Evolution of Air Interface Towards 5G
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Lecture - 31
Waveforms Beyond 5G (Pre - coded GFDM)

Welcome to the lectures on Evolution of Wireless Communication towards 5G. So, what we have discussed still now is the different waveforms which are futuristic which have been investigated and which require even more attention for them to be made mature towards acceptance in the next generation communication systems. And, we have seen a variety of wave forms namely the filter bank multicarrier then unified universal filter multicarrier waveform, we have seen first at the Nyquist we have, we have also started seeing generalized frequency division multiplexing.

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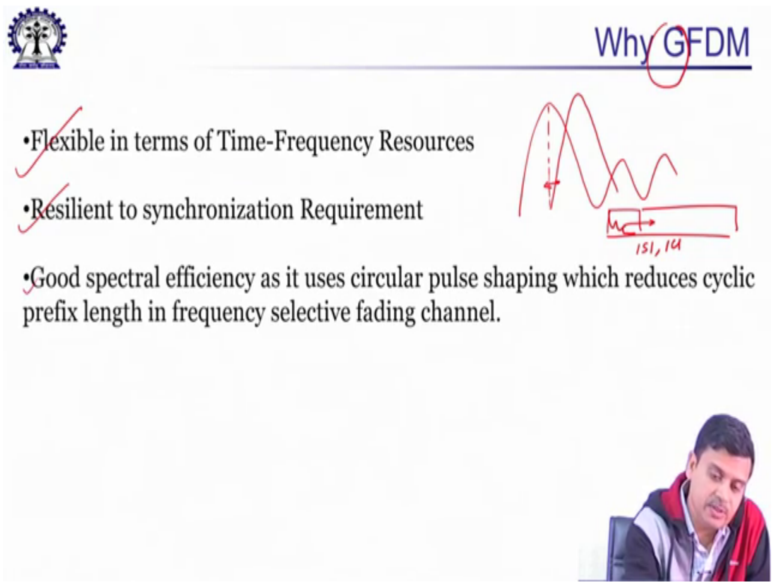

So, we will start looking at GFDM and we will complete the discussion on such different waveforms. So, we have said earlier in the previous class that GFDM is the generalized frequency division multiplexing.

And hence, it is, because of the generalized form it is flexible in the time and frequency resources which we have been seeing will again see today it is also resilient to synchronization compared to OFDM, because in OFDM we have seen that, because they are a sink in nature in the frequency domain. So, although the others are orthogonal that is the peak occurs at the 0 of the neighboring sub carriers, but a slight offset causes a huge penalty, because of inter carrier interference.

And we have also seen that, because there is a guard interval or a cyclic prefix. So, if the guard interval is small then and the channel impulse response extends beyond the CP then there is I C I; I S I and I C I which causes for the degradation in the performance. So, GFDM has a better resilience and it has good spectral efficiency, because it uses circular pulse shape which we have seen in the previous lecture.

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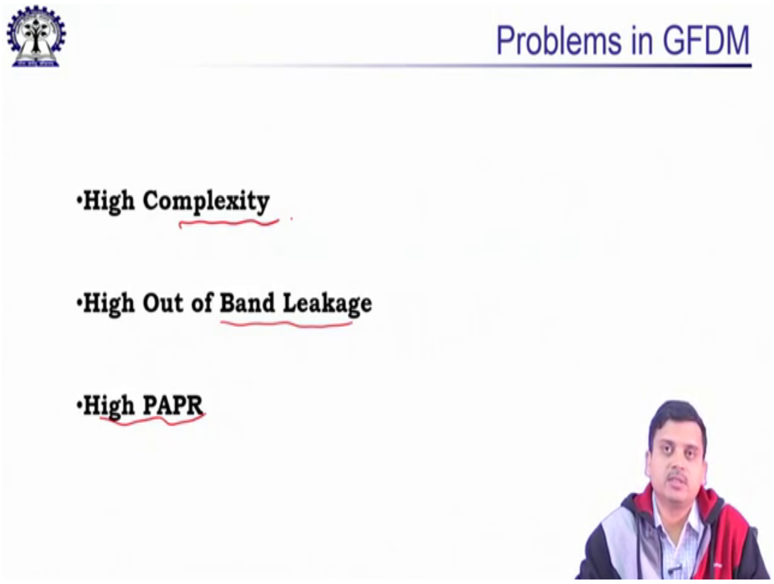

The problems are it has high complexity out of band leakage is still present, but there mechanisms to reduce it PAPR is high, but again there are mechanisms which have been proposed to reduce the PAPR as well as out of band leakage, so, as well as reduction of complexity.

So, with these additional features which have been introduced into this GFDM, it is now in better shape and is a good consenter for the next generation waveforms. So, as you are seeing that we are looking at waveforms trying to build waveforms with certain good characteristics, but once you try to build one good characteristics the other one falls down. So, the, the engineering part of it is to improve over all characteristics of the waveform and provided with better features than what existed in the previous generation systems.

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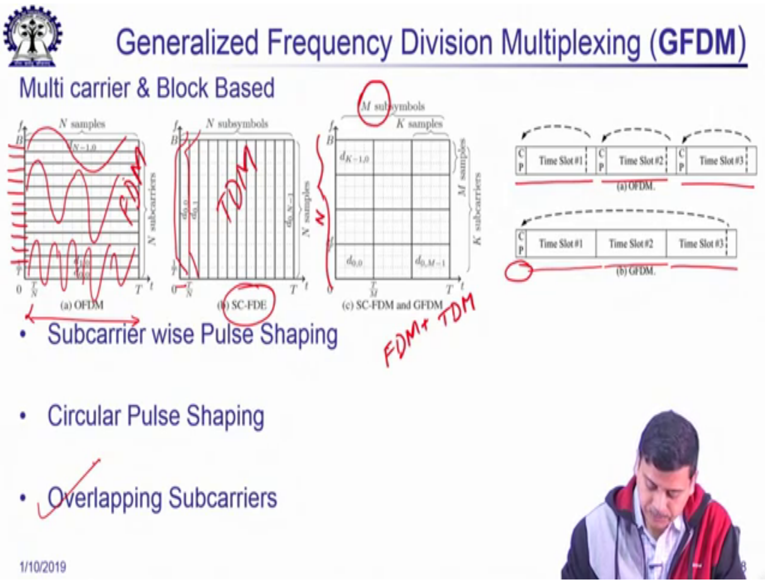

So, GFDM as we had seen in the previous lecture is that we compared with OFDM; that means, with OFDM, we had separate subcarriers. So, these were different, different subcarriers that were present, right and the subcarriers existed for a time duration, right. And each of the subcarriers, who are having different carrier frequencies, right and so on right that is how these were present.

In the single carrier FD, we have also discussed single carrier FDE. since, it is a single carrier it covers the entire band and it is kind of small duration of symbols, because the band width is large.

So, in one case, it is FDM that is what we had written in this case, it is more of ATDM architecture in GFDM, you have both FDM combined with TDM, because it is a block based OFDM, it is a block based system.

So, there are M blocks in time domain and there are N subcarriers in the frequency domain. So, that is how the whole system is and we have compared with OFDM like this is one OFDM symbol duration, this is the second OFDM symbol duration, this is the third OFDM symbol duration. So, here you have blocks of GFDM. And there is one cyclic prefix in front of it there by helping it to have a better spectral efficiency even then OFDM. The subcarriers overlap and hence there is no loss of spectral efficiency.

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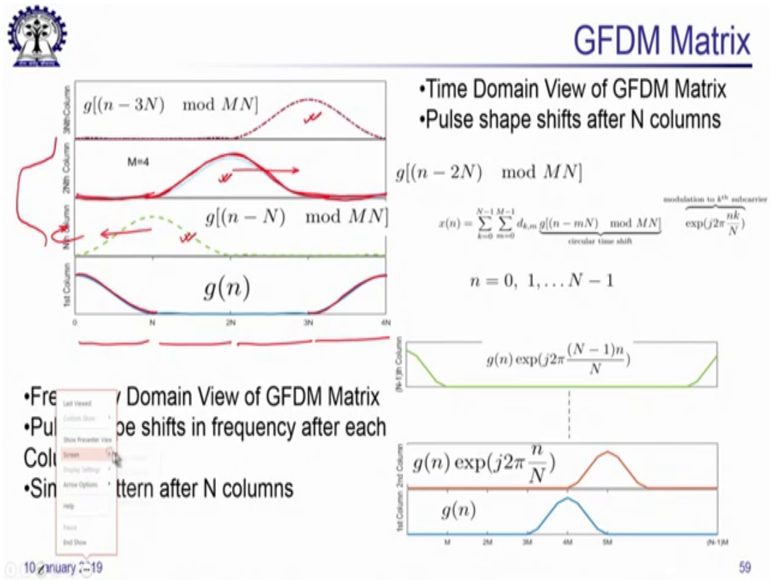

So, these are something's we have seen and this picture also we had explained in the previous lecture where we said that one can think of the pulse shape. So, if we think of this pulse shape which is a spanning over longer duration of time. So, once you have a long duration pulse shape then you naturally can make the spectral occupancy of that particular symbol to be pretty sharp; that means, it is not spreading across in the neighboring sub bands, but now in GFDM what we have seen is it has circular pulse shape.

So, what does that mean is that this pulse shape is, is if you look at the whole thing there is one time block second time block, third time block and the fourth time block, all right.

These are the four time blocks so, there are m is equal to 4. In this system, in this particular diagram and in one time sample in one time block the pulse shape would be this. So, it is basically the pulse shape, it is expanding beyond one time block or one symbol duration. And that one can compare with the partial response signaling where as in OFDM, it finishes within one time block n, n samples. So, as you shift the symbol pulse shape to the left and the right what effectively happen?

Now this is the more simplified diagram, but if you take a extreme shift; that means, for the first symbol. So, if it is the first sample here instead of shifting, it just linearly you do a rapper round or you do a circular shift. If you do a circular shift then what happens is as you get over here pulse shape which looks like this for the first symbol.

For the second symbol, it is this one for the third- third time block, it is this one and the fourth time block it is this one so; that means, if we try to draw it a fresh.

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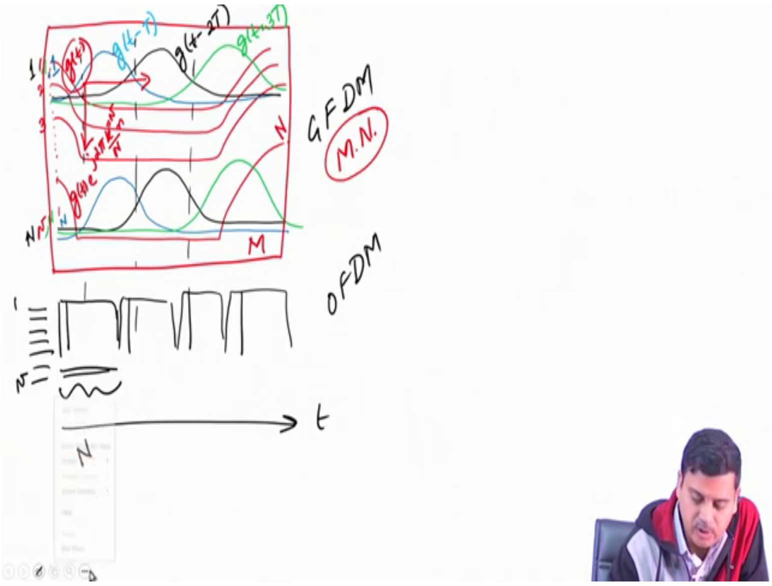

So what we have is in the, in the first time block the pulse shape is something like this. And all the subcarriers are carrying this same pulse shape, subcarrier 1 to subcarrier N, ok. In the next symbol duration the pulse shape I should chose the different color would be for the first subcarrier, it would be something like this for the last subcarrier, it would be like this, ok.

For the next time block, we can change the color to may be black we will have the pulse shape which is going like this. And for the Nth the pulse shape would go like this, right. And then for the last one, we can again change the color to given indication for the first subcarrier it would look like this. For the last subcarrier, it would look like this, right. So, this is how they will be and you can complete the notation, this is the first subcarrier, this is the second subcarrier, the third subcarrier and so on, right and if we chose the different colors to indicate.

So, again you are going to have on the first subcarrier and you will also have in the in the last subcarrier, you will also have this color in the last subcarrier, you will also have the same color the bluish color in the first subcarrier and so on and also for the black, right.

So, that is how the whole structure looks like now if you on would compare this with OFDM, one could say that if we roughly partition this, ok. So, roughly if you think if in, in, in, in, in this manner. So, then HM in OFDM case you are going to have one symbol, two symbol, three symbol and four symbol and each of them are going to have their own CP, right. So, this is for OFDM, right. So, this is how they would contrast against each other time domain all though all the frequency components would be present. So, here all the frequency components would still be present 1 to N.

So; that means, in each symbol duration you are processing N samples whereas, in case of GFDM, what you are doing is you are taking this whole block together. And then if there are N number of such time blocks and if there are N subcarriers. So, you are getting m times n samples to be processed in one go.

So, this is one of the major differences between OFDM and GFDM to look at it generic way, one can also think of putting different pulse shapes. So, basically this is your Gt, ok. And this one, if you write this Gt is your G of t minus let say L t and then we can have the black one as G of t minus 2 T and then the green one, we can have G of t minus 3 T and so on and so forth, right. And all of them are having the same on the same subcarriers. So, if you look at this is G t e to the power of J 2 pi K by N and K is equal to N in this case, all right and that is how you are translating. So, the G t is translated in time as well as it is translated in frequency as in the Gabor frame work what we have defined earlier.

So, this picture might be able to help us to understand the overall picture or overall explanation of a typical GFDM system, ok. So, now, we get back and the, this particular picture is also relevant for us in our discussion. So, if we take this particular pulse shape which is for the first set we will find in frequency. So, these are the corresponding frequencies spectrum occupancy of the different carriers, right.

So, what we have is that here as you as you observe we have the first column then the Nth column. So, first 2 n minus L L to N minus L column of the GFDM modulation matrix would indicate the N subcarriers, ok. The nth to 2 n minus 1 would indicate the same subcarrier. So, 2 n subcarrier is basically the first subcarrier and, and sorry the 2 N subcarrier is the first subcarrier and 2 N, sorry the nth subcarrier is the same as first subcarrier and 2 N minus 1 is the same as N minus 1 subcarrier, right like that.

So, similarly over here in this set what you will also find that 2 N 2 3 N minus 1 and here you are going to find 3 N to 4 N minus 1. So, overall so, again what you will find is that 3 N is the same as the first subcarrier and 4 N minus 1 is the same as the N minus 1 th subcarrier. So, like that they are arranged. So, again first subcarrier first symbol first subcarrier, second symbol, first subcarrier, third symbol first subcarrier forth symbol like that they are arranged first subcarrier second subcarrier first symbol third subcarrier first symbol fourth subcarrier first symbol and like that. So, this is how you have to visualize the entire picture, ok. So, for these set of columns we have the spectrum occupancy given in this particular picture.

So, what do you find is again G N and G N shifted in frequency by the exponential S function? And here we are what you find is G N and then it is shifted in time by this frequencies by, by this time samples and by mod modulo MN, it means there is a circular shift by means of circular shift. You reduce the spreading in time and thereby you have a more compact single in time, but the flip side of it is the out of band naturally becomes, more than if you would let it grow in a linear fashion. So, this is the overall frame work for GFDM that we are looking at, ok.

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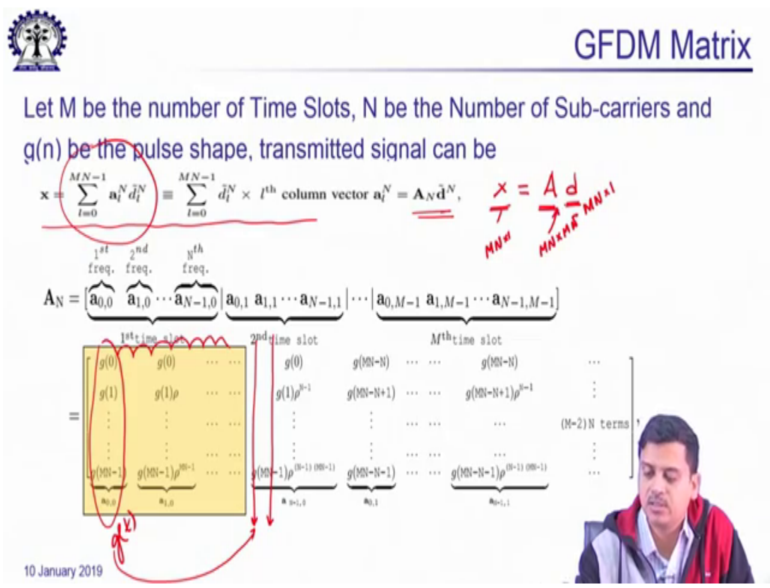

So, this is how these entire set of equations that we have described can be visualized in a graphical manner. And this whole equation that we have over here can be represented in a matrix notation and the a matrix which is the modulation matrix is contained of elements where this G is the first column is the G t, you can think of it as the G t. And then all the columns up to the N minus 1 column or the N th column are basically the frequency is translated 1 then the first column over here is the time translated version of this.

And the second column over here is the time translated and first frequency translated version of the original Gabor atom, ok. That is how one can think of the entire structure and can understand the whole picture, ok.

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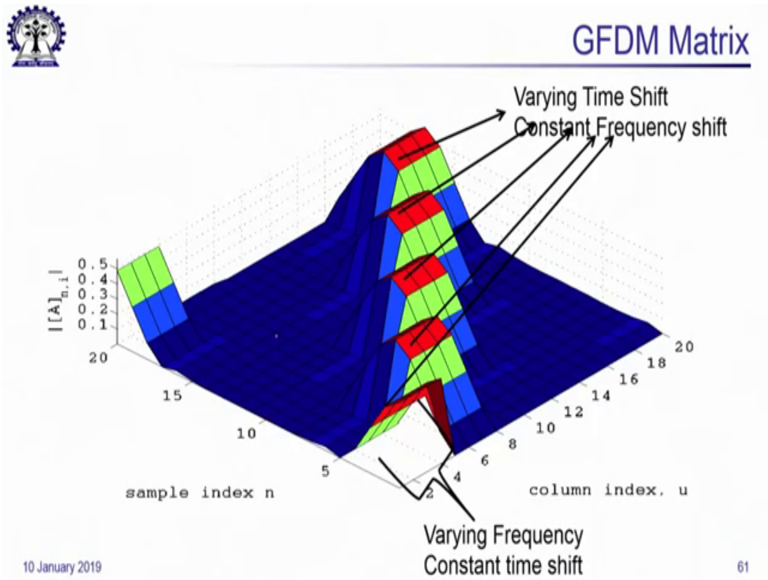

So, this picture we had explained and now you can naturally follow through all the descriptions that we had given earlier.

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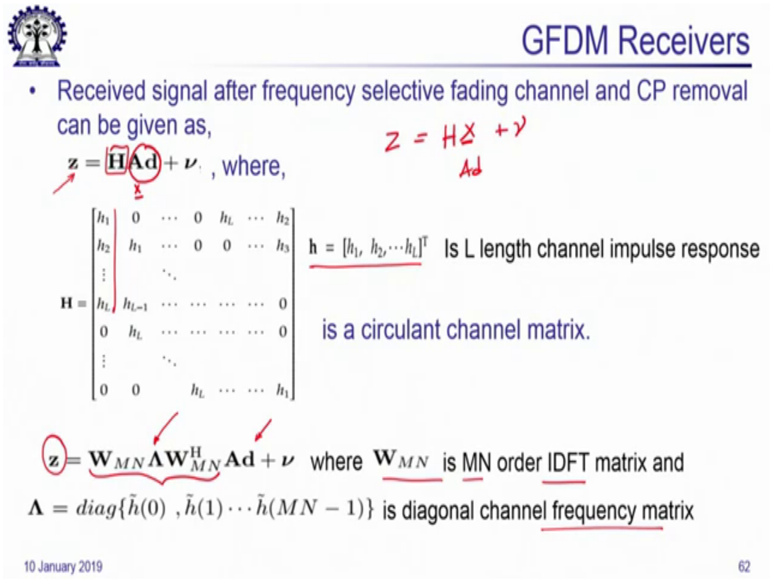

So, if we look at the, the situation now. So, what we see is that X is equal to A d where d is the vector of constellation points A is the modulation matrix and x again is the vector of GFDM modulated symbols. So, here this is MN cross 1. And this again would be MN cross 1 and this would be MN cross MN, right this modulation this GFDM modulation matrix, all right. So, that is how the whole thing. So, MN number of samples are available and so, we have the entire equation now.

So, A d represents the X GFDM modulated signal h represents the channel which is the convolution channel matrix, right. And h is the channel impulse response and we can get the received signal in z and, and this is how we can write the entire set of equations. So, if h is the circulant channel matrix which represents the convolution operation. And so, basically you have X times h is equal to Z and of course, there is noise term which gets added. So, instead of X, we are writing that matrix equation A d, ok. So, now, if you look at Z, we can expand h because it is a circulant matrix in this form, where WMN is a M times M order IDFT matrix, right. This is a this is a natural way of factorization and these are the Eigen values or this in order words, you can think of it as the Fourier transform of these coefficients, ok.

So, this is the diagonal or the channel frequency matrix, right. So, this is how you can expand the whole thing. So, this is what is the received signal. So, what we are now going towards is how do we recover d from this z that we have received.

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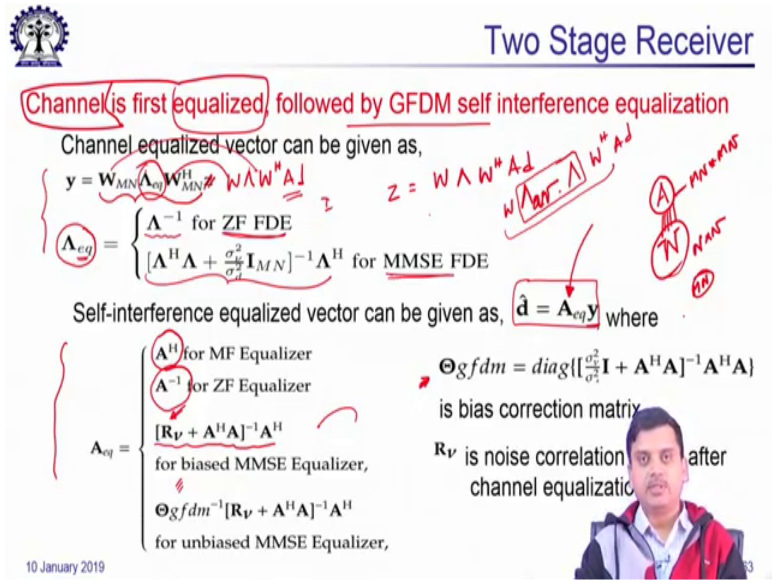

So for receiving there are two ways of doing it one is known as the two stage receiver in the two stage receiver the channel is equalized first, ok. First you equalize for the channel and then you followed by G module GFDM demodulation, ok. We have written GFDM self interference the reason is if you look at this A d, A d is such that everything is mixed up it is, it is not necessarily a orthogonal matrix, it is generally a non orthogonal matrix. And hence there is a lot of self interference due to operation at the receiver. If I do a Hermitian a then it is a mashed filter operation and we are going to get a huge amount of self interference, but at the cost of certain other advantages with GFDM provides.

So, when we are equalizing at the receiver what one needs to do is if you look at the previous expression we have A w lambda W Hermitian. So, we have W lambda W Hermitian, right. And A d, right this is your equal to Z. So, now, what is being done is this is being multiplied, right. So, if you multiply that what will you get is W Hermitian W is identity and then in case of AEQ.

So, this is basically the channel equalization part, ok. So, this is 0 forcing frequency domain equalization. So, in case of 0 forcing frequency domain equalization instead of AEQ the, the lambda equalization EQ indicates equalization, you can take it as lambda inverse. So, the moment you multiply W with W Hermitian with W, you are going to get.

So, basically you replace this Z with W lambda W Hermitian A d, right. So, this will produce an identity and then you have a product of lambda AQ with lambda.

So, this is your choice of equalization matrix at the receiver and you can choose it to be lambda inverse. In that case, this product will again produce an identity matrix then again W W Hermitian will again be identity. And then you are left with A d so; that means, if you are using the lambda inverse of A Q then you are received signal can be written in this manner and then you can recover the whole thing. So, whereas, instead of doing 0 forcing, you can also do AMMSE equalization. So, in case of MMSE equalization your MMSE equalizer A lambda EQ would look in this manner and instead of. So, once you have done this equalization. So, you will be left with so, once you are done with that you will be left with lambda AQ, lambda AQ times A and W Hermitian, sorry lambda EQ lambda this is not a lambda EQ lambda A d and here you have Aw, ok.

So, this whole thing now needs to be equalized. So, generally you would have it in a way that this cancels out the interference either by MMSE procedure or by 0 forcing equalizer. And then whatever is remaining you would have to equalize that with AEQ as the equalizer coefficient for A and then you need to recover the whole signal. So, for match filter operation your AEQ would be equal to A Hermitian and for 0 forcing it will be a inverse, ok. For MMSE, it will be which looks like this where this is the covariance matrix of noise and for unbiased, you have to modify this expression with a factor extra factor which is described in this particular equation.

So, again these works can be these details can be found in some of the references which we will provide in the in the material then you can easily go through the derivations and find out how these kind of things work out.

So, you have to equalize the channel and then you have to equalize for the GFDM modulation matrix. Now, if you compare this with OFDM you will remember that a matrix which is the GFDM modulation matrix is comparable or it is equivalent to IDFT matrix at the transmitter for OFDM. So, if you write W with the IDFD matrix then this will be the ID matrix for case of OFDM and the sizes of course, will be different this is the MN cross MN and this is usually n cross n matrix. So, you can also compare with the MN block size of OFDM to take MN size of samples simultaneously and when you have to equalize both the channel and the GFDM modulation simultaneously. So, what we have is Z is equal to HAD, right that is what you get.

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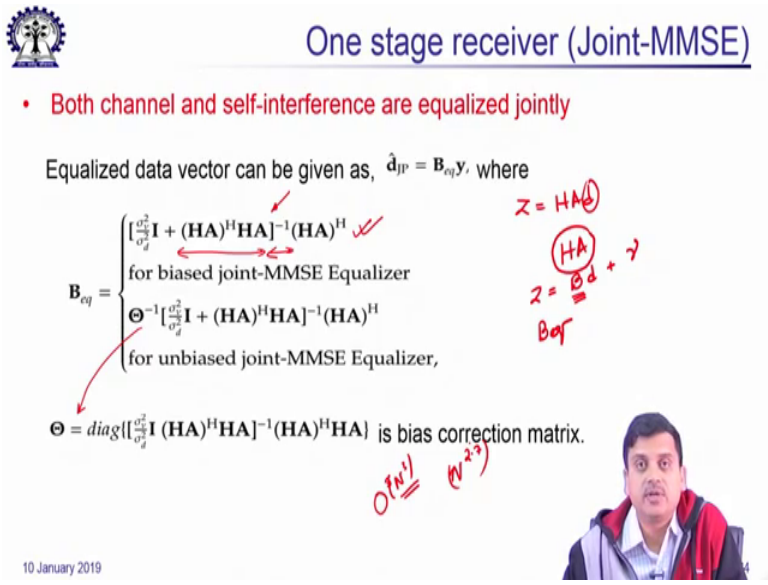

So, Z is equal to HAd, right. So, now, you have to recover d. So, HA together you can assign it to be equal to B. So, you can write whatever you have received as Bd plus noise and then you have to find AB equalizer matrix which can cancel or normalize the problems that has been introduced by the B matrix.

So, here again the M M S E equalization would be in this manner and the unbiased M M S E would be again a modification with this scaling factor which is also described over here and details of this are available again the papers that we can provide to you ok.

All right, so, now, d is receiver operations although they appear straight forward in the mathematical notation, but implementation of MMSE is not a very- very simple job, it is very- very complicated, you can see lot of matrix multiplications that happen over here and there is matrix inversion. So, the order of complexity is very high without doing any reduction of complexity any matrix inversion or multiplication is typically around order of N cube although you can bring it down to N to the power of 2.7 or something like that very still very close to order of N cube let say. So, you can easily imagine the amount of complexity that is involved and then there is a set of low complexity transceiver architectures which again we can provide as reference, but we will not discuss in this particular lecture, because those are special aspects only for those who might be interested in the very great details of such receiver architectures.

So, we will skip few set of things, but it will be made available for those who are interested. So, you move forward and look at another modification of G F D M which is called the pre coded GFDM and will briefly tell you about this.

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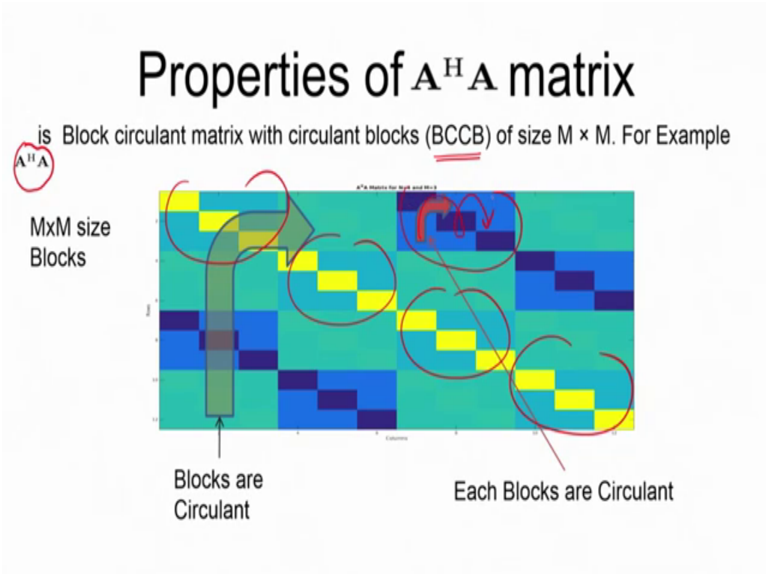

So, in the receiver you basically have a Hermitian A as the operation when you are doing matched filter. So, if you study these characteristics of such matrices, these matrices turned out to be block circulant with circulant blocks. Again details of these are available in reference papers which will provide to you so; that means, there are block structures there are block structures which appear in a circular fashion and within the blocks also you will find that there is a circular symmetricity that is present in it.

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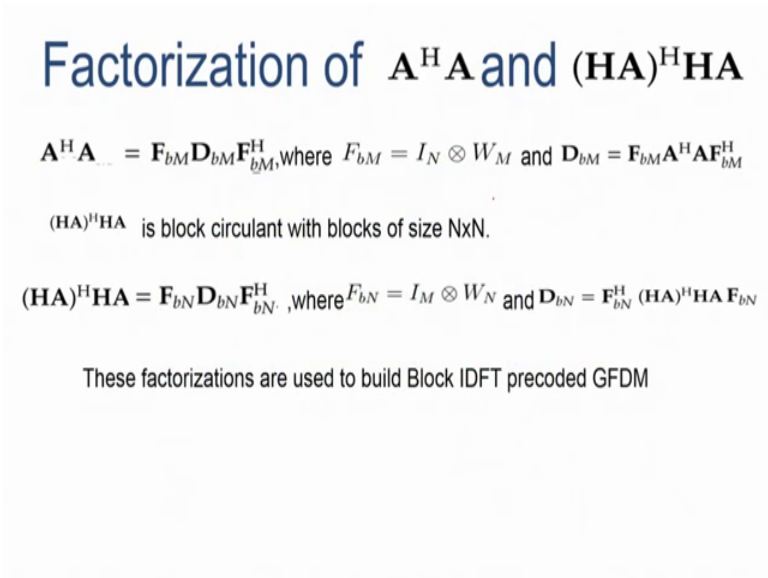

So, that is exploited in factorization. So, a Hermitian A can be factorized in this manner where FbM is the block IDFT matrix and if you are doing H A; that means, channel together with the modulation matrix then you will be again having another similar factorization, but this is of size N and in this case this will be of size M, ok. So, now, if you look at the way this receive this signals are, ok. And this can be factorized with block IDFT matrices, right. So, the good part of all of this is whenever we talk of IDFT, we get low complexity implementation, right. So, once we can realize this in terms of IDFT operations, we will get a great advantage in reducing the complexity.

So, all that we are seeing over here is that in this processing, if we could somehow use this factorization and spilt the processing complexity between the transmitter and the receiver then we can reduce the complexity of operation at the receiver quite a bit, all right. So, in this manner, if you look at the whole factorization matrix, right. So, we should be able to do a lot so; that means, HA Hermitian HA operates. So, if you are at the receiver you would be able to get this whole factorization and then you can do some IDFT operation on this side, IDFT operation on the other side and you would be able to manage the, the operation.

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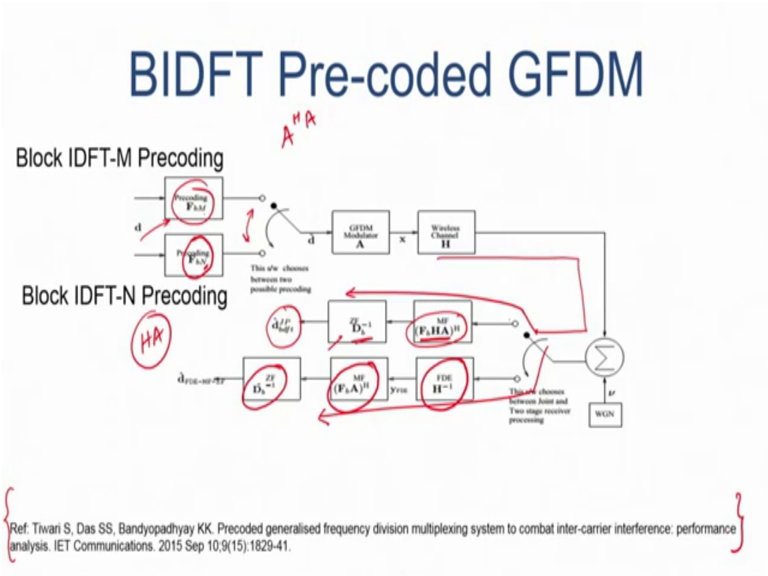

However, from this we have this motivates us to precode the GFDM modulator with FbM, right. If you are, if you are going to operate on A, A Hermitian A matrix and if you are going to operate on this joint processing; that means, if you are going to take h a together then you can do a precoding with FD. So, if you are doing precoding with FbM then your FbM on data, you write FbM on data and then you get this particular part. So, what you see over here is that a Hermitian operation should be done at the receiver. So, if this operation has to be done at the receiver then the receiver sees a Hermitian A, right. So, if the receiver sees a Hermitian A. So, a Hermitian A can be factorized in this form, right and where D is a matrix which, which is of this structure, which is of this structure.

So, if we can get this D matrix, we can demodulate our signal finally, we have to equalize for this D to reduce the complexity you can preprocess at the transmitter with this. And then you only have to process it with the block DFT matrix at the receiver side right the same thing applies in case of the HA based operation. So, what we do is we do a preprocessing with either FD that is block IDFT matrix of size M or Fb of size N depending upon whether we are going for two stage processing or we are going for a joint processing, right.

So, at the transmitter you choose one of the options depending upon a pre agreed a condition and then the signal goes to the channel when it comes to the receiver, if depending upon the mode of operation it would either go via this path or it will go via this path, right.

So, in the two stage receiver case there is a frequency domain channel equalization which happens first and then at the receiver you are doing ADFT based operation. And then D blocks equalization happens at the receiver in the other case as you can see over here in the matched filter reception you are doing A F B H A multiplication at the, at the receiver side followed by this A D block, this D matrix based inversion in order to get the particular detection symbol out. So, what are the benefits of doing this precoding is what we are going to briefly discuss and again all these details are available in this particular paper which is again easily available, ok.

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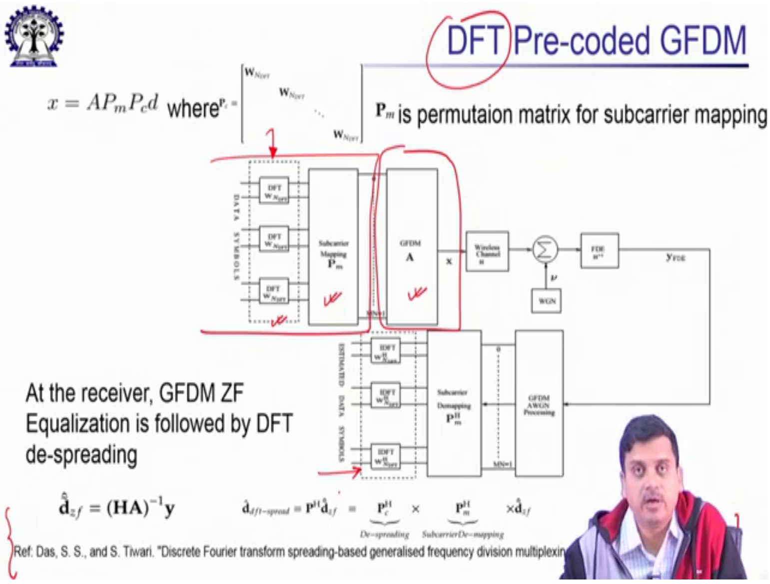

So, with this, with this we also move to another form of precoding which is called the DFT precoding, and this is also available in this particular paper. So, one should feel free to down load and get into the details.

So, here the idea is relatively simple here we do a precoding with DFT spreading matrix. In a similar fashion that DFT spread OFDM or kind of systems or S C F D M A kind of systems are handled. So, with this DFT precoding then there will be a subcarrier mapping matrix followed by the GFDM operation. So, there is this is the precoding that happens in case of DFT precoding and here these are the possible precoding options that happen in case of GFDM.

So, here on you are seeing that the GFDM operation is happening. So, here also you are seeing that GFDM operation is happening. So, these things are happening before at the transmitter and at the receiver accordingly you have to do the reverse processing and then you get back your data.

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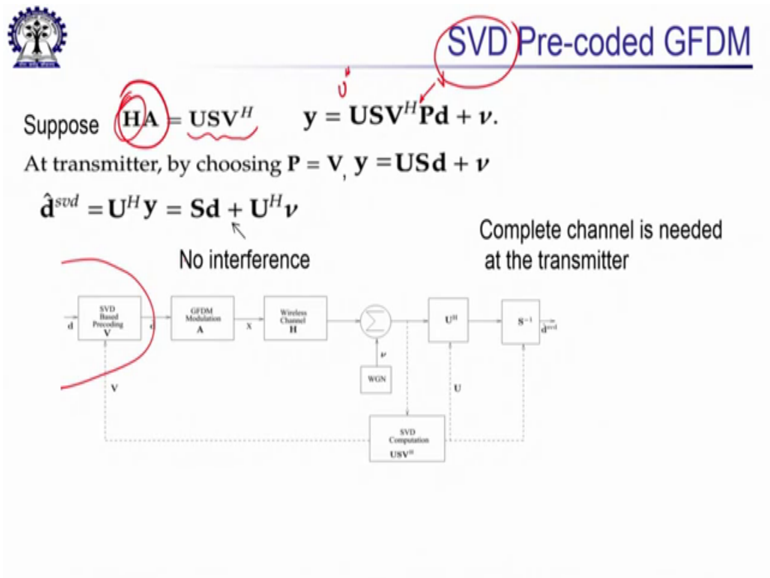

Similar, manner in a similar manner, because you have HA H is a matrix a is a matrix you would be able to also decompose into a singular value decomposition based a structure. And in that case, you would be able to a preprocess at the receiver like at the transmitter side you could pre-coded with A V and at the receiver side you would multiply with the U Hermitian. And then you can get back your original data and this kind of a system you would call A S V D based pre-coded GFDM system.

So, here you have A S V D based pre-coded GFDM system, but this depends upon the channel matrix, right. Here also this one there is no dependence on the channel. In this case, there is dependence on the channel only when you are doing the channel joint, joint processing if you are not doing the joint processing then there is no dependence on the channel over here. In this case, there is no dependence on the channel in F, in SVD there is some more dependence on the channel, right.

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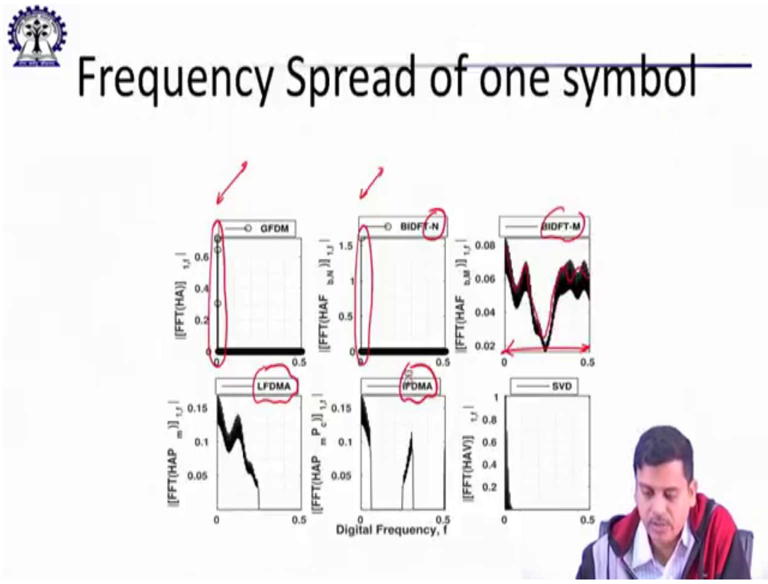

So, what we see over here is the frequency domain view of the signals. So, in case of GFDM, we see that these signals are concentrated around one frequency band. In case of the B I D F T N based pre-coding. Again we see a similar structure, it is concentrated in one frequency band and hence it is spread in time, right, where as in BIDFTM mode, we are seeing that the signal structure is having spreading coefficients across the entire set of signal bands, right. We also have seen some more variants one is the L F D M A L F D M A; that means, when we do the DFT, we do the DFT over a set of subcarriers which are next to each other collocated, ok. Whereas, we can do I F D M A; that means, interlay frequency division multiple access.

In that case this frequency distributed all over, they are instead of having localized frequencies one can have one frequency component here, here, here. And there and the next frequency component would go there, there, there and so on and so forth.

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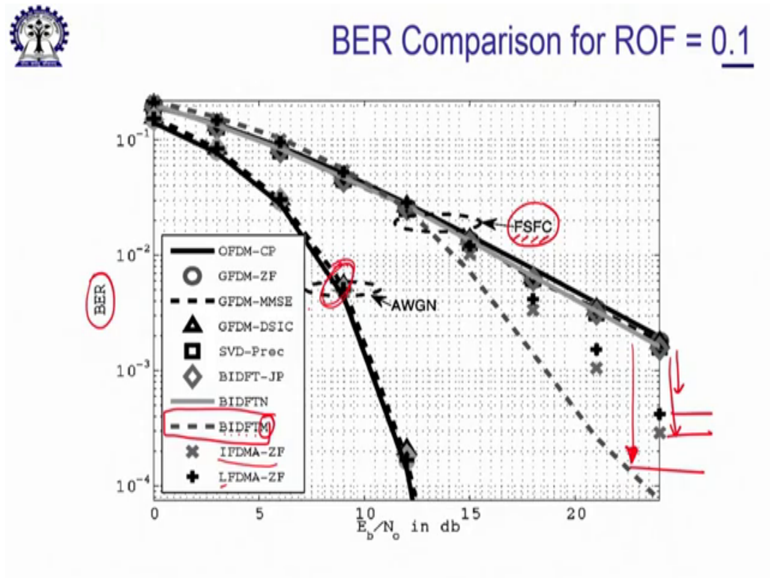

So, what we find? That is interesting results over here. In case of AWGN channel when we look at the BER there is no difference in performance. So, that clearly indicates that these methods does not degrade the performance of a typical GFDM systems these the non- degrading pre-coding techniques. Now when we go to frequency selecting fading channel frequency selective fading channel what we find is that when you are doing this BIDFTM; that means, block inverse discrete furrier transform of size M; that means, of the two stage receiver case you find that we are getting a huge amount of performance benefit, right. The, the bitter rate is becoming better and what we also see is that the IFDMA with 0 forcing also has some good performance and. So, is LFDMA, but in the reverse order.

So, this is the best then this then this compared to other typical GFDMA systems. So, so why does it happen one can understand, because there is a frequency domain diversity gain that one that one gets in this scheme, in this scheme. And in this scheme the more the order of diversity the more is the gain and that is what is reflected over here, right in this particular picture. So, what we gain is that by precoding there is no loss in DER in case of GFDM. So, these are non distortion techniques; however, when we look at frequency selective fading channel this is; obviously, provides a much better improved performance than other known GFDM techniques.

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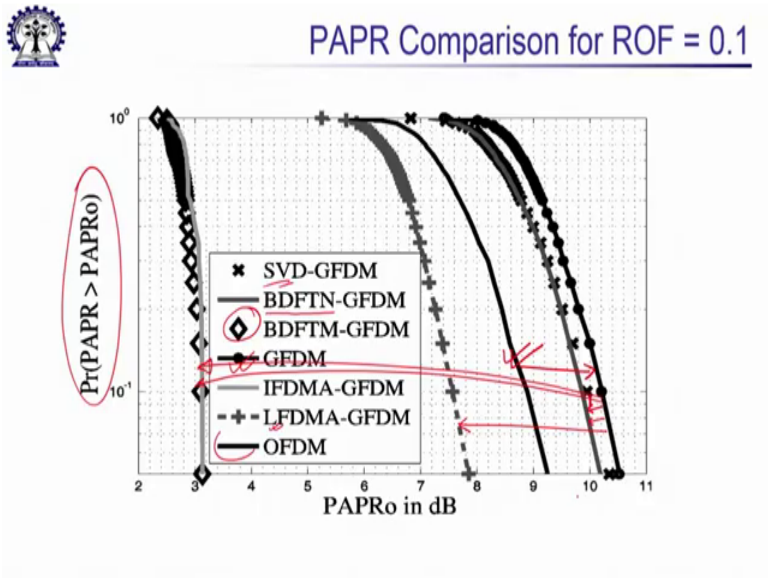

Interestingly, there is also very important performance matrix which is the peak to average power ratio. So, in peak to average power ratio what we find is that OFDM which is here has a standard peak to average power ratio.

A standard form of GFDM has increased peak to average power ratio whereas, if we are doing a let us say SVD based GFDM, there is a reduction if you are doing block DFTN there is a similar reduction, ok. So, this is for SVT based this is for block DFT based and if you are doing BIDFTM, sorry if you are doing LFDMA, right. So, we get a reduction over here and if we are doing BIDFTM. So, we see a huge reduction in the peak to average for ratio and which is also the same as in IFDMA.

So, what we find is that this IFDM and BIDFTM they provide a similar gain in performance. Although BIDFTM produces a better BER performance than other systems, right. So, what we conclude is that pre-coded GFDM system provides a very low PAPR which is a very big advantage for providing for low complexity devices for up links procedures and all other things. And they also improve the BER performance in a frequency selective fading channel.

So, finally, will conclude this particular lecture over here and what will do is because of time constraint we could not add it. In this particular, lecture we will briefly look at the performance comparison of this different waveforms. In the next lecture and then we will move on to newer things different techniques of fifth generation communication system from the next generation from the next lecture onwards.

Thank you.