Power System Dynamics, Control and Monitoring
Prof. Debapriya Das
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 07
Power System Stability (Contd.)

So, we are back again. So, whenever starting this your next lecture, just starting from the beginning of the previous one, right. So, some just said that things will be easier for all of us.

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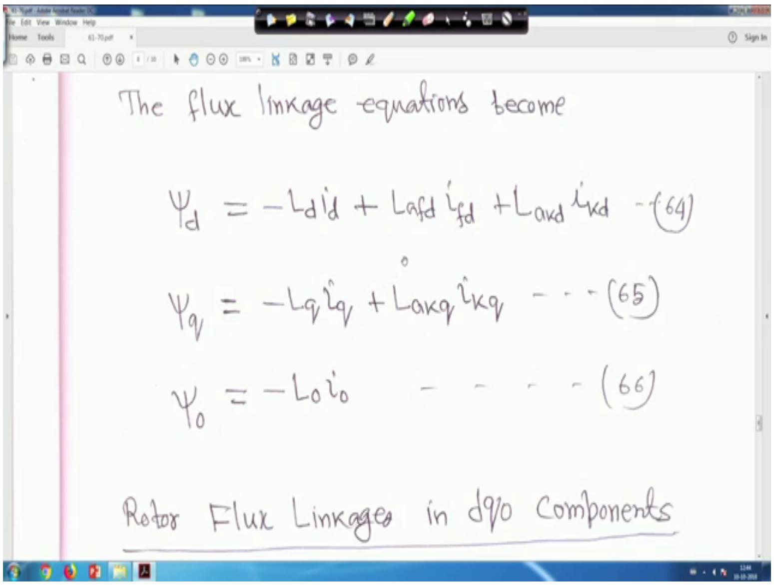

So, the flux linkage equation then psi d, psi q and psi 0 this already we have seen in the previous lecture, but just we have to move further. So, psi d will be minus L d i d plus L afd i fd plus L akd i kd after your transformation. Then psi q also will be minus L q i q plus L akq i kq and psi 0 will be minus L 0 i 0, right.

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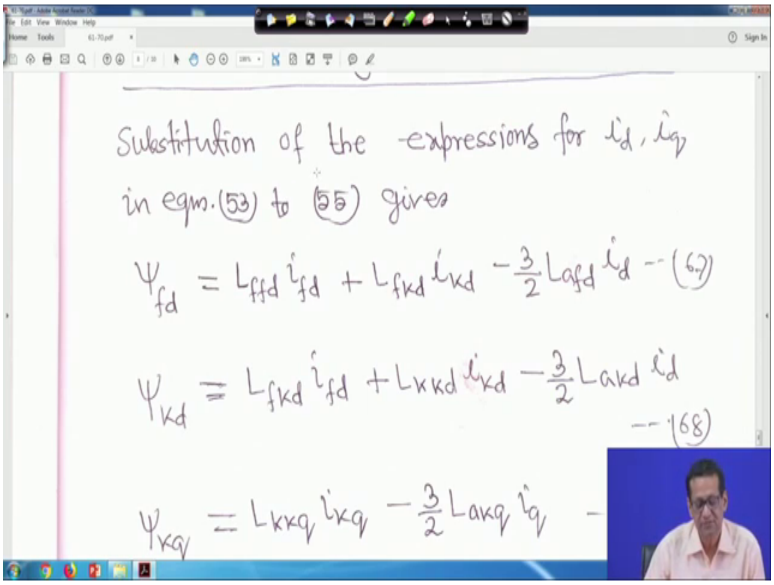

So, now next we will see the rotor flux linkages in d q 0 component, right. So, substitution of the expression for i d, i q in equations 53 to 55 if you make everything then we will find the psi f d will become L ffd i fd plus L fkd i kd minus 3 by 2 L afd i d.

What I suggest that whenever you will listen to the lecture you please keep the list of nomenclature video because, so many terms are there, right. Like you are mutual inductant between stator and rotor, then self inductance of stator self inductance of the rotor circuit, right. So, similarly your psi k d will be L fkd ifd plus L kkd ikd minus 3 by 2 L akd i d and similarly psi k q will be L kkq i kq minus 3 by 2 L akq i q.

Now, question is that if we put everything in the 3 into 3 matrix form and simplify you will get like this, but as to save the time directly I have written all these things, right otherwise it will take long time. But, as an exercise you please try yourself you will find hardly it will take few minutes to derive that.

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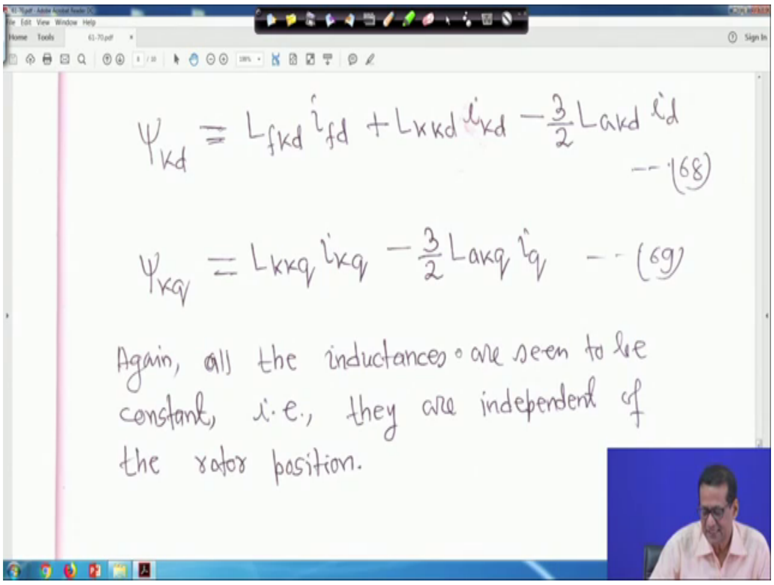

So, again all the inductance is are seen to be constant that is they are independent of the rotor position. Here this psi fd, psi kd and your psi kq they are independent of theta, right.

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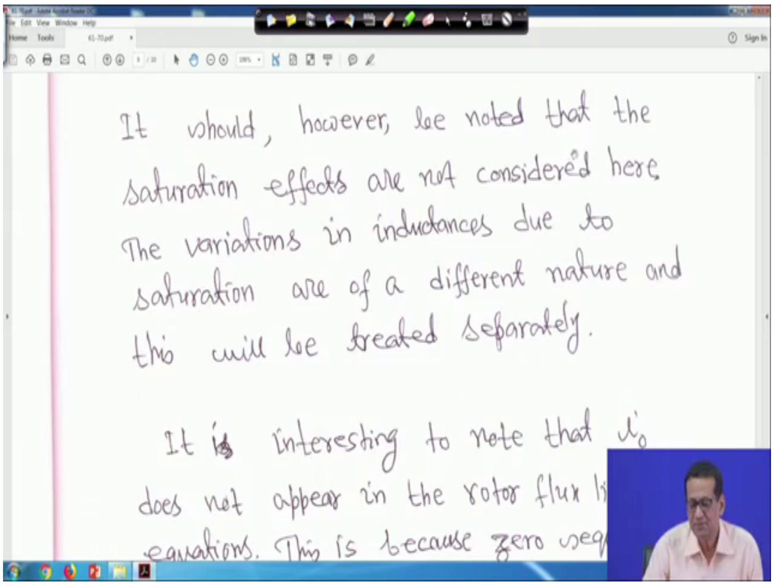

So, it should however, we noted that the saturation effects are not considered. We will not consider the saturation effects throughput right.

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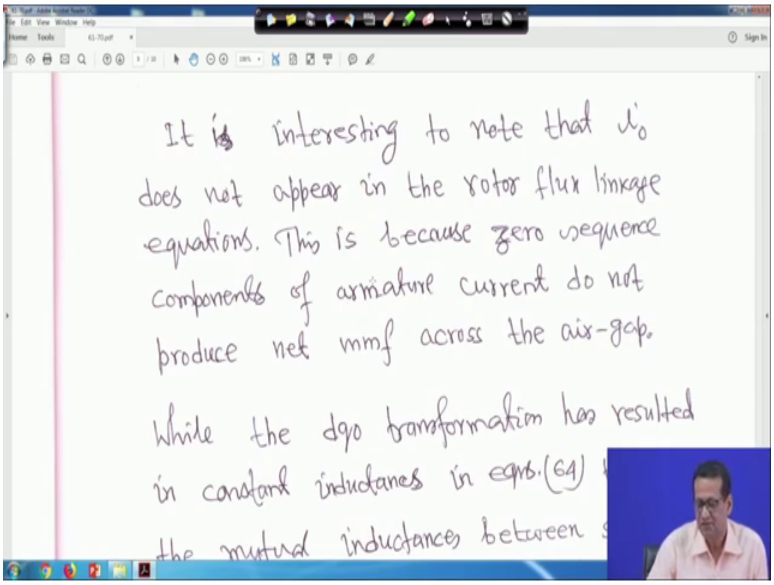

So, now it is interesting to note that i 0 does not appear in the rotor flux linkage equation. This is because 0 sequence component of armature current do not produce the mmf across the air gap this should you this you should keep it in your mind.

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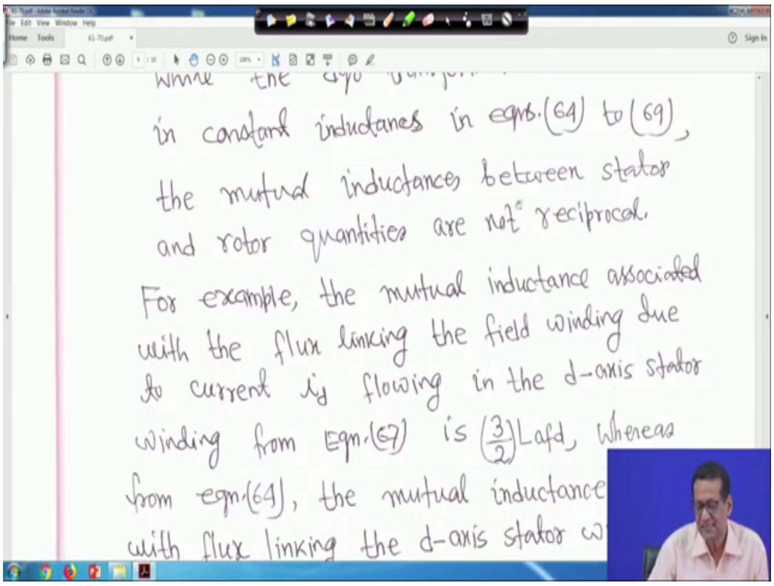

Now, while the dq 0 transformation has resulted in constant in that sense equation 64 to 69, right they are independent of theta. The mutual inductances between stator and rotor quantities are not reciprocal. Reciprocal means for example, when you have studied say transformer when you make per unit values then either it is refer to primary or refer to secondary it is same per unit values, but in this case it is not, right. So, later you will see you will see some mechanism such that both can be you are made it same I mean either to the stator side or to the rotor side, right when you make, the per unit system analysis.

For example, the mutual inductance associated with the flux linking the field winding due to the current id flowing in the d-axis stator winding from equation 67 is 3 by 2 L afd, right.

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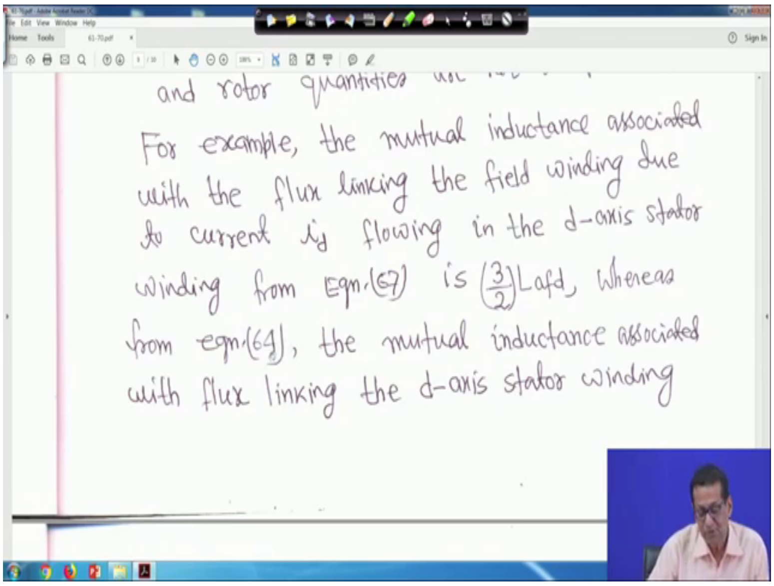

Whereas, if you look into equation 64 you will find the mutual inductance associated flux linking the d-axis stator winding due to filed current is L afd, it is 3 by 2 L afd and it is L afd. So, it is not reciprocal, reciprocal means they are not same, right.

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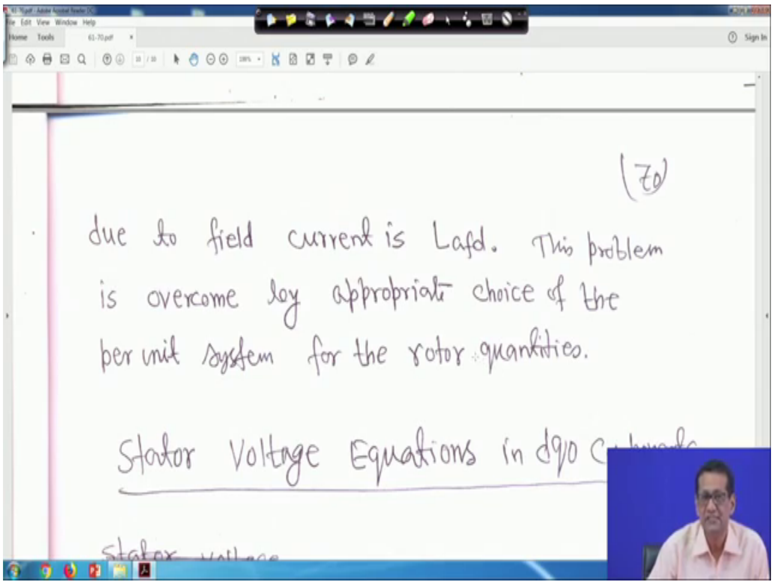

So, this problem is overcome by appropriate choice of the per unit system for the rotor quantities that we will see later.

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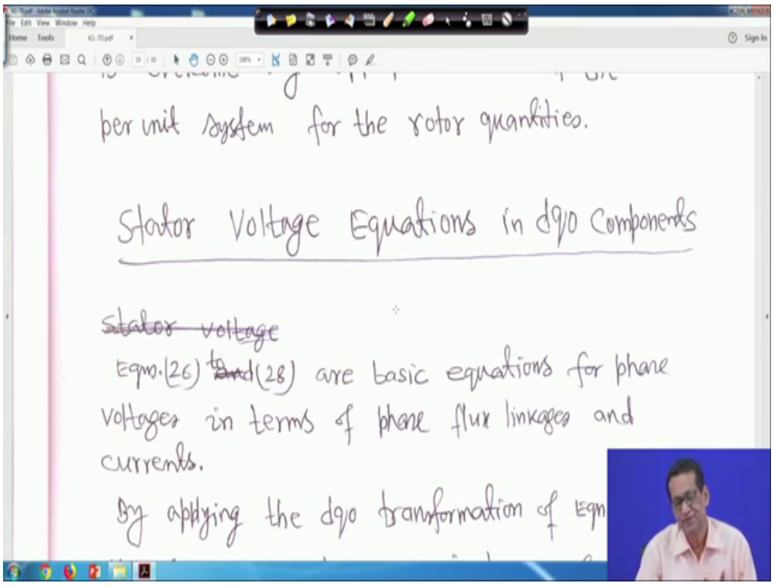

Now, next is this already we have seen in the previous lecture, but started with this only. So, stator voltage equation in dq 0 component now equation 26 to 28 are basic equation for phase voltages, right that we have seen earlier in terms of phase flux linkages and current.

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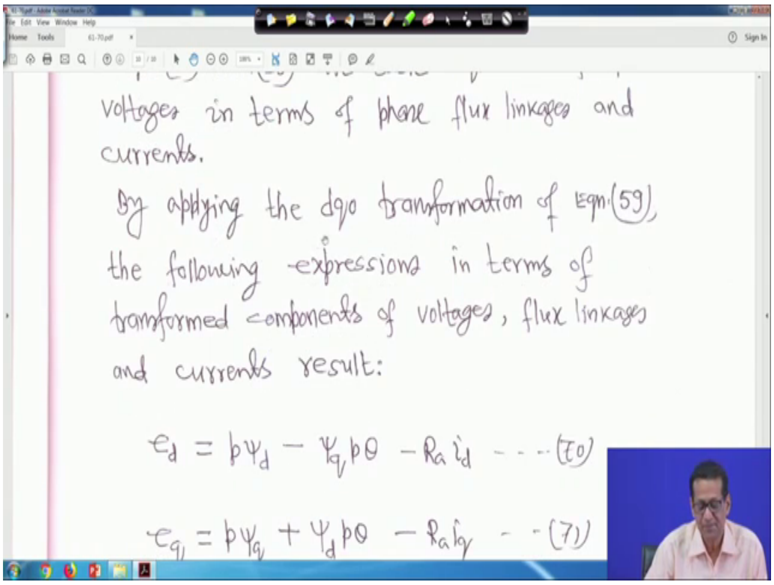

By applying the dq 0 transformation of equation 59 the following expressions in terms of the transformed components of voltage flux linkage and currents results. I mean all the derivations not putting it here because it is say it will I told you again and again that it will take long time. I suggest you try to make yourself and certain things you should keep it in your mind. For example, this should you keep it in your mind. In this case what will happen e d will become p psi d minus psi q p theta minus R a i d, right.

So, this is p ddt so, it is d of dt psi d, right. So, this is later it is explain. This is actually one term called psi or transformer induced voltage this is p theta that is d theta by d t that is speed voltages, right. And similarly e q is p psi q plus psi d p theta minus R a i q and e 0 will be p psi 0 minus R a into i 0 this is equation 70, 71 and 72. This is actually keep it in your mind but try to derive of your own. All derivations are not given here because it will consume then many hours, right.

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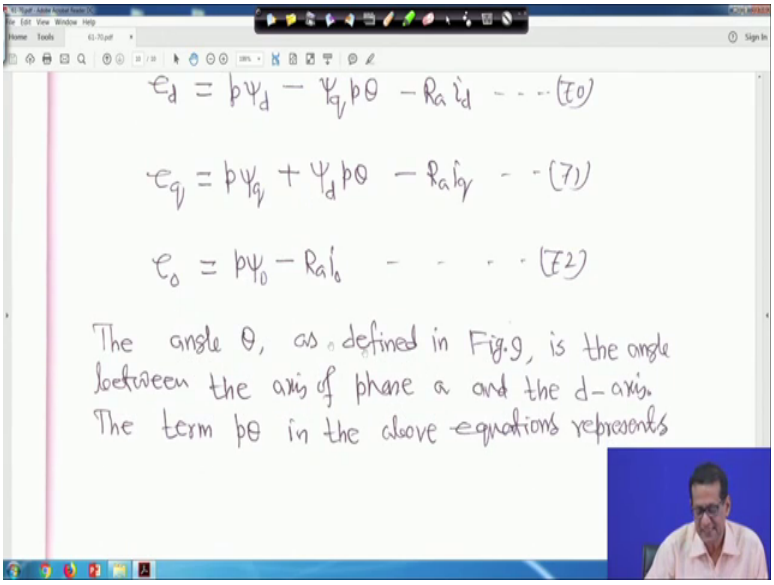

So, the angle theta as defined in figure 9 earlier I told you, right. And you see the figure 9 also everything is drawn there, that angle between the axis of phase a and d-axis the term p theta in the above equation actually it will it represent I will come to that; I will come to that, right.

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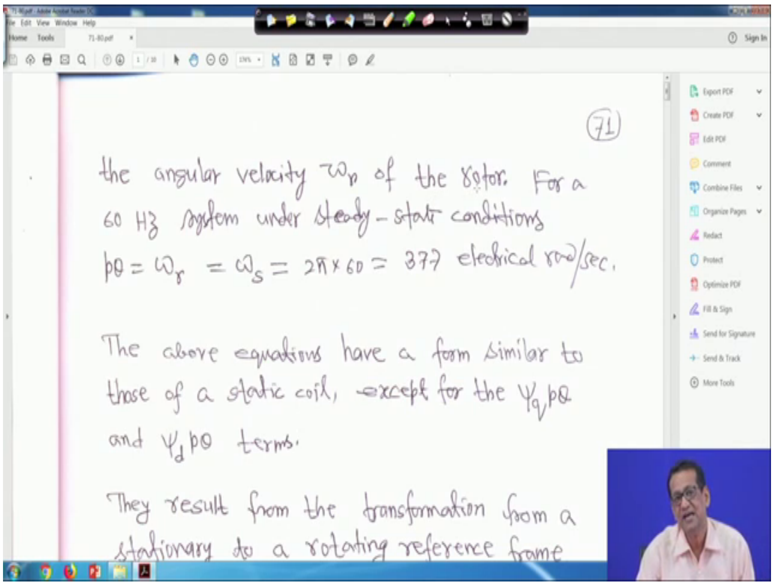

That your, the angular velocity omega of the rotor that is b theta upon dt is equal to omega r and omega r will be basically omega s because it is synchronous machine, right. So, for a 60 hertz system under steady state condition p theta will be omega r is equal to omega s because it is your both are same so, it will be 377 electrical radian per second. Now, the above equation have a form similar to those of a static coil except for the psi q p theta and psi d p theta terms, right.

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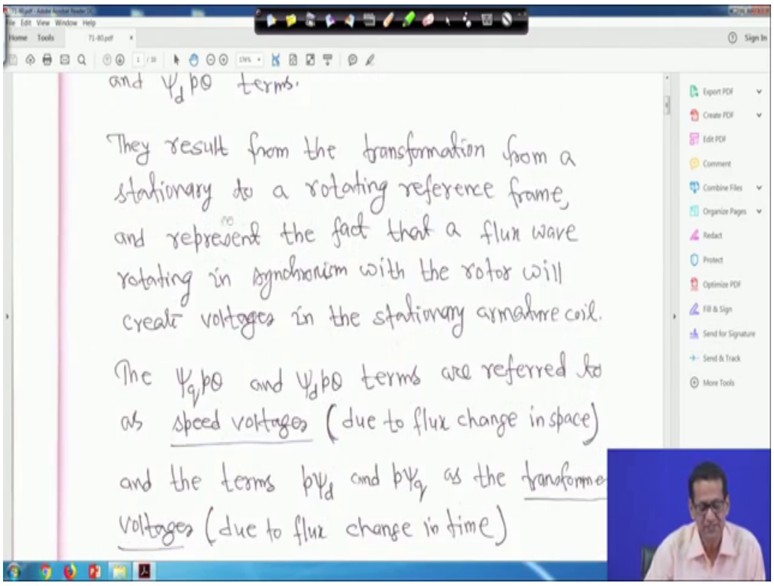

The result from the transformation from a stationary to a rotating reference frame and your, what you call the represent the fact that a flux wave rotating in synchronism with the rotor will create voltages in the stationary armature coil, right.

Similarly, this for example, the psi q p theta and psi d p theta psi q p theta mean psi q into d theta dt that similarly psi d p theta means psi d into d theta dt, right. A term referred to as a speed voltages because it is d theta dt in general, right due to the flux changes in phase voltages. And that is your due to the flux and your p psi d and p psi q equal transformer voltages due to the flux change in time because we know d psi by dt. So, it is p psi d means your d psi d dt and this is d psi t and dd psi q dt, right.

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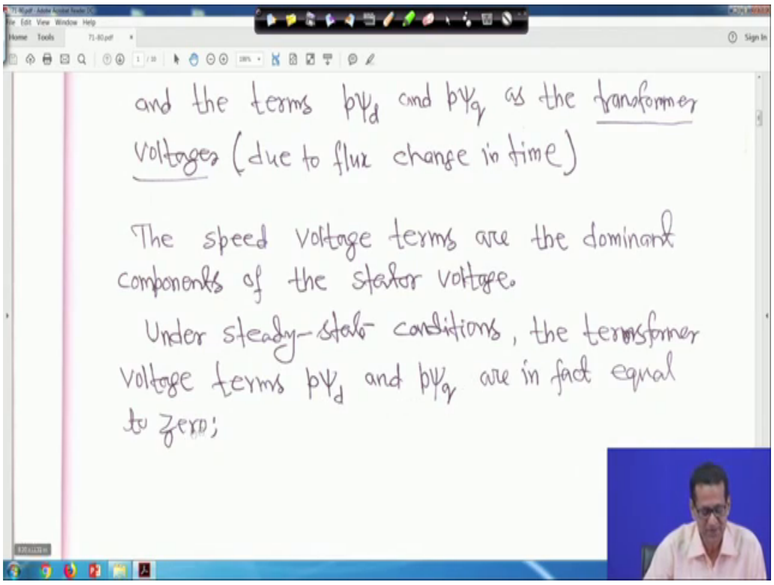

So, the speed voltage terms are the dominant components of the stator voltage that d theta by d theta under steady state condition the transformer voltage terms p psi d and p psi q are in fact, equal to 0, right.

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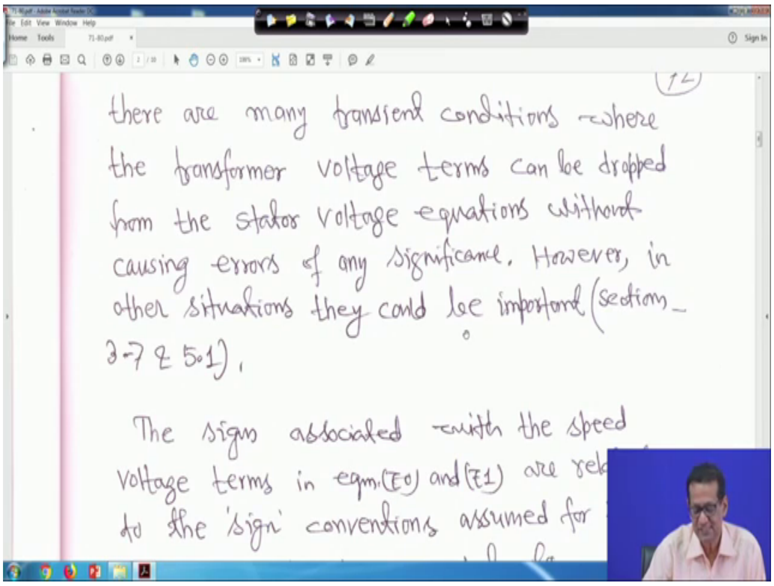

So, there are many transient conditions where the transformer voltage terms can be dropped from the stator voltage equation without causing error of any significance, right. However, in other situations they could be important, right. This is actually just hold on so, just hold on; just hold on. So, in this case what happen that you are however, in other situations that is that is this is something written section 3.7 5.1 it is not for you, right.

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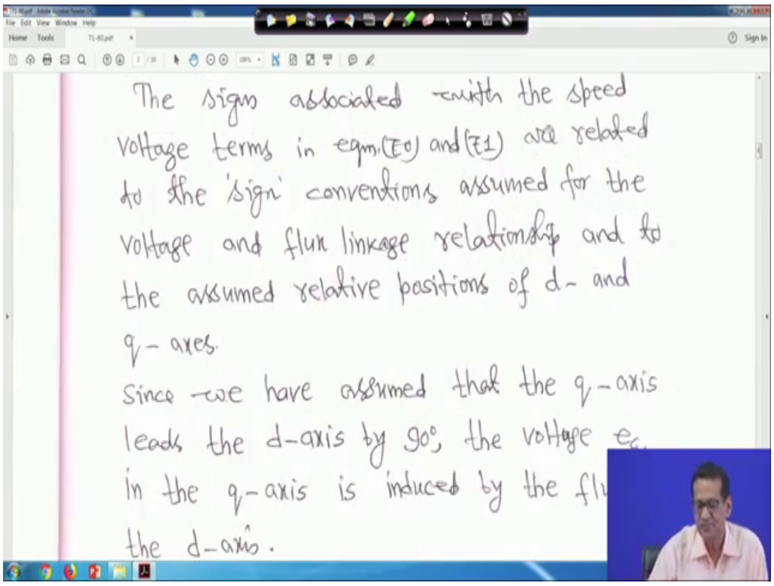

Now, signs associated with the speed voltage terms in equation 70 and 71 are related to the sign conventions assumed for the voltage and flux linkage relationships and to the assumed relative position of d and q axis, right. We have seen the q-axis actually leading the d-axis by 90 degree, the voltage e q in the q-axis is induced by the flux in the d-axis, right.

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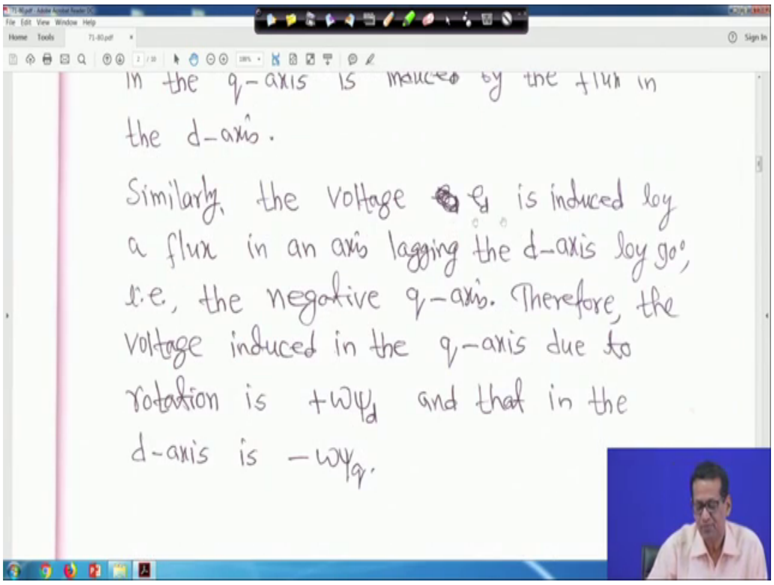

Similarly, the voltage e d is induced by a flux in an axis lagging the d-axis by 90 degree that is the negative q-axis. Therefore, the voltage induced in the q-axis due to rotation will be plus omega psi d and that in that d-axis will be minus omega psi q this should you keep it in your mind.

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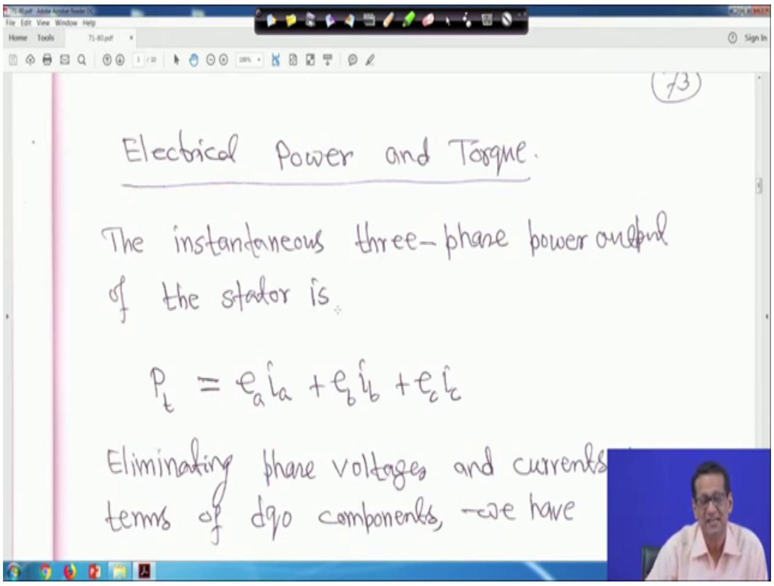

Now, electrical power and torque; now, the instantaneous 3 phase power output of the stator is that generally we know P t is equal to e a i a plus e b i b plus e c i c. Now, if you if you make this all these things your transforms are d and e q at your dq 0 transformation eliminating phase voltage and currents in terms of dq 0 components I mean you substitute, all right.

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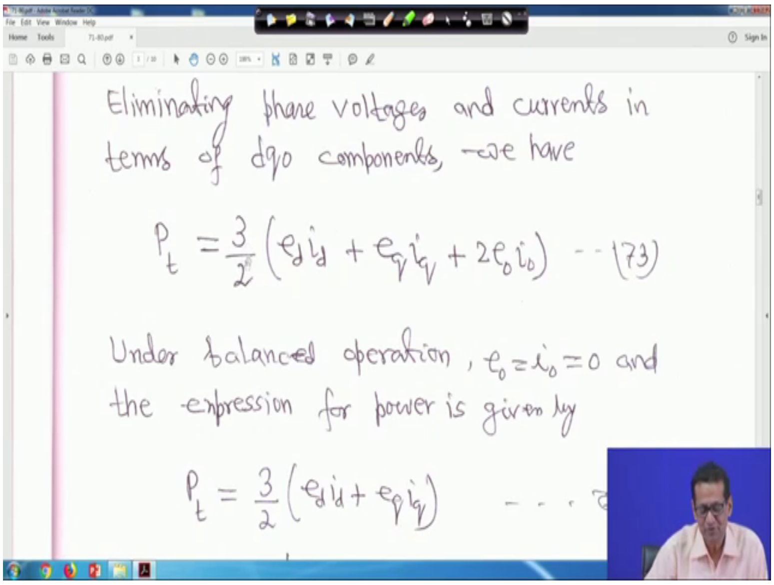

Wherever, e a i a, e b i b, e c i c all the in terms of your dq component and if you simplify then you will get P by 2 will be 3 by P t will be 3 by 2 into e d i d plus e q i q plus 2 e 0 i 0, 73. This also you should keep it in your mind it will be 3 by 2 into e d i d plus e q i q plus 2 e 0 i 0. After making all these all the you substitute all and you simplify then only you will get this one, right. And under balance operation e 0 i 0 is equal to 0 therefore, the expression per power is given by 3 by 2 e d i d plus e q i q, right.

Now, using equation 70 to 72, right to express the voltage component in terms of flux linkage and currents by recognising omega r as the rotor speed is d theta by d t and rearranging we have.

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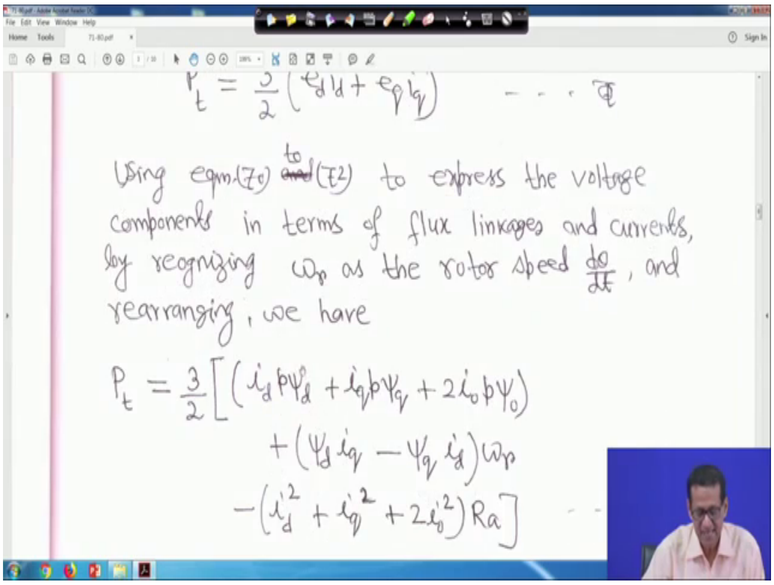

Now, here what you do that e d e q expressions are known to you put everything in this expression because that is equation your 70, 71 and 72 put every everything and you substitute here.

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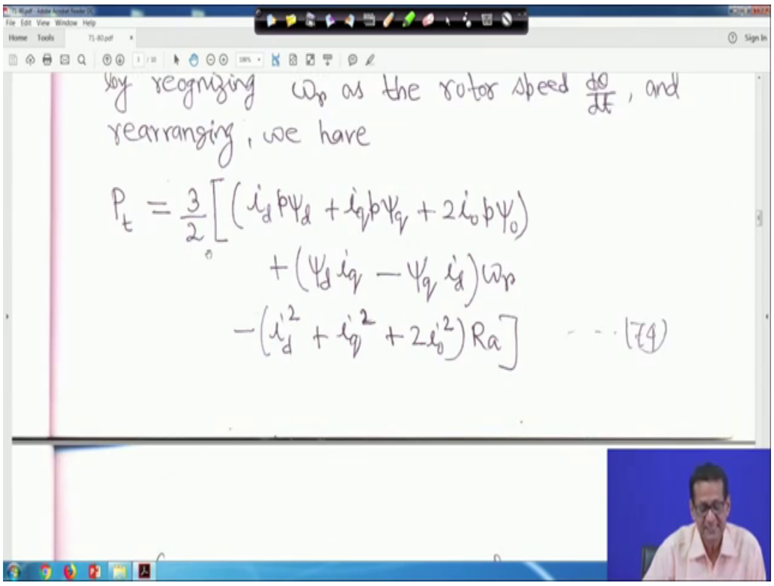

If you do so, you will get this term 3 by 2 into your id p psi d plus i q p psi q plus 2 i 0 p psi 0 plus psi d i q minus psi q i d into omega r minus in bracket i d square plus i q square plus 2 i 0 square bracket close into r a this is equation 74. This expression you will get, but each term has separate meaning. For example, the first term that is your i d p psi d plus i q p psi q plus 2 i 0 p psi 0 this is actually rate of change of armature magnetic energy, right. This is the 1st term.

Now, 2nd term is more important second term is psi d i q minus psi q i d into omega r this is actually second term is the power transferred across the air gap. This transfer this term is the power transfer your transform across the air gap. And third one that is i d square plus i q square plus 2 i 0 square into r a this is nothing but the armature resistance loss.

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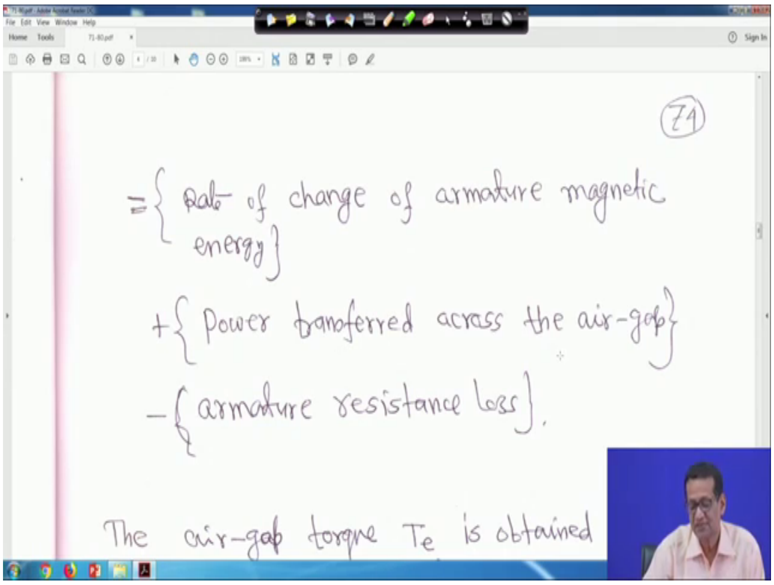

So, this is the first term has some significant, second term also, third term also that is why it is written first term is rate of change of armature magnetic energy plus second term is power transferred across the air gap of the machine and third term is the armature resistance loss, right.

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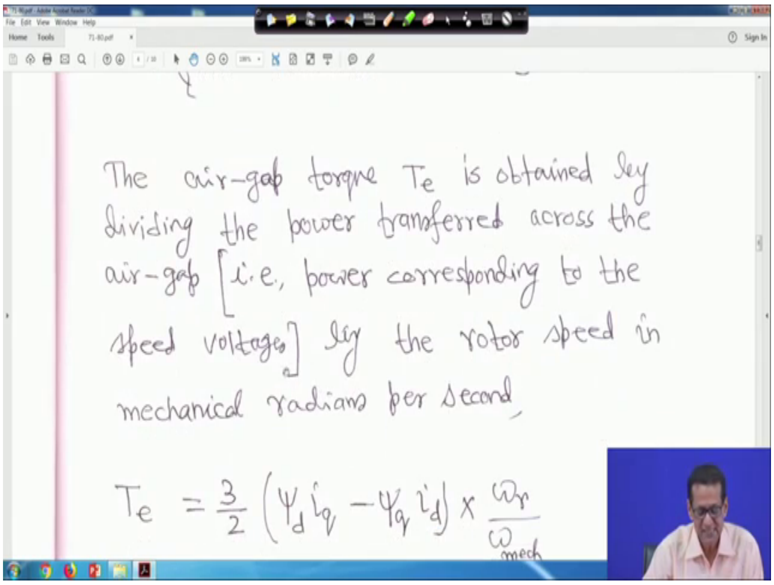

Now, therefore, the air gap torque we know p in general power is equal to torque into angular speed that we know, right. Therefore, the air-gap torque T e is obtained by dividing the power transferred across the air-gap that is power corresponding to the speed voltages that means, this term this is the second term is the power; second term is the power.

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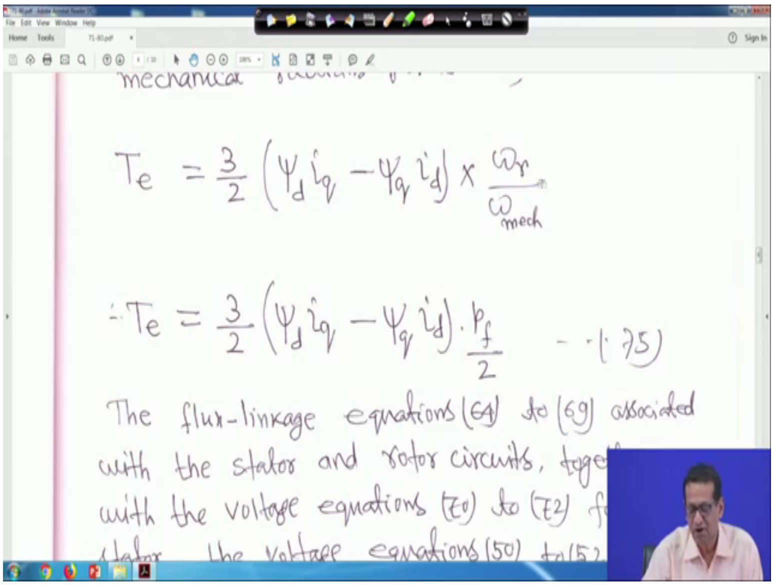

Then if you want to make it torque, so it will be T is equal to 3 by 2, psi d i q minus psi q i d into omega r upon your omega mechanical because this is your power and power generally we know torque into speed. So, T into omega mechanical is equal to 3 by 2 psi d i q minus psi q i d omega that means, this term; this term this second term, this second term, right this second term, so that means, this is my torque equation.

So, or you can simplify omega r upon omega mechanical earlier we have seen this is nothing but the number of field poles by 2. So, it will be 3 by 2 psi d i q minus psi q i d into p f by 2, right this is equation 75.

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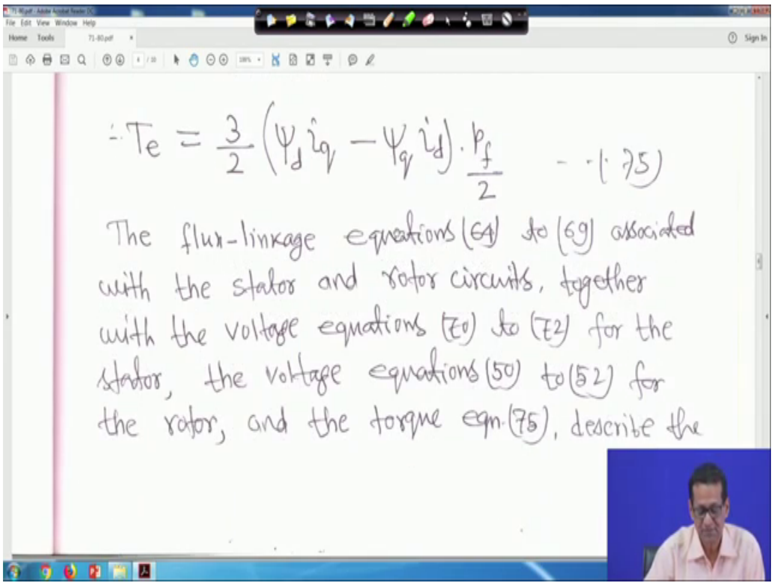

Now, the flux linkage equation that is 64 to 69 associated with the stator and rotor circuit. Together with the voltage equation that is 70 to 72 for the stator and the voltage equation 50 to 52 for the rotor and the torque that is equation 75 actually describes the electrical dynamic performance of the machine in terms of dq 0 component.

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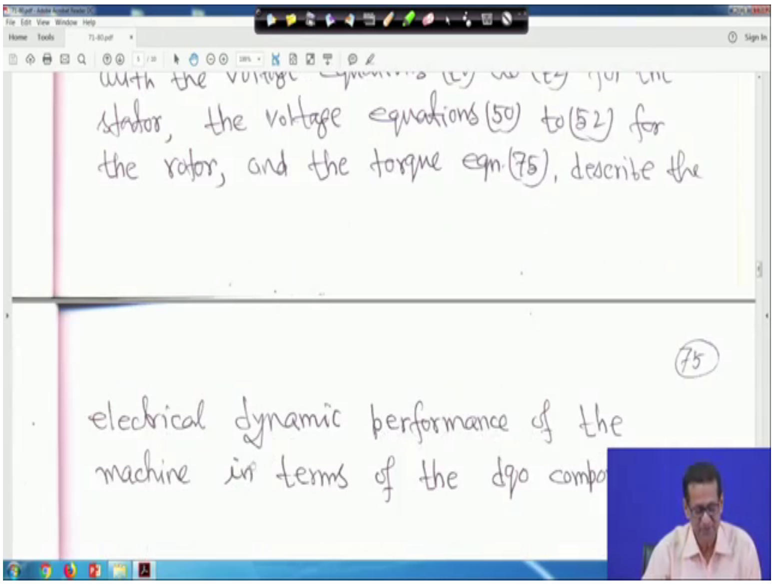

So, all these equation basically 64 to your 75 all these equations actually represent the dynamics of the equation related to the dynamics of the machine much more detail we will see later, all right.

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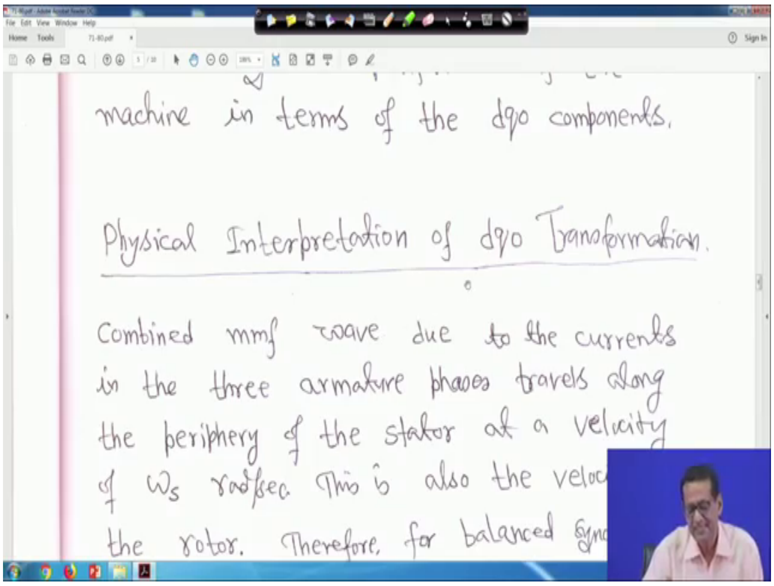

Now, physically interpretation of dq 0 transformation, why you do so; that is the physical interpretation. Now, combine mmf wave due to the currents in the 3 armature phases travels along the periphery of the stator at a velocity of omega s radian per second, right.

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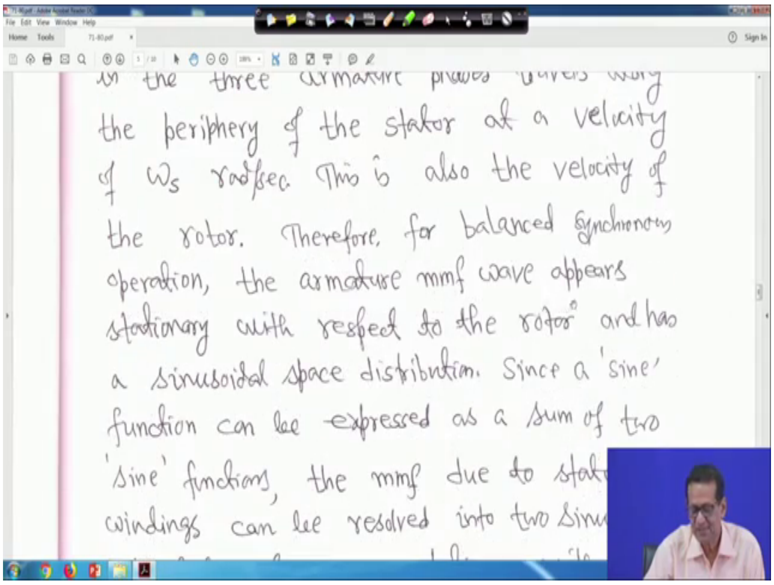

This is also the velocity of the rotor because the rotor in synchronous phase. Therefore, for balanced synchronous operation the armature mmf wave appears stationary with respect to the rotor and has a sinusoidal phase distribution this also we have discussed before, right.

Since, a sine function can be expressed as a sum of 2 sine functions, right the mmf this also we have seen the mmf wave mmf d-axis, mmf q-axis, right the mmf due to stator windings can be resolved into 2 sinusoidally distributed mmf wave stationary with respect to the rotor.

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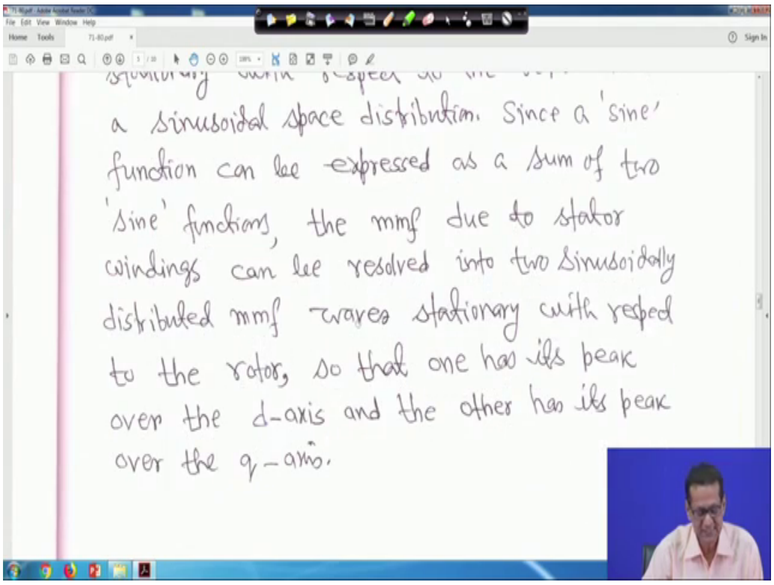

So, the one has it is peak over the d-axis and the other has it is peak over the q-axis this diagram we have seen earlier also, right.

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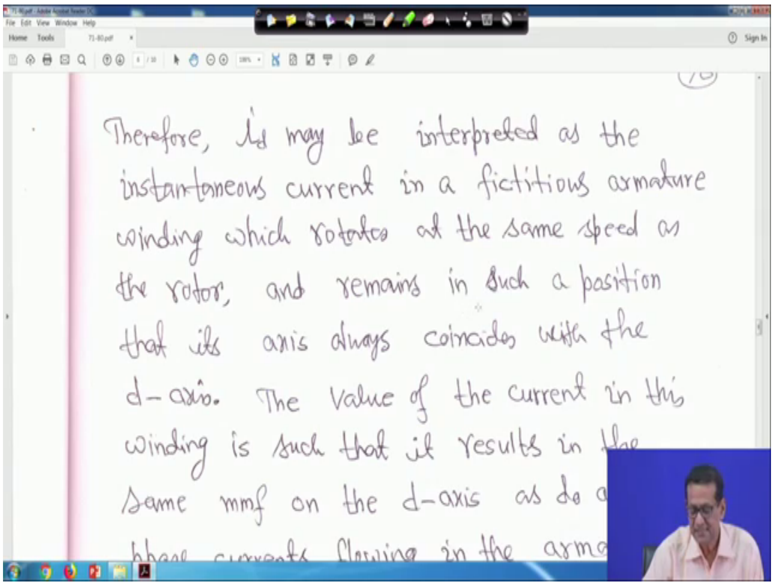

Therefore, it may be interpreted as the instantaneous current in a fictitious armature winding which rotates at the same speed as the rotor and remains in such a position that is axis always coincide with the d-axis. The value of the current in this winding is such that it results in the same mmf on the d-axis as do actual phase current flowing in the armature windings.

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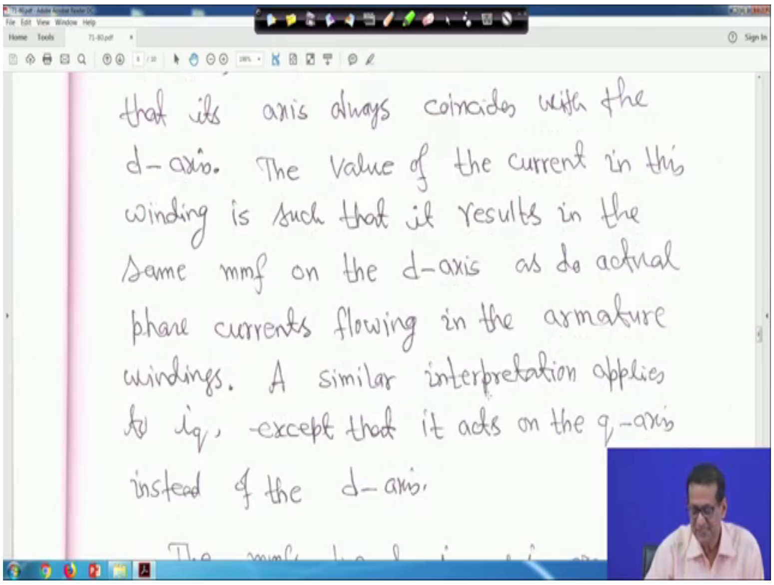

A similar interpretation applies also to i q except that it acts on the q-axis instead of the d-axis. This certain thing this is the physical interpretation.

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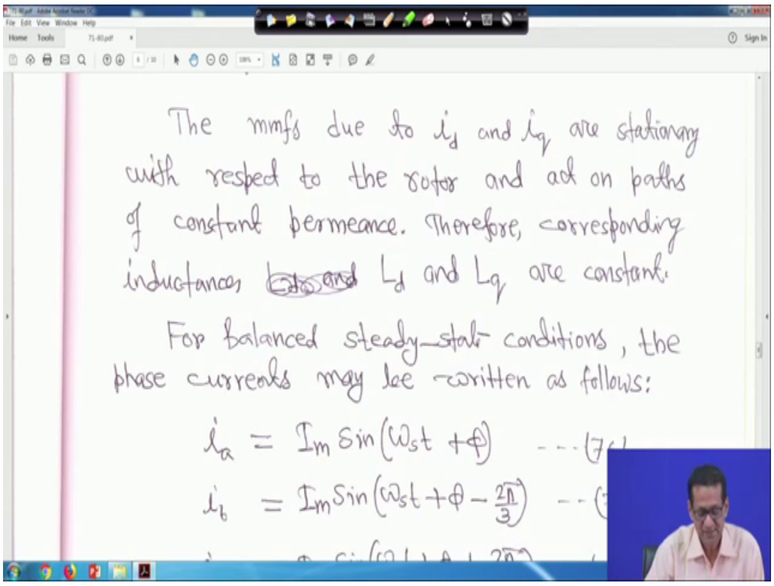

The mmf due to i d and i q are stationary with respect to the rotor, right and act on paths of constant permeance. Therefore, your corresponding in your it inductances L d and L q then d-axis and q-axis are constant. For balance steady state conditions the phase currents may be written as follows, right.

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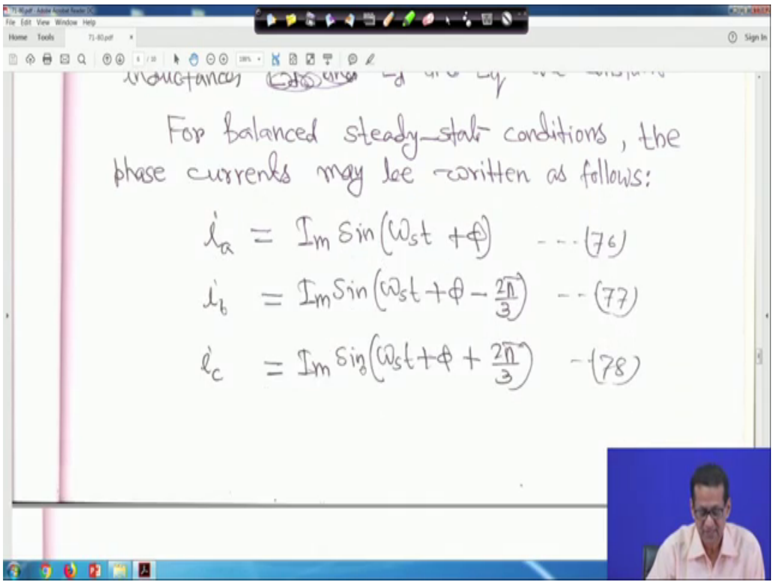

We can write i a is equal to I m sin omega s t plus phi, i b is equal to I m sin omega s t plus I phi minus 2 pi by 3 and i c is equal to I m sin omega s t plus phi plus 2 pi by 3. This is 76, this is 77 and this is 78, right.

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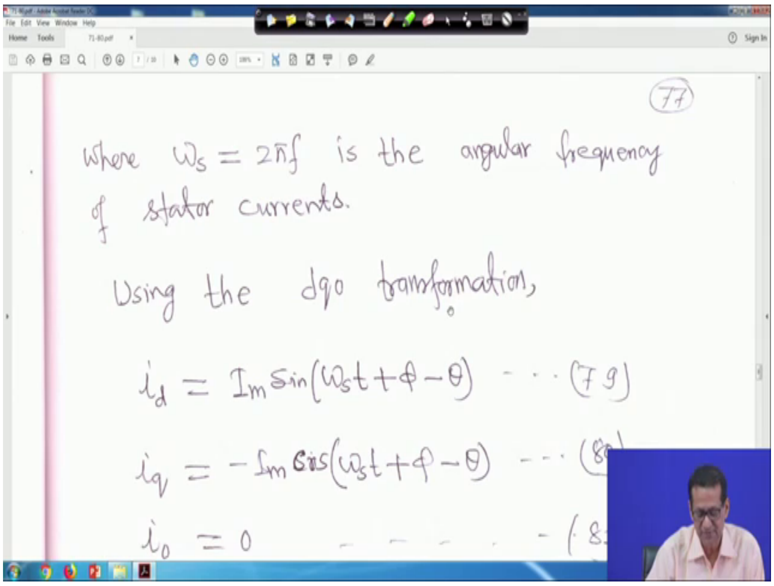

Now, omega is, omega s is equal to 2 pi f that is that angular frequency of the stator current, right using the dq 0 transformation. So, this one if you go for a dq 0 transformation that this equation i d can be written as I m sin omega s t plus phi minus theta because here also your omega s t plus phi, right.

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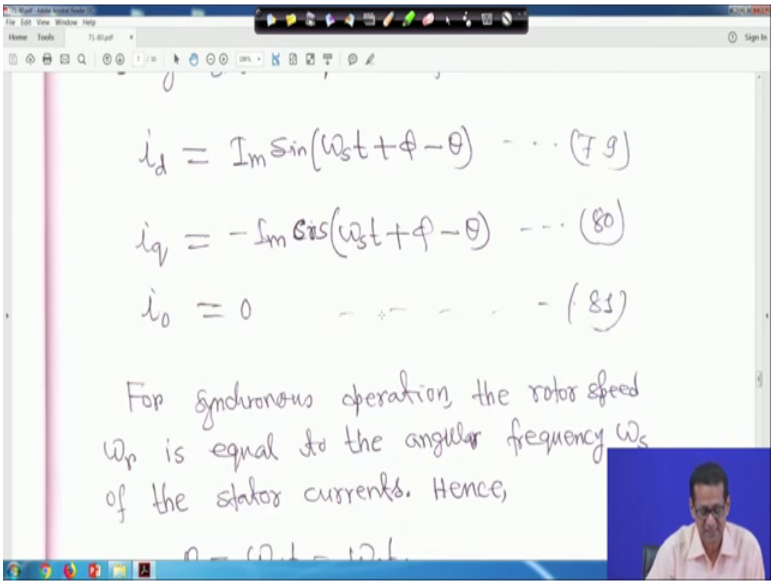

Now, even you go for d q transformation i d will be I m sin omega s t plus phi minus theta similarly i q will be minus I m cos omega s t plus phi minus theta this is equation 80 and i 0 will be 0 that is equation 81. Now, we know that because it your what you call machine rotates in say synchronous speed. So, basically theta is equal to omega r t is equal to omega s t.

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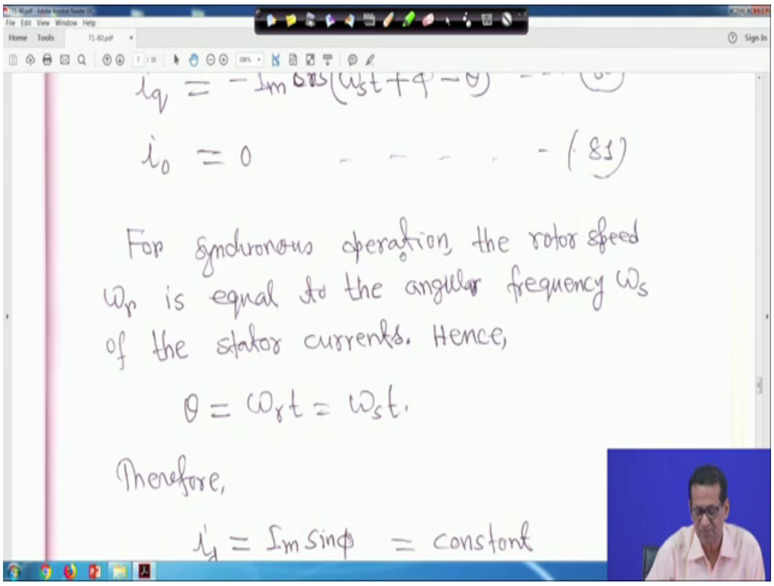

That means, for synchronous operation the rotor speed omega r is equal to the angular frequency omega s of the stator current, right same thing. Therefore, theta is equal to omega r t is equal to omega s t that means, here if you put theta is equal to omega s t theta is equal to omega s t then omega s t omega s t will be cancel only sin phi will be there and cos phi will be there.

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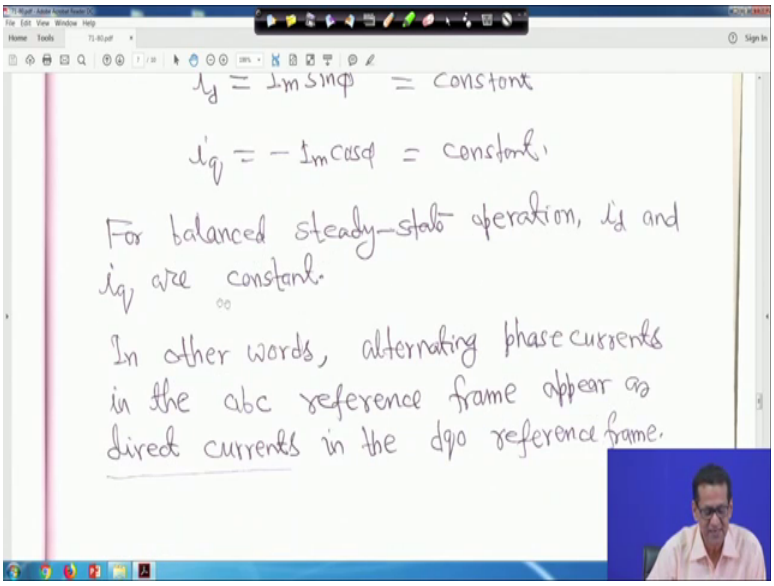

So, i d is equal to I m sin phi. So, it is a constant and similarly i q is equal to minus I m cos cos phi that is also constant as if their DC current; as if their DC current, right. For balanced steady operation i d and i q are constant, right. So, in other words alternating phase current in the abc reference frame appear as direct currents in the dq 0 reference frame, right. This is your, this is that physical interpretation of your dq 0 transformation, right.

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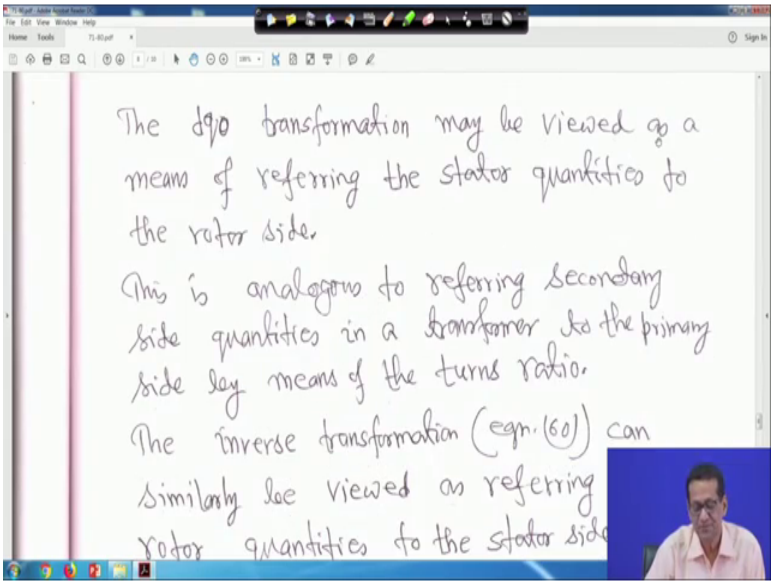

Now, the dq 0 transformation may be viewed as a means of referring the stator quantities to the rotor side. This is advantageous to referring secondary side quantities in a transformer to the primary side by means of the trans ratio.

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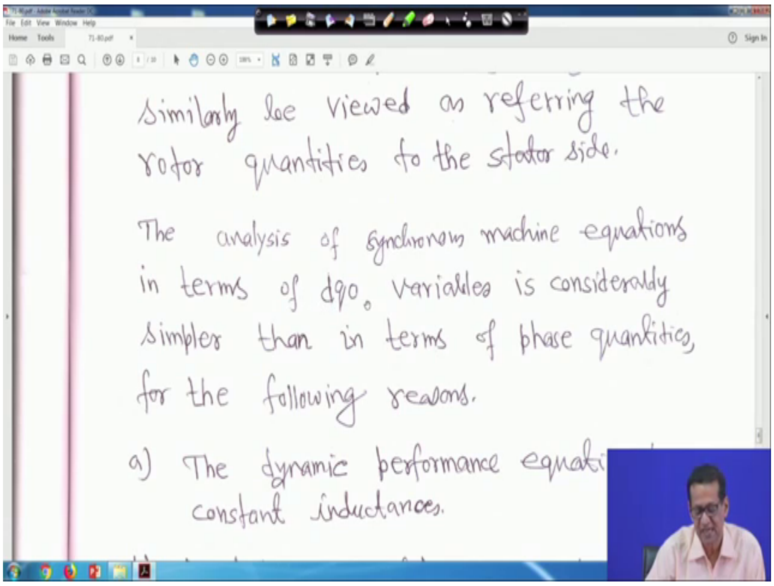

So, the inverse transformation that is equation 60 can similarly be viewed as a refereeing to the rotor quantities to the stator side, right. The analysis of synchronous machine equations in terms of dq 0 variables is considerably simpler then the terms of phase quantities in the following reason. Some advantageous are there for dq 0 transformation.

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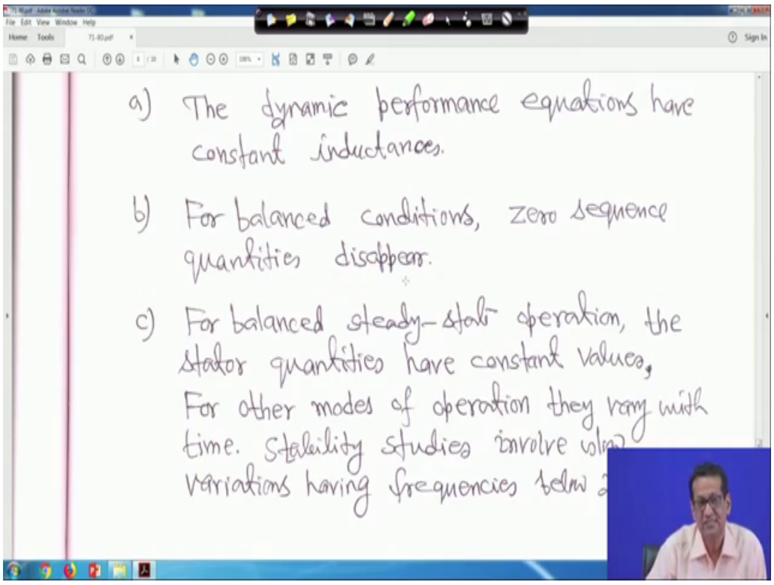

First one is the dynamic performance equation have constant inductances this is the major advantage, right. Next is for balanced conditions 0 sequence quantities disappear, right.

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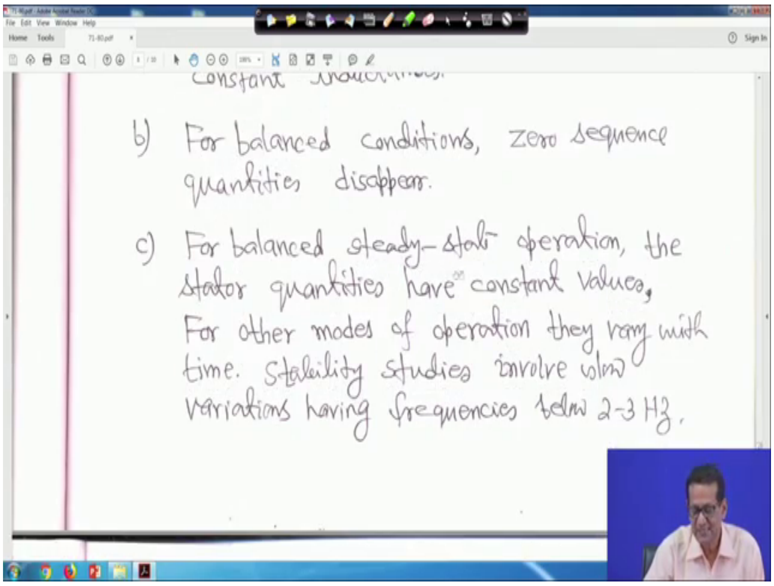

For balanced steady state operation the stator quantities have constant values for other modes of operation they may vary with time and stability studies involved your slow variations having frequencies below your 2 or 3 hertz, right.

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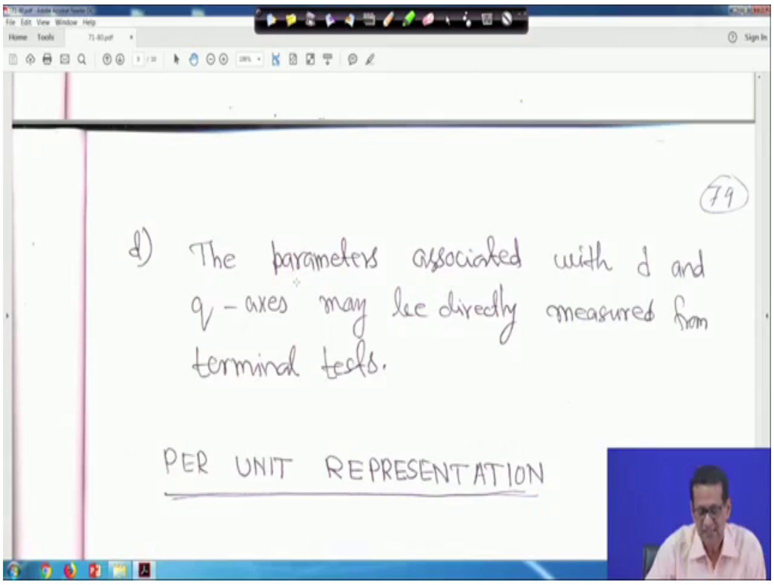

And fourth one the parameters associated with d and q-axis may be directly measured from your terminal tests, right. So, these are the 4 major advantages for dq 0 transformation.

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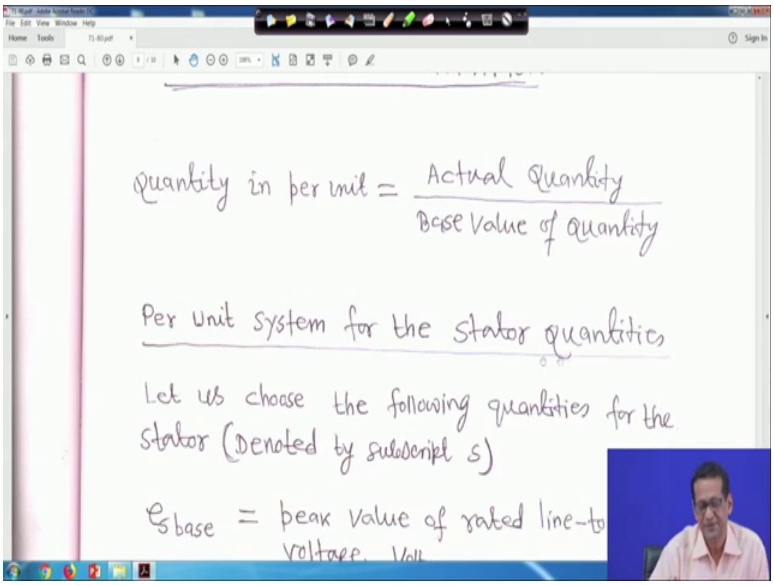

Next is the per unit representation. In power system analysis we have already studied the per unit representation, right, but here for synchronous machine although will follow the same thing, but per unit representation will be slightly different, right. And whole heartedly we will try to understand this certain things I will tell you certain things small derivation I leave up to you also to save some time, but just see how things are the per unit representation. Now, your quantity in per unit can be defined as actually quantity by base value of the quantity this you know. Now, per unit system for the stator quantities, right.

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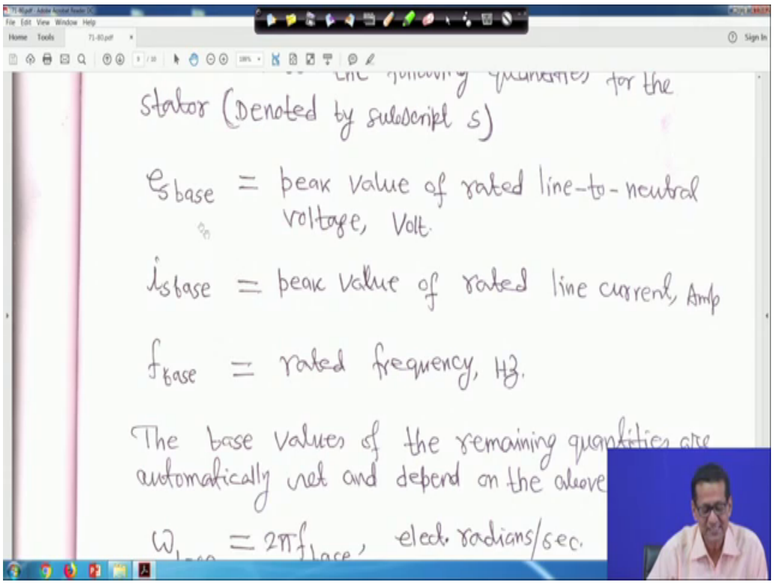

Now, let us choose the following quantities for stator denoted by subscript s, right. So, we have to choose some your what you call your base quantities. For example, e s base this is actually peak value of rated line to neutral voltage in volt, right. Then i s base it is peak value of rated line current that is ampere and f base that is rated frequency in hertz, right, so this base value. Now, the base values of the remaining quantities are automatically set and depend on the above as follows.

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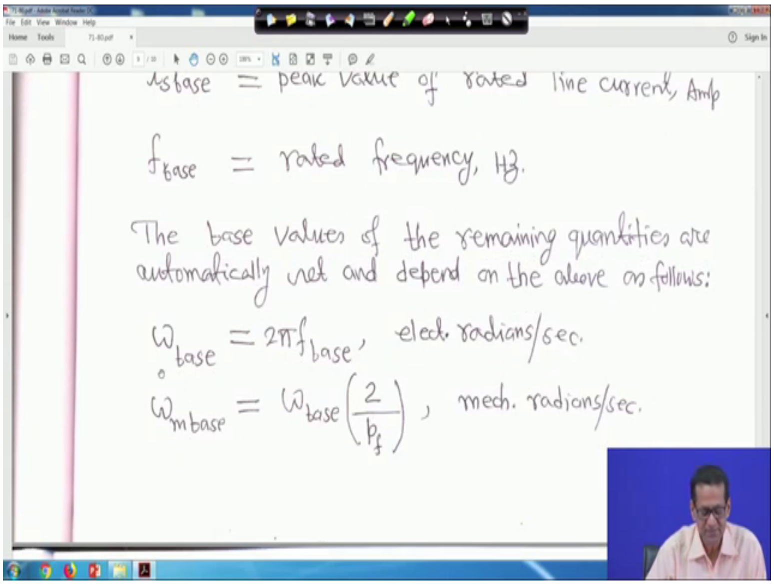

For example omega base here also we need some omega base it will be 2 pi f base that is electrical radian per second then omega mechanical base, right that is omega base into 2 by field force, right p f. So, that is your mechanical radian per second.

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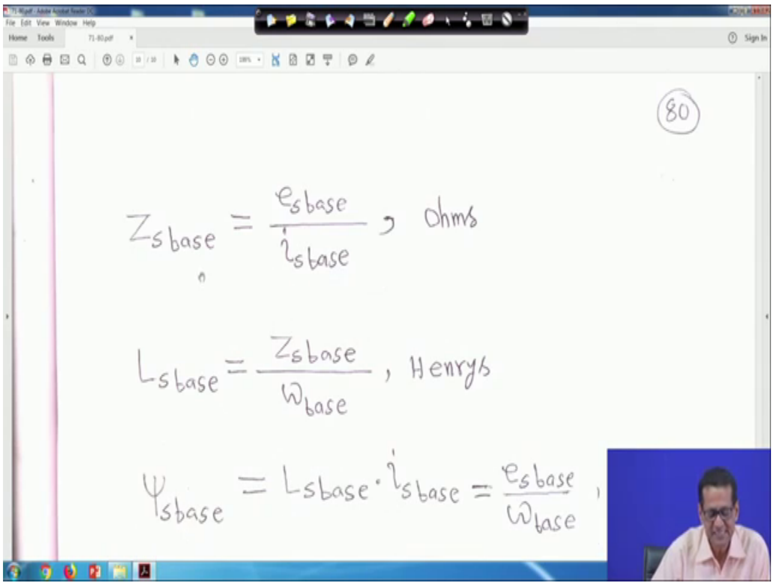

Now, if it is so then Z s base will be now e s base upon i s base that basically Z is equal b by i you know. So, Z s base is equal to e s base upon i s base this is ohm, right. Now, L s base L s base will be your Z s base upon omega base Henry’s, right. So, actually when you find the actually we are going to making the base quantities when you try to find out say reactants of a line, right.

What we do? We make L omega, right L omega is the reactants. So, here also if you multiply L s base and omega base that actually in per unit we want to represent. So, you represent L s base and into omega base is equal to Z s base, right. Instead of your reactance type we will this is actually Z s base, right dimensionally we have to see that correct thing. So, L s base will be Z s base upon omega base this is in Henry's.

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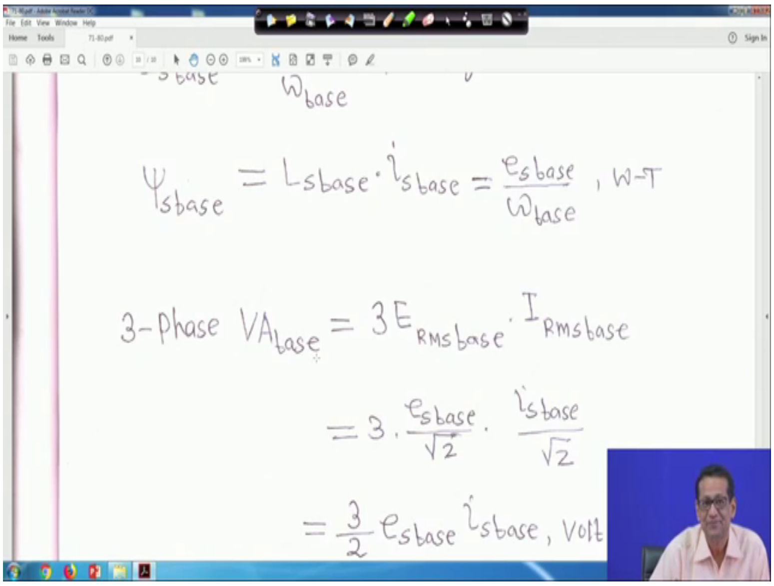

Now, psi s base that is the flux linkages; we know psi is equal to Li, right that means, psi s base will be L s base into i s base, right and this is your L s base and another thing is that can be written as e s base upon omega s base, right. So, this is also this is also your known to known to you, right because you know e is equal to your d psi by d t in general dimension wise, right. So, this one we make e s base upon omega s. So, L s base into i s base is equal to e s base upon omega this is omega turn, right.

Now, next is 3 phase volt ampere base. So, 3 phase volt ampere base means we will take 3 into E RMS base into I RMS base that is the RMS value of the voltage and RMS value of the current this capitally RMS, right. So, this can be written as 3 into E RMS base will be e s base upon root 2 because we have chosen this your e s base the peak value of the rated line to neutral voltage. Similarly, for the current peak value of rated line current.

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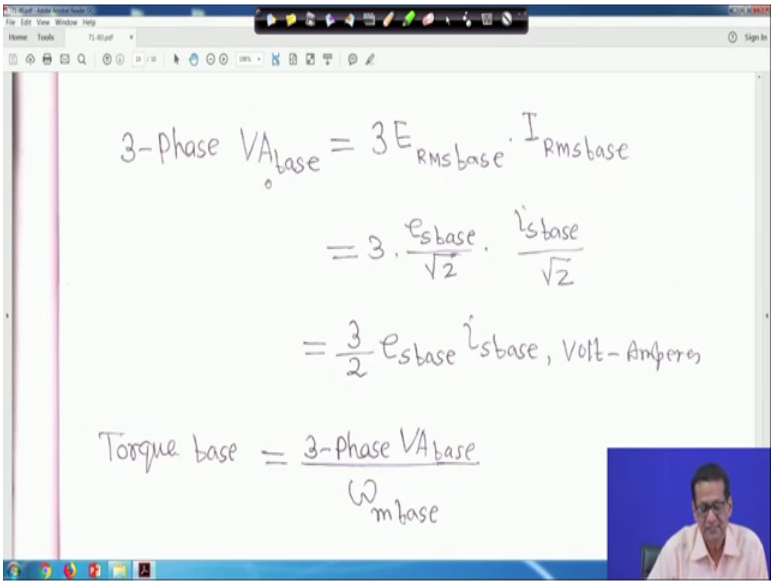

So, when you take capital E RMS, right, basically this is this E RMS base we can write e s base upon root 2 and I RMS we could write i s base upon root 2. So, root 2 root 2, 2 will be 2. So, 2 2 will be your what you call it will be then 3 by 2 e s base into i s base volt ampere. This is your 3 phase volt ampere base, right.

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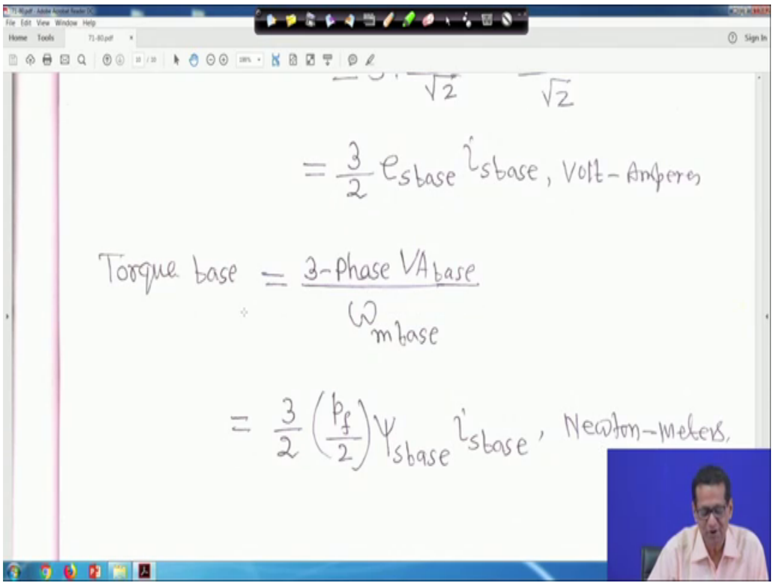

Now, torque base, we know torque base is equal to in general we know power is equal to speed into torque, right, but we are trying to represent the base quantities. So, torque base will be 3 phase volt ampere base divided by omega mechanical base, right. So, if you and e s this is your what you call this e s i s base, right here it is in your what you call here it is given that your this is your just hold on. This is your psi s base is equal to given that is L s i s base into i s base and it is equal to e s base suppose omega base.

So, here also when you write torque is equal to your 3 phase VA base by omega mechanical base. So, in this case this 3 phase volt ampere base is 3 by 2 e s base i s base you substitute here 3 by 2 e s base i s base and this is your omega mechanical base. Just simplify this one, just simplify this one you will get 3 by 2 p f by 2 psi s base into i s base that is Newton your meters, right.

So, here already it is given something is given that your what you call psi s base is equal to L s base i s base everything is given, right. Just you please just to substitute and just to bring this one that it will be psi s base into i s base just. Just little bit you manipulate you put this one, you put this one e s base and i s base and just to meet this one omega base your what you call omega r upon omega s base relationship you put it here and then little bit you simplify you will get 3 by 2 p f by 2 psi s base psi s base Newton per meter. This is your what you call an exercise small exercise for you, right. So, this is your torque base.

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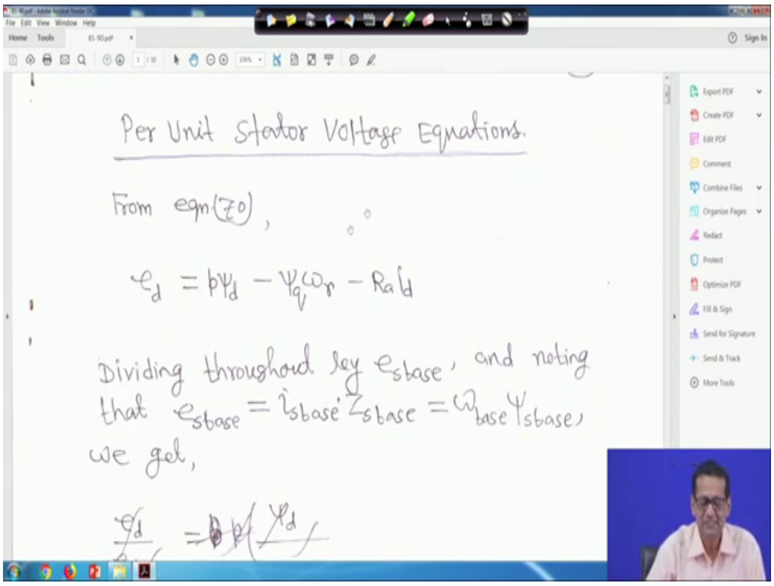

Next is, per unit stator voltage equation we will come now after making all these small thing we will come to this one. So, from equation 70 this is the equation e d is equal to p psi d minus psi q omega minus R a i d. Now, we have to make the per unit stator voltage equation, right you have to convert it. So, you what you can do it dividing throughout e by e s base and noting that e s base is equal to i s base into Z s base is equal to omega a base into psi s base.

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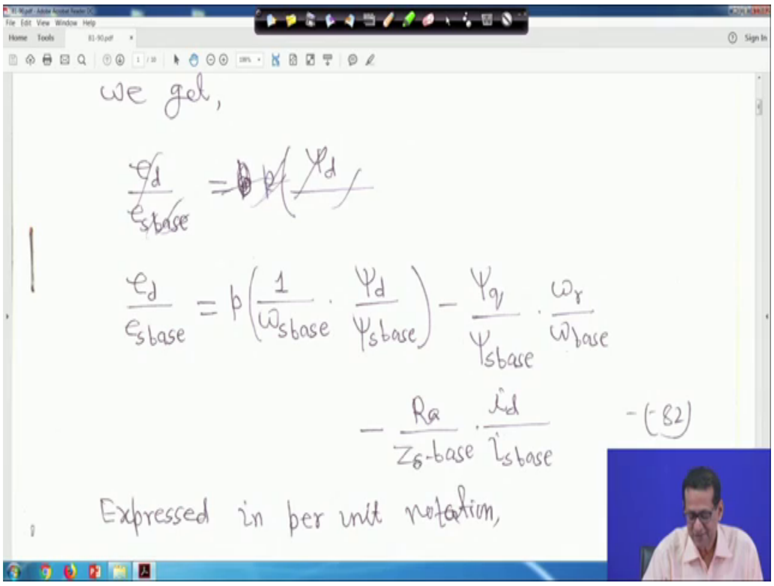

So, if you divide both side e d by e s base. So, this side also you can divide both side that e s base is equal to you know that i s base Z s base. So, that is your what you call that your this is my p psi d. So, both side we are dividing by e s base. So, e d by e s base that this term when you are taking p psi d we are making it as i s base your what you call omega base then psi s base. So, this term divided by omega s base here I have made e is a s here right. So, your this thing is omega s base psi s base.

And for the second term; second term psi q omega r here also we are making psi s base omega s base and last term it is a voltage drop R a i d, right what we are doing is we are making your z s base i s base because e s base is equal to Z s base. So, this term also e s base this term also e s base, but we have to make it per unit form and this relationship, we are using this relationship we are using, right.

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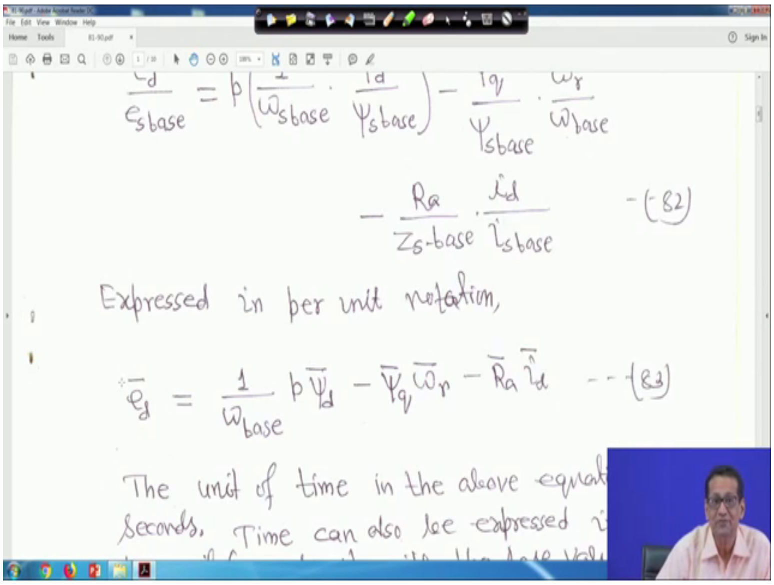

So, with this with this if you do, so then what we will see? We will see that this e d, e d upon e s base now e d upon it is a per unit. So, we are making e d bar. So, this is actually e d bar. After making all the per units again we will remove the bar and from that onwards we will understandable that everything is in per unit, that I tell you later e d bar. And this is your psi d upon psi s base. So, this is psi d bar p into psi d bar then, right and into 1 upon omega base. And this one psi q upon psi s base this is basically psi q bar and omega r upon omega s base that is basically omega r bar and this 1 minus R a upon Z s base, so per unit, so R a per unit is R a bar and i d upon i s base this is i d bar.

So, the all these things your what you call e d bar is equal to this equation your what you call the per unit. Whenever putting bar means these are per unit values, right because ultimately you will find at the latest stage things are very simple and we will make it all everything in per unit.

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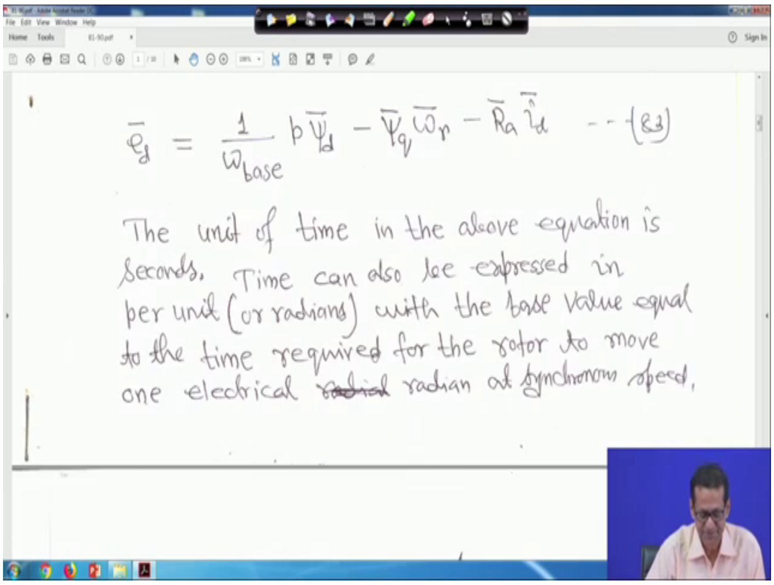

The unit of time in the above equation is second, right, but most literature also we will find we will generally use time in second, but I will show you if you want per unit also time can be represented by per unit time, all right. So, time also can be expressed in per units with the base value equal to the time required for the rotor to move one electrical radian at synchronous speed, right.

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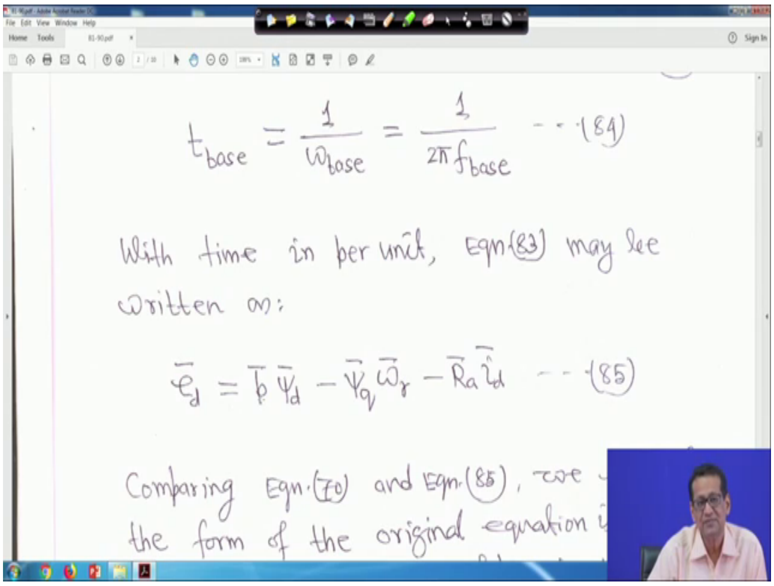

So, what we can do is that t base actually 1 upon omega base because generally omega is a radian per second radian is a dimensionless quantity so, it is radian per second. So, t base we will take omega 1 upon omega base that is nothing but your second, right that is 1 upon 2 pi f base, right, this is equation 84. With the time in per unit equation 83 it may be written as e d bar is equal to p bar psi d bar minus psi q bar omega r minus R a bar i d bar. Even time also p ddt that is also we are converting in per unit how I will show you. Next lecture I am coming back, ok.

Thank you very much, we will back.