Power System Dynamics, Control and Monitoring
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Lecture - 06
Power System stability (Contd.)

So, we are back again. So, similarly you can write now flux linkages of phase b and phase c.

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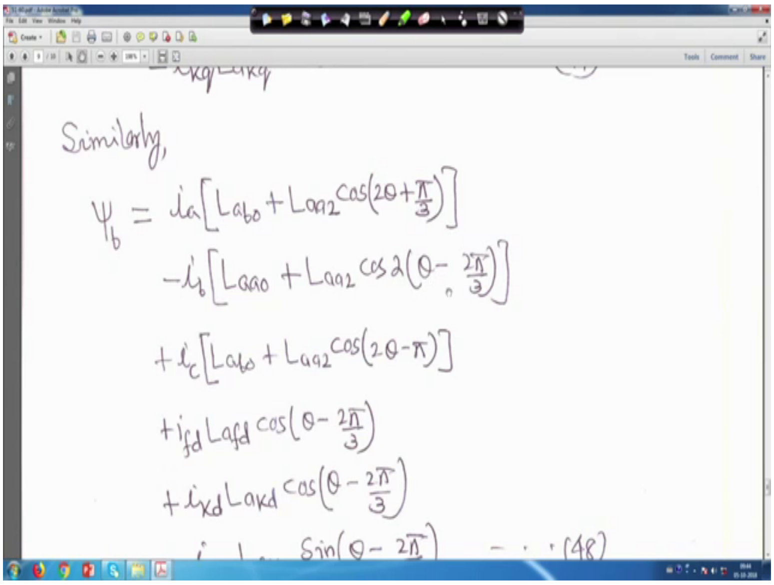

So, similarly you can write that for your phase b psi b is equal to i a into L a b 0 plus L a a 2 cos 2 theta plus pi by 3. These all we have seen, just you put it now. Then minus your i b into L a a 0 plus L a a 2 cos 2 into theta minus 2 pi by 3.

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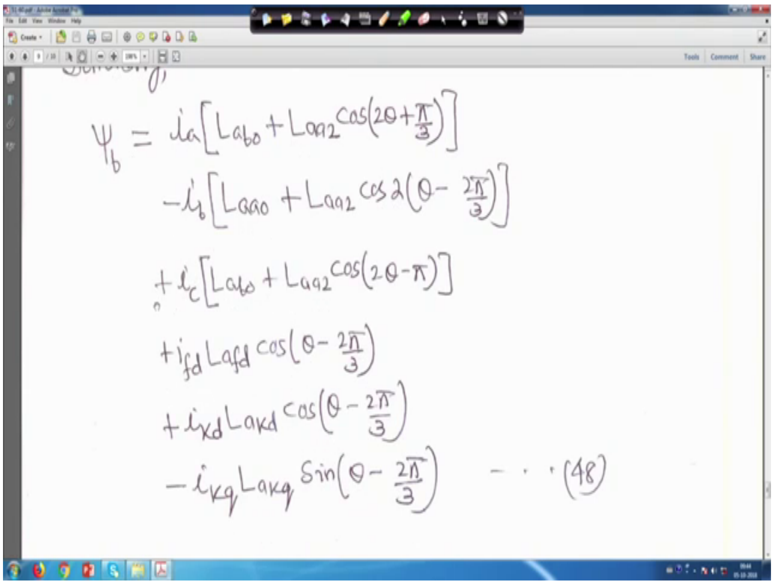

Then, plus i c into L a b 0 plus L a a 2 cos 2 theta minus pi plus i fd L a f d cos theta minus 2 pi by 3 plus i kd L a kd cos theta minus 2 pi by 3 minus i kq L a kq sin theta minus 2 pi by 3.

Whenever you are going through this equation and these things first thing is that figure 9 should be there in front of you. Second thing is all the nomenclature also should be there in front of you and then you will find initially perhaps so many terms are there, but initially perhaps you will find things are little difficult, but it is not difficult. You keep the figure 9 in front of you and keep the your all the nomenclature in front of you and then just write step, see and write see and write you will find things are simple, right.

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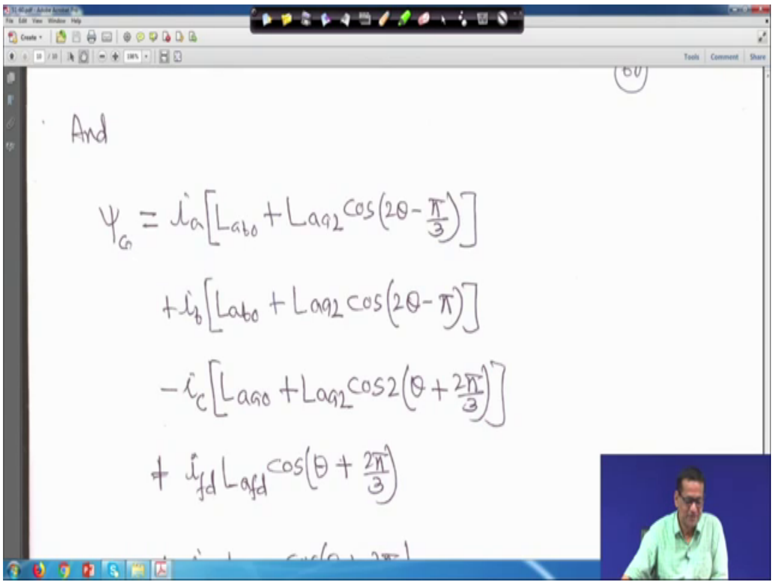

And similarly for your psi c will be i a earlier only earlier in equation time we where equation 29 we wrote only psi a psi b psi c is not written. So, now, directly you are substituting, but they are also looking at those figure 9, looking at figure 9 you can write for psi b and psi c, but directly we are writing here, right.

So, psi c will be i a into L a b 0 plus L a a 2 cos 2 theta minus pi by 3 plus i b L a b 0 plus L a a 2 cos 2 theta minus pi minus i c L a a 0 plus L a a 2 cos 2 into theta plus you are what you call pi by 3, right.

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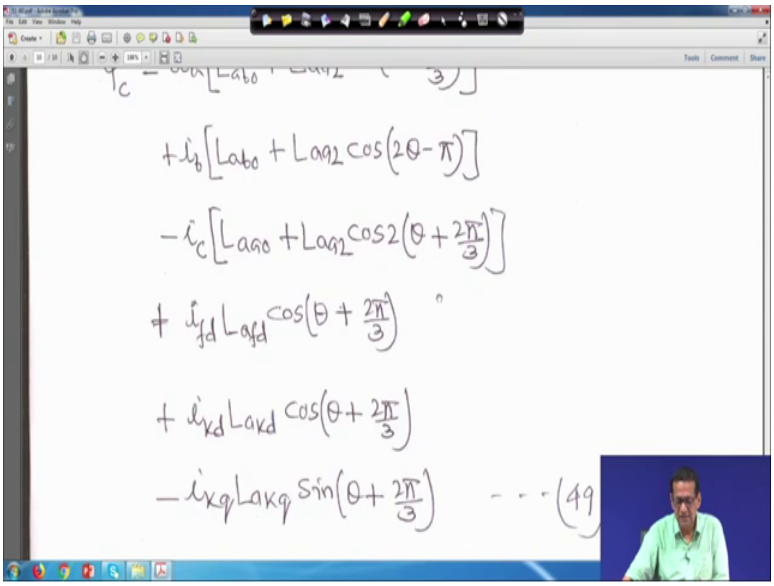

So, this one your what you call your plus i fd L a f d cos theta plus 2 pi by 3 plus i kd L a kd cos theta plus 2 pi by 3 and minus I k q L a a kq sin theta plus 2 pi by 3. So, now flux linkage for your psi b, psi c and all these things, right all these things are your what you call derived, right.

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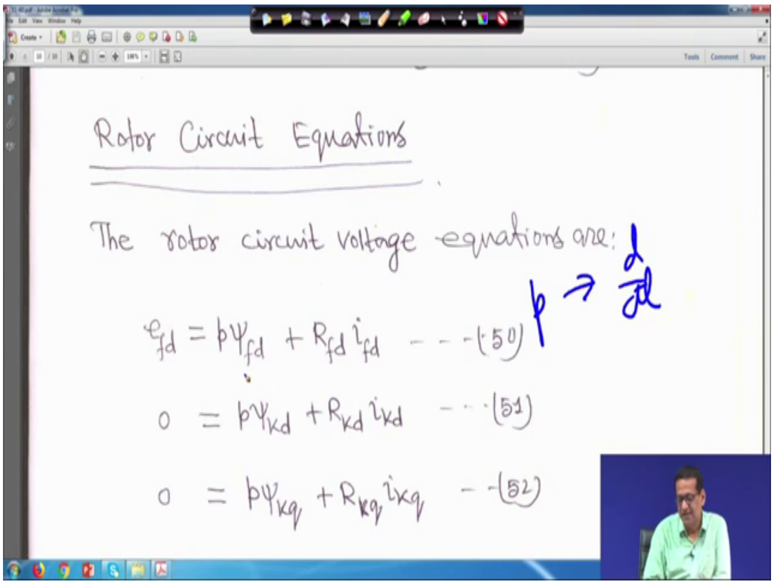

So, only thing is that I will tell one thing that as if this course is a particularly asynchronous machine part as its you know many you know mathematics are involved. So, if you find any writing error from my side, you please your just send me a mail or put it in the forum. If any error you find particular in this equation is writing I rectify that, right. Hope everything is correct, but if you find any error or if you have any doubt please put a question in the forum, right.

Now, stator part now is over stator rotor mutual inductance all these. Now, rotor circuit equation will come because we have this is the rotor circuit means on the d axis you have field, field winding as well as armature winding similarly on that your q axis you have only armature winding. So, the rotor circuit voltage equation will be filed voltage will be p is actually that is your d by d t, right. So, this p is equal to d by dt. So, rotor circuit equation actually your e f d is equal to p psi fd plus r fd i d this is equation 50 and your on d axis you have armature winding and there it is your what you call circuit is a close circuit, so no voltage there.

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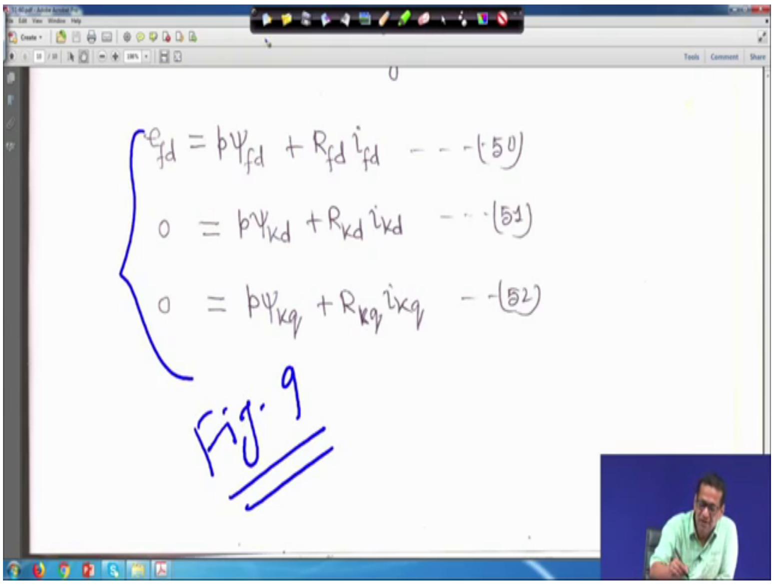

So, 0 is equal to your p psi kd plus R kd i kd this is equation 51 and your q axis you have another armature winding. So, 0 is equal to because it is a close circuit, so 0 is equal to p psi kq plus R kq i kq all these things all these things please refer to figure 9, right. So, just see the figure 9 this is simple equation, right. So, this is your, what you call rotor circuit equation. So, just hold on.

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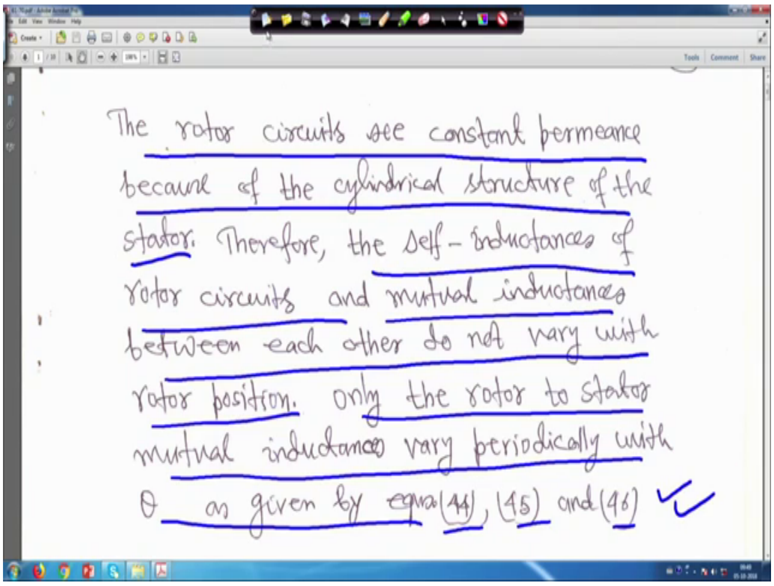

So, the rotor circuit see constant permeance because of the cylindrical structure of the stator, right. So, the rotor circuit see constant permeance because of the cylindrical structure of the stator, right. Therefore, the self inductances of rotor circuit and mutual inductances between each other do not vary with rotor position. Only the rotor to stator mutual inductances vary periodically with theta as given by equation 44, 45 and 46 these we have derived, right.

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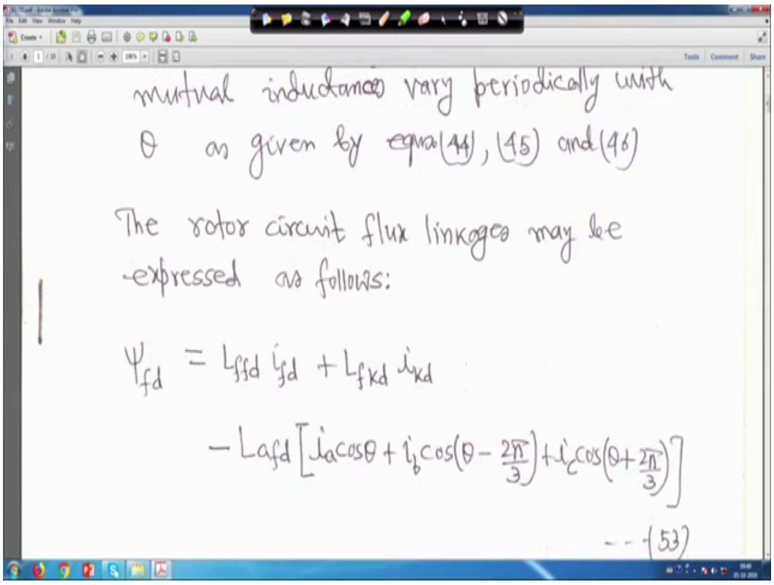

So, that means, that your rotor circuit flux linkages may be express as follows.

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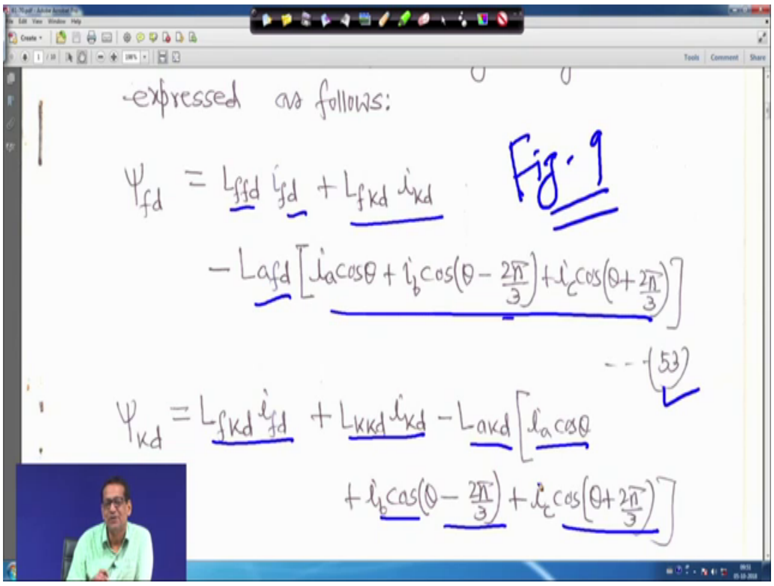

So, psi fd looking at again whenever I am writing this equation whenever I am writing this equation all the time please see the figure 9, right, figure 9.

So, in this according to the your direction of the current and how the flux linkages all have been told in the beginning, right So, L ffd i fd plus L fkd i kd, right minus L afd into take the projection of all the stator current is all phase a, b and c. So, it will be i a cos theta plus i b cos theta minus 2 pi by 3 plus i c cos theta plus 2 pi by 3 this is equation your 53, right.

This your because this L afd actually it is that mutual inductance between the stator and the your rotor, right. So, this that means, this lf L ffd i fd plus L fkd i kd and minus this your all these things this L afd into simple thing i cos theta plus i b cos theta minus 2 pi by 3 plus i c cos theta plus 2 pi by 3, right.

Similarly for psi kd because you have armature winding on the d axis also again referring to figure 9 again and again. So, there also L fkd i fd plus L kkd i kd minus L kkd same thing i a cos theta plus i b cos theta minus 2 pi by 3 and plus i c into cos theta plus 2 pi by 3. So, it will be L fkd i fd plus your L kkd i kd minus L akd into this bracket the same term i a cos theta plus i b cos theta minus 2 pi by 3 plus i c cos theta plus 2 pi by 3. From the symmetry we are writing actually equations it is equation many terms are there, but just see the winding and just put it that is all very simple, right. Just hold on.

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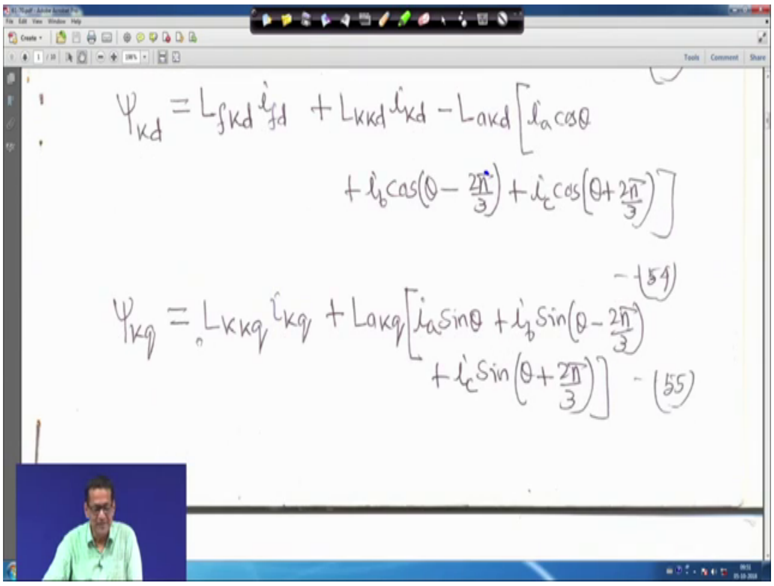

So, similarly you have just hold on. Similarly you have your armature winding on the q axis, so there it will be L kkq i kq plus L akq it will be i a sin theta plus i b sin theta minus 2 pi by 3 plus i c sin theta plus 2 pi by 3 because the angle between your what you call d axis and q axis it is 90 degree apart. So, that is why L kkq i kq plus L akq i a sin theta plus i b sin theta minus 2 pi by 3 plus i c sin theta plus 2 pi by 3 this is equation 55, right.

Now, next is because these all flux linkages then we your equations we have seen in that senses everything, but this direct analysis is this slightly difficult.

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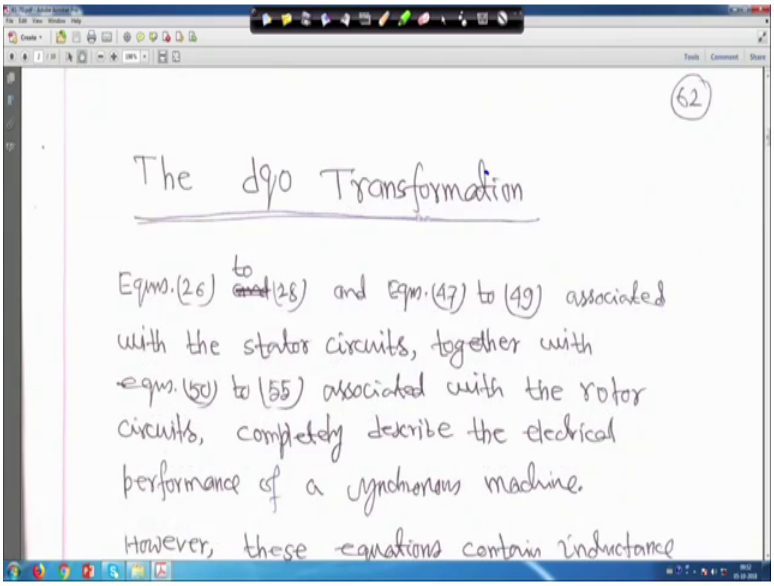

So, we will make some your, what you call that dq0 transformation, right direct axis coordination axis and stationary axis this dq 0 transformation.

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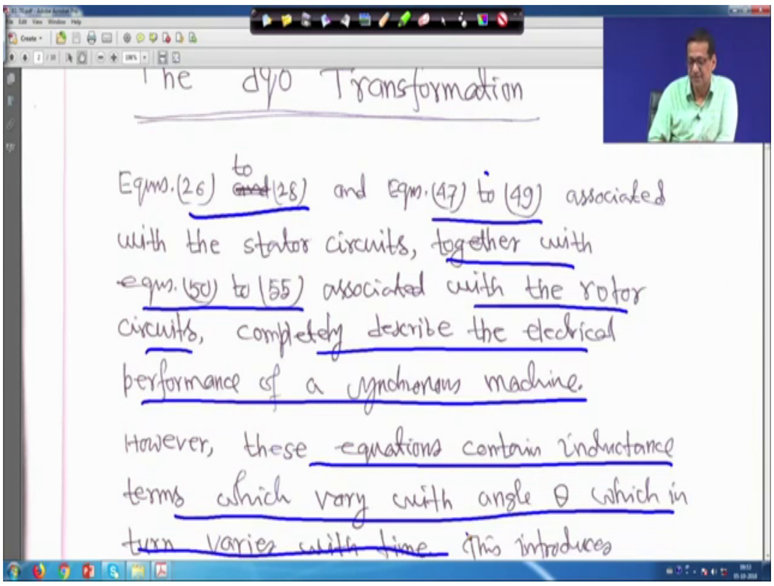

Now, equation 26, 27 and 28 and equation 47, 48 and 49 associated with the stator circuits together with equations 50 to 55 associated with the rotor circuit, right. This actually completely describe the electrical performance of synchronous machine, right. So, equation 26 to your 28 then 47 to 49, right and together with your equation 50 to 55 associated with the rotor circuit completely describes the electrical performance of a synchronous machine, right. However, these equations contain inductance terms which vary with angle theta, right which actually in turn varies with time, right.

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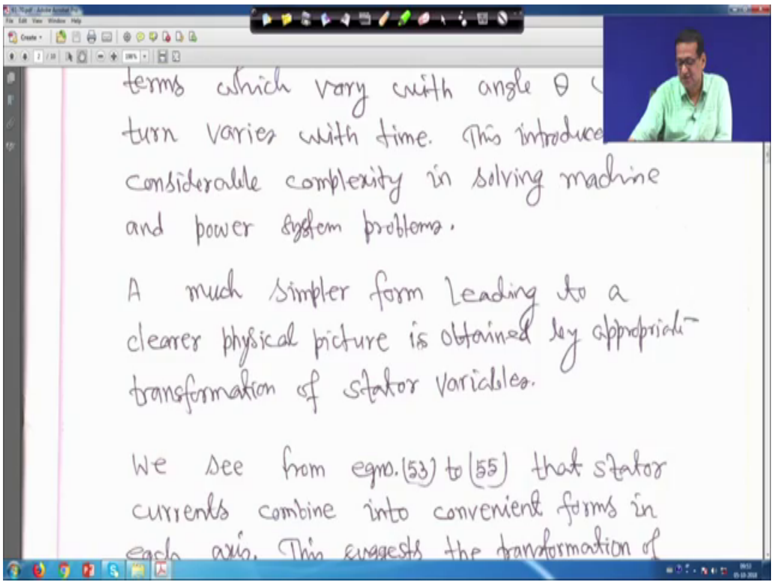

So, we have to find out from methodology such that things you know your what you call that analysis will be easier. However, this equation contains inductance terms which vary with angle theta which in turn varies with time. This introduces considerable complexity in solving machine and power system problem

Therefore, a must simpler form leading to your clearer physical picture is obtained by appropriate transformation of stator variable. One thing is there that physical significance of dq, this dq0 transformation after doing this every your, what you call a your, what you call a elaborate explain your explanation will be given, right. So, a must simpler form is that one, right.

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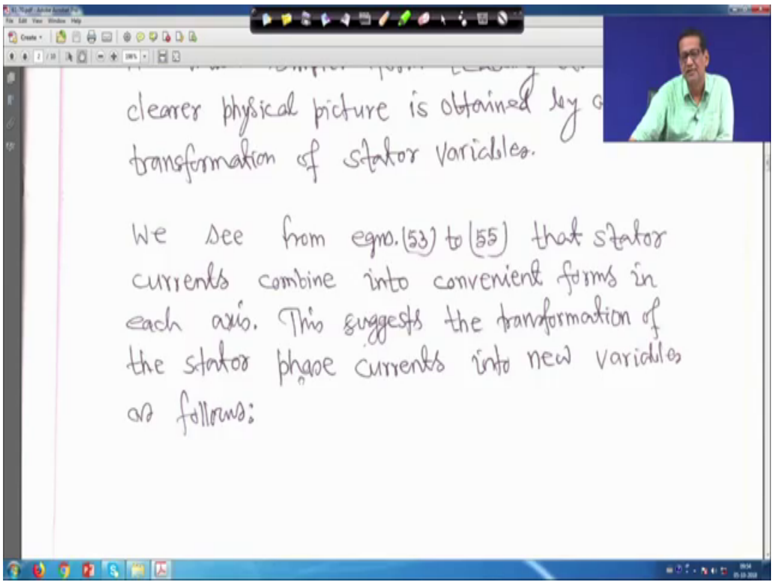

Now, we see that combining equation 53 to 55 that is 53, 54, 55 the stator current combine into convenient form in each axis. This suggest the transformation of the stator phase current into new variables as follows, right. So, we were going for dq0 that is your direct axis, coordination axis and stationary axis with transformation.

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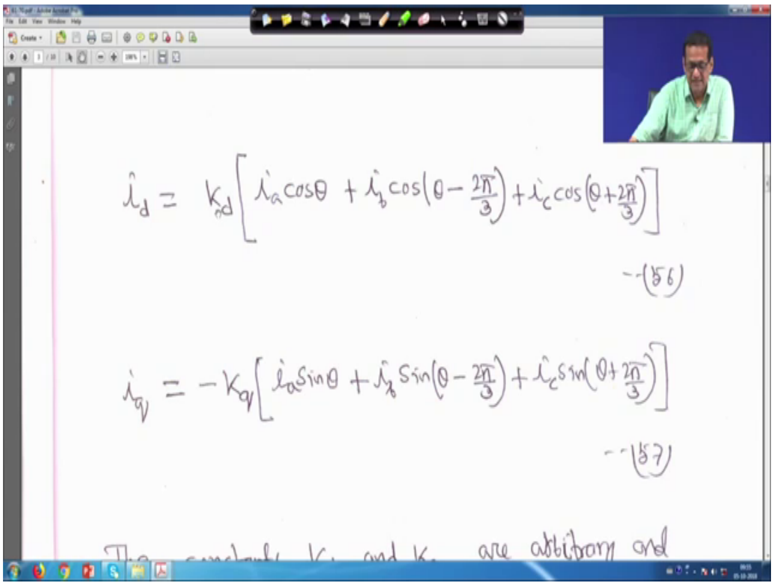

Now, if we represent suppose if we assume that i d is equal to because here we are training that from 50 equation 53 54 and 55, right that is your these 3 equation that is your this is your it is 53, this is your 53, this is 54 and this is 55, right. So, in this case your, what you call that your we can we see from equation 53 to 55 that stator currents combine into convenient form in each axis this suggest the transformation of the stator phase current in to new variables as follows.

Suppose, if we assume say i d is equal to some constant k d into i a cos theta plus i b cos theta minus 2 pi by 3 plus i c into cos theta plus 2 pi by 3 say this is equation 56. Similarly, i q is equal to say minus k q into i a sin theta plus i b sin theta minus 2 pi by 3 plus i c sin theta plus 2 pi by 3 this is equation 57, but what will be value of k d and k q, right, that we will see.

Now, the constant k d and k q are arbitrary and there values may be chosen to simplify numerical coefficient in performance equation.

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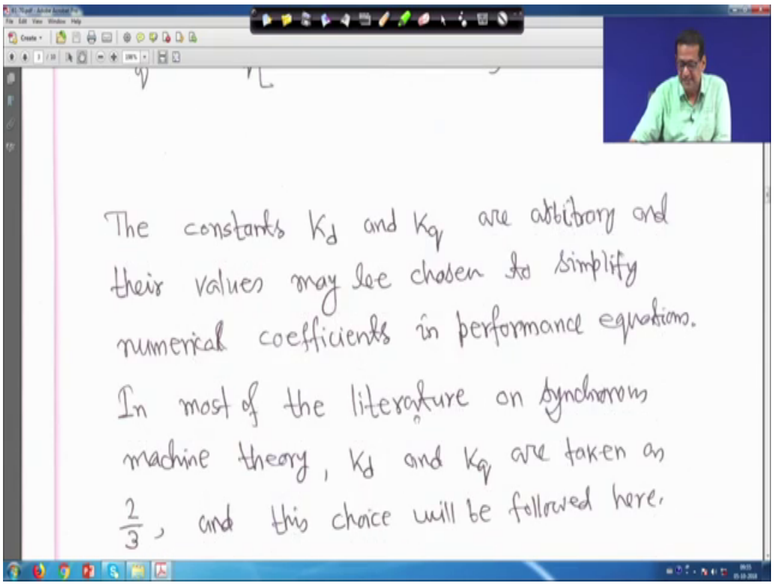

In most of the literature you will find on synchronous machine theory k d and k q are taken as 2 by 3 and this choice will be followed here, right. So, k d and k q will be 2 by 3, right.

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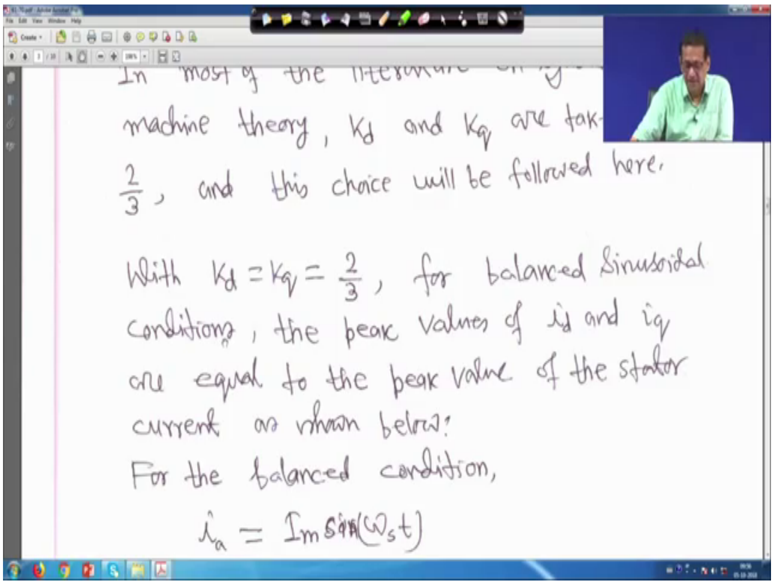

With k d is equal to k q if you take 2 by 3 for balanced sinusoidal condition the peak values of i d and i q are equal to the peak value of the stator current as shown below. I mean if we choose k d is equal to k q is equal to 2 pi 2 by 3 then the balanced sinusoidal condition the peak values of i d and i q are equal to the peak value of the stator current as shown below.

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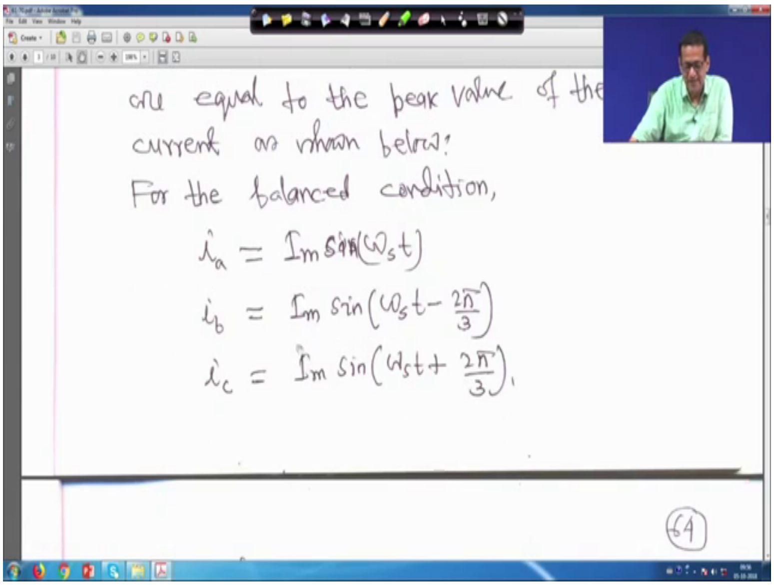

For the balanced condition we know that i a is equal to I m sin omega s into t, I m is the peak value. So, i b is equal to then I m sin omega s t minus 2 pi by 3 and i c is equal to I m sin omega s t plus 2 pi by 3, right. This we know.

Now, if you substitute all these thing substituting in equation 56, I mean if you substitute in this equation suppose in equation 56 you substitute i a, i b, i c expression you substitute and then simplify. If you simplify that in this thing then this will be substitute, all i a, i b, i c in equation 56 and expression will be like this.

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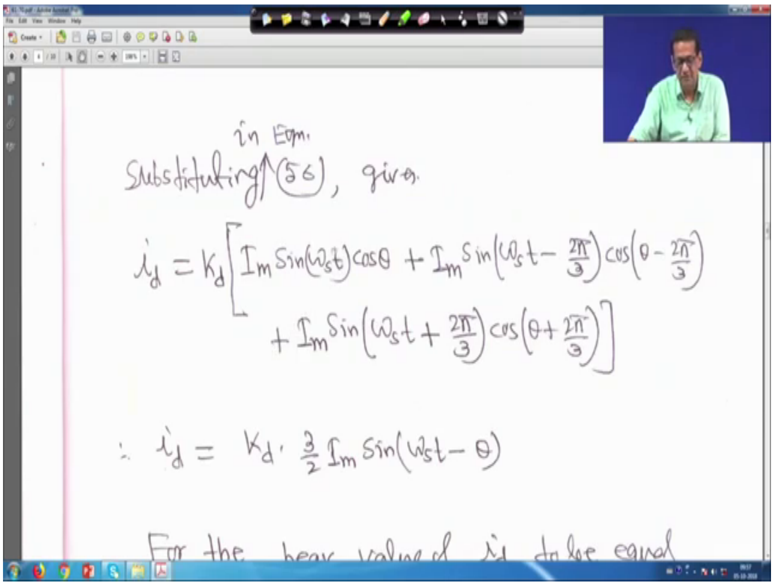

And if you simplify, right from trigonometrical this your what you call trigonometrical application point if you simplify you will find i d will be is equal to k d into 3 by 2, I m sin omega s t minus theta, right.

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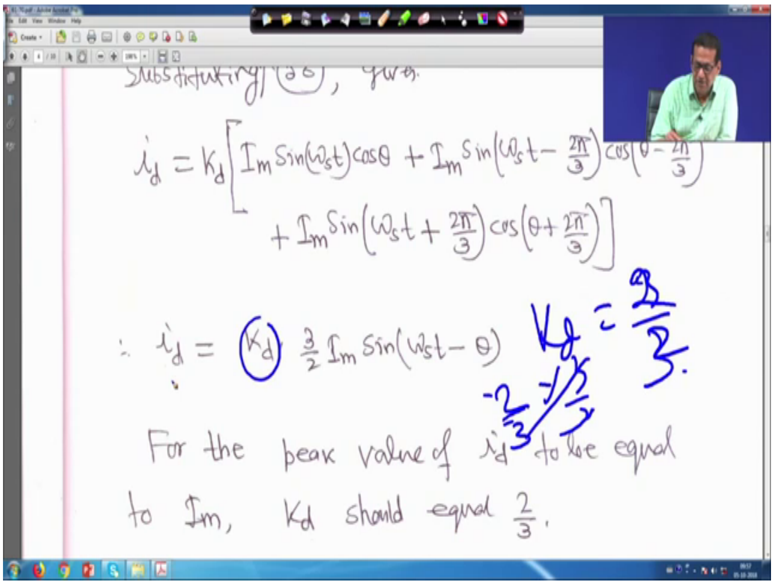

So, now if you choose k d is equal to 2 by 3 then this will be then this product will be unity, right. If you choose k d is equal to 3 by 2, right therefore, your if you choose k d is equal to 3 by 2 sorry k d is equal to 2 by 3, right then it is 2 by 3 into 3 by 2. So, it will be unity only, right. Therefore, i d will become I m; I m sin omega s t minus theta, right.

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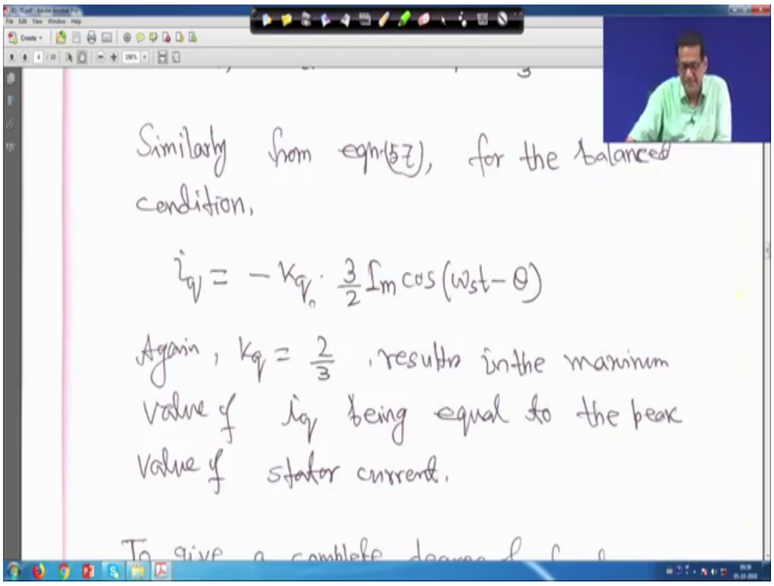

So, for the peak value of i d to be equal k d should be equal to 2 by 3. Similarly equation 57, in equation 57 here in this equation also if you substitute i a, i b and i c, right then you will get this expression that your i d will become sorry i q will become minus k q into 3 by 2 I m cos omega s t minus theta. Here also k q if you choose 2 by 3 then your i q will become minus I m cos omega s t minus theta, right.

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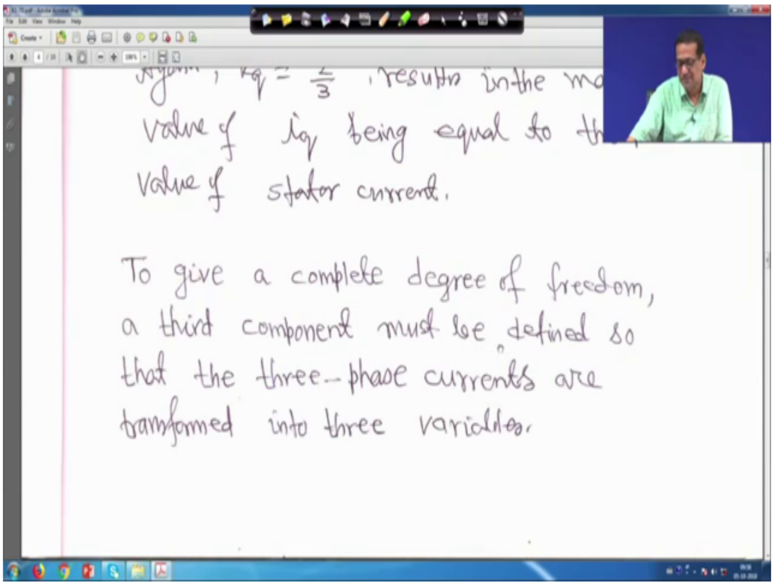

Therefore it results in the maximum value of i q being equal to the peak value of the stator current. Then if both i d and i q their maximum value will become peak value of the stator current that is I m, right. To give a complete degree of freedom a third component must be defined so that the 3 phase current are transform into 3 variables, right.

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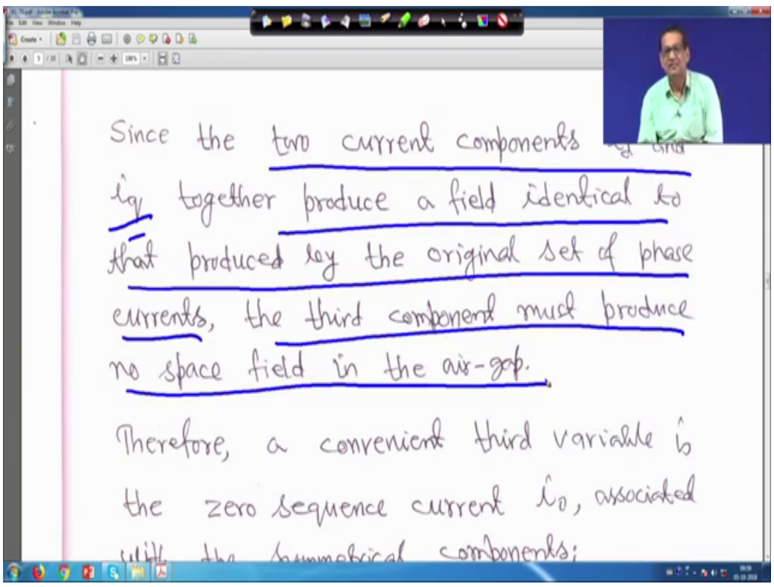

So, if you do so since the 2 components of i d and i q together produce just a minute the since the 2 current components i d and i q, right, together produce a field identical to that produce by the original set of phase currents, right. Therefore, the third component must produce no space field in the air gap, right.

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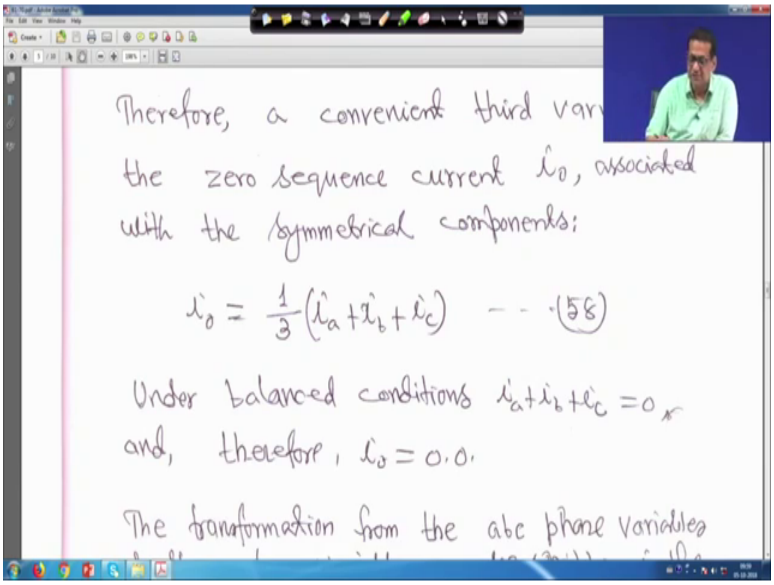

So, they therefore, therefore, a convenient third variable is the 0 sequence current, right. So, 0 sequence current i 0 associated with the symmetrical component that means, i 0 is equal to one third i a plus i b plus i c. But for the balance condition i a plus i b plus i c is equal to 0 that you know that means, i 0 will this that means, i 0 will become 0, right.

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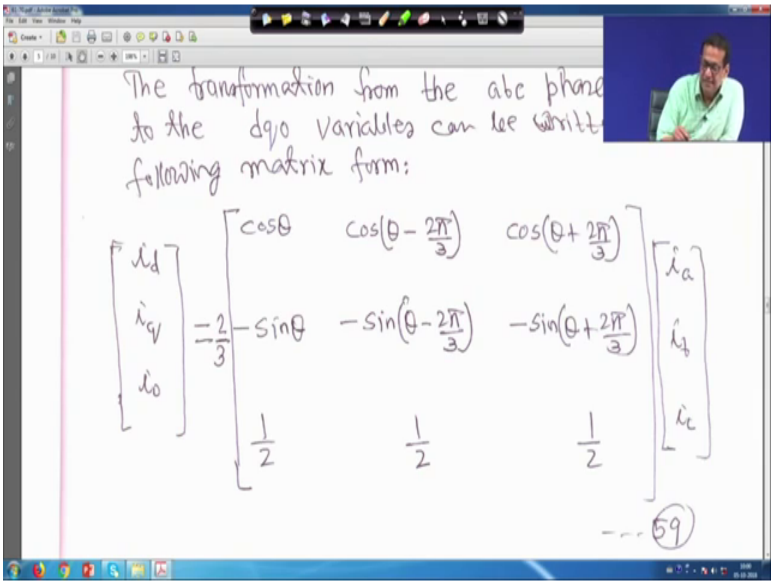

Therefore, the transformation from the a b c phase variables into the dq0 we call variable can be written in the following matrix form. Therefore, we can write that id is equal to your this matrix cos theta cos theta minus 2 pi by 3 and cos theta plus 2 pi by 3. And second it is 2 by 3 is multiplied, just one minute. 2 by 3 is multiplied, right this is 2 by 3 into that first row cos theta cos theta minus 2 pi by 3 cos theta plus 2 pi by 3, right. And similarly your what you call second row minus sin theta minus sin theta minus 2 pi by 3 minus sin theta plus 2 pi by 3.

And third row half, half, half and I actually if you multiply 2 by particular a third row 2 by third into half, half, half means it will be one-third i a plus i b plus i c and i d expression we have given that if you take k d is equal to k q is equal to 2 by 3 there will be it will be 2 by 3 cos theta i a plus this cos theta minus 2 pi 3 pi by 3 i b plus cos theta 2 pi by 3 ic similarly for i q, right. And this is half, half, half is taken because two-third then into half half i a plus i b plus i c. So, basically it will become one-third i a plus i b plus i c that is nothing, but is equal to i 0. This is actually equation 59, right.

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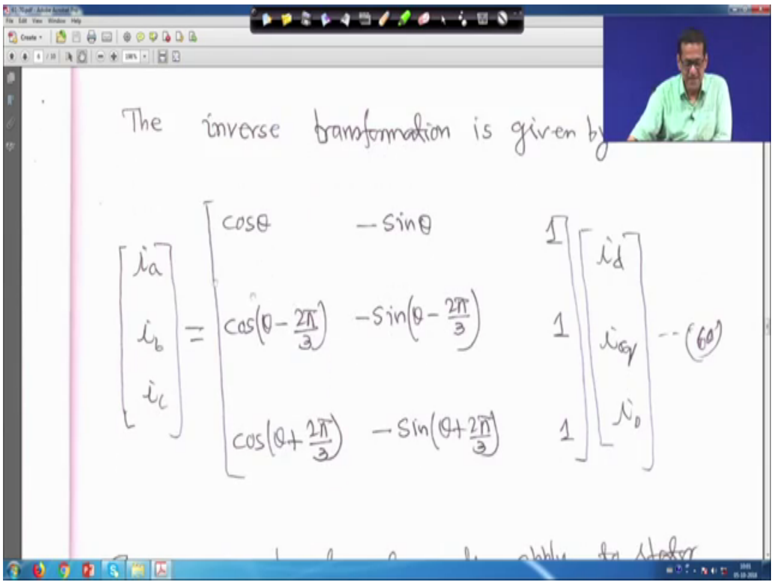

So, if you take the inverse transformation of these, if you take the inverse, if you take the inverse transformation of these then it will become that i a, i b, i c will become basically it is cos theta minus sin theta first row one then cos theta minus 2 pi by 2 minus sin theta minus 2 pi by 3 1, then cos theta plus 2 pi by 3 minus sin theta plus 2 pi by 3 1, right. That means, you will see that i a will be is equal to i d cos theta minus your i q sin theta plus your i 0.

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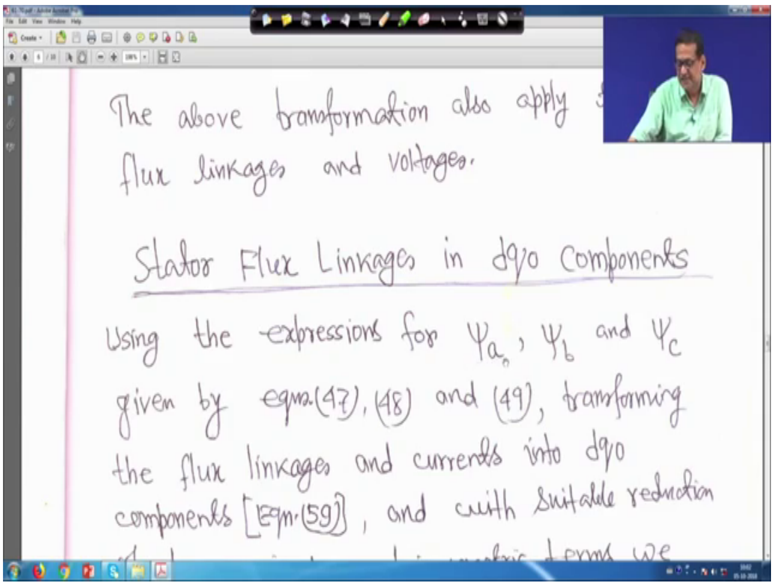

Similarly, i b will become your id cos theta minus 2 pi by 3 minus i q sin theta minus 2 pi by 3 plus i 0 similarly i c will get, right. So, if you make the inverse if you take the inverse transformation, I mean if you take the inverse of this matrix, right. The above transformation also apply to the stator flux linkages and voltage this is same for a stator flux linkages as well as the voltages, right.

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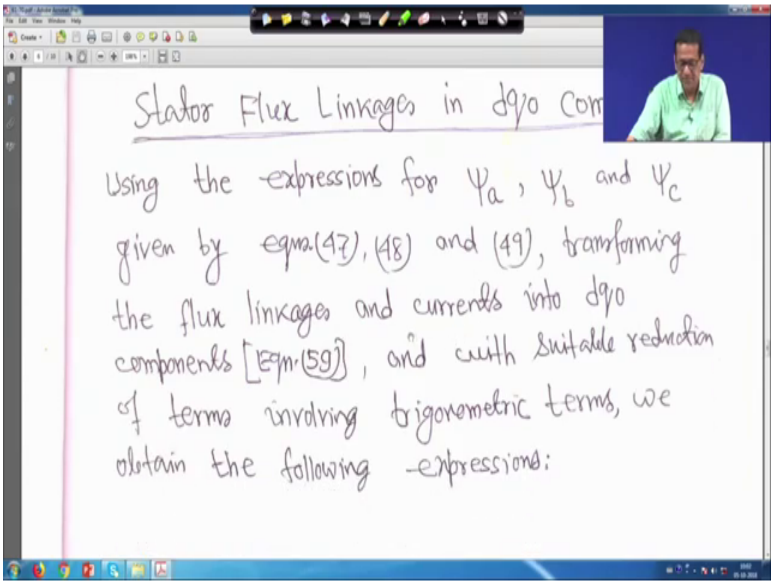

So, using the expression for psi a, psi b and psi c giving by equation 47, 48 and 49 that we have seen, right; transform the flux linkages and current into your dq0 your component, right and with suitable reduction of your terms involving trigonometric term we obtain the following expression. And if you look here if we try to derive all these mathematics, right, all the derivation then it will consume lot of time. So, just certain things we have to keep it in our mind, right.

So, now flux linkages also voltage induced also all expressions will be similar, right only variables are different.

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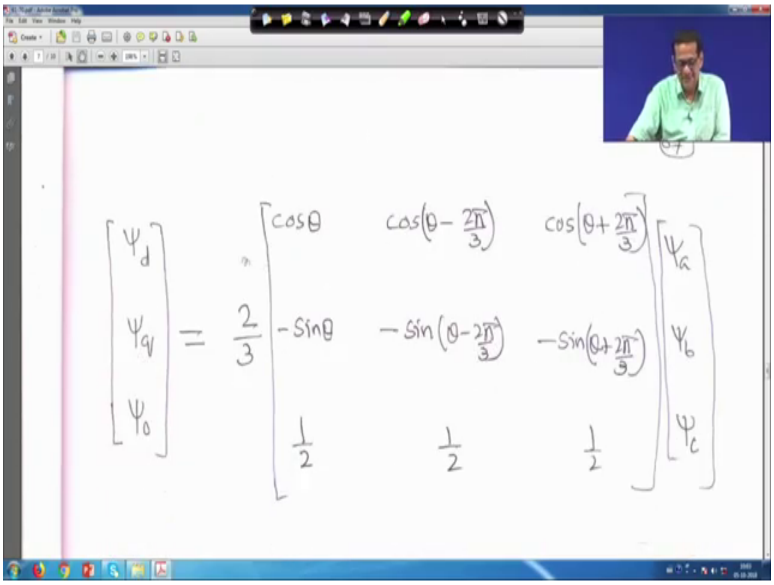

So, if you do so then you will find that psi d, psi q, psi 0 same as before same as before like your this one your i d, i q, i 0 two-third into this one same thing same thing for flux, right only variables is flux only psi d psi q psi 0 two-third this is your cos theta cos theta minus 2 pi by 3 then cos theta plus 2 pi by 3, right. And second row is minus sin theta minus sin theta 2 pi minus theta minus 2 pi by 3 minus sin theta plus 2 pi by 3 and it is half half half sum again and this is psi a, psi b, psi c, right same for flux linkages.

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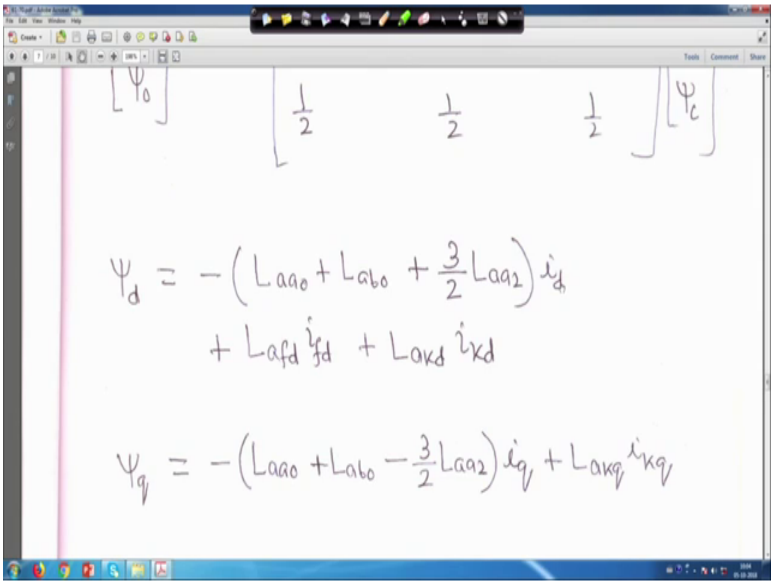

And therefore, we can write psi d is equal to minus L aa0 plus L ab0 plus 3 by 2 L aa2 i d, right.

So, question is that that psi d expression we are given then we know psi a, psi b, psi c all expression that we have to substitute psi a, psi b, psi c we have got it that you substitute and just simplify. You put expression of you take expression of psi a, psi b, psi c put it here, right and just simplify then you will get psi d is equal to minus L aa0 plus L ab0 plus 3 by 2 L aa2 i d plus L afd i fd plus L akd i kd, right. Only thing is that from previously we have derived psi a, psi b, psi c in terms of all these things you put it here and just simplify you will get this equation, right.

Similarly, psi q will become minus bracket L aa0 plus L ab0 minus 3 by 2 L aa2 i q plus L akq i kq.

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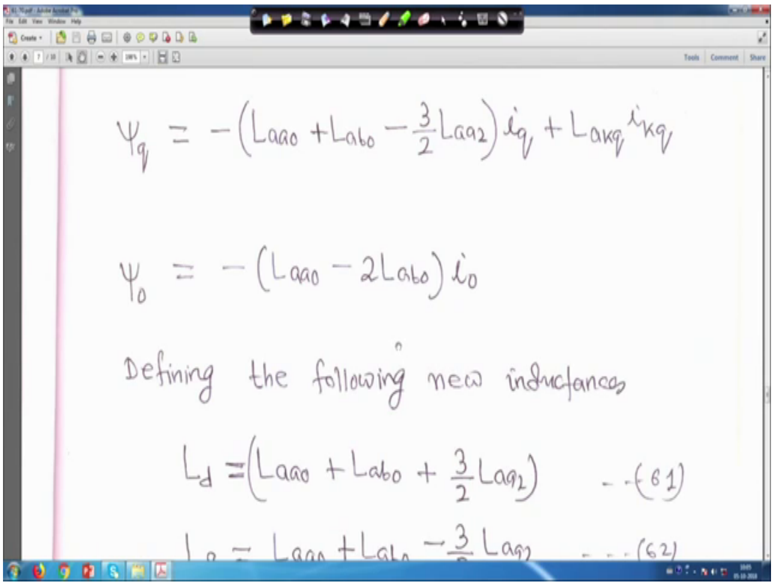

Similarly, psi 0 we will get minus in bracket L aa0 minus 2 L ab0 i 0. Only I will request all of you those who are your listening to these and when we will your go through these just you put psi a, psi b, psi c expression and just try little bit, right. Here I have to save some time.

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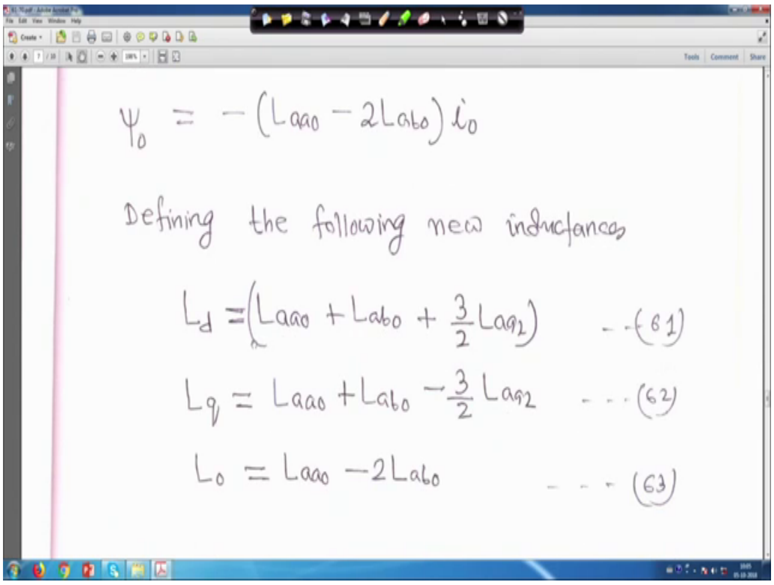

So, in this case now defining the following new inductances; suppose if we define L d is equal to L aa0 plus L ab0 plus 3 by 2 L aa2 this is equation 60. (Refer Time: 22:22) defining this is L d because it is related to psi d, right that is we are defining. Similarly L q define L aa0 then L ad0 minus 3 by 2 L aa2. This is equation 62. And similarly you define L 0 is equal to L aa0 minus 2 into L ab0. This is equation 63.

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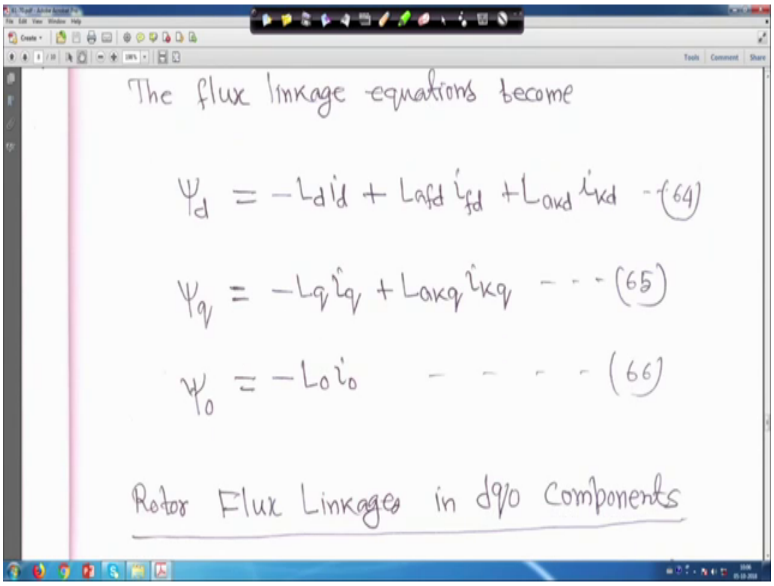

Now, if you do so then the flux linkage equation will become minus L d i d that is psi d is equal to minus L d i d plus L afd i fd plus L akd i kd this is equation 64, right. Similarly psi q is equal to minus L q i q plus L akq i kq this is equation 65. Similarly psi 0 will be is equal to minus L 0 i 0 this is psi d, psi q and psi 0 equation also we have seen i d, i q, i 0, it is psi d, psi q, psi 0 flux linkages equation, right.

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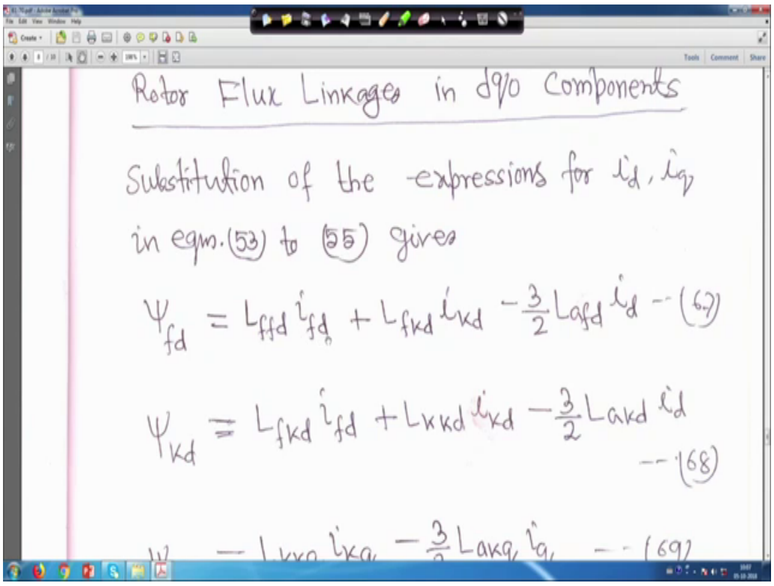

Next is total flux linkage into in your again your d q 0 component, right. So, here also a substitution of the expression for i d i q in equation 53 and 54 and 55; you just put the expression of your what you call i d and i q. Then what you will get? You will get this expression this I request you to derive, right again and again I am telling because it will take then few more pages, right. So, just have patience and just do this and this one actually psi fd will be L ffd then i fd plus L fkd i kd minus 3 by 2 L afd i d. This is equation 67, right.

Similarly, psi kd will be L f kd i fd plus L kkd i kd minus 3 by 2 L akd i d. This is equation 68.

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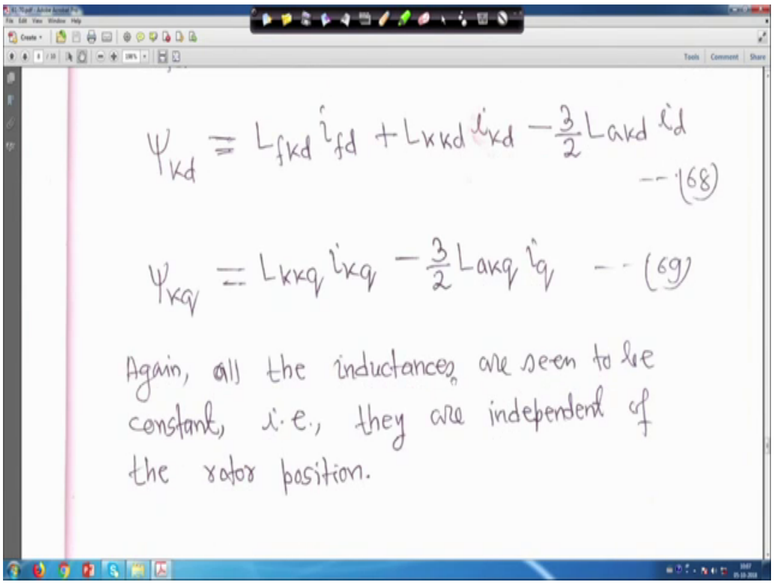

And similarly psi kq will become L kkq i kq minus 3 by 2 L akq i q 69, right. Therefore, again all the inductances are seen to be constant because here no question of theta, right that is they are independent of the rotor position because theta is equal to omega rt or omega st, right because synchronous machine omega r is equal to omega s, right.

So, again all the inductances are seen to be constant that is your, they are independent of the rotor position.

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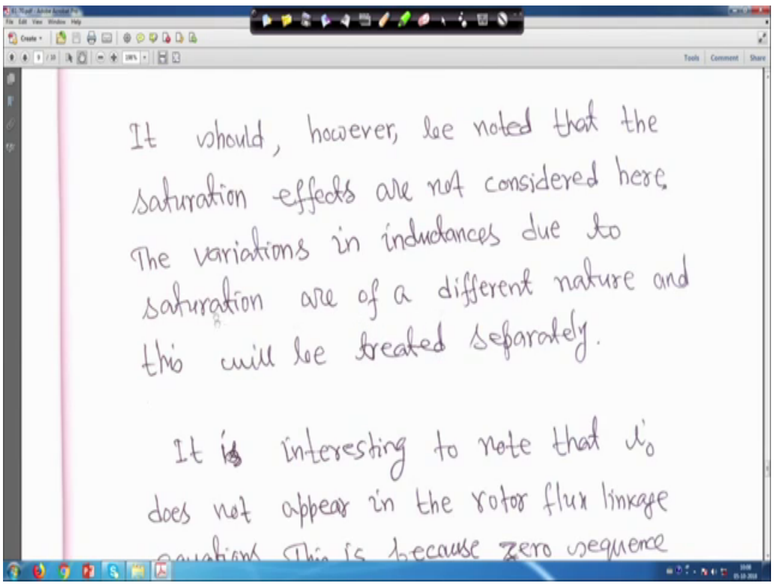

So, it should however, we noted that the saturates actually we are not consider the saturation effect, and throughout this course we will not consider that that things will be more your more complicated, right. So, are not considered here the variation in inductances due to saturation are of a different nature and this will be treated separately. So, we will not consider that saturation effect here, right.

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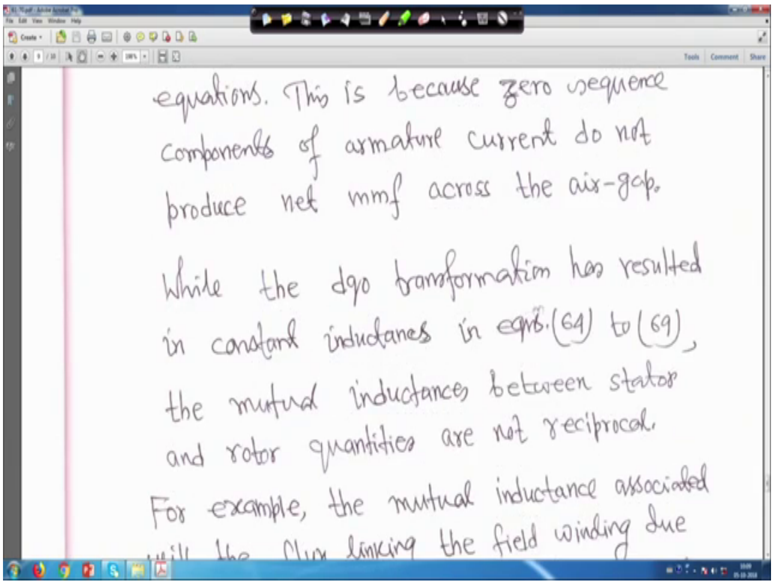

So, it is interesting to note that i 0 does not appear in the rotor flux linkage equation this is 2 we have seen it. This is because your 0 sequence component of armature current do not produce net mmf across the air gap, right. While dq0 transformation as resulted in constant inductances is equation 64 to 69 the mutual inductances between stator and rotor your quantities are not reciprocal. The mutual inductances, which is stator and rotor quantities are not. Reciprocal means for example, something like this actually if you say a 1 2 not is equal to a 2 1, right. So, that can be made reciprocal, right. When we when we will go for per unit system for synchronous machine with appropriate base quantities, right, so that we will see later in detail.

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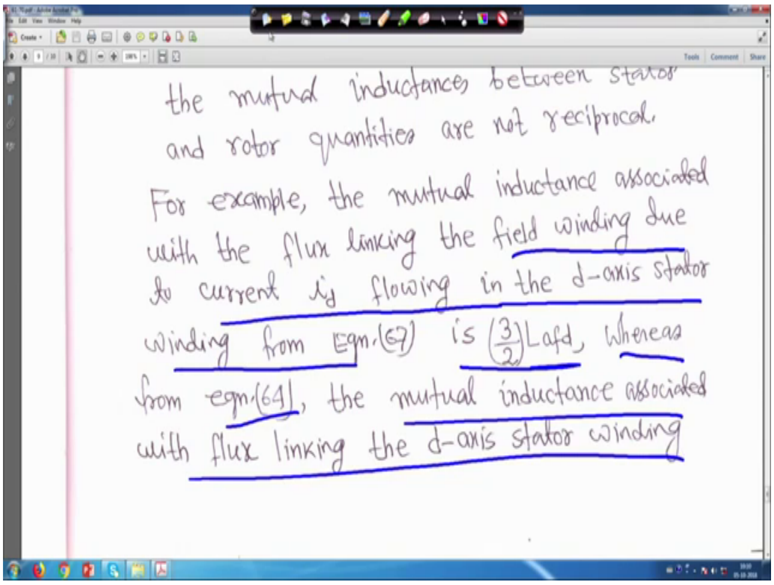

So, for example, the mutual inductance associated with the flux linking the field winding due to the current i d, right your that field winding due to the current i d flowing in the d axis stator winding from equation it is 3 by 2 L afd, right. So, it is 3 by 2 L afd whereas, equation 64 the mutual inductance associated flux linking the d axis stator winding, right. Just see this, this d axis stator winding it is due to field current L afd here it is 3 by 2 L afd from one side to another if you take mutual one and this is your L afd so they are not reciprocal. Reciprocal means they are not same, I mean I mean meaning is something like that m 1 2 not is equal to m 2 1 that kind of things.

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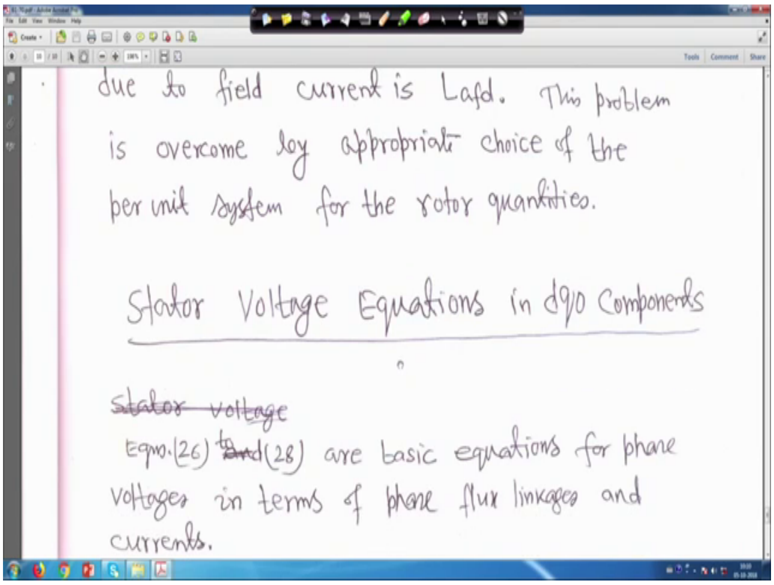

So, we probably choose an appropriate base such that we can transform in the same per unit values that we will see later, right. So, this problem is overcome by appropriate choice of the per unit system for the rotor quantities that we will see later, right.

Similarly, for stator voltage equations in that same your transformation dq0 component, right. Equation 26 to 28, are basic equations for phase voltages in terms of phase flux linkage and current that we have seen, right.

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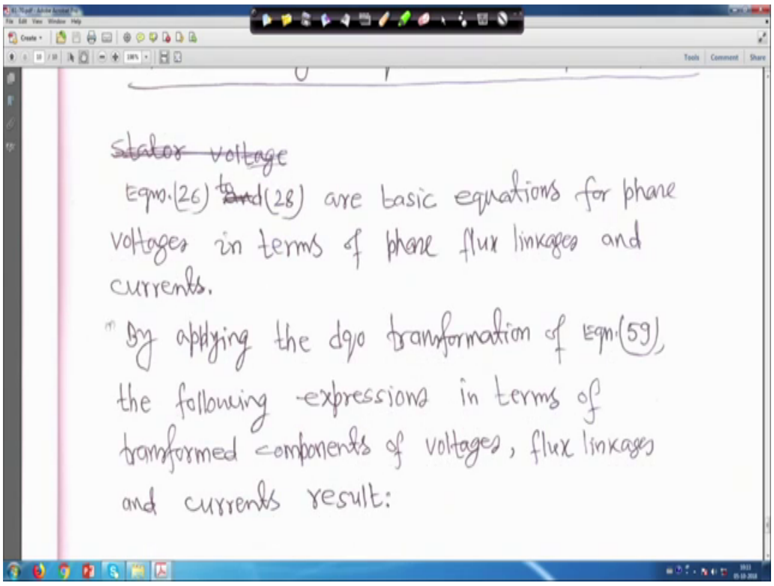

So, by applying that dq0 transformation in equation 50 of equation 59 the following expression in terms of transformed components of voltage flux linkage and current will result. So, you will get, I mean what you will do that equation 26 to 28 the basic equations which transform into your dq and 0 your what you call transformation, right and use equation 59, right.

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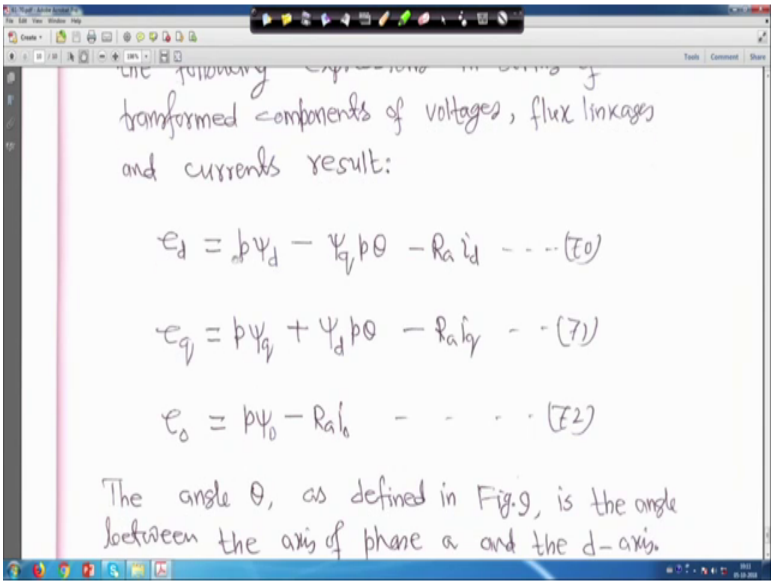

And if you do so you will get this expression: e d is equal to p is d by dt, e d will get p psi d minus psi q p theta minus R a i d. This we will we will explain all the meaning of p psi d and psi q p p theta little later, right minus this is equation 70.

Similarly, e q is equal to p psi q plus psi d p theta minus R a i q, I mean p psi d means it is ddt of psi d and p theta means d theta dt, right because p is ddt. And similarly here e q is equal to ddt of psi q plus your ddt of your psi d ddt of d theta dt and this one will be ddt of psi 0 minus R a into i 0. This is 70, 71 and 72, right.

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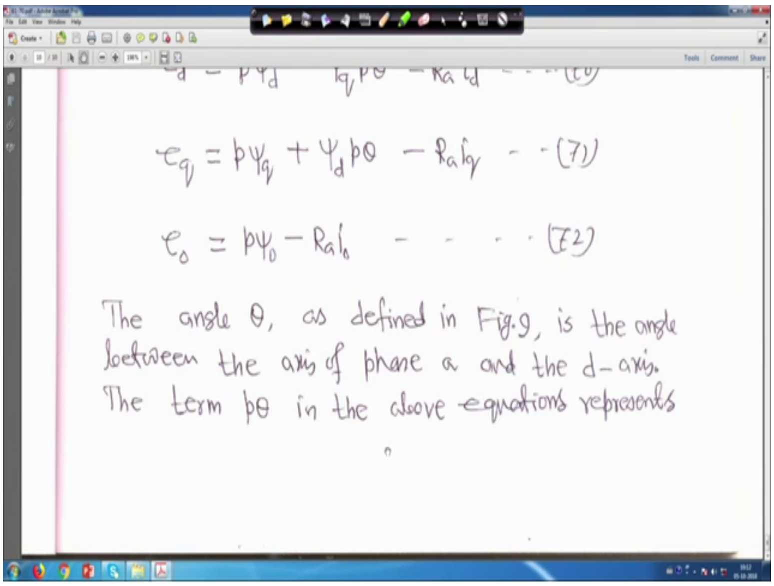

Angle theta as defined in equation figure 9, again your, what you call figure just see that your figure 9 is the angle between the axis of phase a and the d axis, right. So, question is the term p theta in the above equation actually it represent actually d theta by dt that is the rotor speed, right. So, we will explain it later in detail all these things.

Thank you very much. We will be back again.