Evolution of Air Interface towards 5G
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Lecture - 39
Mimo Signal Processing (Capacity & Massive Mimo)

Welcome to the lectures in Evolution of Air Interface towards 5G. So, in this journey so far we have discussed various things of the 5th generation communication system starting from the requirements the evolution of different standards. And then we have talked about different mechanisms by virtue of which the new requirements can be made.

We have also talked about various waveforms then we looked at the propagation characteristics how the modals are available what kinds of documents you refer to. And then we are looking at a very important class of techniques which would help in improving the spectral efficiency, which is been there and would also continue in 5th generation or in general in any broadband wireless communication systems wherever high spectral efficiencies requirement and that is MIMO.

We have talked about the diversity mode of communication which helps in improving the beta rate and we have started looking into the capacity which enhances the spectral efficiency in bits per second per Hertz. So, we have discussed about one particular method in enhancing capacity by sending parallel data streams. So, we have identified that the MIMO signal systems where there are M T number of transfer antennas and M r number of receive antennas can open up r which is the rank of the channel matrix number of spatial SISO data pipes.

So, one can imagine like parallel pipes going on and each can communicate and each can have a capacity of a SISO link with a power proportional to the lambda which is the eigen value of the HH Hermitian or sigma squared which is the singular value the sigma is the singular value of the H matrix that is the MIMO channel matrix. So, now, let us get back to that particular discussion.

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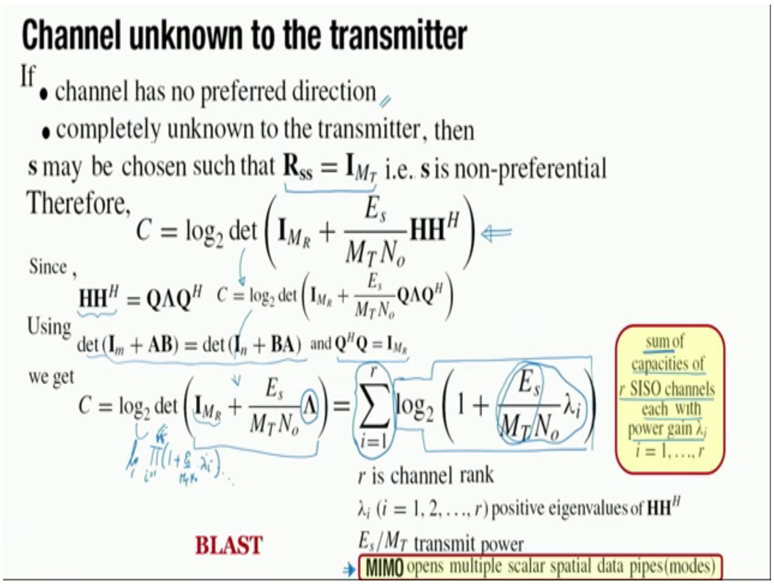

So, what we said is that the situation when there is no preferred direction; that means, when we do not have channels state information then the best we can do is set the R SS that is the covariance matrix of the transmit power to I M T and based on the expressions that we have discussed earlier the capacity would be as given in this particular expression.

And then we also saw that HH Hermitian which is a Hermitian matrix can be expanded in terms of Q lambda Q Hermitian with the property that Q Hermitian Q is M R and the expression simply translates to this and using the determinant identities and the relationship of Q we find that further the expression can be written in this form. So, what we see there is an lambda which is the diagonal matrix, this is also a diagonal matrix and hence this whole thing is a diagonal matrix is what we have discussed.

And determinant of diagonal matrix would be the product of the diagonal terms log product. So, log product of diagonal terms this is what we had written. We had assumed let M be the number of links or rather i is equals to 1 2 r and then what we have got is since there is a log, it can be expanded as summation of logs. So, this expression if you look at it carefully contains within it a SISO link capacity and we are adding up this r number of SISO link capacity and hence we have said it is a sum it is adding up the capacities of r SISO links each with the power gain of lambda i.

So, that is what we see that the SNR over here is this and E s by M T is by virtue of dividing the transmit power equally amongst MT number of transmit antennas. So, what we concluded was that this is the MIMO system which opens up r spatial pipes. And how to use this if we have to see that let us open up a new screen.

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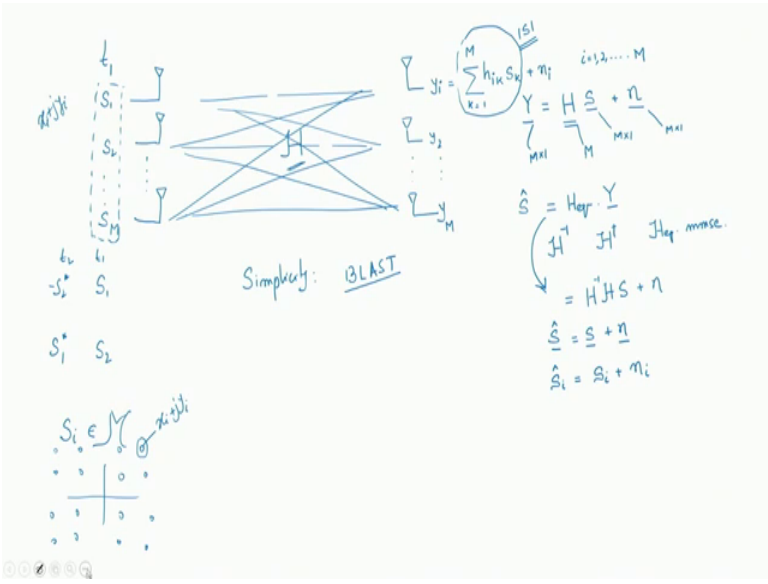

And what we will find is that suppose you have a certain number of antennas at the transmitter and there are a certain number of antennas at the receiver and the H matrix lies in between. So, in that case if we are sending signal S 1 S 2 up to S r let us say or S M called simplicity when it goes to the channel the signal that is received in each of the antennas are y we could write it as M R or M T in this case.

So, we would write Y is equal to H which is H matrix times s plus noise right. So, now, what we see is that this is the M R cross M T matrix and you can for sake of simplicity let us take it as n for the non square matrix of course, you could do the pseudo inverse and this is M cross 1 and this is also M cross 1 this is also M cross 1. So, let us say all both the sides of M.

So, now, what we are effectively doing is sending different signals now if you compare this with the earlier mechanism of STDC we had sent S 1 and S 2 and again we had sent in 2 time intervals t 1 and t 2 whereas, here in the same time interval t 1 you are sending different symbols each of this S i element of the constellation point. So, each of them can be selected from the complex constellation right.

So; that means, each one would be a complex value x i plus j y i indicating one of these values and they will be going out. Channel as we have already said it will be again complex values noise is also same. So, now, in order to recover S S cap we can use various different mechanisms. So, we would say H equaliser times Y and one of the simplest equalisers in this case would be H inverse if it is a square matrix otherwise, it would be a pseudo inverse right which would take us through that.

So, if we execute this expression what we get is that H inverse H S plus n since we have taken square and assumed it to be invertible, we would get it as S plus n this vector so; that means, each of the elements that we receive are the transmitted element plus the noise element right.

So, we must remember that each y i is sum over H if this is one H 1 k let us say H 1 k k equals to 1 to M T, but I am writing M over here S k plus noise 1 and then you could simply generalise it by writing Y i is equal to H i k so; that means, each of the received signal i equals to 1 2 up to M r, but I am writing it as M so; that means, each contains inter symbol interference as you can clearly see. But if this channel is invertible then we would be able to recover everything and there is no ISI and this kinds of mechanism is kind of 0 forcing where we are forcing the interference to 0 and getting a coefficient of 1 against the desired symbol.

Instead of using H inverse one can use H pseudo inverse we have also said H e q could be also based on m m s e criteria, but otherwise one can go for advanced non-linear processing also at the receiver end, but the scheme is very famous because of its simplicity and this usually known as the V BLAST or the vertical BLAST.

Blast the name coming from bell labs layered space time communication system. So, this is the very simple system of communicating where we send parallel data streams or parallel symbols which go through this channel and they can be recovered depending upon the channel characteristics right.

So, this is what matches with the definition of whatever we have discussed over here right. So, let us see that. So, we have discussed about a very common method of transmitting and sending in parallel data pipes while channel is not known at the transmitter. So, then we move further to the situation where the channel may be known at the transmitter.

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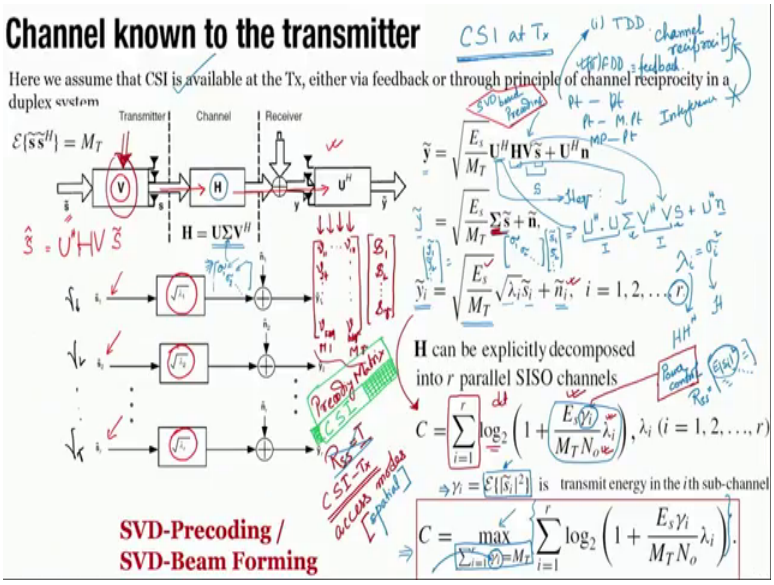

So, that is CSI is available at the T x, we have already explained how CSI can be made available at the CS at the transmitter. So, there could be various method just to summarise one could be in time division duplexing mode assuming channel reciprocity and in FDD mode that is Frequency Division Duplexing it could be through feedback.

Now, these all sound good, but this sounds to be very very easy and very nice way of doing it, but we must remember that we are talking about 0.2 point link. So, in case of point to multipoint or multipoint to point or multipoint to multipoint there is the notion of interference which is not reciprocal in both the directions ok. So, in case interference it may not be reciprocal in both the directions. And hence we may not be able exploit this kind of thing and there this explicit feedback is very very important. Explicit feedback means you have to give a feedback although there is a term called as explicit feedback which is used in different ways.

So, here we assume that CSI is available at the T x either through feedback or the principle of reciprocity and this is what we do over here. So, the channel we have seen this mode of analysis earlier when we said dominant ideal mode this H can be decomposed into its singular value decomposition as we had earlier U sigma V Hermitian. So, sigma contains the eigenvalues sorry the singular values and U and V are unitary matrices which contains the vectors corresponding to these singular values.

So, now, if you look at the received signal the received signal is H times s, but at the transmitter we do some precoding as you can see there is this V vector that is the precoding ok. So, S is the signal V times S produces the S which we had been using in our earlier notation and this kind of precoding can be called as SVD based precoding.

And similarly at the receiver what you see is that this is corresponding to H e q in the previous discussion that we had. So, now, if we concentrate on the set of equations we have U Hermitian from this from this we are getting U Hermitian multiplied by if we look at H H get expanded as U sigma V Hermitian and V is the precoding at the transmitter plus U Hermitian noise because of receiver processing.

So, now what you see is that U Hermitian U is identity V Hermitian V is an identity we have left with sigma times S. So, that is what we get over here sigma times S. So, sigma times S sigma is diagonal matrix as we are stated over here right. So, we have sigma 1 sigma 2 and so on and S is what we have S 1 S 2 so on. So, effectively what it means and of course, this is y so, y 1 y 2 and so on. So, if we break down the elemental equations we will find y i is equal to of course, this is the scaling because of the energy normalisation times, we have stated earlier that lambda i is equal to sigma square i where, the lambda i are the eigen values of H H Hermitian and sigma belongs to H a singular a values of H.

So, from instead of writing sigma we will write it as square root of lambda times S i plus n i tilde n i is the noise for that particular received signal path. So, now what you see is that the deceived signal can again be broken down into r number of spatial data pipes once again and hence in a similar manner you would be able to get the capacity expression as we are discussed earlier is a sum.

So, we are getting the sum operation over here of log because we had log determinant inside the log we are getting a diagonal because look at this is the diagonal this is causing the diagonal matrix there was i MR that would have been there. So, now, the expression is S because of this because it is taking a SNR expression the square root is one where N naught because of the noise part ok.

And we are saying lambda i they were square root. So, now, it is lambda i, but we have included gamma i as the new entry into the system if you compare this expression with a earlier expression earlier expression we had used R SS is equal to identity matrix. But here since CSI is available CSI is available at the transmitter we have the option of accessing the modes. Modes means the spatial modes ok.

So, we are able to access the spatial modes, but in order to access the spatial modes this is the power control or the power sharing you can say for each of the mode. So, this is one of the resources which have been used and what we see is that, here it is mentioned gamma i is E of expected value of the sigma power of the power in the i th branch.

So, now what you will find is that R SS which was set as I earlier this is not what we will use in this particular case earlier we have used because CSI is not available at the transmitter. So, now, what we will get back to is R SS would be the diagonal matrix which will contain these elements which is E of S i square like that these entries will happen; that means, R SS if we recall the expressions that we had discussed earlier.

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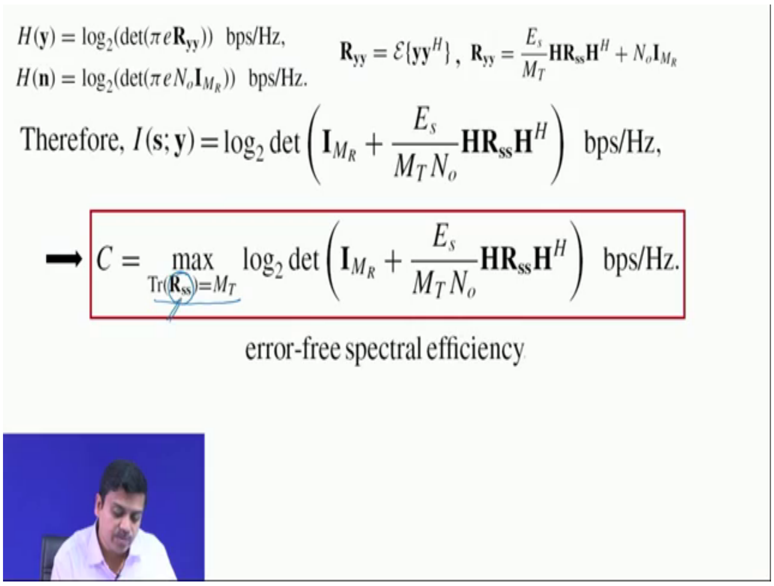

We said that you maximise over the choice of R SS subject to this constraint when CSI is not here not available we had set R SS to identity, but now CSI is available. So, there is a particular choice of R SS which can be determined which can be used. So, R SS is basically covariance of the received signal. So, this is here and of course, the signals are independent hence these elements would be 0, but I mean we are getting a diagonal values which are the powers of the each of the paths.

So, what we have as the capacity expression finally, is the maximum value of this such that this constraint is held true. So, we had to we are left with finding the power distribution within the powers constraint that is what is the problem. So, this in this expression we had simply quoted power control, but still the values of these have been found out. So, that is the next stage of operation while implementing this.

So, just statement before you move forward these kind of mechanism are called S VD based precoding and as you can see in the diagram the S signal is multiplied with the V vector. So, basically when we write down we write it S multiplied by V and then it is going to the channel where it is getting multiplied by H and then at the receiver you are multiplying by U Hermitian and then what you all getting it is S tilde cap correct.

So, that is what we are having and this, what we are given over here each of the data pipes have this kind of amp it should a power and we are left with the amount of power that we dedicate to each of this links.

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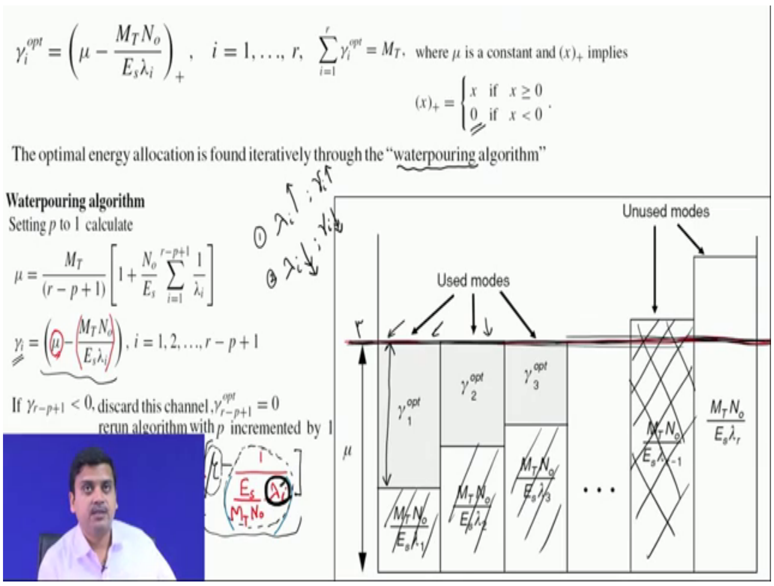

So, the gamma i optimal can be found in this manner where this plus sign indicates positive values; that means, if x is greater than 0, if this argument is greater than 0, then you take the value if it is less than 0 then you just apply 0 over there and this is for all the r streams because we have r data pipes k and of course, this constraint is maintained.

So, we are not going through the details of how to find this out this is available in the reference material. So, the optimal energy allocation is found through iterative waterpouring ok. So, we will just explain what it means. So, what you do is that set p to 1 through iterations and you calculate this expression. Once we have calculate this expressions then you can feet this mu into this expression which is derived from the expression above ok.

So, now we will go through this iterations and you can find out gamma i in case the solution is less than 0 we are not going to use it if it is only positive, then we going to keep it. So, that is what it says if this value is less than 0 discard this channel put this equal to 0 rerun the algorithm p implemented by 1. So, get slowly the non negative values or the values which are greater than 0 as your remaining streams.

So, let us see how does it look like. So, if we look at the expression we have a mu take away something. So, mu is the constant term. So, we have the mu threshold across the different spatial modes and each of the spatial mode has M T N naught by E S of a lambda. So, if you look at that particular term M T N naught by E S lambda i. So, this you can easily write it as 1 upon E S by M T N naught times lambda i.

So, now carefully look at the expression here this expression is the channels strength. If we go back and see this expression here or if we go back and see the expression there; that means, if we see this expression we said it is the channels strength that is what we have to mentioned in this particular statement there.

So, what we see is that we are taking away this M to T from mu. So, let us write mu over here mu minus this. So, inverse of the SNR. So, inverse of the SNR for that particular link inverse of the S NR what is the meaning of the inverse of the SNR? So, what we mean over here that if gamma of lambda i is very large then this value is small right.

So, that means, we give the power which is as close to mu as possible if lambda is small; that means, when channel power is small then this entire thing is very large if this number is very large this entire expression is very small; that means, the power you are allocating is very small. In other words if lambda i is high you allocate gamma i which is high if lambda i is small then you allocate gamma i which is small and this is the famous waterpouring algorithm.

So, what we see is that this quantity indicates the noise. So, larger the channel strength lesser is the noise more is the power to be allocated. So, with respect to some threshold that is what we have said as mu the amount of power that we are going to fill in is proportional to the channels strength right.

So, after you do everything only on the total amount of power that is going into the channel of the signal proved noise ratio of the total amount of power they maintain the same value which is mu and whatever is in the denominator term is a significance of amount of noise.

Now, if the channel weak very very weak like these channels then you will going to get a negative value in these expression if it is a negative value do not use that channel; that means, a channel is too weak to be used. So, this is the another level of mechanisms that has to be added on to this.

So, now if we get back, so, we have got the gamma i values this is gamma 1 gamma 2 to gamma r. So, this is fine you can calculate the capacity, but in both the schemes; that means, whenever you are doing.

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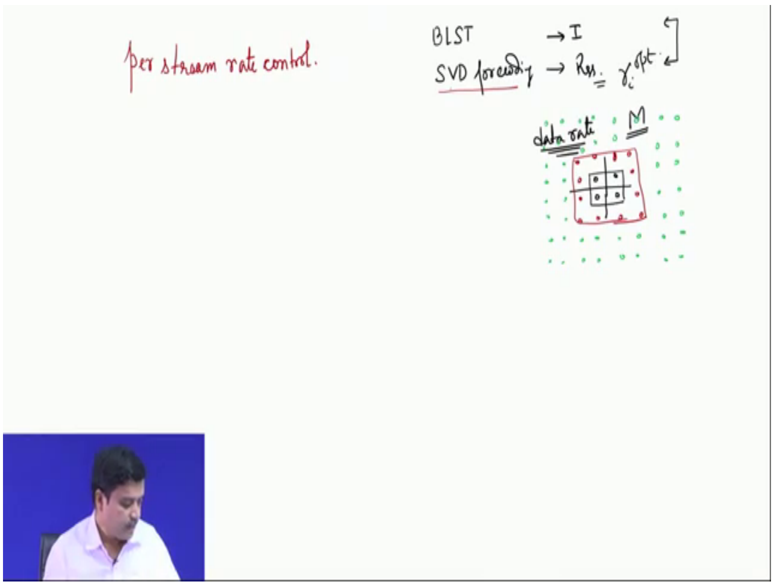

Either you go for BLAST or you go for SVD precoding. So, in this case you use an identity matrix here you use R ss R ss is determined through the gamma i opt to the algorithm we have described and now we have to see is that in each of the cases in both the solutions we had only talked about the power.

Now, once the power is known data rate also have to be decided for practical communication systems; that means, the set of constellation that you are going to choose has to be decided; that means, depending upon the SNR whether you are going to use a q p s k or whether you are going to use a 16 qam or whether you are going to use a 64 qam that depends upon the signal to noise ratio of the particular link.

So, what we mean to say is that, depending upon the particular links SNR which is determined by the earlier expression you can choose different constellation for different streams. So, this is called per stream rate control right ok. You have got per stream rate control and together you can achieve high spectral efficiency. Now one important thing I would like to mention over here in SVD based precoding if we go back what we find is that, there is this precoding matrix V which we are talking about.

So, if you look at this precoding matrix you have S vector which is nothing, but s 1 s 2 up to s r; that means, r data sets each. So, we select r number of columns over here. So, basically v 1 1 v 1 2 let us say v 1 M t like that v 1 r up to v 1 sorry v this should be v 2 1 this should be v M 1 and this should be v M r.

So, we select r such columns. So, this is the precoding matrix and this is dependent on channel state information. So, this is absolutely dependent on channel state information. So, if we have channels state information the precoding matrix weights are decided based on the H. So, if H is known at the transmitter then only we can achieve this scheme and if H is not known one can go for blast now these two mechanisms are extreme ends on one end we do not have any information at the transmitter the other end we have full information at the transmitter, but now what we see is that something else can also be done. So, let us see what can be done.

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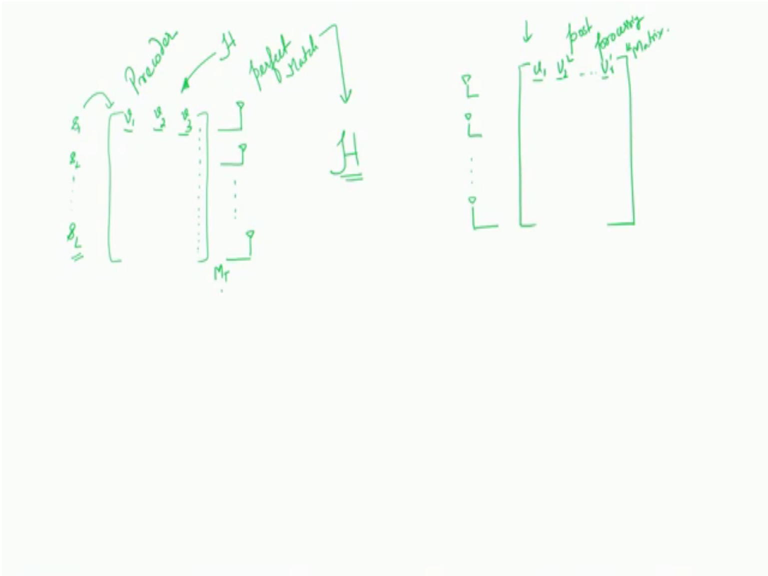

What can be done is we have the transmitting antennas we have the receiving antennas right and what is happening is H is a matrix. So, we have the symbols s 1 s 2 up to a certain number let us say L and then they are getting processed through the precoder. So, we have selected in the previous case the column from the v matrix which are generated from the H matrix ok.

So, this means there is a perfect match with the channel coefficience because at the receiver you are again using a linear processing; that means, you are using a post processing is the post processing matrix where you are taking U 1 vector U 2 vector of course, you have the Hermitian up to U r vector let us say So, now the number of entries or number of elements is equal to the number of antennas where as the number of data streams is less than or equal to the rank of the channel.

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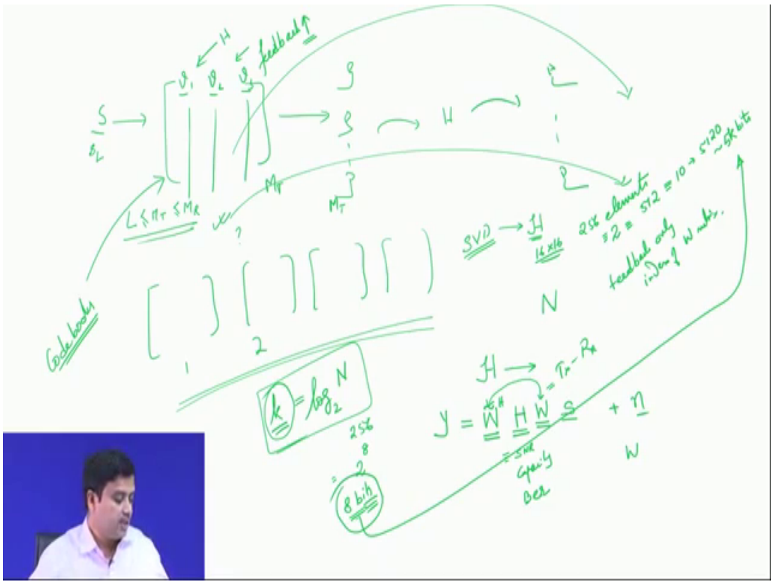

So, we have this S vector we have that goes into the precoder which goes to the antennas this is what we have drawn this goes to the channel and what we mentioned is these columns contain entries which are of M T length; that means, if there are M T number of transmit antennas and S has S L number of symbols where S is less than or equal to M T which you can put as less than or equal to M R and then everything would hold out.

So, now this we have said depends entirely on the channel the other case is extreme where there is no preference. Now the question is can we do something better I mean why something better because the amount of feedback is very high in this case is extremely high.

So, now what people have thought of intelligent ways of doing things is that we could have various precode books designed; that means, we would call them as code books which would contain complex weights and there could be various such matrices ok. And if we have let us say capital N number of such matrices we would require log of N with the base of 2 number of bits to be used in identifying what operation is to be done at the receiver and these are not dependent on the channel these are predefined right.

So, what the receiver does based on H. So, if we say that the received signal can be written as S vector with some precoder matrix W then it goes to the channel and then there is noise. So, the receiver can think of doing a processing in this manner and the receiver needs to find out the index of this matrices which would maximise the SNR or maximise the capacity or would minimise the B e r depending upon the requirements.

Now, once the index of the W matrix is identified because the transmit and received in both close these matrices. The receiver has to feedback only the index of W matrix; that means, which matrix is to be used the transmitter would use that matrix at the transmitter. The receiver would use a corresponding matrix of the receiver a very general thing would be there would be pair of the matrices W and W and W tilde we can say that right.

And one could use in either way. So, effectively in the previous case when we are doing SVD in the SVD case you are sending back h matrix now if it is let us say 16 cross 16; that means, there are 256 elements right. You know 256 elements each elements would contain 2 entities because of complex so; that means, you have 512 real entries. And if you would sent at even 10 bits you would have 5120; that means, nearly 5 kilo bits of information we fed back for processing whereas, if you compare with the scheme over here if I have N number of such precoding matrix I just have to use log N plus 2.

So, even if I have let us say 250 if I have 256 let us say then this is equal to 2 to the power of 8 ok. So, if it is 2 to the power of 8 yeah. So, then I will require only 8 bits of information so; that means, I require only 8 bits of feedback over here compare to the huge amount of feedback over there. So, of course, with the reduced feedback the quality of SNR and the channel capacity will be significantly reduced, but this is one important strategy which is necessary to realise large scale communication systems ok.

So, with this let us move on to see some of the code books that are used in LTE and similar things are extended in the newer generation. So, I will just show you a few samples of such code books that are used.

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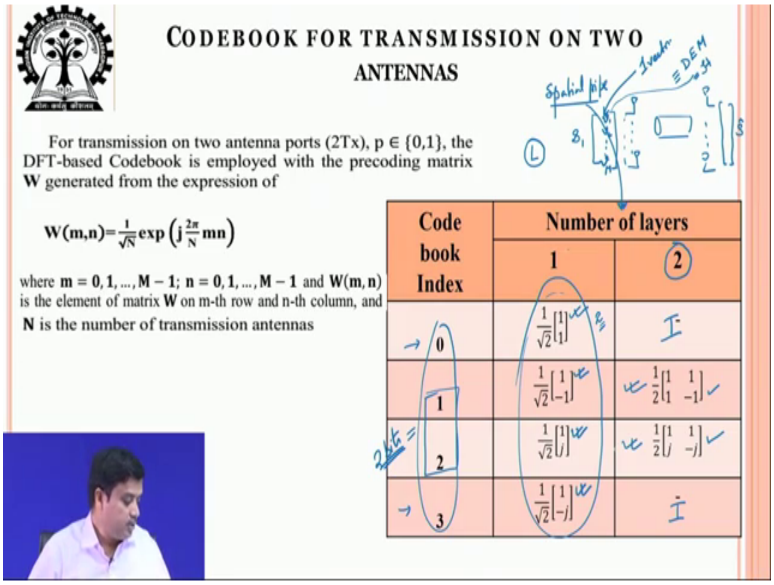

So, let us look at this particular slide which talks about code book index number of layers; that means, if there is only 1 layer 1 layer meaning that 1 layer means sorry let us take a right pen 1 layer means that you have only one spatial pipe right.

So, oh sorry this is the number of layers so; that means, it is the number of spatial pipes so; that means, L what we have indicated earlier is basically the number of layers. So, although we can have a large number of antennas we may use choose to use only one data pipe. So, when we choose to use only 1 data pipe what do we do that precoder should contain only 1 vector and there would be only 1 signal over here.

So, this signal gets multiplied by all the elements of this vector and then they go through the transmitter similarly you have the vector processing at the receiver and you extract signal at the receiver, this particular case is similar to the dominant Eigen mode, but there this vector is chosen from the channels SVD, but here we are saying that you can have different channel vectors. So, these are the different channel vectors.

So, if we have to send back channel vector, we would require at least two elements to be feedback and I mean the number of complex bits would be significantly large compared to the option that we are having over here. So, sees with choosing between only 4 you require only 2 bits of information to the feedback if there is any 2 layers this particular example has code book defined over there.

So, if I am using the code book over there. So, if I am using code book index 0 or three we are actually not using any codebook. So, I think of an identity matrix where as if you are using codebook value of 1 or 2 you are choosing this or this matrix respectively right. So, if we move further.

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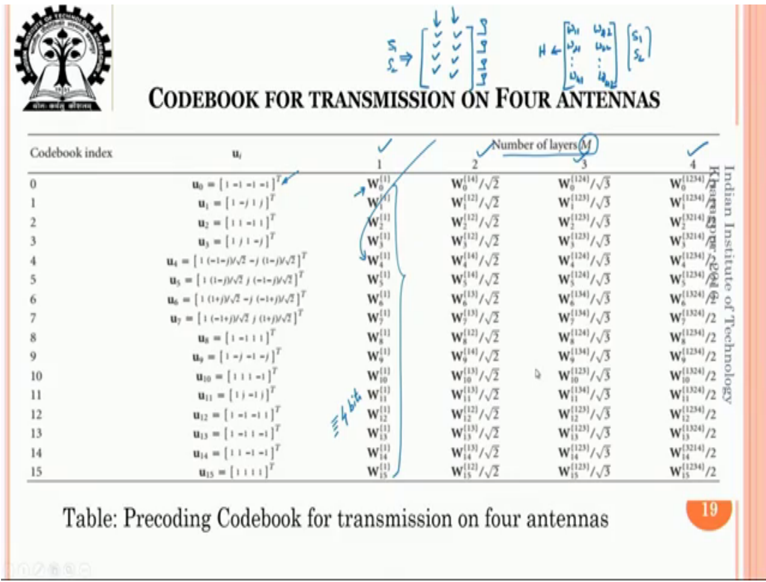

What we find is that this contains the different code books that are available. So, number of layers is denoted as M. So, it could be 1 layer 2 layer 3 layer or 4 layer and then there is a particular mechanism of generating these matrices through these u i vectors and then all one has to choose is which particular matrix one is sending. So, out of 16 possible options if one has to choose then one requires only 4 bits to send them.

So, now just imagine this that you have a matrix where you have 2 columns only so; that means, and if there are 4 antennas. So, you will taking basically 2 vectors which take 2 data inputs and send them out over 4 such antennas. So, when you write down the expression you have S 1 S 2 as a vector multiplied by these W 11 W 21 up to W 41 and W 12 W 22up to W 42 and this goes into the channel. So, that is how the processing is done and how one chooses this particular weight matrix is the proprietary mechanism by which one can do it. So, in this way one can take advantage of exploiting the different MIMO spatial data pipes that means.

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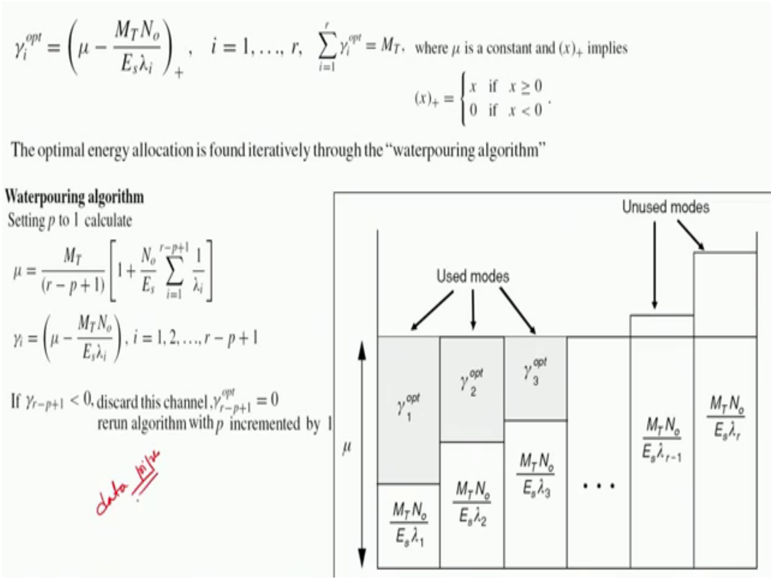

One would take advantage of the data pipes and one can expand the spectral efficiency.

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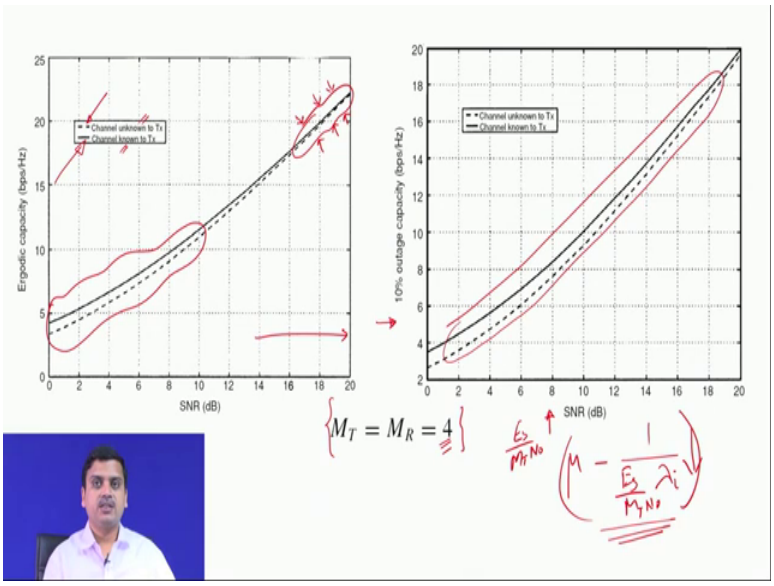

So, in these two figures we see the result of the case when channel is unknown to the transmitter and when channel is known to the transmitter for the situation of M T equals to MR equals to 4 as your number of antennas increase you get a different kind of result, but what we see over here is from low SNR conditions there is a gap in the channel known to the transmitter compare to the channel known, but for high SNR condition there is not much difference between this. So, high SNR condition means that ES upon MT N naught is very high.

So, that is very high 1 upon ES by MT N naught this whole thing is very low which means mu minus this is very high and there is not much difference between each of the modes that we are going to use and as your SNR increases there is not much a gain that one would get if one uses full CSI at the transmitter. So, it is something to be remembered.

We also talked about the 10 percent outreachcapacity. So, there we see that the difference is much more notable, but I mean depending upon the situation one has to choose the appropriate mode of operation. So, when we use the code book it is not a full CSI, but a partial CSI information about the channel.

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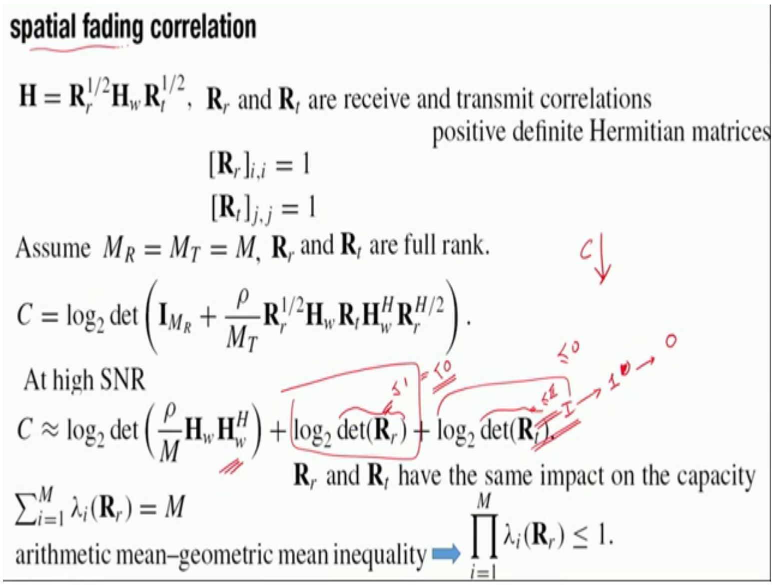

The next important thing that we would like to see is the effect of spatial fading correlation we had seen such a thing for the diversity performance we will now see it for the capacity results. So, H can be expressed in this form we had discussed this discussed these things earlier and we assume M R equals to M T equal to M and then what we see is that HH Hermitian that we had over here. Remember we had HH Hermitian. So, if you expand H it is R half H w R t half right and if you look at this that will be R t half H w Hermitian R r half Hermitian right. So, R t half R t half Hermitian would give you R t that is what you have and then you have H w H w Hermitian rest of the terms you can easily figure out into this expression.

At high SNR conditions; that means, when the row is very very large this can be represented in this manner with a log determine because we are neglecting the effect of I 1 over here in the diagonal elements which is very very large and you have R r and R t through I mean determinant identities you can just swap these things and you get this R t over here sorry R t over here and R r half and R r half Hermitian would come there. So, what you would see is that the capacity can be approximated to the capacity of the spatially white channel, but there are this additional terms of log determinant if R r and log determinant of R t.

This would mean that R r and R t would affect the capacity in a similar manner if you look at it the affect is similar. So, we study one of them like before we have to put this constant that sum of lambda i of R is equal to M using arithmetic mean geometric mean inequality which we had seen earlier for the diversity case what we get is this result this was also seen earlier.

So, now, if the determinant of R r is less than 1 log determinant. So, let us clear out everything this entire thing is less than or equal to 1; that means, thing is only less than 0. Similarly, if this is also less than or equal to 1 with determinant; that means, this would be less than or equal to 0.

Now, at when these are identity in that case the determinant would turn out to be 1 and in that case the log of determinant will be 0 and will be left with the capacity of a H w channel which is true when R t or R i R r equals to 1 identity when they are not. So, since log determinant of them are negative, we see that the capacity drops in case of spatial fading correlation.

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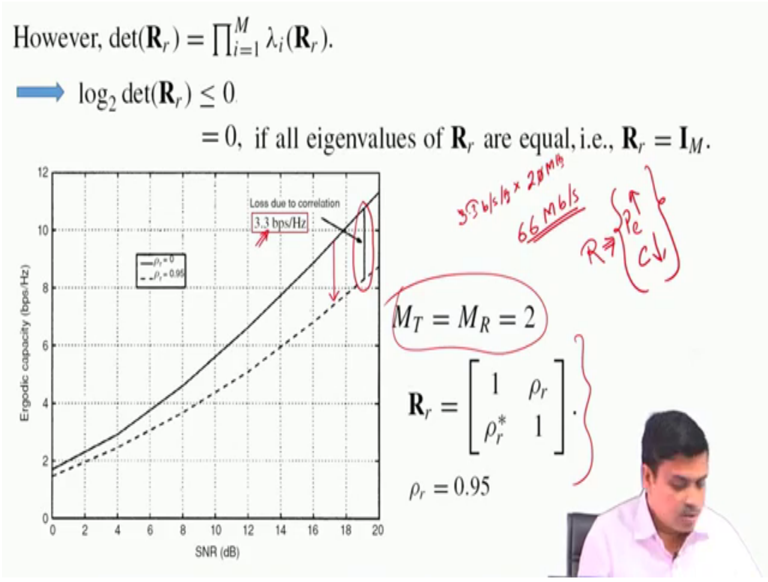

So, what we see over here is that for a particular configuration that we are showing the results we find that the spatial correlation has a detrimental effect on the capacity. So, the capacity falls significantly 3.3 bits per second per hours if you just have 3.3 bits per second per Hertz now you multiply let us a 20 mega hertz right. So, you can get almost 66 mega bits per second loss in this particular situation right. So, what we see is that correlations increases the probability of the error and decreases capacity. So, net effect uncorrelated channel gives you much much better performance than a correlated channel.

So, correlation only effecting us detrimentally if we know the details, then we can take advantage of designing the transmission mechanism schemes which can provide us with the higher spectral efficiency.

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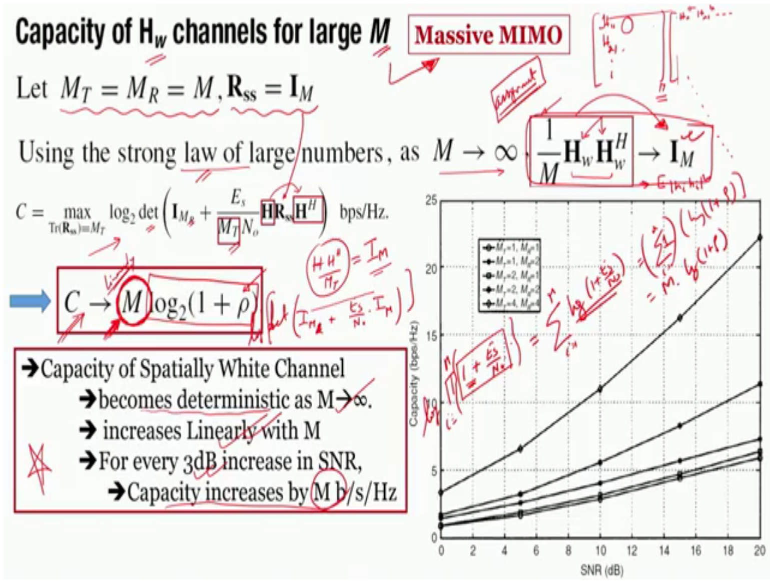

So, last thing that I would like to show you before we conclude in a in a minute from now the capacity of spatially white channel for large M this is the motivation for studying massive MIMO. So, what we see is that if we let M T equals to M R equals to M again for simplicity and also we let R SS equals to identity of size M.

Using this strong law of large numbers as M tends to infinity you can get this thing. If you open up these things you will be get H 11 H 21 this whole matrix and H Hermitian would be H 11 H 21 and so on Hermitian. So, when you multiply these matrices you will get diagonal terms which contain the product of the streams and the off diagonal would be the cross terms. If it is spatially white with large size of M each of the diagonal entries would turn out to be almost same as expected value of h i h i whole square and hence this matrix would turn out to be a identity matrix, this particular part you can check as an assignment for yourself.

So, now, if you look at the capacity expression we have HH Hermitian with M T below and R SS has been assumed to be set as identity. So, now, what we see is that inside you have HH Hermitian by M T HH Hermitian upon M T we said we will get it as i m t which you can check out during the derivation for yourself and it is not very difficult it is few steps and you can get it very easily using the central limit theorem or law of large numbers. So, you are going to use the law of large numbers you will get that and then with this i m t. So, inside what do we have I m r plus ES by N naught plus of course, we are talking about M we are talking about M this thing determinant and a log.

So, this is an entire diagonal matrix and if you see each of the entries each of the entries are 1 plus ES by N naught. So, this is the SNR of an AWGN or of a SISO link one could say right and of course, one is going to get the yeah one is going to get the if we have made this assumption there is no other problem only left with ES by N naught. So, this expression is the capacity or the spectral efficiency of the SISO link that is what we get from here.

And now since we were talking about determinant. So, we basically have the product of these things because that is the diagonal matrix determinant is the product of these things 1 to M and you have a log. So, this would be summation of log 1 plus ES by M naught since this is the constant term i equals to 1 to M, you need to get summation i equals to 1 to M multiplied by inside of course, 1 log 1 plus row. So, this would result in M multiplied by log 1 plus row and that is what you have.

So, what you get finally, is M times the capacity of a SISO link, this is the biggest motivation one of the biggest motivation of massive MIMO that is as you keep increasing the number of antennas one would for an H w channel of course, we are talking about the H w channel one would tend towards a situation that you have S SISO links and you are getting M times the capacity without any increase in transmit power.

So, transmit power remains the same as SISO, but you have an M factor increase. So, all the spectral efficiency that we required can be simply got by linear scaling of the SISO capacity with simply large number of antennas. So, this means as M tends to infinity the channel becomes deterministic we will discuss that simply because of this identity operation and it increases linearly with M which in other words mean than for every 3 dB increase in SNR capacity increases by M bits per second per Hertz instead of logarithmic increase.

So, it is huge importers in providing huge amount of spectral efficiency requirements for all future techniques and hence in the 5th generation system massive MIMO is one of the major communication techniques which we briefly overlook in the next lecture along with some more advanced mechanisms.

Thank you.