Power System Dynamics, Control and Monitoring
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Lecture - 59
Subsynchronous oscillation

So, this will be the last two lectures right; so, we will start with Subsynchronous oscillations right and after this we will see that wind up and non-wind up limiters those things and with this we will close this course. So, whatever we have seen previously that we have considered that whole turbine generator altogether it is combined a single mass right. And we try to find out the oscillations with respect to other generators, but here we will do little bit separately and this is actually subsynchronous oscillations right.

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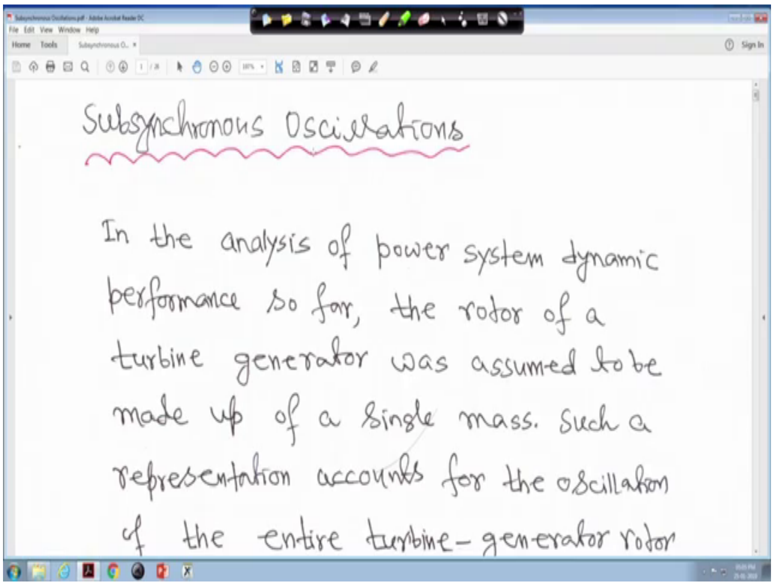

So, sorry in the analysis of power system dynamic performance so, far the rotor of a turbine generator right was assumed to be made up of a single mass that we have seen so far right. We just consider that it is a single mass I mean we combine together. Such a representation its accounts for the oscillation of the entire turbine generator rotor with respect to other generators right.

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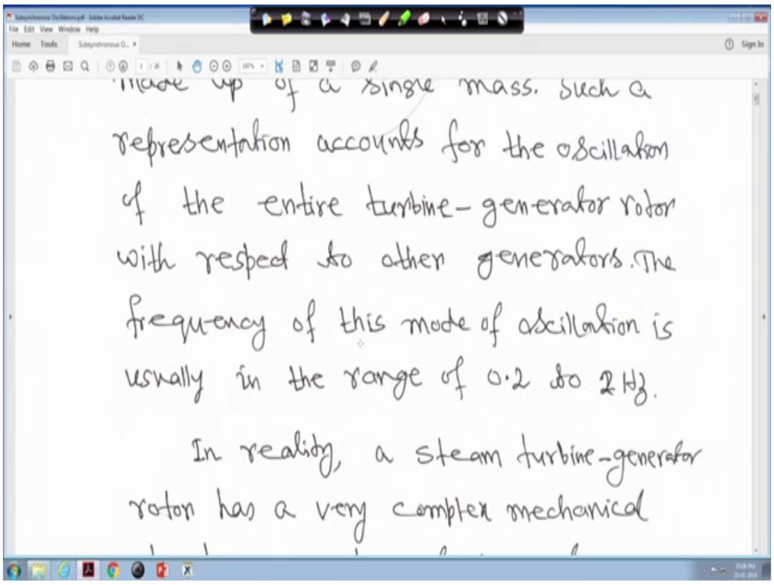

The frequency of this mode of oscillation is usually in the range of 0.2 to 2 Hertz right this we have also discussed before.

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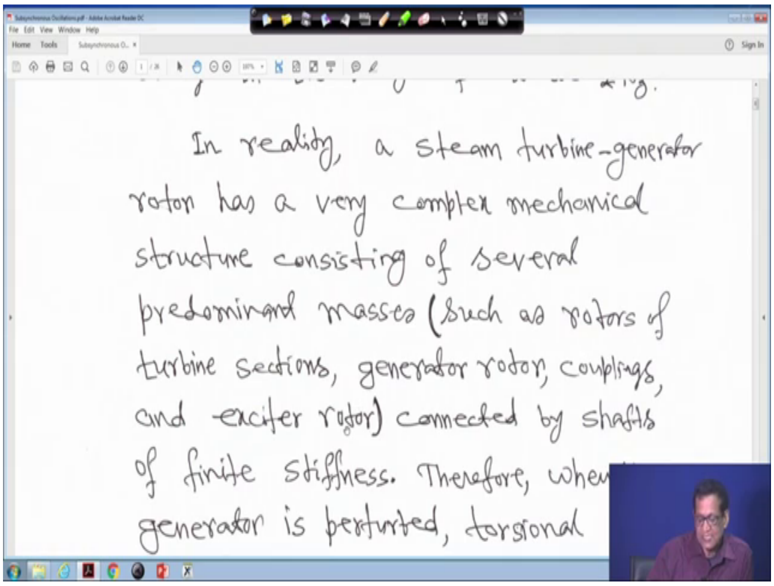

Now, in reality a steam turbine generator rotor has a very complex mechanical structures consisting of several predominant masses such as rotors of turbine sections, then generator rotor, couplings if it is there right then an exciter rotor right. So, connected by shafts of finite stiffness; so, the stiffness of the shaft we will assume.

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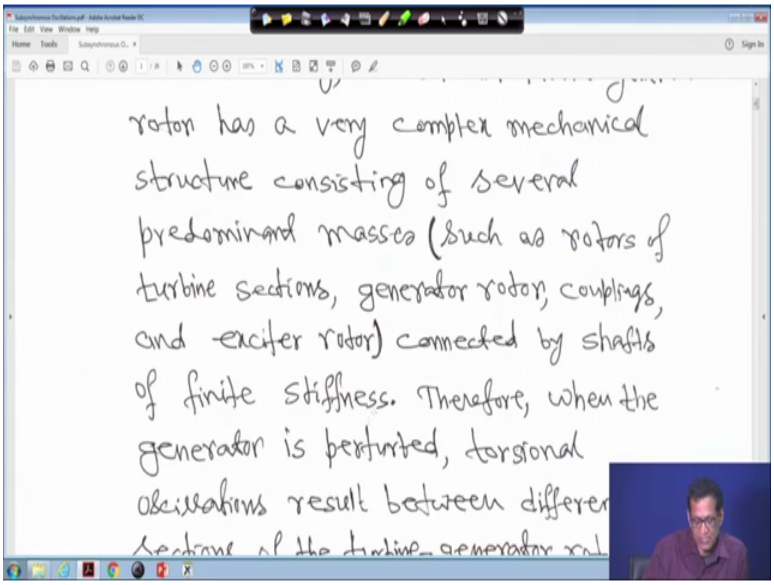

Therefore when the generator is perturbed that torsional oscillation results between different sections of the turbine generator rotor. Because, in the your what you call turbine case you have a high pressure portion of the turbine, low pressure also intermediate pressure low pressure they may have different section right. So, earlier whenever we have considered these we considered this turbine generator as a combined mass, but here we will see it separately right.

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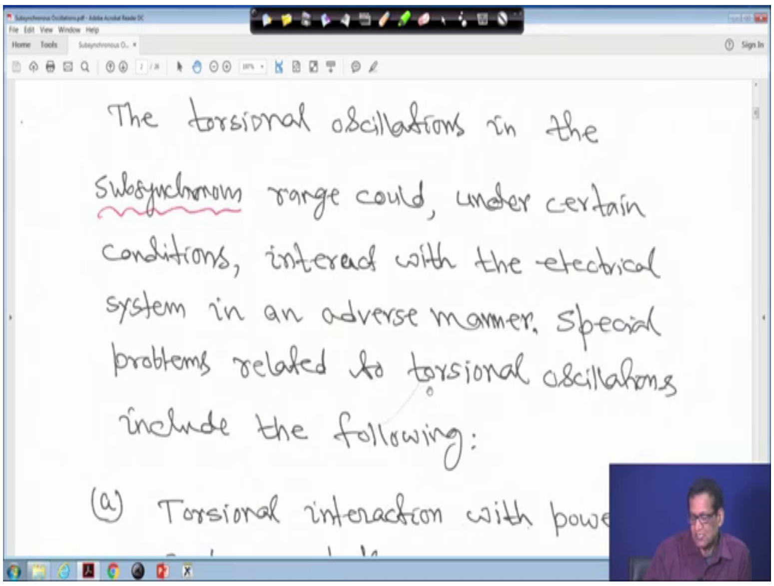

So, the torsional oscillations in the subsynchronous range could under certain conditions interact with the electrical system in an adverse manner. So, the special problems related to the torsional oscillations include the following.

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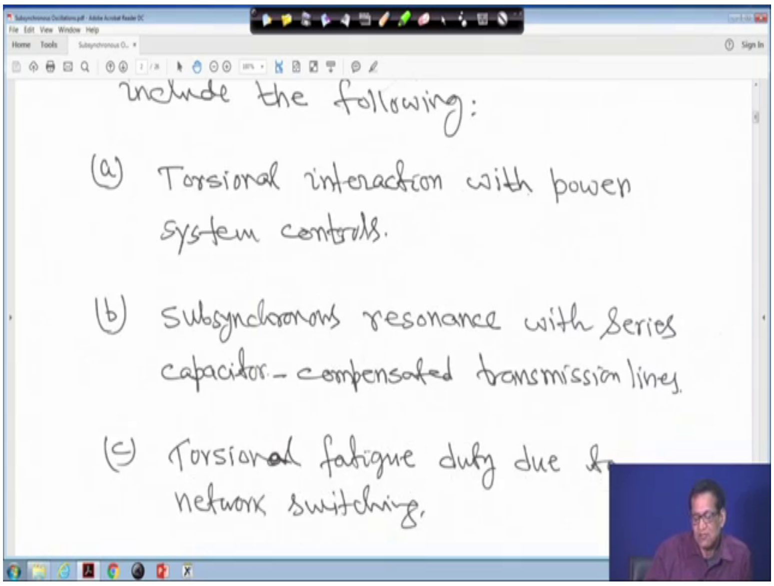

Now, a the torsional interaction with power system controls, subsynchronous resonance with series capacitor compensate transmission line and torsional fatigue; fatigue duty due to network switching right.

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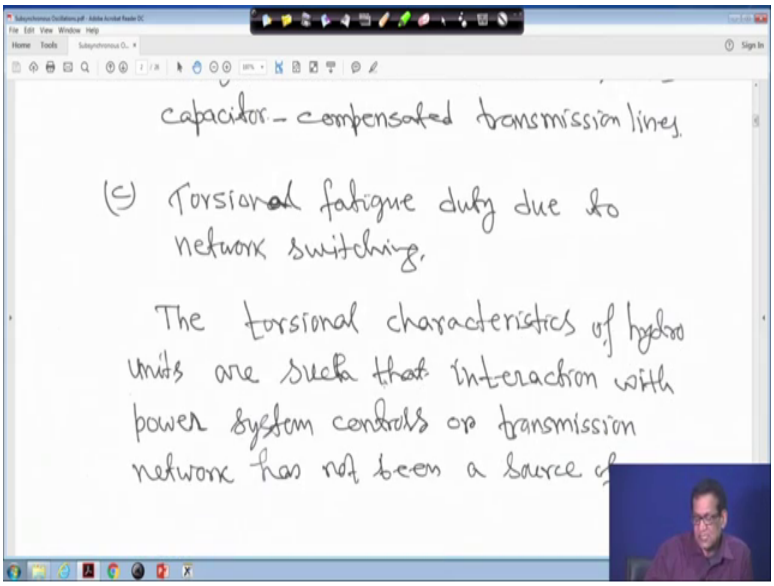

So, the torsional characteristic of hydro units are such that interaction with power system controls or transmission network has not been a source of concern right.

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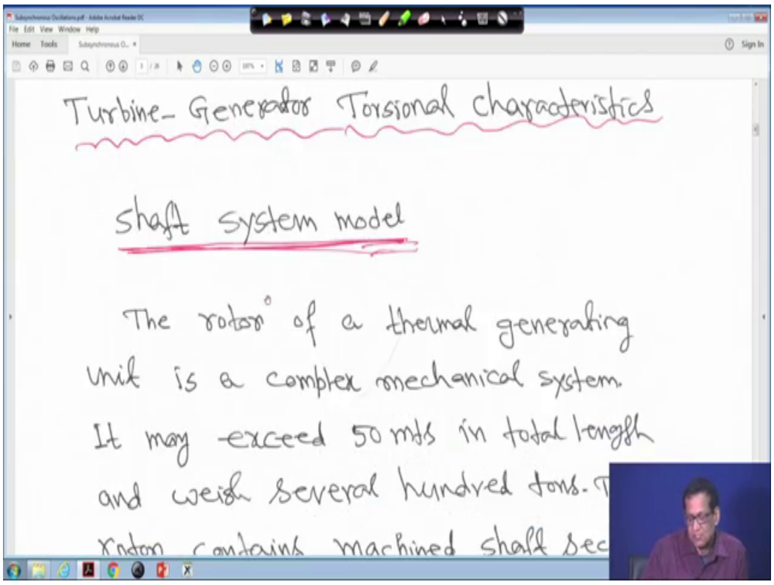

So, now turbine generator torsional characteristics right; ao first is we have to see the shaft system model because, we have turbines we have your what you call high pressure, intermediate pressure, low pressure right and that is also coupled with the generator. So, we have to see that how we can model it and we will go for simplest model. So, the rotor of a thermal generating unit is a complex mechanical system. It makes it 50 meters in total length and weight several hundred tons.

I mean I do not know whether you have seen the open rotor particularly for thermal power plant or not. If you see the rotor thing then you will find that your everything together I mean it is very rotor is very long right. And, and your what you call and it makes it 50 meters in total length and weight may be several tons because, we have turbine then generator its coupled right. So, it will be it is a very very complex system and weight will be very heavy right.

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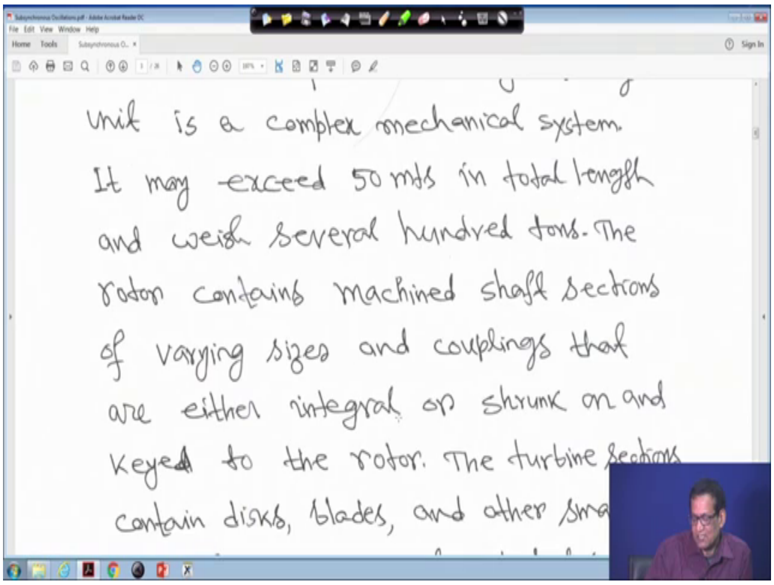

The rotor contains your mechanical your machine shaft sections of varying sizes and couplings that are either integral or shrunk on and key to the rotor right.

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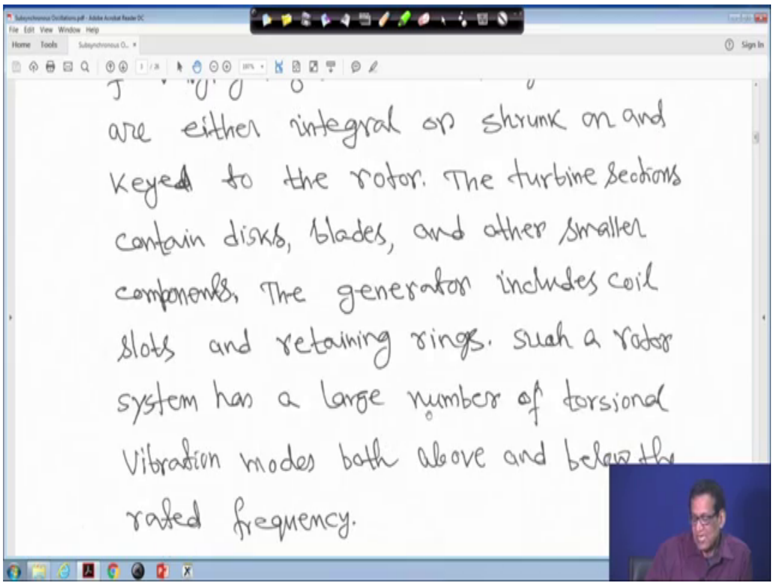

The turbine sections contain disks, blades and other smaller components. So, the generator includes coil slots and retaining your what you call rings; such a rotor system as a large number of torsional vibration modes both above and below the rated frequency right.

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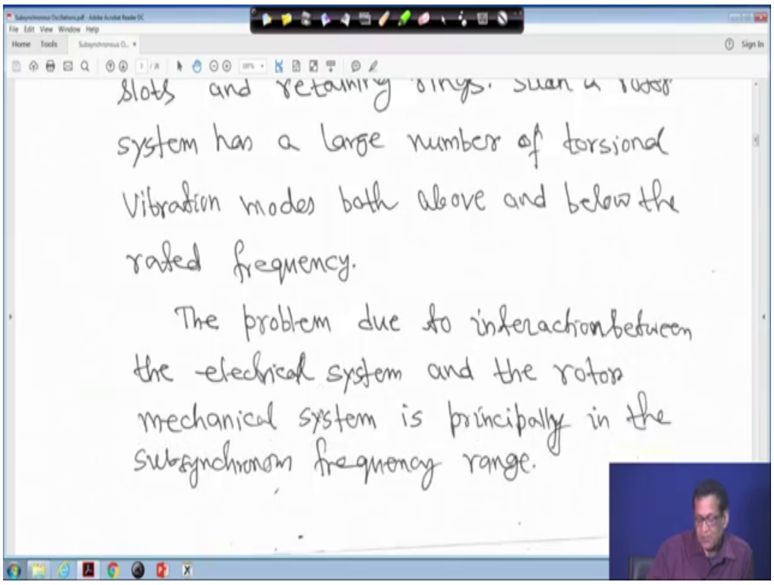

So, the problem due to interaction between the electrical system and the rotor mechanical system is principally in the subsynchronous frequency range right. So, these are the these are the things certain things studies by many researchers in the past right and only little bit we will study in this lecture.

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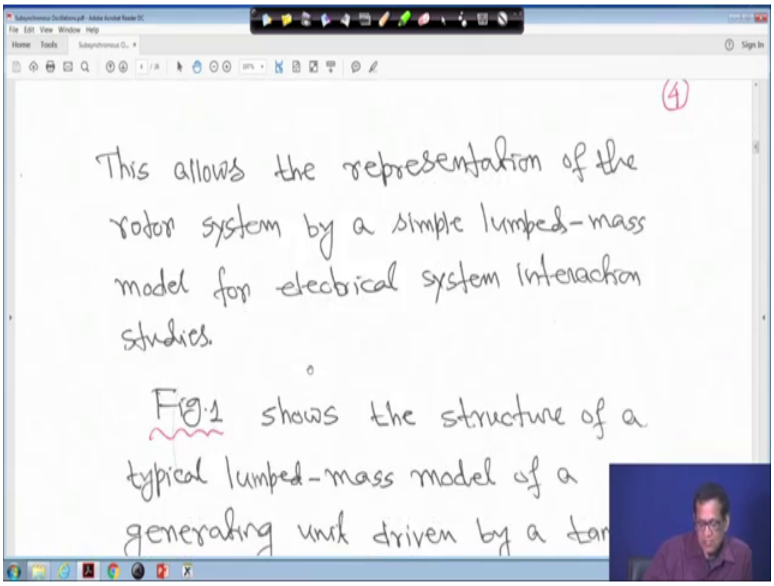

So, this allows the representation of the rotor system by simple lumped mass model or electrical system interaction studies right. So, now after this we will come to your what you call that your figure 1 that how represent. For example, figure 1 I am coming figure one it shows the structure of a typical lumped mass model of a generating unit given by tandem compound reheat turbine.

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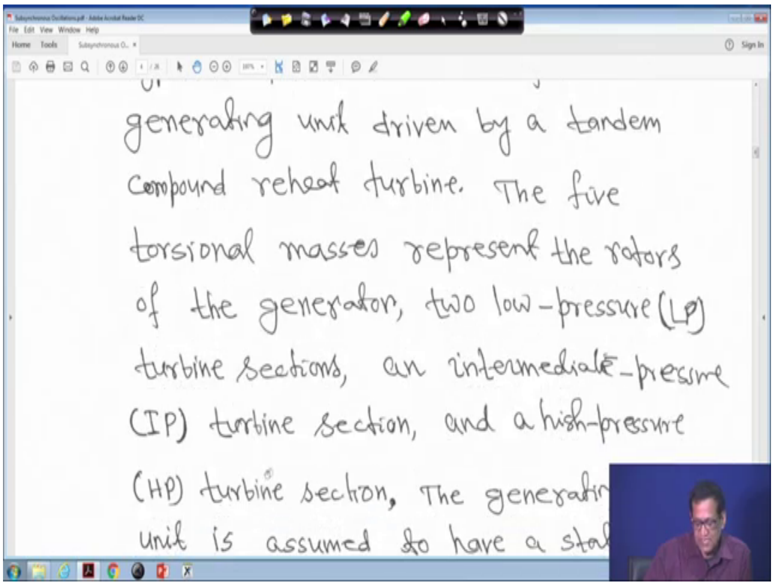

We have assumed that is a tandem compound reheat turbine. The five torsional masses represent the rotors of the generator two low pressure turbine sections and interconnect an intermediate pressure that is IP turbine section and a high pressure turbine sections right.

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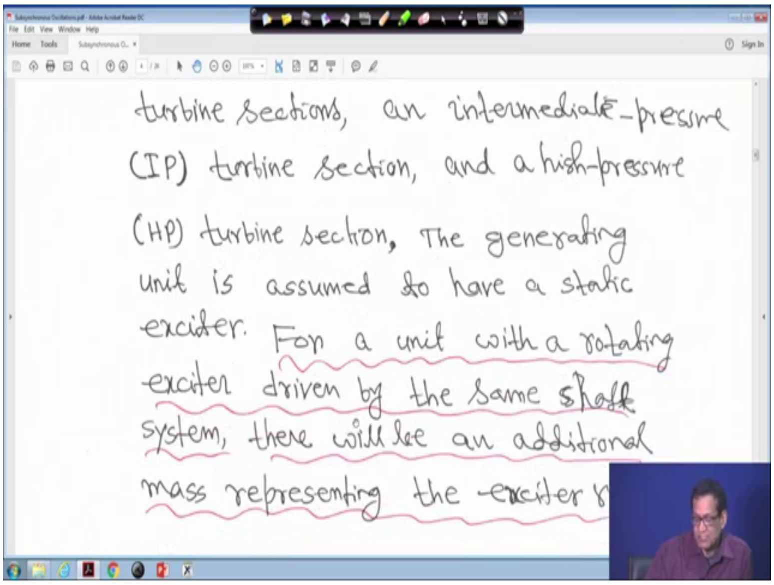

So, the generating unit is assume to have a static exciter right. For a unit with a rotating exciter driven by the same shaft system there will be an additional mass representing the exciter rotor. But, here is assuming the static exciter system right otherwise if you assume rotating exciter then you have to your what you call you have to represent it by an additional mass.

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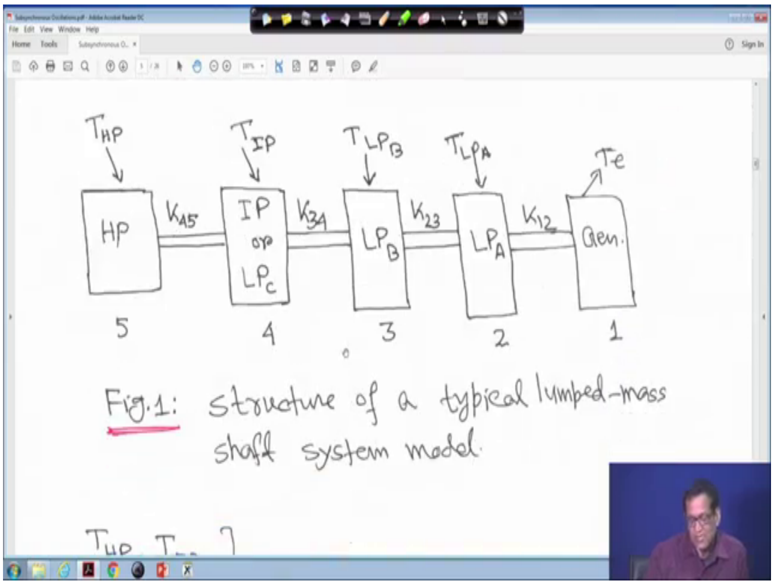

So that means, we will come to say figure 1. So, it is a simple like your connecting one after another model. So, this is regenerator is output torque is T e and it is marked as 1 and this is your low pressure turbine there are say two sections of low pressure turbine L P A and L B P two sections are there say. So, this is your section 2 L P A and L P B right and here torque is T L P A and T L P B and then intermediate pressure. Here it is marking on the section 4 and this is your T I P right all torque representation and this is section HP high pressure portion we are making numbering at 5 and this is your T H P.

Now, the stiffness of the shaft right; so it is 1 and 2 is connected we are representing as a K 12 right. Similarly 2 to 3 we are representing as K 2 3, similarly 3 to 4 we are representing as K 3 4 and similarly 4 to 5 we are representing this as a K 4 5; these are the stiffness of the shaft right we have to mathematically we have to relate this. So, this is actually a high pressure intermediate pressure to low pressure sections and this is the generator and this is a structure of a typical lumped mass shaft system model.

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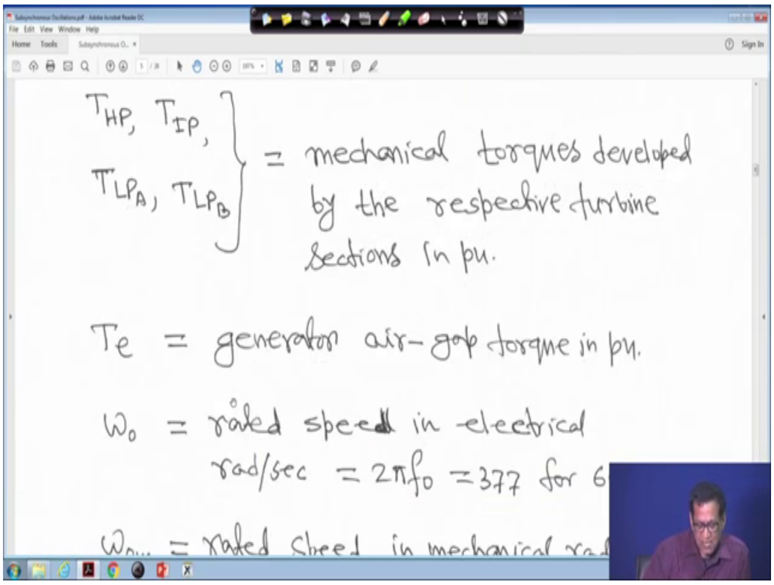

So, now nomenclature is here the T H P T I P T L P a T L P B there is a mechanical torque developed by the respective turbine sections in per unit right. T e is the generator air gap torque in per unit right. These all these things while studying synchronous means in this T omega 0 all have been defined right. And, omega 0 is equal to rated speed in electrical radian per second is equal to 2 pi f 0 and it will be 377 radian per second; if f 0 is equal to 60 Hertz right.

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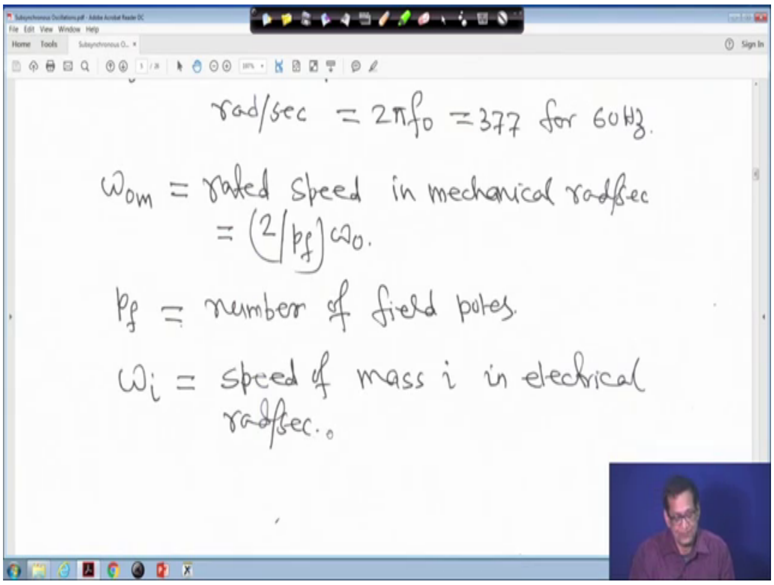

And omega 0 m that is your rated speed in mechanical radian per second this also we have seen before that is to upon p f into omega 0 right. So, p f is the number of field poles; this we have seen when we are in the beginning when we are studying your a synchronous machine right. And, say omega i is equal to speed of mass i in electrical radian per second right.

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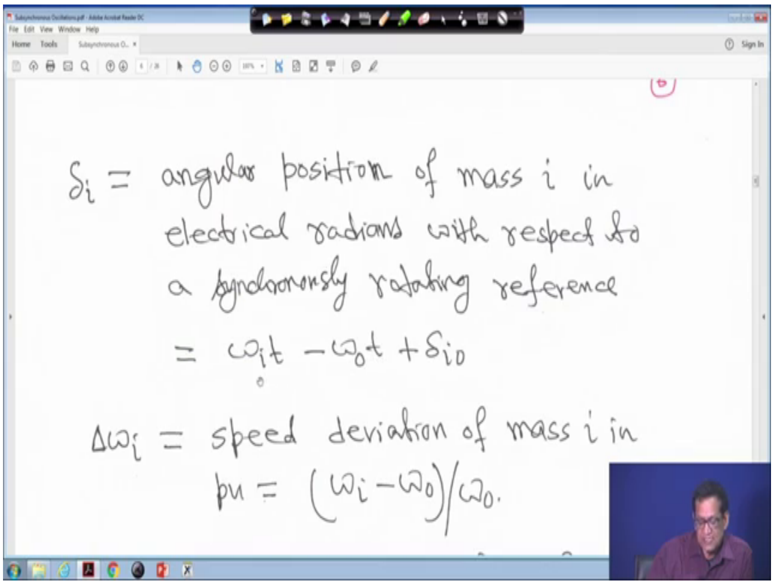

So, and delta i actually angular position of mass i in electrical radians with respect to a synchronously rotating reference right; that is we can write delta is equal to omega i t minus omega 0 t plus delta i 0, delta i 0 is the initial angle right. And, omega 0 your what you call omega 0 it rated speed in electrical radian per second this is omega 0 right. So, it is minus omega 0 t and this is omega delta is equal to omega i t right. And, delta omega i is speed deviation of mass i in per unit that is omega i minus omega 0 divided by omega 0 right.

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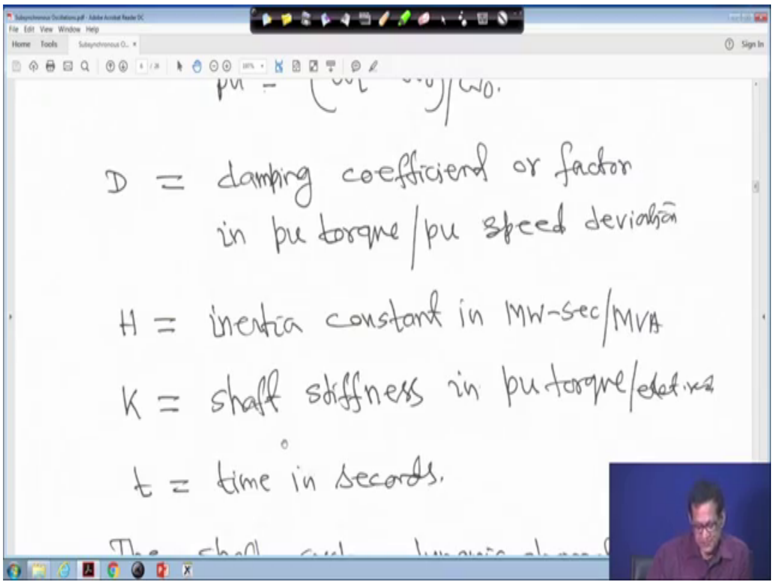

So, and D actually we call damping coefficient or factor in per unit torque per units speed deviation. This we have also seen that D is the damping coefficient right and H is equal to your inertia constant in megawatts second upon MVA; that will be H in second. This megawatt by make MVA it is a dimensionless it is second right, but we are writing megawatt second per MVA. So, H actually is your; what you call that in second right and K is equal to shaft stiffness in per unit torque per electrical radian right and t we know the time in second right.

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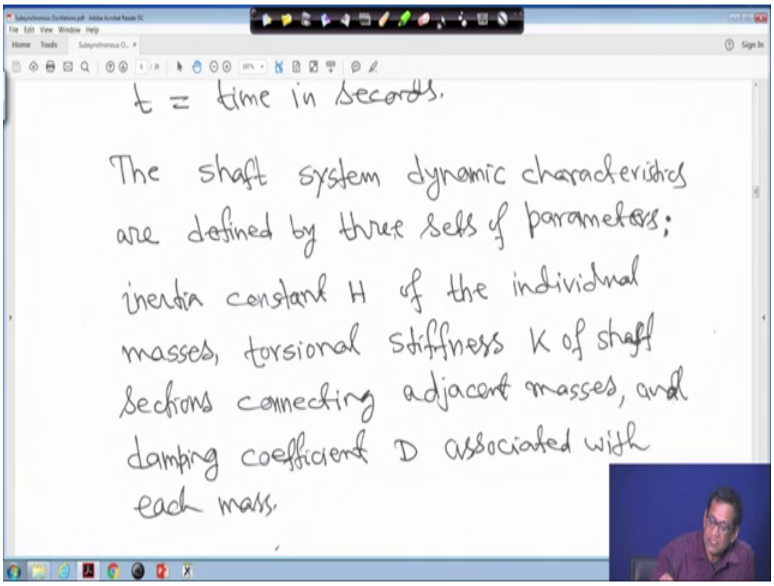

So, the shaft system dynamic characteristics are defined by three sets of parameters right that is your that is your just hold on. So, three sets of parameter that is your this is one is inertia constant H, this is one of the individual masses. Then torsional stiffness your stiffness is K of the shaft right sets that is shaft different section 1 to 2, 2 to 3 we have given you 1 to 2 K 1 2 in the figure 1 right and connecting adjacent masses and the damping coefficient D right. So, these are the actually what you call that three sets of parameters we can define it by three sets of parameters.

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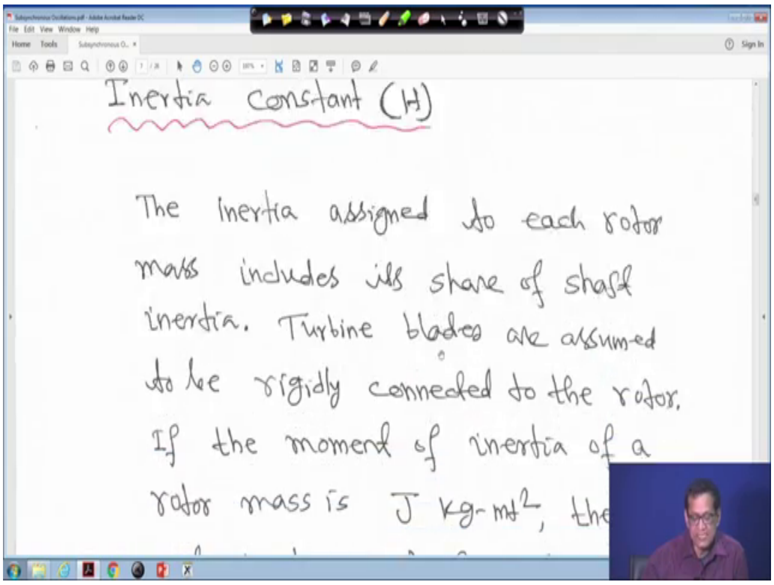

So, if it is so then inertia constant H right. So, this early earlier we have seen that your what you call that your inertia constant H for synchronous machine modelling when you are developing; we have already seen the inertia assigned to each rotor mass includes to its share of shaft inertia. So, turbine blades are assumed to be rigidly connected to the rotor right.

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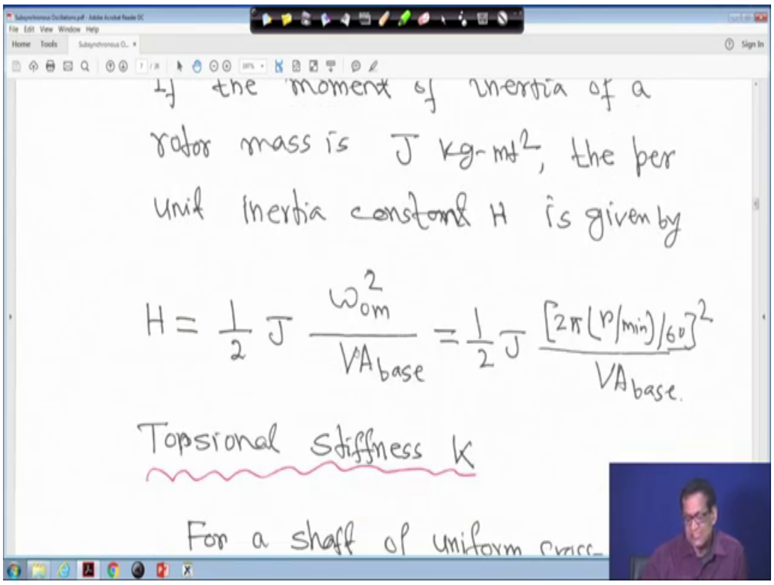

Now, if the if the moment of inertia of a rotor mass is J kg per meter square right; the per unit inertia constant H is given by this earlier we have derived this for synchronous machine chapter right. So, H is equal to half into J into omega 0 m square divided by volt ampere base right and this one, this one we can write half J in bracket 2 pi that is your RPM right 2 pi r that is per minute divided by 60 whole square divided by VA base right. This already we have your; what you call we have derived for your synchronous machine case the inertia right.

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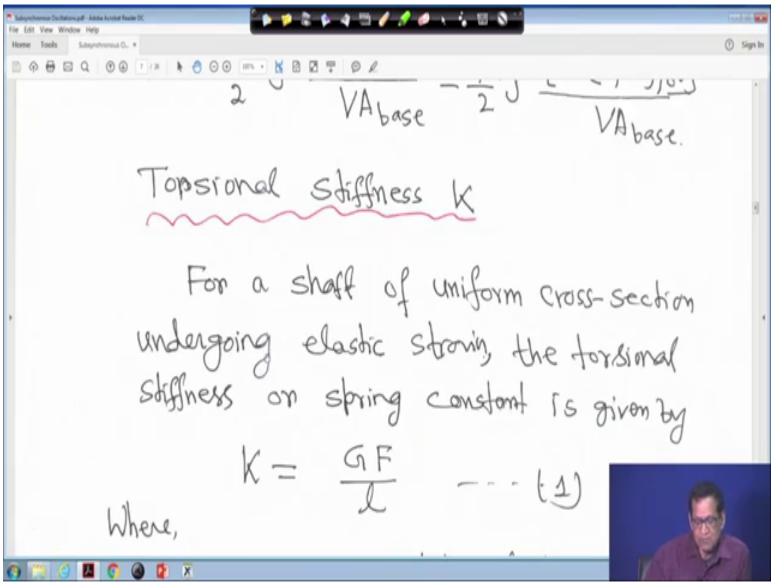

Now, next is the torsional stiffness K; for a shaft of uniform cross section undergoing elastic strain the torsional stiffness or spring constant is given by this is actually little bit from physics right.

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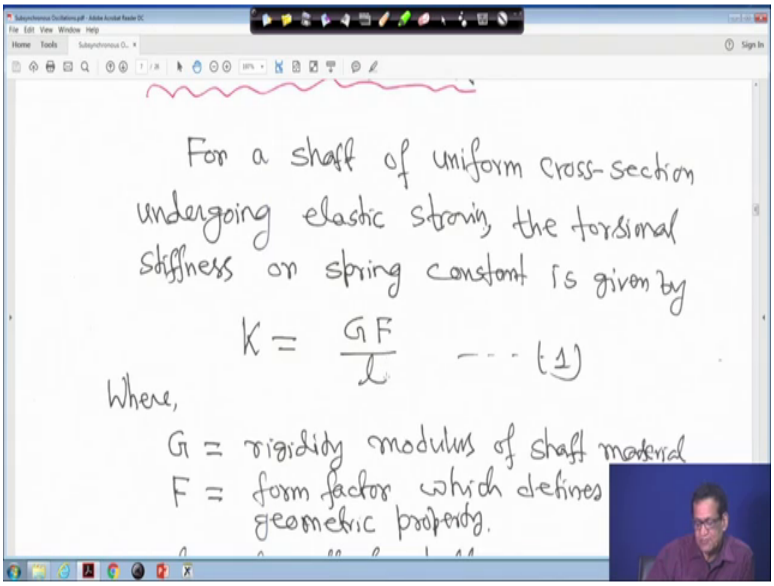

So, its spring constant or your what you call torsional stiffness we K is equal to G into F upon l directly we can write that right where, G is equal to rigidity modulus of shaft material, F form factor which defines the your geometric property and l is the length of the shaft.

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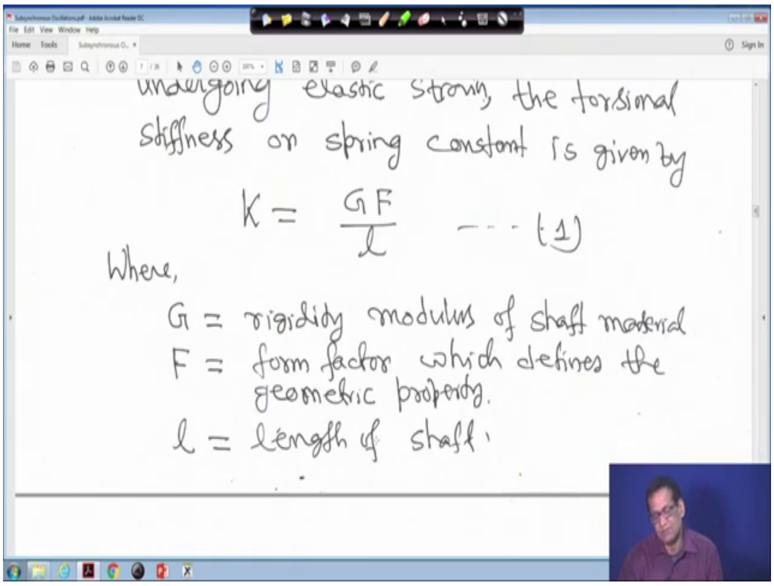

Therefore, K is equal to G F upon l this is equation 1 right. Now, for a solid shaft of circular cross section say with diameter d, we can write this is your F this is your F F is equal to form factor which defines the geometric property.

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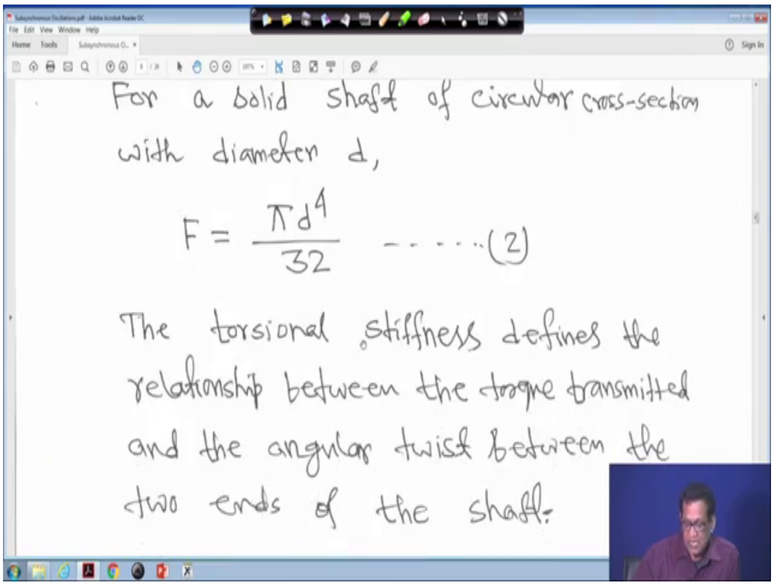

So, F is equal to actually pi d 4 by 32 right this is equation 2. The torsional stiffness defines the relationship between the torque transmitted and the angular twist between the two ends of the shaft right.

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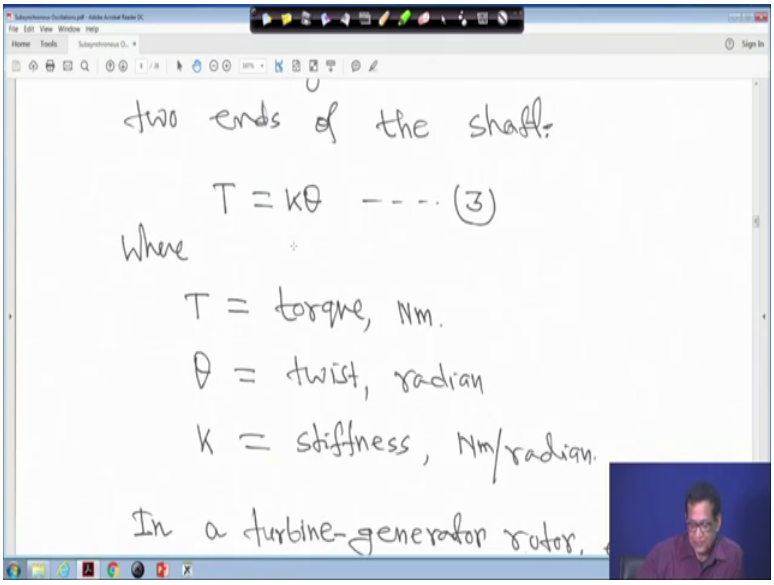

That means we can write the in general the T is equal to K into theta this way we can write where, T is equal to torque in Newton meter. And, theta is equal to twist that is in radian and K is equal to stiffness that is Newton meter per radian torque is equal to we can write K theta right.

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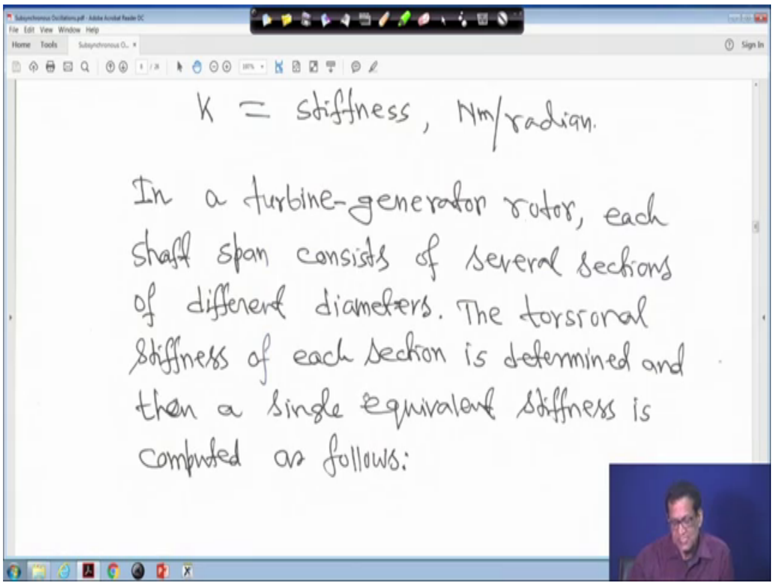

So, in a turbine generator rotor each shaft span consist of several sections of different diameters. The torsional stiffness of each section is determined and then a single equivalence stiffness is computed as follows.

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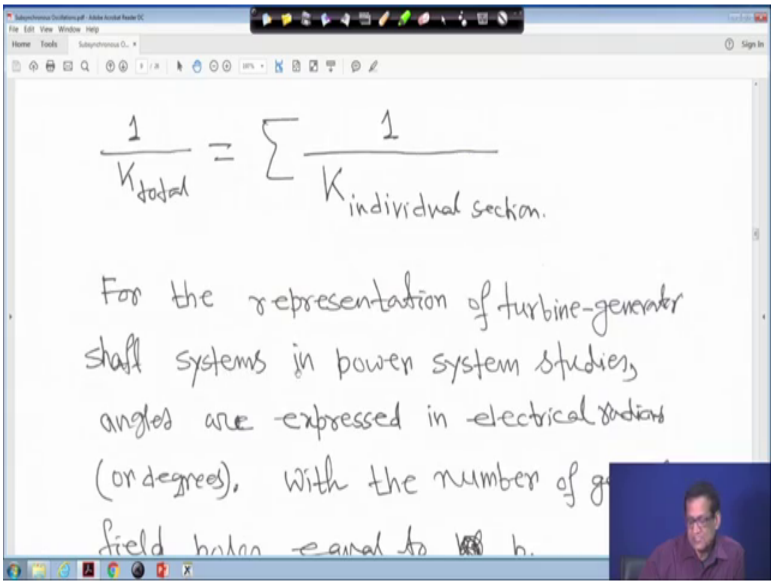

So, how we will do this that 1 upon K total that is the total stiffness is equal to sigma 1 upon K individual section, that just you have to this is the relationship that 1 upon K total is equal to sigma 1 upon K individual section. For the representation of turbine generator shaft system in power system studies angles are expressed in electrical radians or degrees right with the number of generator field poles equal to p f right.

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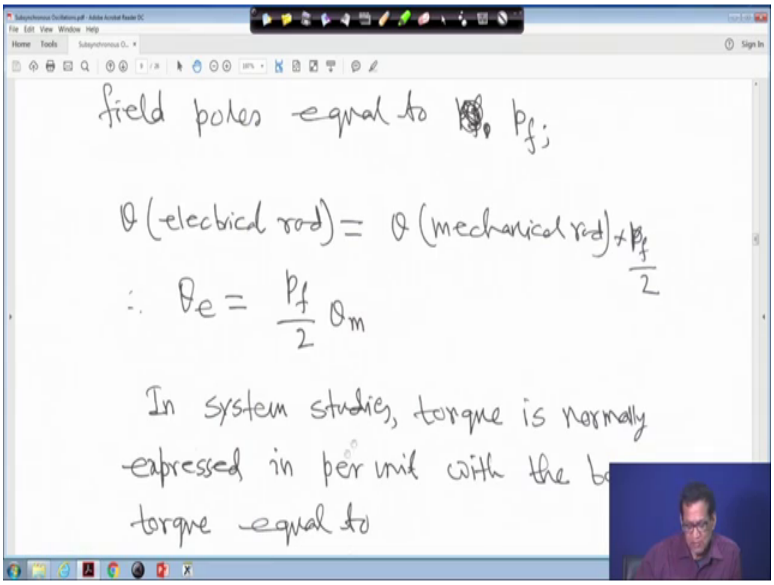

So, we know this relationship you know the theta electrical is equal to your what you call theta mechanical into p by 2 generally relationship you know. So, theta electrical radian is equal to theta mechanical into p f by 2; p f is the number of field pole right or we can write theta electrical is equal to p f by 2 theta m this relationship also we know right. In system studies torque is normally expressed in per unit with the base torque equal to.

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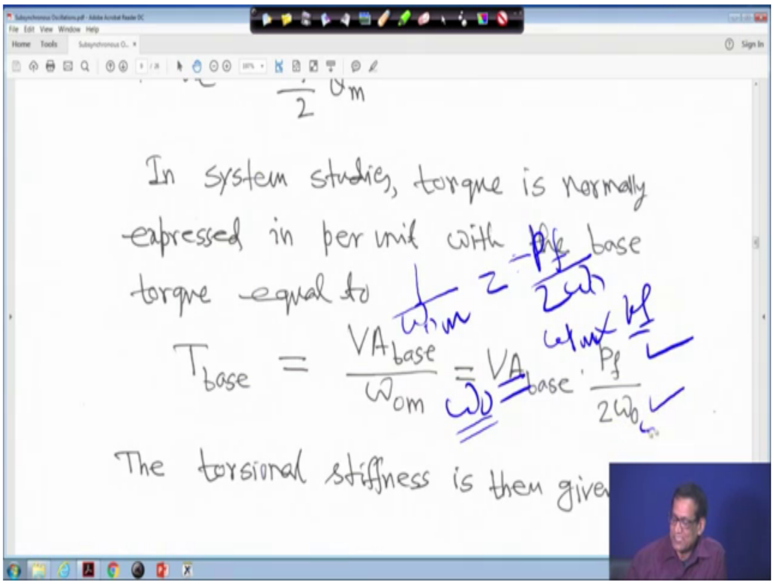

So, this base torque T base is equal to VA base upon omega 0 m right, this one we have already derived in synchronous machine chapter right sorry synchronous machine that your modelling development of the modelling.

So, this is VA base and omega 0 m and omega 0 the relationship also know you substitute, it will be actually your p f VA base into p f upon 2 omega 0 right. So, that the your what you call this relationship that your what you call this omega 0 m that mechanical radian per second and electrical radian per second same. So, already the already we have made it. So, this omega 0 m we are replace it by your what you call that 1 upon omega 0 m replaced by p f upon 2 omega 0.

That means, this one we have already done it know that your I am making it here suppose this omega 0 m whatever we are writing here, that is your just to this thing that is your p f divided by 2 omega 0. That means, omega 0 right is equal to your what you call that omega 0 m into p f by 2 right. So, we know theta is equal to p f by your what you call p by 2 into theta m. So, therefore, omega 0 is equal to omega 0 m into p f by 2 that relationship we are using here right. So, so this is already this thing already we have derived in synchronous machine topic right.

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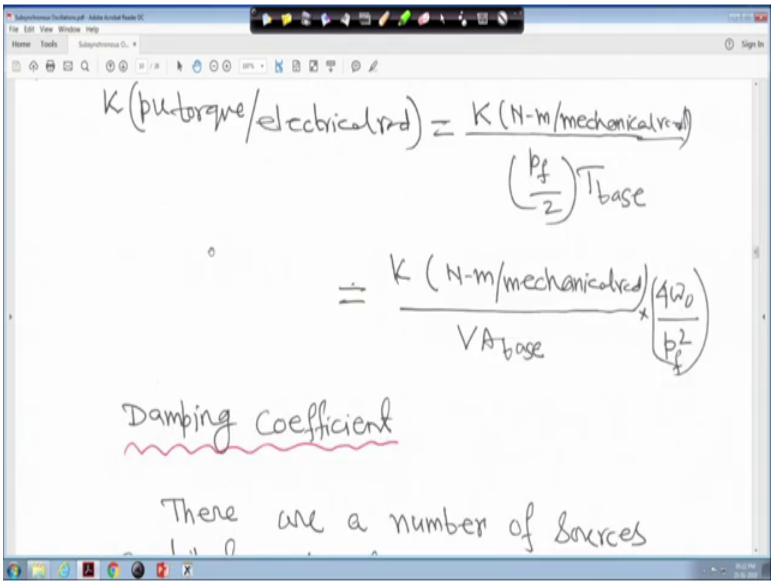

So, the torsional stiffness is then given by that K that is the torsional stiffness it is per unit torque per electrical radian right. So, in general it is a dimensionless quantity is equal to K that is Newton meter per mechanical radian divided by p f upon 2 into T base right this way we can represent, you can try also simple thing right. So, it will be K that you define the your unit it is Newton meter per mechanical radian divided by p f by 2 into torque base.

So, if we substitute the torque base expression. So, this will become actually K that is your Newton meter per mechanical radian that is that units also writing in the bracket for easy understanding. And, then T base you replace right T base whatever you have got from here T base you replace by this one right. If you do so, if you do so then it will become K Newton meter per mechanical radian divided by V A base into 4 omega 0 upon p f square right. So, this is actually expression for K; next is the damping coefficient.

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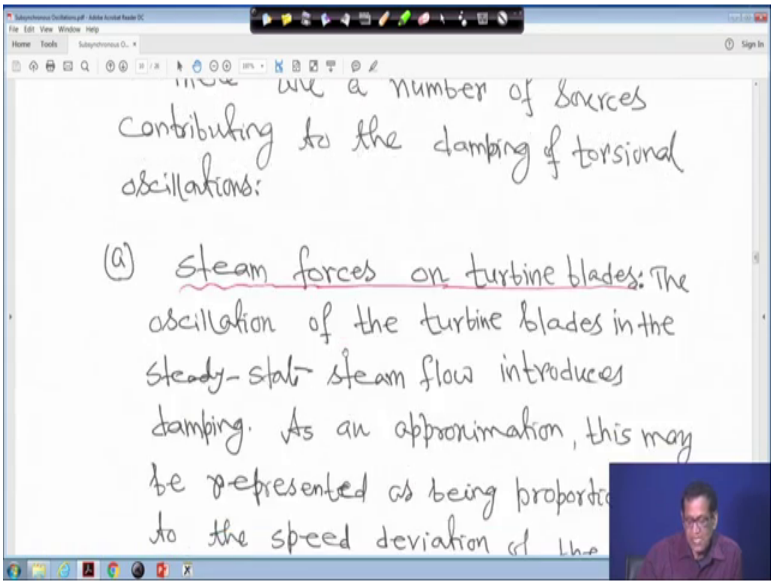

So, there are a number of your what you call sources contribute in to the damping of torsional oscillations right. So, number a that that is that is steam forces on turbine blades right that is because, that steam actually inject on the turbine blades. Therefore, the oscillation of the turbine blades in the steady state steam flow introduces damping right.

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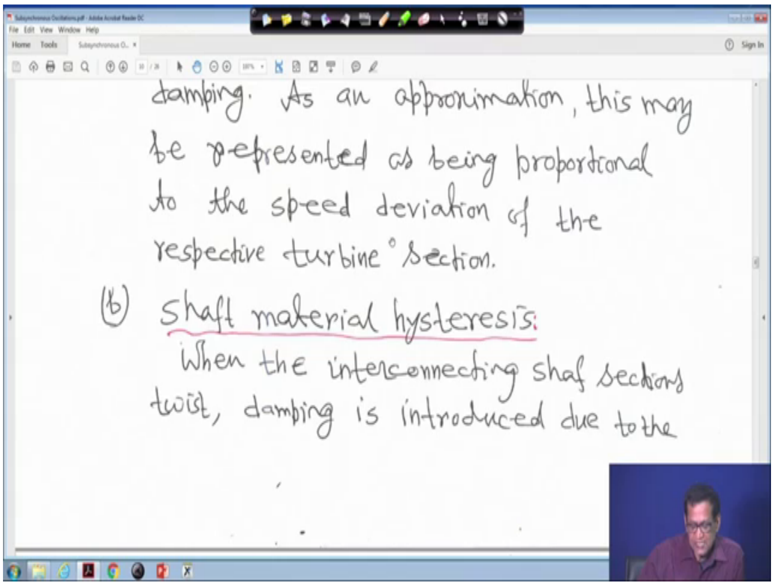

As an approximation this may be represented as being proportional to the speed deviation of the respective turbine section right. This is for the your what you call that is the steam forces on turbine blades right. Now, number second one b that is the shaft material hysteresis when the interconnecting shaft section twist damping is introduced, due to the mechanical hysteresis of the shaft material as it undergoes cyclic stress strain variations right.

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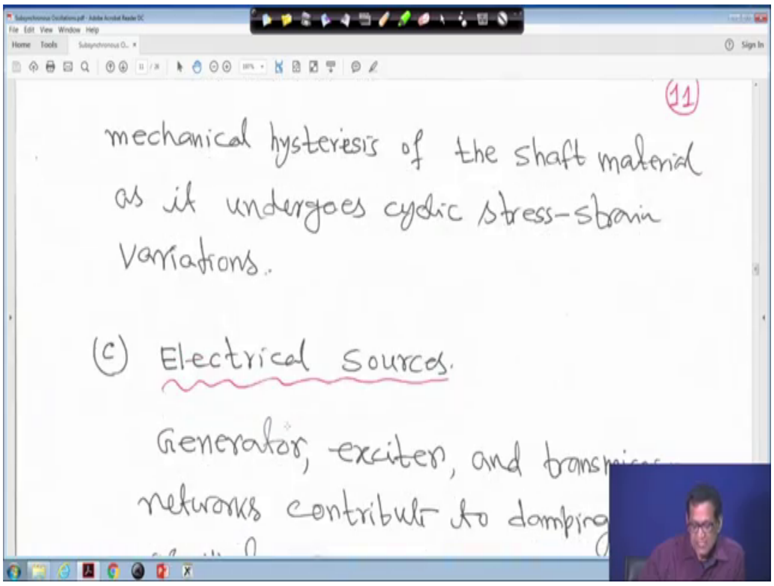

So, this is number 2 then number 2 the electrical sources that is generator, exciter and transmission network contribute to damping of oscillations.

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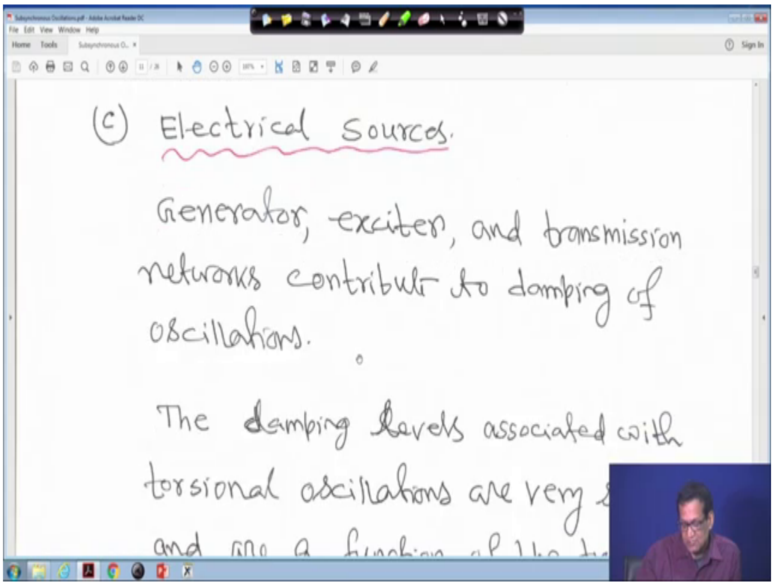

Therefore, that your what you call that your there are a number of sources contributed to damping. One is the steam forces on turbine blade, then b is that shaft material hysteresis and c is last one is electrical sources; electrical sources means generator exciter and transmission networks contribute to damping of oscillation right.

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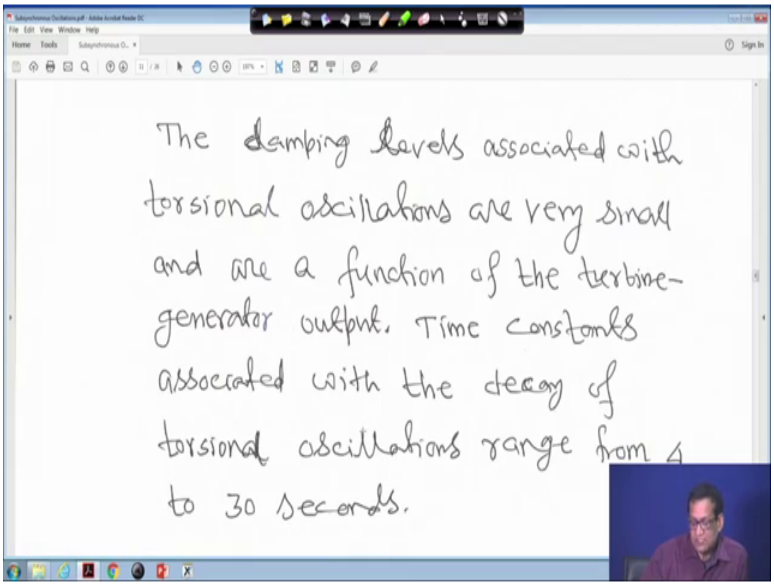

The damping levels associated with torsional oscillations are very small and are a function of the your what you call turbine generator output right. So, time constant associated with the decay of torsional oscillations range from 4 to 30 second right. So, these are the time constant associated with the decay of torsional oscillations right so, it range from 4 to 30 second.

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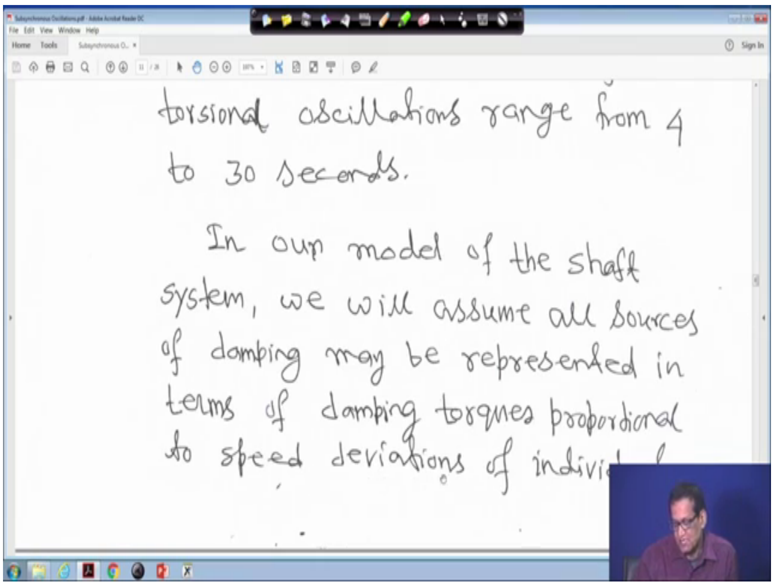

In this model the your so, that is of the shaft system we will assume all sources of damping may be represented in terms of damping torques proportional to speed deviation of your individual masses right.

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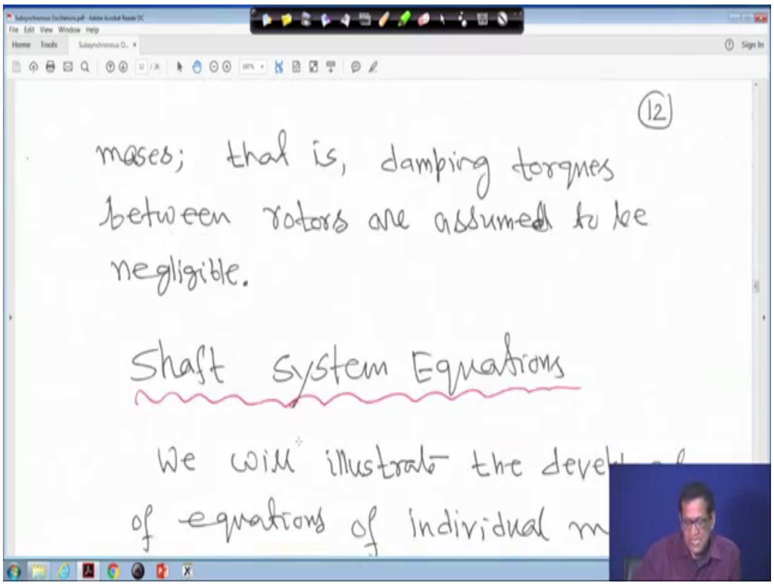

So, we represent it in a simplest manner we represent it in a simplest manner, that is damping torque between rotors are assumed to be negligible right.

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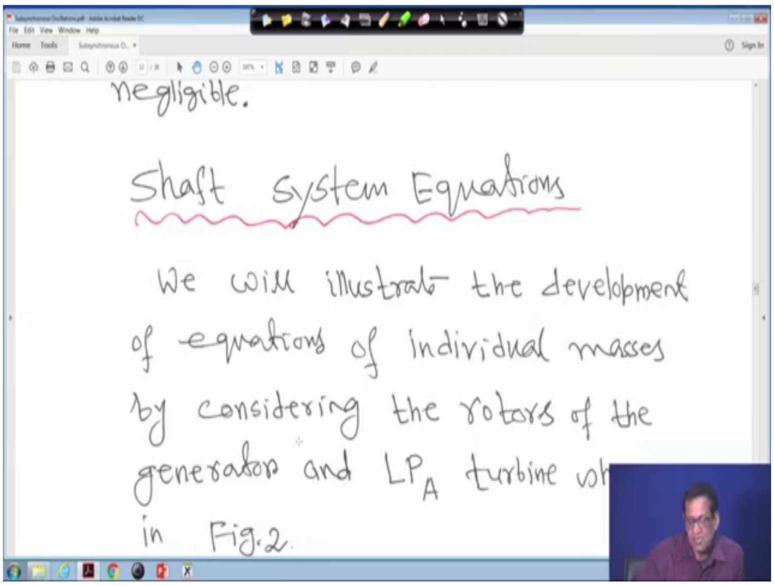

So, now shaft system equations. Now, because shaft system we have seen you have generator, you have low pressure section of the turbines, then intermediate pressure, then high pressure right. So, we will illustrate the development of question of individual masses by considering the rotors of the generator and L P A turbine shown in figure 2 right. So, this is actually this is a only we have considered whatever we have whatever we have seen in figure 1, this is actually section 1 this is actually 1 right.

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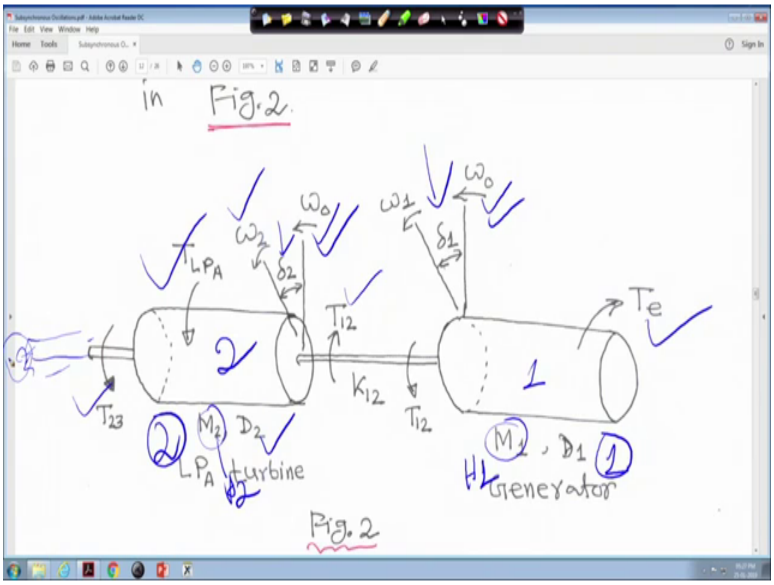

And this is actually 2 only this these two parts we have considered similarly, 3 4 like this. So, this is actually inertia return is M what we will take as a H 1 right and is damping for this section is D 1, this is also instead of M 2 we will take as a H 2 right and this is D 2.

And, and this is section 1 and this is section 2 right and here that stiffness that is K 12 and this is the toque T 12 right acting on the shaft. And this is your omega 0 right, this is your omega 0 and this is actually omega 1. So, your what you call that deviation in between omega 0 and omega 1 that is your delta 1; omega 0 is the rated speed right that is the rated speed.

But we are assuming that it cannot be on rated speed maybe some your what you call it is not ideal so, some variation will be there. So, that is why this is your angle is delta 1. Similarly, this if this is omega 0 this one your rated speed. So, for the your L P A that is that low pressure one of the low pressure section; next it is showing omega 2 right and this is your delta 2. Although they are what you call they are connected together right, but shaft has different stiffness. So, naturally the all this variation omega 1, omega 2 may be your what you call not your what you call not much difference, but some difference will be there it cannot be neglected right.

And, this is the electrical torque we have define and T L P A also define and T your T 12, then section 2 to 3 where this is section 2 this side is section 3 is also connected right section 3 is also connected with this. So, that is why it is show T 23 So, this way only what we have done is we have only considered for section 1 and section 2.

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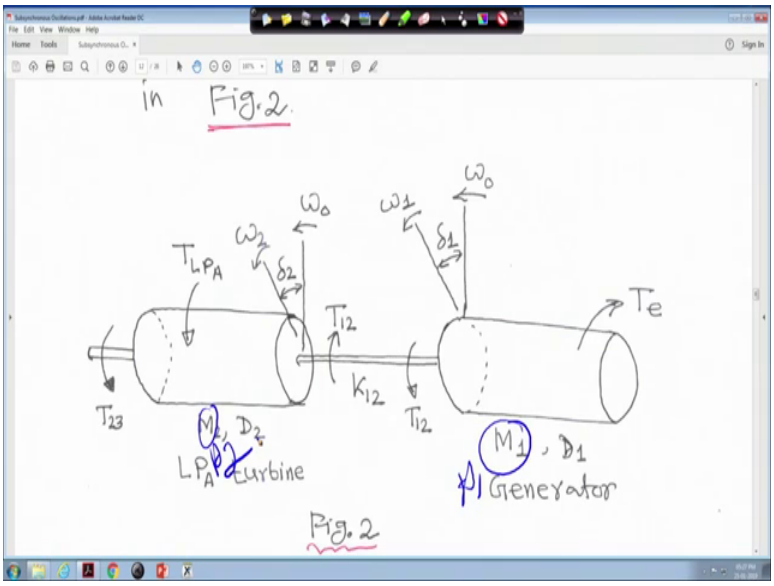

And this one again I am telling this is actually H 1 and this is actually H 2 right instead of M we write H because throughout this course we have taken H right. So, if you now write down the equation one after another; so, from our your intuition we can write those equations right.

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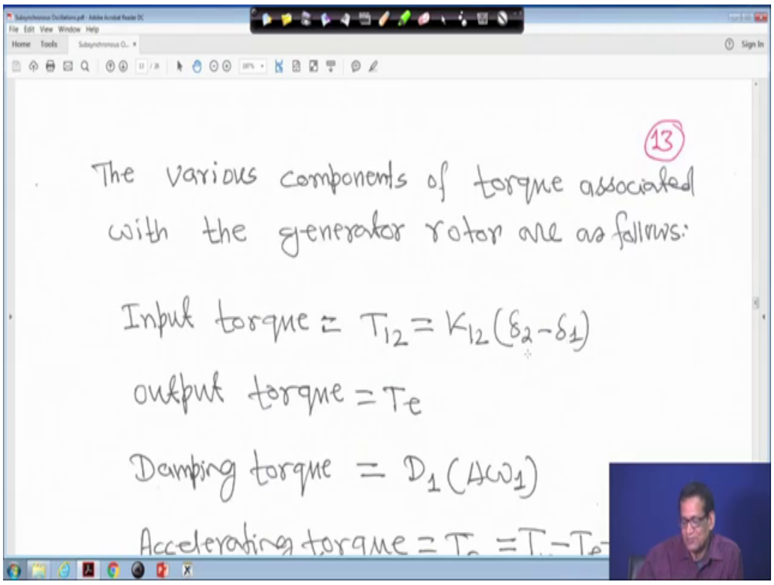

So, for example, the various components of torque associated with the generator rotor are as follows that input torque T 12 will be K 12 delta 2 minus delta 1. So, this is input torque T 12 I mean to that your generator side right input torque T 12 will be your K 12 the stiffness right into your delta 2 minus delta 1 that is the angle different; this delta 2 minus delta 1 and that is your T 12. So, this is the input torque right, similarly output torque is equal to T e this is my output torque, this is your T e right.

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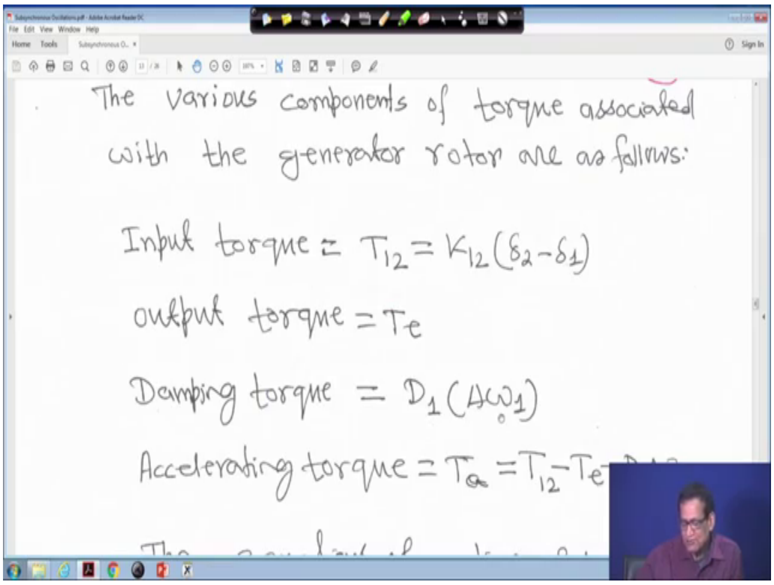

Now, damping torque generally D 1 into delta omega 1 right. So, this is actually D 1 and it will be delta omega 1 right. So, later we will see this. So, accelerating torque then T a will be is equal to T 12 minus T e minus D 1 delta omega 1 right. So, this is how we write the acceleration torque T a is equal to right. So, the equations of motion of the generator rotor then can be written as the 2 H 1 d dt delta omega 1 is equal to T a; that means, for this part that means, this is my generator part this is my generator part right.

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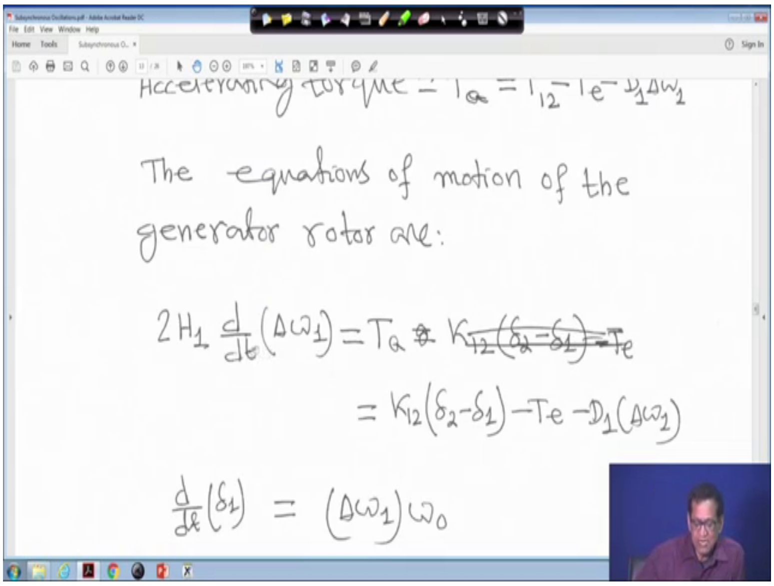

So, this equation earlier also we have seen this and 2 H 1 d dt of delta omega is equal to T a is equal to K 12 delta 2 minus delta 1 minus T minus D 1 delta omega 1 right.

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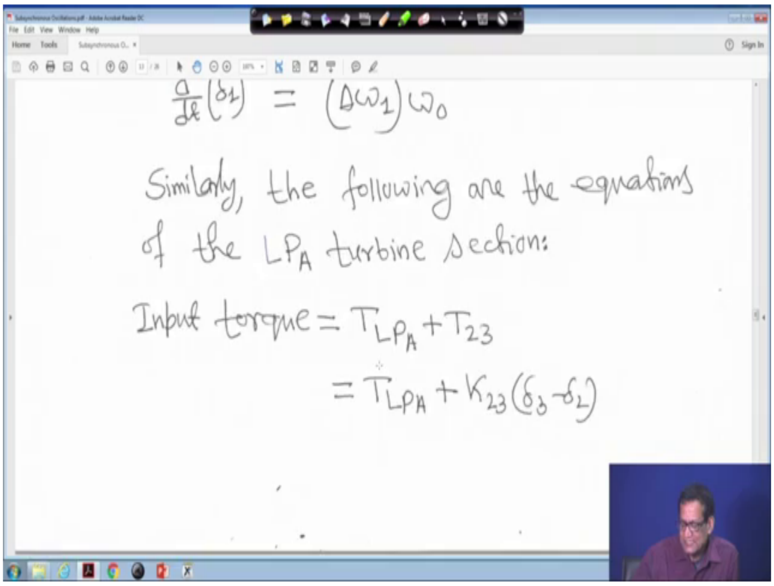

Similarly, for delta 1 d dt of delta 1 this equation we have already seen before that for synchronous machine generator for single machine infinite bus system. So, there we wrote know d dt of delta is equal to delta omega into omega 0, but it is for generator your section 1 that is the generator that is why we write d dt of delta 1 is equal to delta omega 1 into omega 0 right. So, this part therefore, this part for this generator right for the generator.

Now, next one similarly the following your are the equations of the L P A turbine section. Now, input torque to the L P A section will be your if this is the L that is the L P A section, it will be your T L P A plus T 23 this is the input torque to this right. Therefore, here it is your here it is that input torque will be T L P A plus T 23 right. So, it is equal to T L P A plus K 23 delta 3 minus delta 2 because this side although two sections are shown right.

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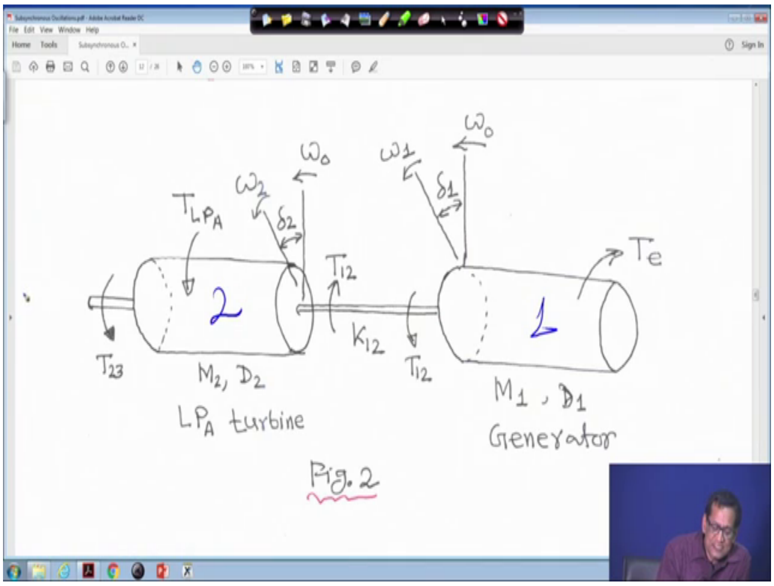

Then this is my section 1, this is my section 2 that other side also it is connected know just hold on the that section 3 also that L P B part is connected right.

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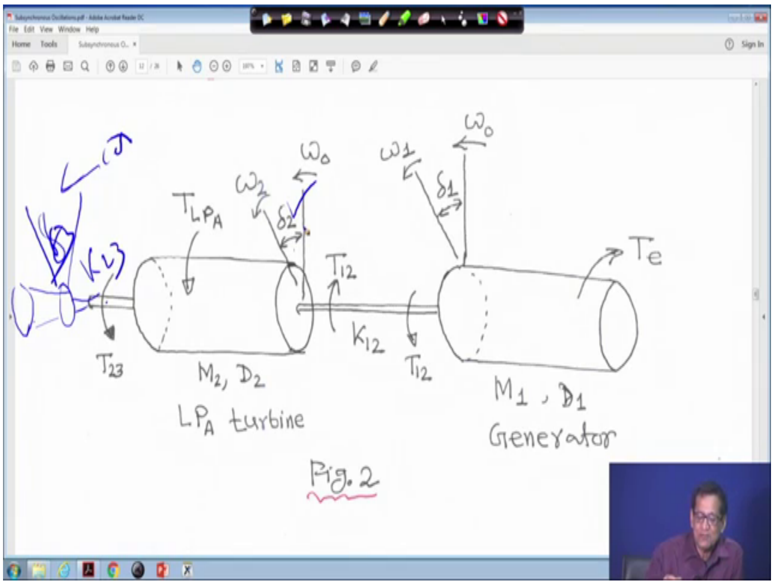

So, this is the shaft so, T 23. So, then here it is angle delta 2 for here also your this thing also it is omega 0, this angle is delta 3 right. So, that is why we are writing and stiffness for this one for this shaft it will be K 23 that is why we are writing K 23 into delta 3 minus delta 2. So, this is delta 3 minus delta 2. Similarly, for section 4 section 5 this way from your intuition or from the inspection you can write it right. So, that is why we are writing this equation that T L P A plus K 23 delta 3 minus delta 2 right that is the input torque.

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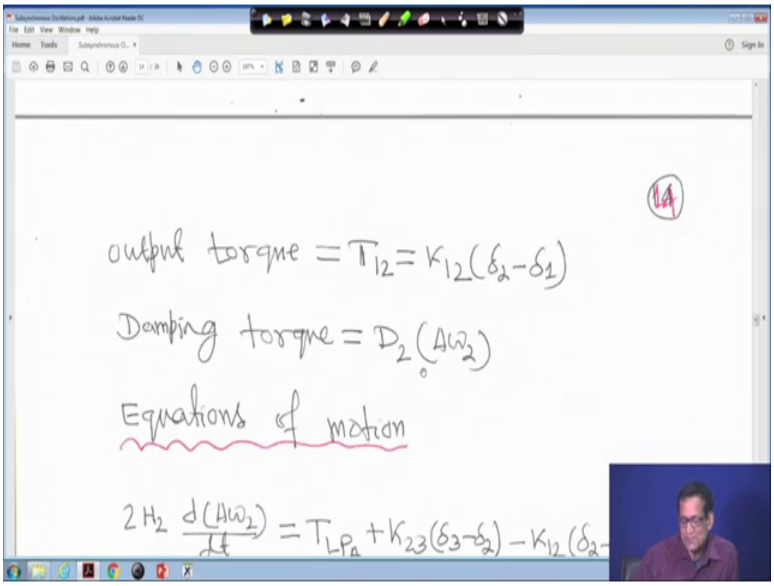

Now, output torque that is T 12 therefore, this section if you come for this part the if this case its output torque is T 12 which is input to the generator right. So, out T 12 is equal to you know K 12 then delta 2 minus delta 1. So, this output torque is the T 12 is equal to K 12 delta 2 minus delta 1 and damping torque is equal to same as before; for the section 2 it will be D 2 into delta omega 2 right.

Thank you very much, we will back again.