltmeter and ammeter has some error uncertainty in their reading. So, we do not know the true value, we only have an estimated value measured value ok.

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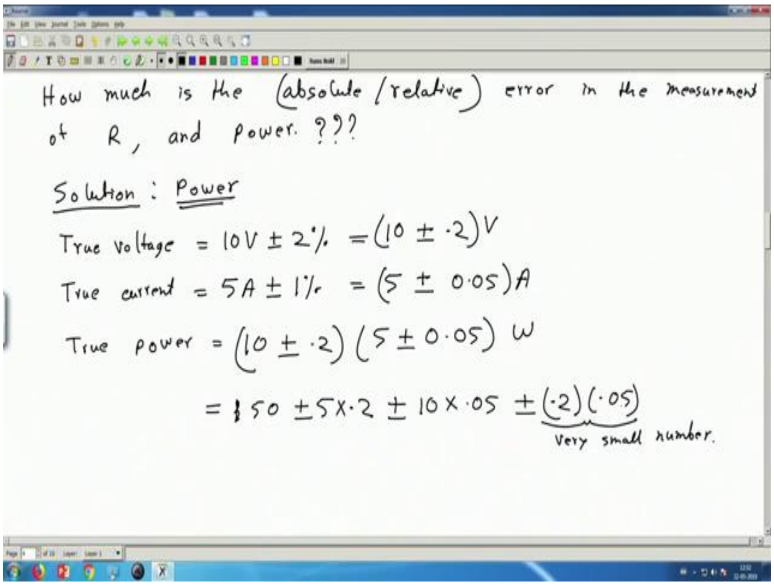

Now, the question is how much is the; so, how much is the error absolute or relative error in the measurement of R and power resistance and power? So, this is the question ok. So, the solution; first say power, let us do for the power. Now true voltage is voltmeter reading

True voltage = 10 $V \pm$ 2% = (10 $\pm$ 0.2) V

True current = 5A $\pm 1\% = (5 \pm 0.05)A$

So, Volt multiplied by ampere this will be Watt ok.

Now, it is possible that when this error is positive for voltmeter this error is also positive for ammeter, but it is also possible that when this error is positive for Volt meter, this error is negative for ammeter. Similarly other possibilities are also possible that when this is minus this error is plus when this error is negative this error is negative all 4 possibilities can happen ok. So, now,

True power = (10$\pm$0.2)(5$\pm$0.05) W

$$= (50 \pm 5x0.2 \pm 10x0.05 \pm 0.2x0.05)\ W$$

Since these numbers are small; these are small numbers and the product of 2 small number is going to be further small. So, we will ignore this term, this is a very small number ok. So, for ease of calculation we can ignore this.

$$= (50 \pm 1.5)\ W$$

Absolute error = 1.5 W estimated power= 50 W

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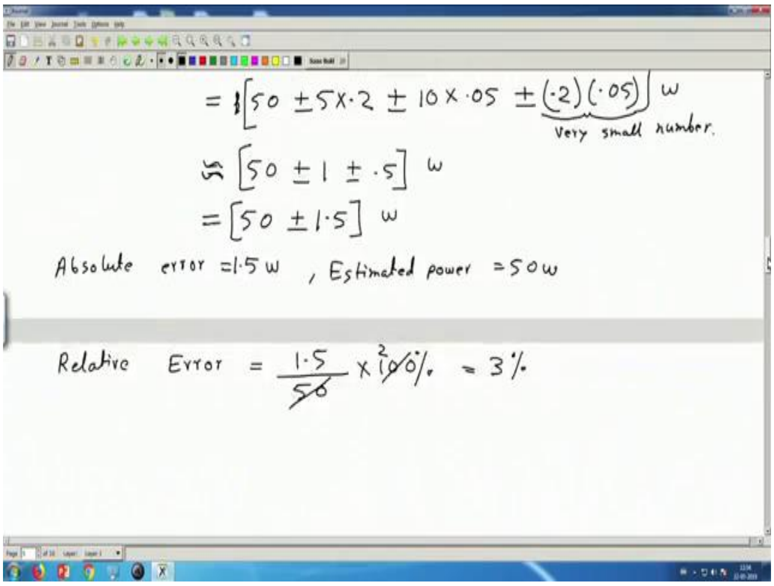

Relative error = $\frac{1.5}{50}$ X 100 = 3%

just observe that the error in the voltmeter was 2 percent and error in the ammeter was 1 percent if we sum these 2 and 1 2 plus 1 is 3 ok.

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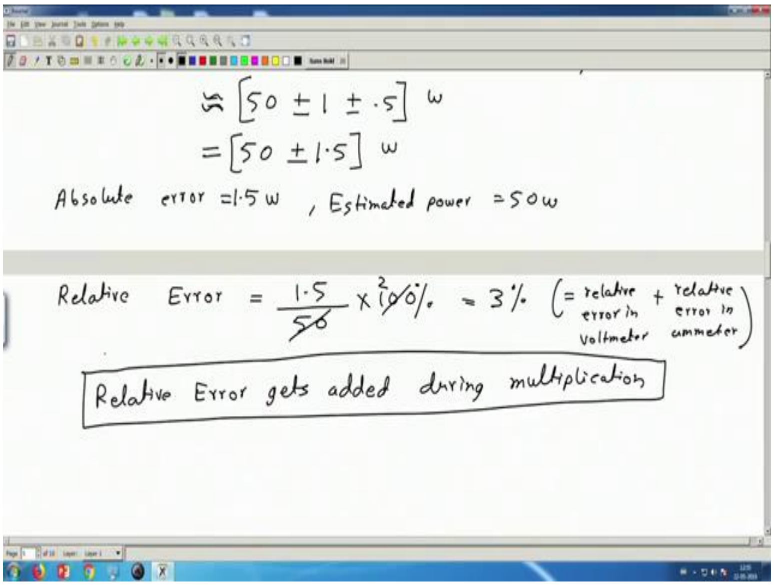

So, this is same as relative error in voltmeter plus relative error in ammeter ok.

So, relative error is gets added; relative error gets added during product or during multiplication. So, we have multiplied voltage with current and the relative error in the product is the sum of the individual relative errors multiplication. So, this is one rule you should know. So, we can prove it in general ok.

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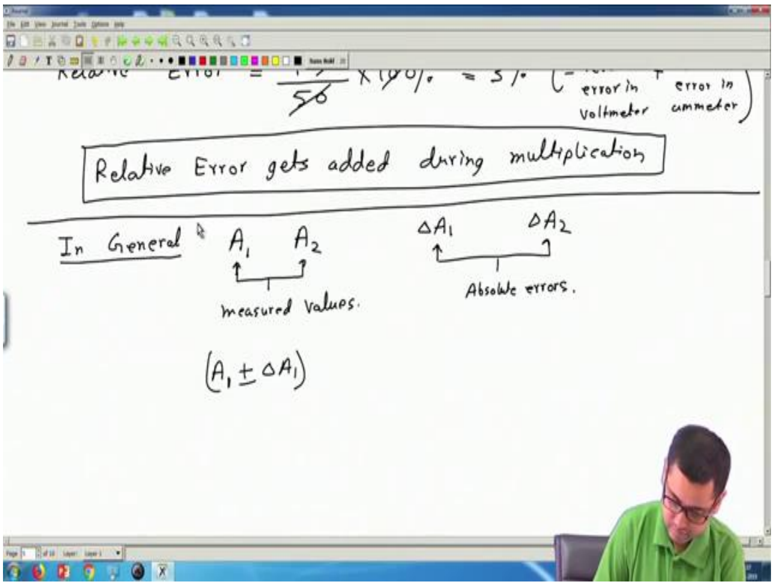

So, if I have So, in general if I have two quantities say call them A 1 and A 2 ok. So, in general if A 1 and A 2 are measured values, these are measured values and the they are errors ok, they are percentage error or relative error and call them delta A 1 and delta A 2. So, these two are the relative errors.

So, delta A 1 is the relative error for A 1, delta A 2 is the relative error for A 2. So, these are the relative errors then it means the true values; so, we can write the true values, so, true value of A 1 will be A 1 sorry, let me call this as absolute errors. So, then the true value of A 1 will be A 1 plus or minus delta A 1. So, this is the true value of the first quantity. Let me make a table.

	Measured	Absolute error	Relative error	True value
Quantity 1	A_1	ΔA_1	$\frac{\Delta A1}{A1}$	$(A_1 \pm \Delta A_1)$

Quantity 2	A_2	ΔA_2	$\frac{\Delta A2}{A2}$	$(A_2 \pm \Delta A_2)$
Quantity 1 X quantity 2	A_1 X A_2	$A_2\Delta A_1 + \Delta A_2 A_1$	$\frac{\Delta A1}{A1} + \frac{\Delta A2}{A2}$	$(A_1 \pm \Delta A_1)(A_2 \pm \Delta A_2)$ $= A_1 A_2 \pm A2\Delta A1 \pm \Delta A2\ A1$

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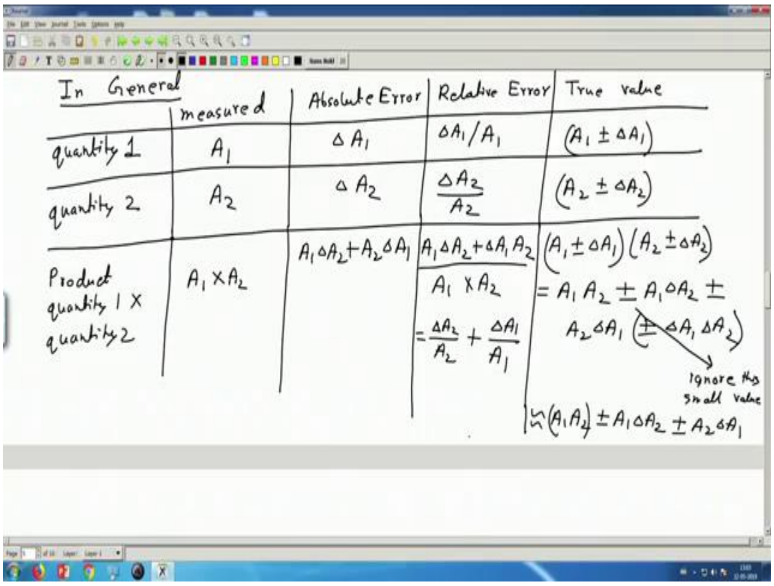

So, you see that the relative error in the product is the sum of the individual relative errors this plus this. So, this quantity plus this quantity is this quantity. So, this is true in general, now this is also true for division ok.

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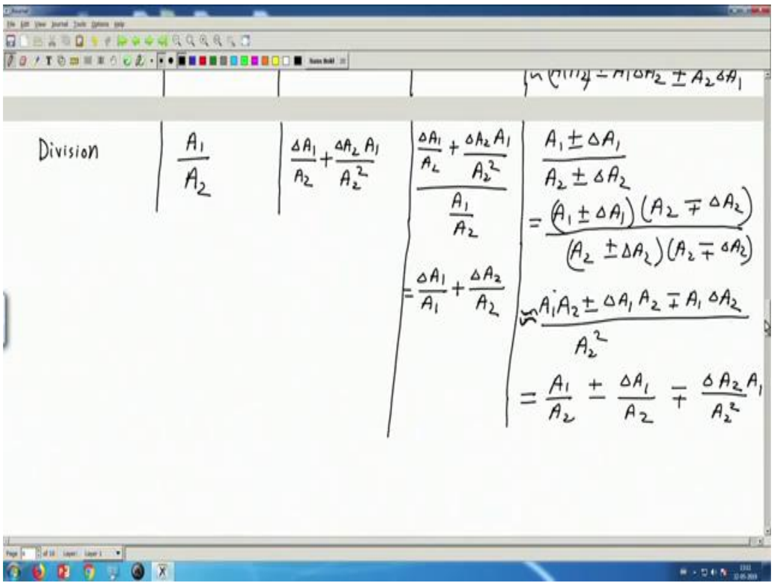

So, let us consider. So, let us write let us take another row, let us take the division suppose we want to find the ratio of A 1 and A 2. So, this is what we want to find,

	Measured	Absolute error	Relative error	True value
Division	$\frac{A1}{A2}$	$\frac{\Delta A1}{A2}+\frac{\Delta A2 A1}{A2^2}$	$\frac{\frac{\Delta A1}{A2}+\frac{\Delta A2 A1}{A2^2}}{\frac{A1}{A2}}$ $=\frac{\Delta A1}{A1}+\frac{\Delta A2}{A2}$	$\frac{A1 \pm \Delta A1}{A2 \pm \Delta A2}$ $=$ $\frac{A1}{A2} \pm \frac{\Delta A1}{A2} \mp \frac{\Delta A2 A1}{A2^2}$

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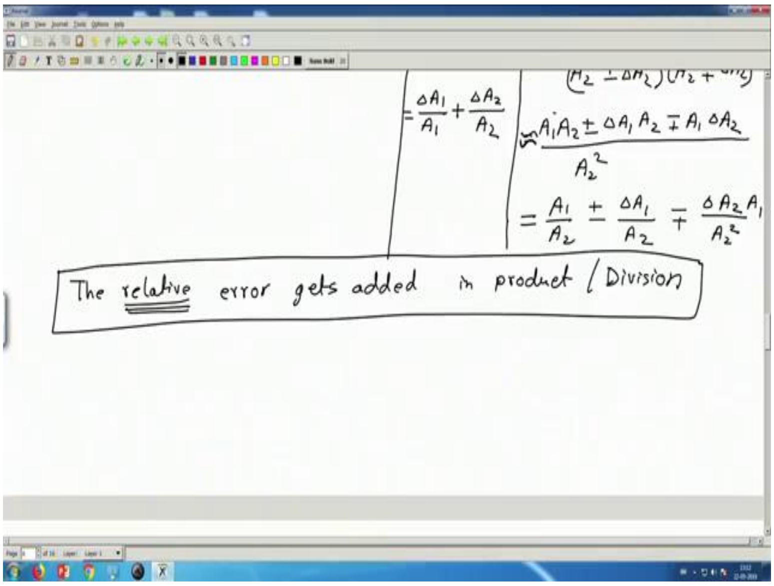

So, here you will see this is once again this is the relative error in A 1, this is the relative error in A 2. So, in for division also the relative error gets added. So, the note; so, the note the important result is that, the relative error gets added in product or division ok. So, it is added for both product and division, it is never thing that is subtracted, it is added by magnitude ok, but its true only for the relative error, it is not true for the absolute error. Absolute error gets added for summation of quantities as we have seen in the first example; in this example the first example that we have dealt with here there the absolute error gets added ok, but for product and division only relative error gets added. So, this is an important result ok.

So, now let us finish the question that we have said; so, what is the error in estimated R or power? So, power we have already computed the estimated, relative error in power was 3 percent. Now for resistance in this question going back to the question original question so, the resistance estimated value it was here 2 ohm voltage was 10 Volt plus minus 2 percent, current was 5 ampere plus minus 1 percent.

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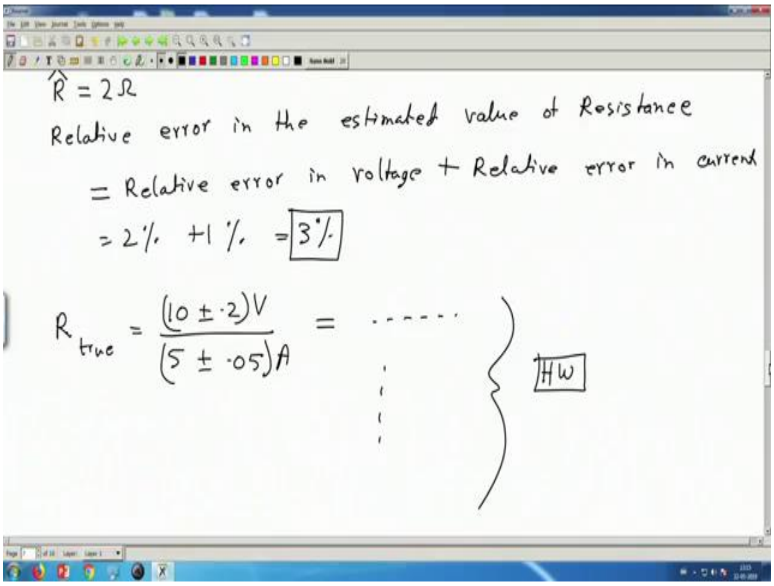

So, estimated value of R was 2 ohm and relative error you can compute it thoroughly, but now since we already have developed a formula, let us use the formula directly relative error in resistance; in the estimated value of resistance, it will be the sum of the relative error in voltage plus relative error in current. So, this will be 2 percent plus 1 person is equal to once again 3 percent.

You could have done it thoroughly also like the way that R true is equal to 10 Volt plus minus 2 percent is 0.2 Volt divided by 5 ampere plus minus 1 percent is 0.05 ampere and then you expand what this says find the absolute error and then find the relative error you can complete it and finally, we will see that the relative error comes out to be 3 percent. So, I will be happy if you find the relative error with a thorough computation like this so, I put this as a homework once again.

So, thank you for watching.