Power System Dynamics, Control and Monitoring
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Lecture - 57
Hydraulic turbine modelling

So, in the previous lecture that we have seen that if u is a upper triangular then directly that it can be solved for u, right since U is upper triangular, right.

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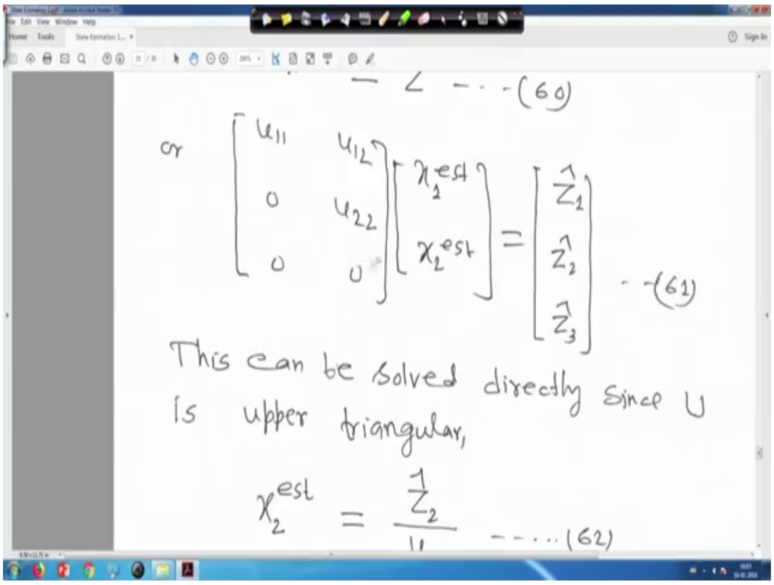

So, we have seen that your x 2 estimated is Z 2 cap upon u 22 and x 1 estimated is equal to 1 upon u 11 into Z 1 cap minus u 12 into x 2 your what you call estimated, right.

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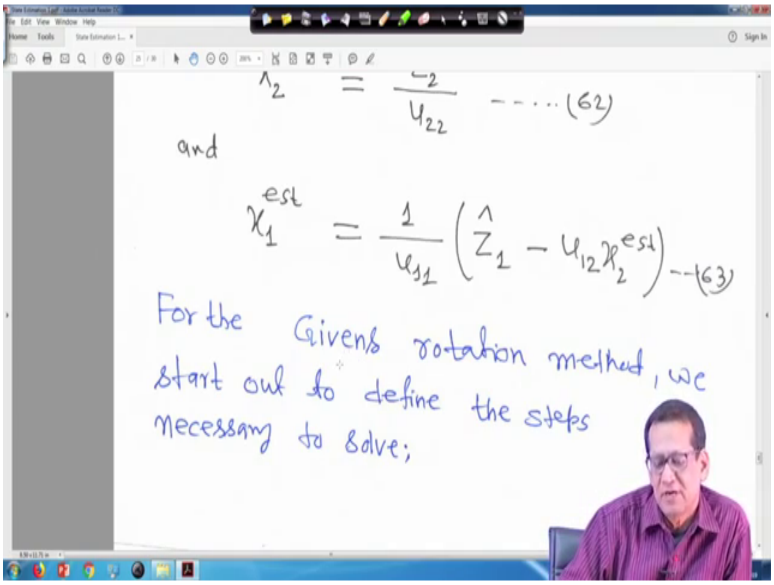

So, for the given rotation method we start out to define the steps necessary to solve. Actually here that givens rotation method, right we will not this thing cover that one because I have to minimise the time, but just give you some idea that one article is there Simoes-costa, A Simoes-costa and V Quintana, right.

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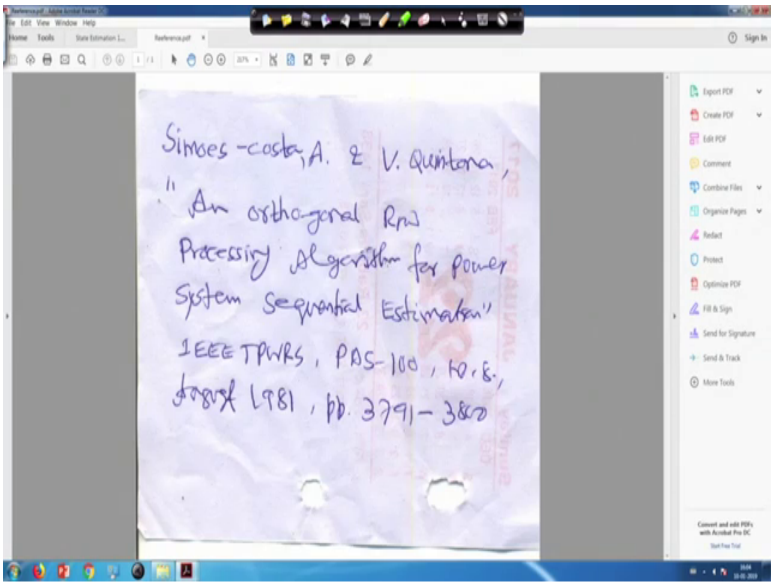

So, just if you look into the article it was published in 1981, right that IEEE transaction on power system TPWRS, right. And if you have anything is there that your, right now I cannot provide you from here that this article where rotation your we will find that givens rotation method, but during the conduction of this course, right, at that time if you put ask something we will upload it to the in the forum, right. So, this is actually this is the article.

So, here we will not go in detail, right. So, what we will do that just give you some favour on the given rotation method. So, we start out to define the step necessary to solve, right.

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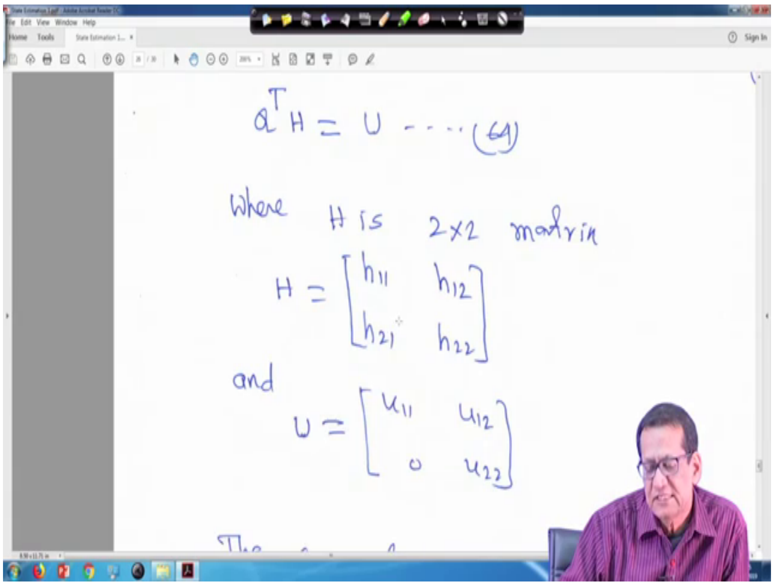

So, let us say Q transpose H is equal to U this is equation 64. Now, where H for example, say H here is a 2 into 2 matrix. So, H is equal to the small h 11, h 12, h 21, h 22, right and as U is upper triangular, so u 11, u 12, 0, u 22, right.

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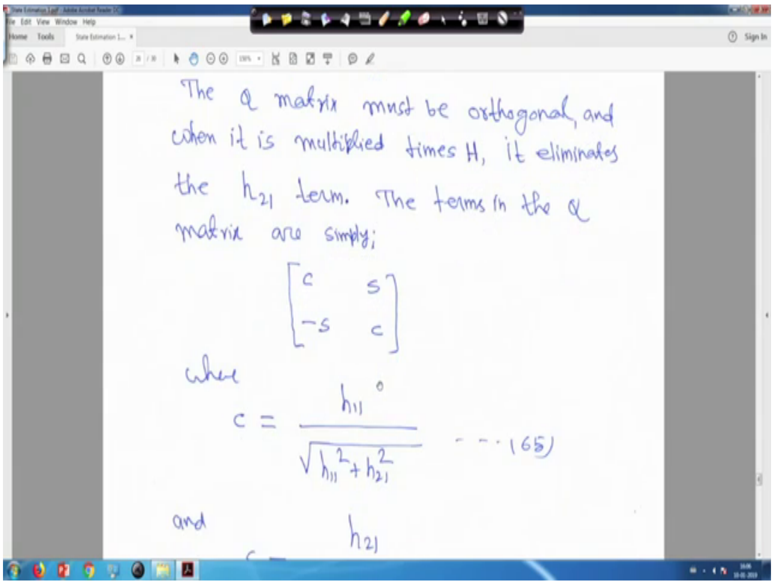

So, the Q matrix must be orthogonal when it is multiplied times H, right just hold on I will reduce that size. So, when it is multiplied times H it eliminates the h 21 term. The terms in the Q matrix are simply, it is actually c, s, minus s, c, right. So, where c is equal to define h 11 root over h 11 square plus h 21 square and s is equal to h 21 root over h 11 square plus h 21 square.

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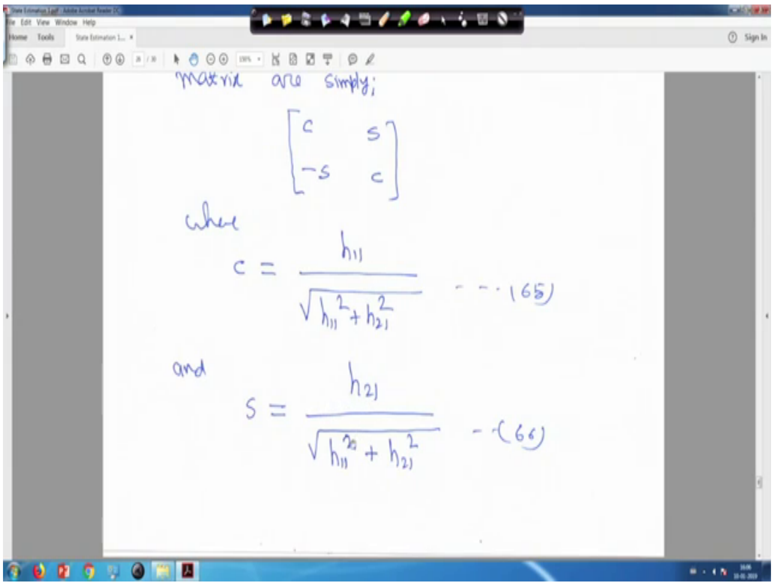

So, if you take the determinant of this matrix it will be basically c square plus s square because c square minus of minus s square, so c square plus s square. So, if you take c square plus s square then it will be 1, right if you add these two it will be 1. So, this way the terms of the Q matrix are simply this one, right where c and s are defined like this.

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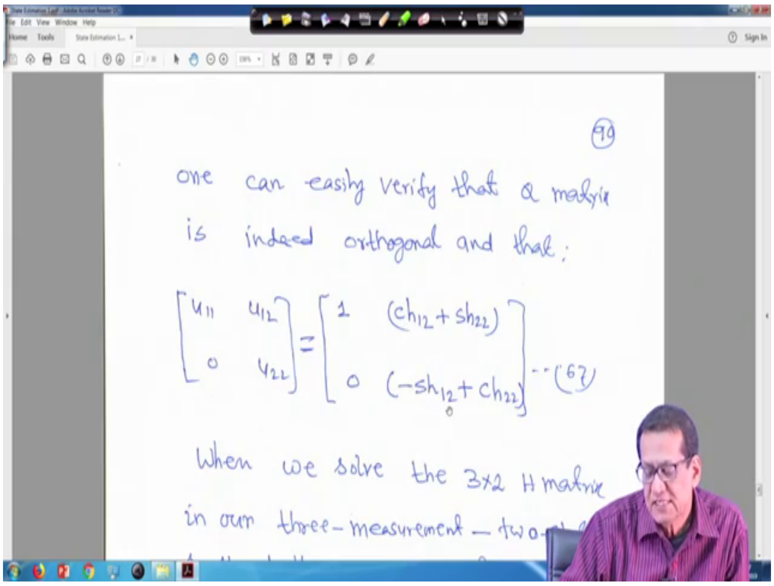

So, one can easily verify that Q matrix is indeed orthogonal and this one u 11, u 12, 0, u 22 will be 1, ch 12 plus sh 22, 0, minus sh 12 plus ch 22, right. So, here we have assumed that your Q transpose H is equal to U, right. So, U is this one H is given, right. So, and your Q matrix is this therefore, your u 11, u 12, 0, u 22 that is equal to 1, ch 12 plus sh 22, 0, then minus sh 12 plus ch 22. This is equation 67.

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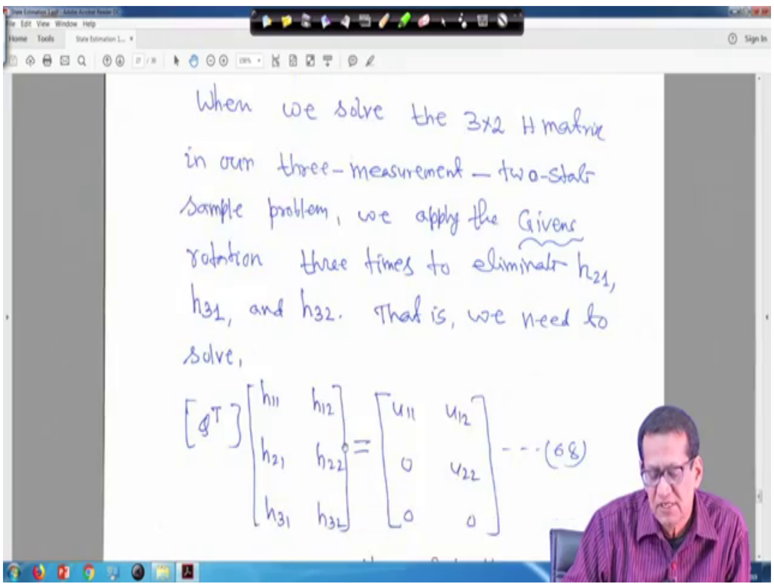

Now, when we solve the 3 into 2 H matrix, in our 3 measured measurement two state sample problem that we have seen before, right whereas, 3 measurement values, but ultimately, we are evaluating two values like delta 1 and delta 2 and bus 3 it was a slag bus, right. So, you apply the givens rotation 3 times to eliminate h 21, h 31 and h 32 that is we need to solve that Q transpose into H. So, H is h 11, h 12, h 21, h 22, h 31, h 32 is equal to u 11 it is upper triangular u 11, u 12, 0, u 22 and 0, 0. So, this is equation 68.

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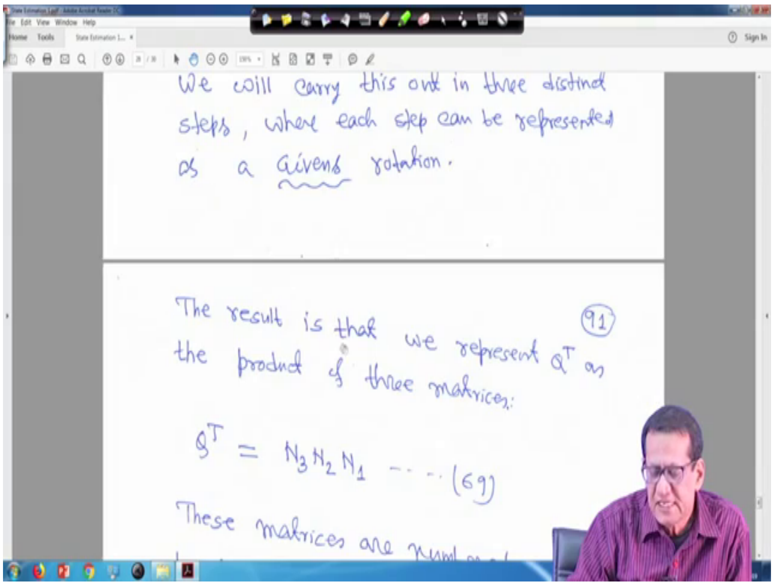

Now, we will carry this out in 3 distinct steps, right when each step can be represented at a given rotation. So, detail all these things I will not cover, I will just simply tell you that methodology, right. So, the result is that we represent Q transpose as the product of 3 matrices that is Q transpose is equal to say N 3 into N 2 into N 1. So, this is equation 69.

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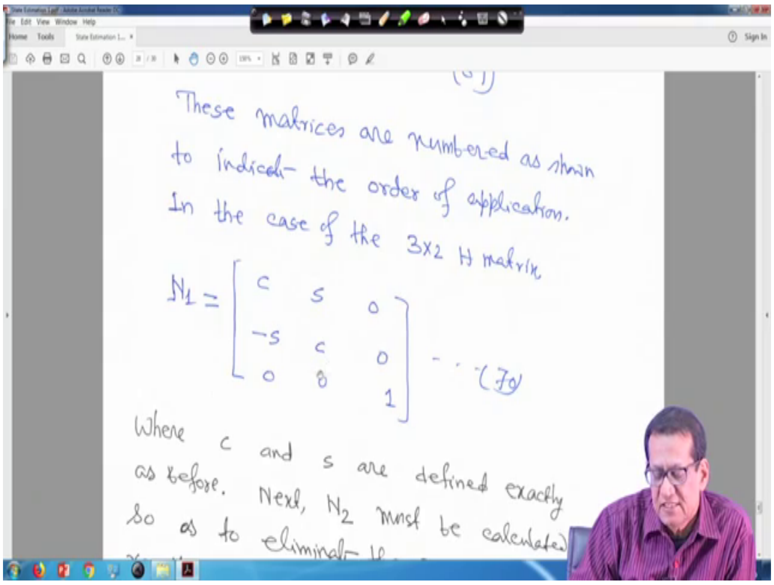

These matrices are numbered as shown to indicate the order of applications. In the case of 3 into 2 H matrix, your a here it is N 3 N 2 N 1, so N 1 will be c, s, minus s, c and this one is 0, 0 and this one is 1, that is your N 1. This is equation 70, right, where c and s are defined exactly as before.

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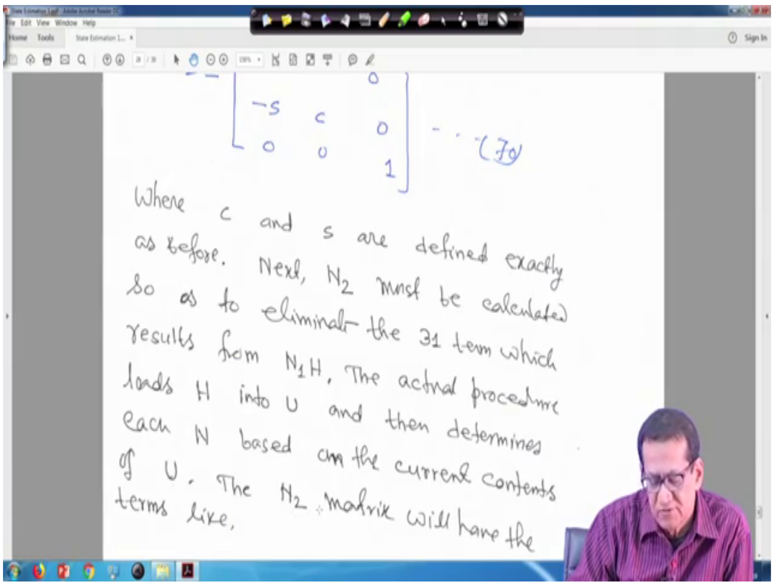

So, next N 2 must be calculated. So, as to eliminate the 31 term that is h 31 term, right 31 term which results from N 1 into H. The actual procedure your loads your loads H into U and then determines each N based on the current contents of U. So, the N 2 matrix will have the terms like this.

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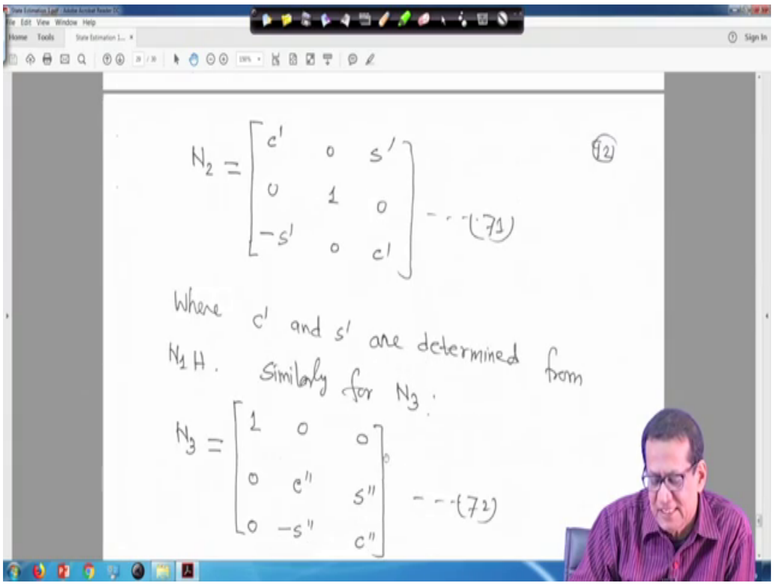

So, the N 2 matrix will be N 2 will be c dash, 0, s dash, then 0, 1, 0, minus s dash, 0, c dash, right here it was c, s, minus s, c, 1, 0, 0, 1, right. So, this one you I mean otherwise you see any control system book or anything just little bit go through it of your own here I have to minimise the time, right. So, N 2 is equal to c dash, 0, s dash, 0, 1, 0 and minus s dash, 0, c dash, right where c dash and s dash have determined from N 1 H. Similarly, for N 3 if you look into that here it will be 1, 0, 0, 0, c double dash, s double dash, 0, minus s double dash, c double dash, right. So, this way you have to evaluate.

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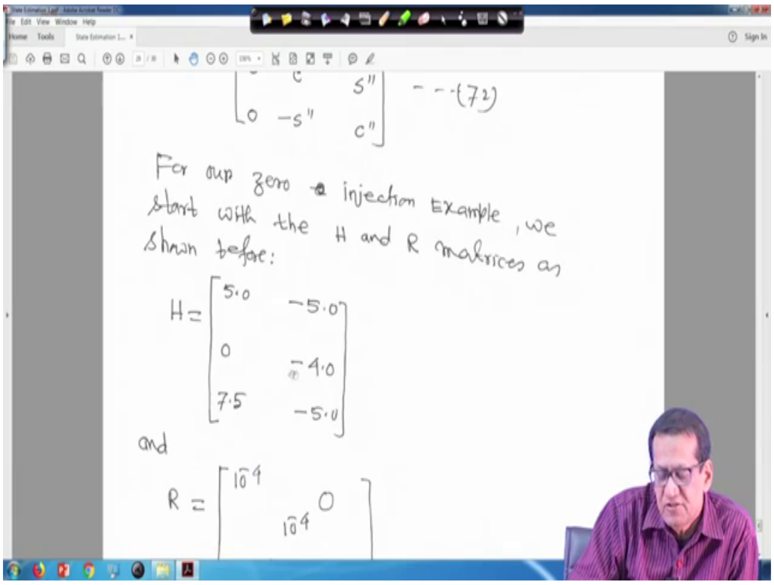

So, for our 0 injection example which start with the H and R matrices as shown before So, we knew H is equal to 5 minus 5 from the previous data whatever example we have seen, then 0 minus 4 and 7.5 minus 5. And R is equal to 10 to the power minus 4, 10 to the power minus 4 and this one 10 to the power minus 20, right.

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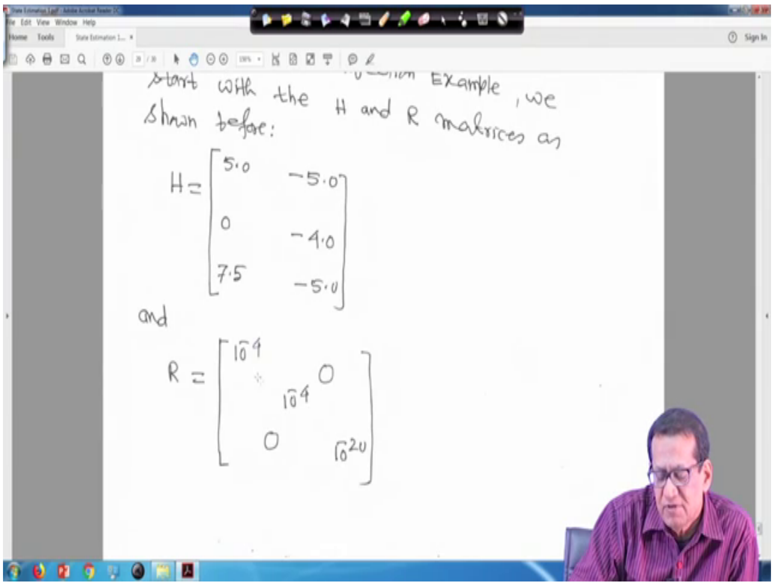

Therefore, that H dash matrix is actually it will become 5 into 10 square, then minus 5 into 10 square it means 0, minus 4 into 10 square, then 7.5 into 10 to the power 10 and minus 5 into 10 to the power 10. And the measured vector Z cap is 32, 72, 0, right.

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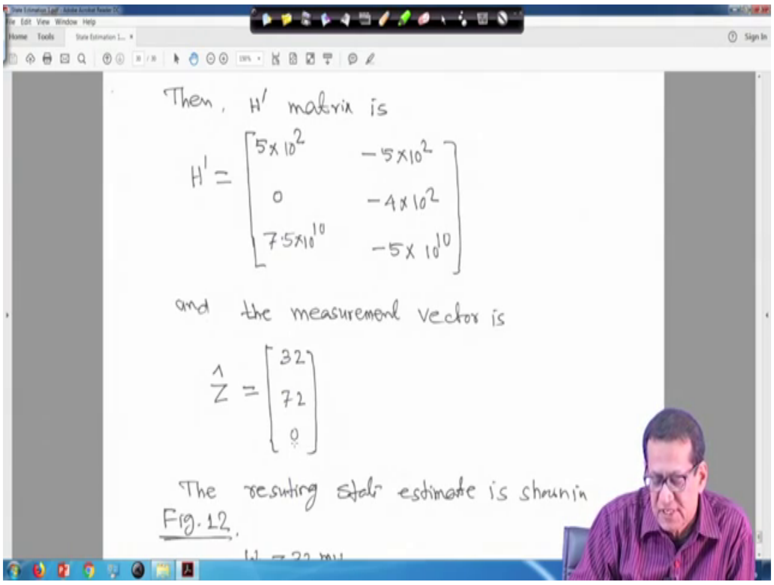

The resulting state estimate I will, after making all this calculation. I am showing you only the final result, right.

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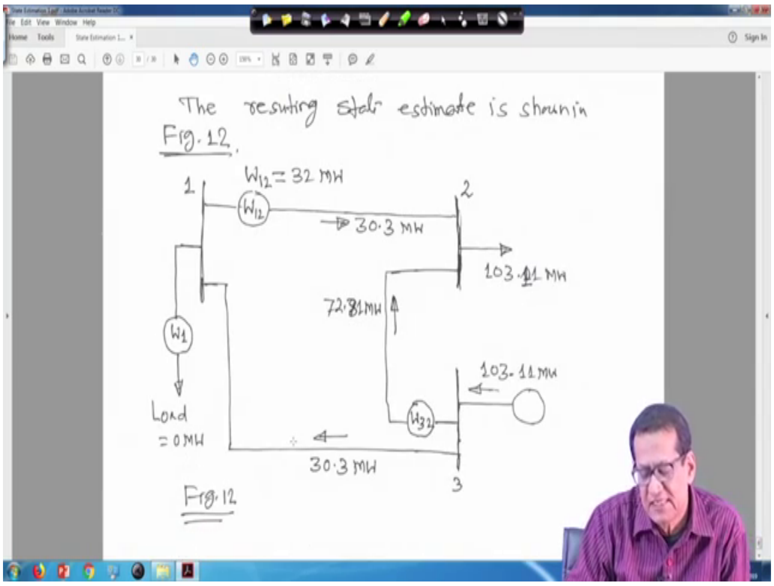

If you look into that here actually it is 0 megawatt and see it is a perfect your what you call there is no error. Here if you look into that the 72.81 megawatt and 30.3 megawatt, total is 103.11 megawatt, here also 103.11 megawatt and here also 30.3 megawatt because here it is 0 load is 0 30.3 megawatt, so it going through this also 30.3 megawatt, right.

So, this makes the your what you call calculation accurate calculation. And another thing I told you that for orthogonal decomposition if matrix is singular it case problem, so it helps to solve the problem, right. So, with this that state estimation many things are there whatever little bit we have studied is ok, but many things are there, but detailed cannot be covered in a video course, right, it takes lot of time. And particularly and from the your new your assignment or exam point of view we have to look into that the problem which you can solve in the classroom, right. So, that is why few things I have skipped here.

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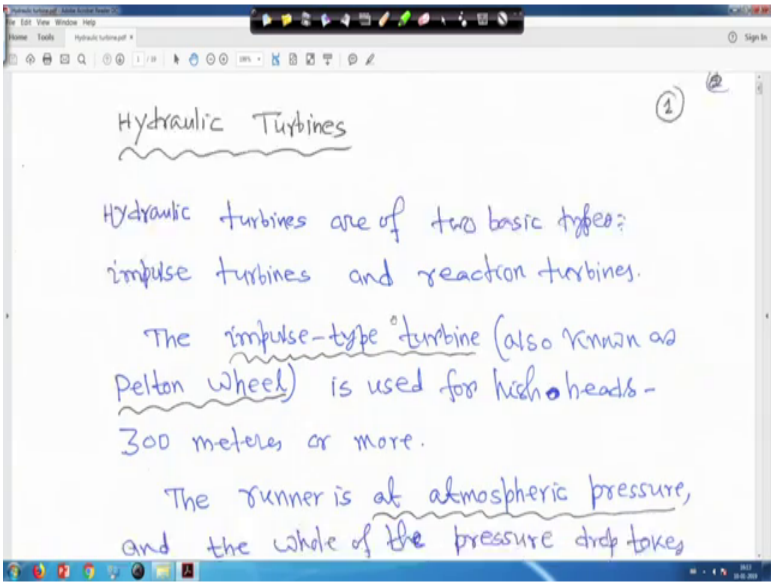

So, another thing now with this your; with this your just hold on; just hold on. So, now during this your AGC type when deregulated environment and conventional and deregulated thing I told you at that time we saw that your detailed modelling more or less detailed modelling of governor and your what you call steam turbines, right. So, at that time hydraulic turbines we did not take because I was thinking whether I will get time or not.

So, as we have some time, so we will start with hydraulic turbines, but not hydro governor. Hydro governor modelling again it will take some time. So, only hydraulic turbine, the classical hydro turbine modelling and just see how things, right. And another thing is little bit ideas will be given at the end I will regarding your limiters different type of limiters, right.

So, now hydraulic turbines actually are of two basic types, the impulse turbines and reaction turbines, but their detail will not be explained here, right. So, detail will not be explained here, but it just some brief description because our objective is that you to find out the your what you call the classical turbine model hydro turbine model, right. So, the impulse type turbine also known as Pelton wheel, right is used for high heads that is 300 meters or more, right.

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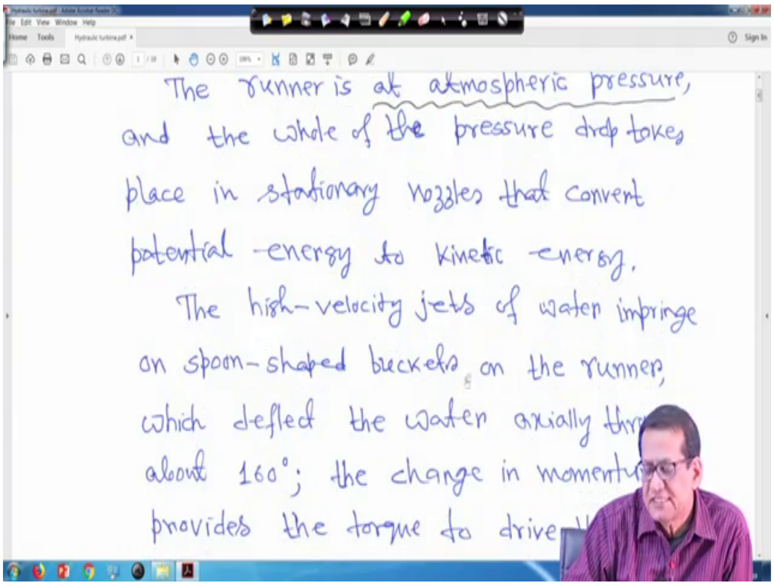

The runner is at atmospheric pressure and the whole of the pressure drops take place in stationary nozzles that convert actually potential energy to kinetic energy, right. The high-velocity jets of water impinge on spoon-shaped buckets on the runner which your deflect the water at axially through about 160 degree, right. The change in momentum provides the torque to drive the runner the energy supplied being entirely kinetic.

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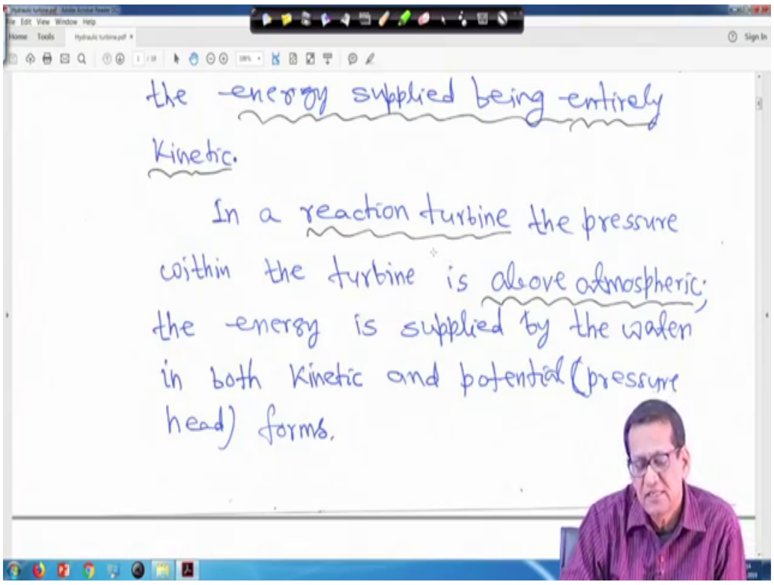

Now, in a reaction turbine the pressure within the turbine is above atmospheric, the energy is supplied by the water in both kinetic and potential that is your pressure head forms, right, both kinetic and potential forms, right.

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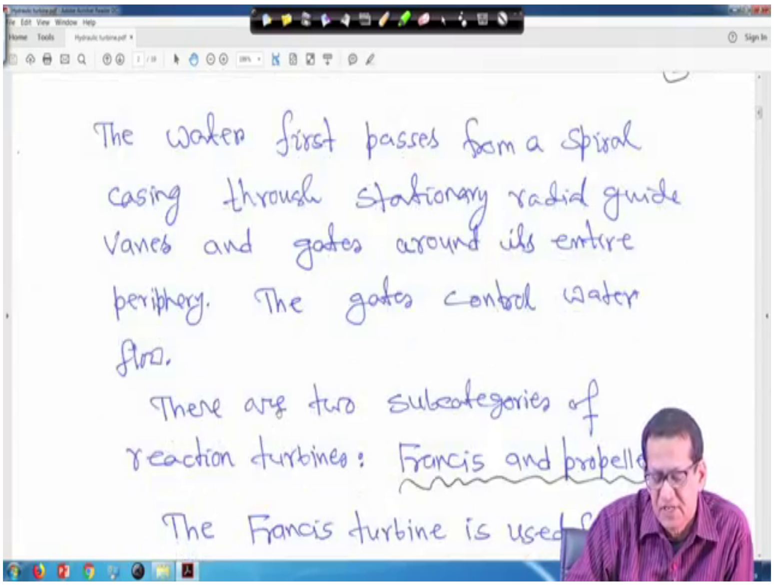

The water passed passes from a spiral casing through stationary radial guide vanes and gates around its entire periphery, right. The gates control water flow. That is you know, I think from your power plant engineering little bit you have studied. There are two sub categories of reaction turbine that is Francis and propeller.

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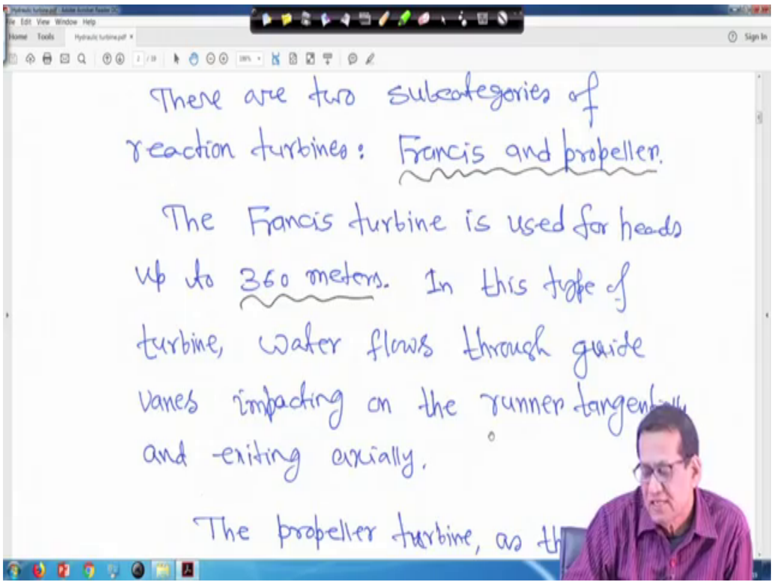

The Francis turbine is used for heads up to 360 meters. So, in this type of turbine water flows through guide vanes your what you call impacting on the runner tangentially and exiting axially, right.

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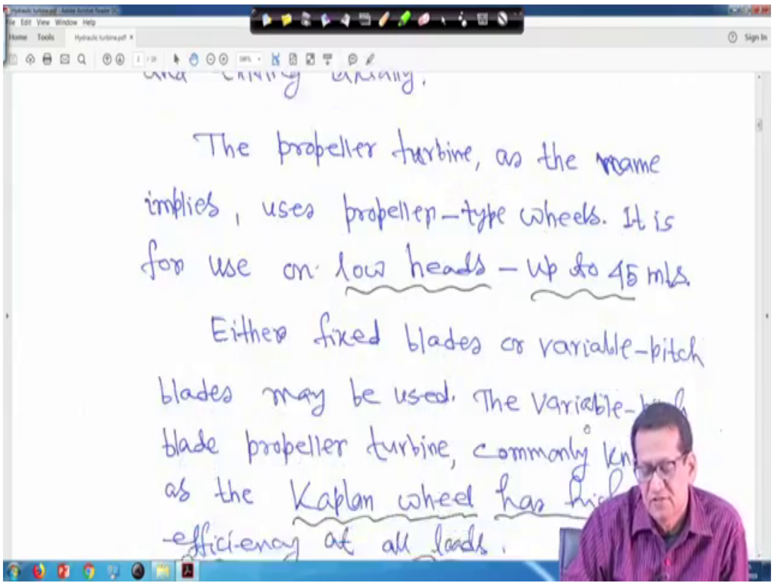

The propeller turbine as the name implies use propeller type wheels. So, it is for use on low heads up to 45 meters, right. So, either fixed blades or variable pitch blades may be used. So, the variable pitch blade propeller turbine commonly known as the Kaplan wheel has high efficiency at all loads, right. So, this little bit ideas you have.

Now, the performance of a hydraulic turbine is influenced by the characteristics of the water column feeding the turbine. These include the effects of water inertia, water compressibility and pipe wall elasticity in the penstock, right.

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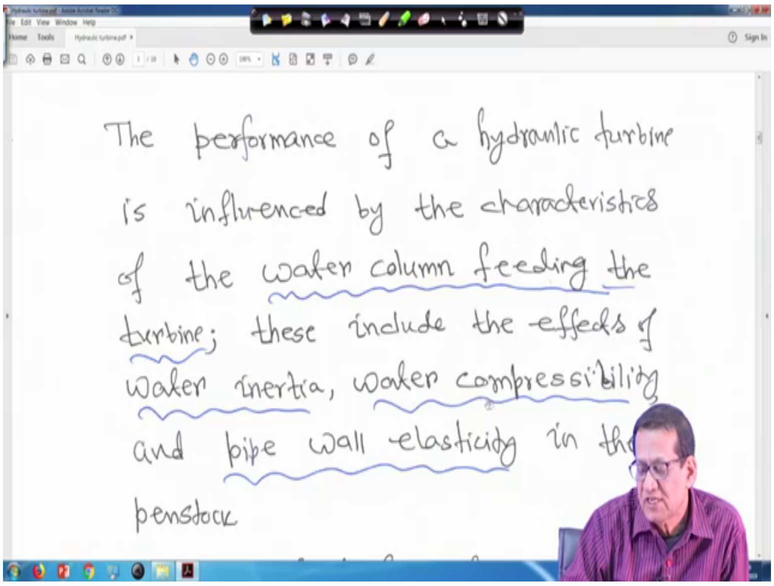

So, this is that thing. The performance of a hydraulic turbine is influenced by the characteristic of the water column feeding the turbine; this includes the effect of water inertia, water compressibility and pipe wall elasticity in the penstock, right.

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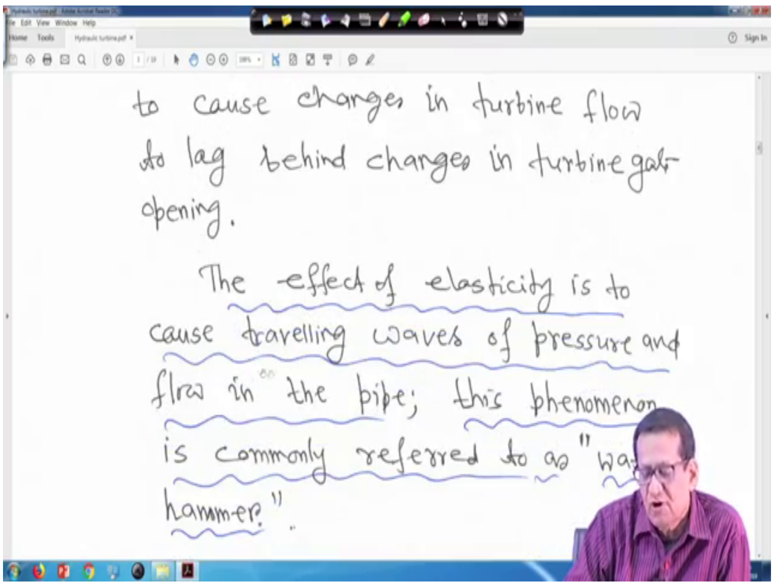

So, the effect of water inertia is to cause changes in turbine flow to lag behind changes in turbine gate opening, right. So, the effect of elasticity is to cause travelling waves of pressure and flow in the pipe, right. This phenomenon is commonly referred to as water hammer effect, right. So, the effect of elasticity is to cause travelling waves of pressure and flow in the pipe, this phenomenon is commonly referred to as water hammer effect, right.

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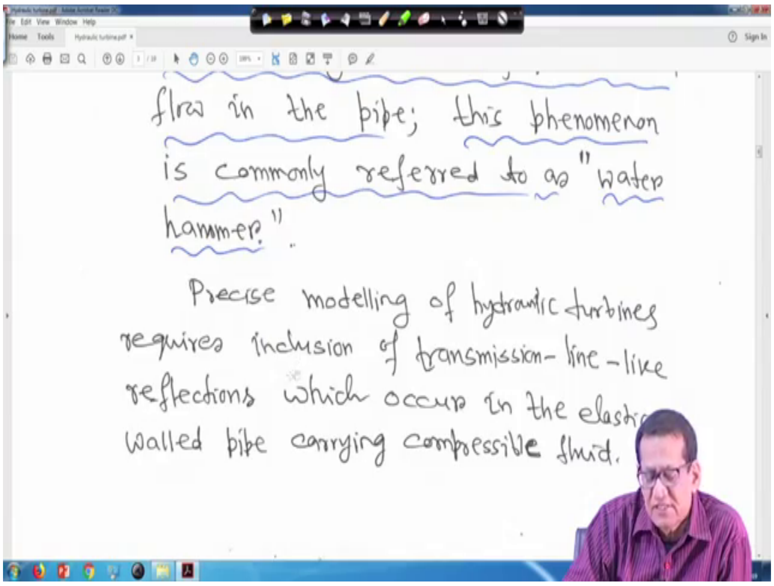

So, precise modelling of hydraulic turbines requires inclusion of transmission line like reflections which occur in the elastic walled pipe your what you call carrying compressible fluid, right. So, but we will see the simplest model, we will see the simplest model.

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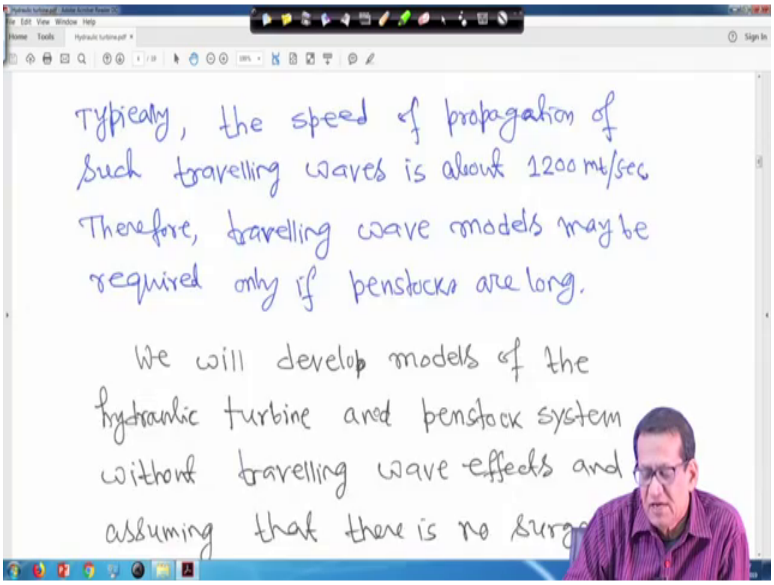

So, typically the speed of propagation of such travelling waves is about 1200 meter per second therefore, travelling wave models may be required only penstocks are long, right. We will develop the models of the hydraulic turbine and penstock system without travelling wave effects and assuming that there is no surge tank. So, based on this assumptions, we will try to your what you call model the hydraulic turbine. So, we will develop models of the hydraulic turbine and penstock system without travelling wave effects and assuming that there is no surge tank, right. So, hydraulic turbine transfer function.

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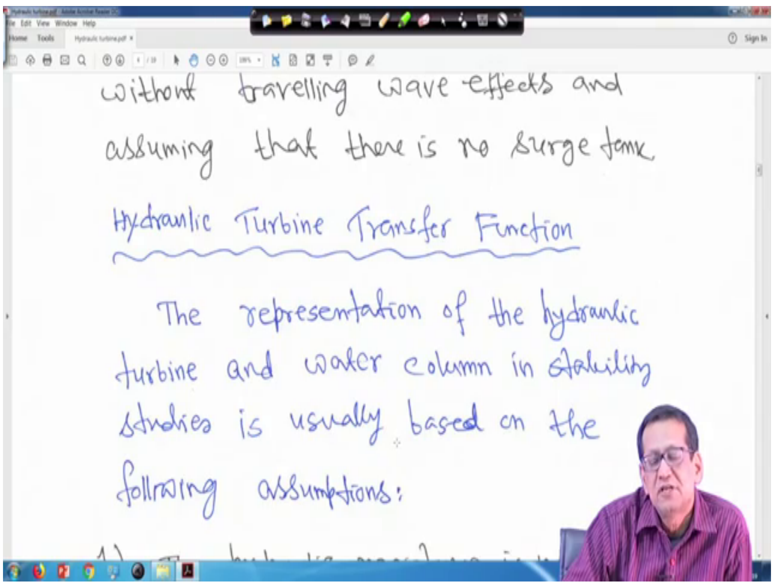

Now, we have to find out there what is our steam turbine transfer function we have seen, but for hydraulic turbine transfer function it is something different. The representation of the hydraulic turbine and water column, right in stability studies is usually based on the following assumptions. So, we will make some assumptions, right. So, though based on that only we will try to your derive the hydro turbine modelling.

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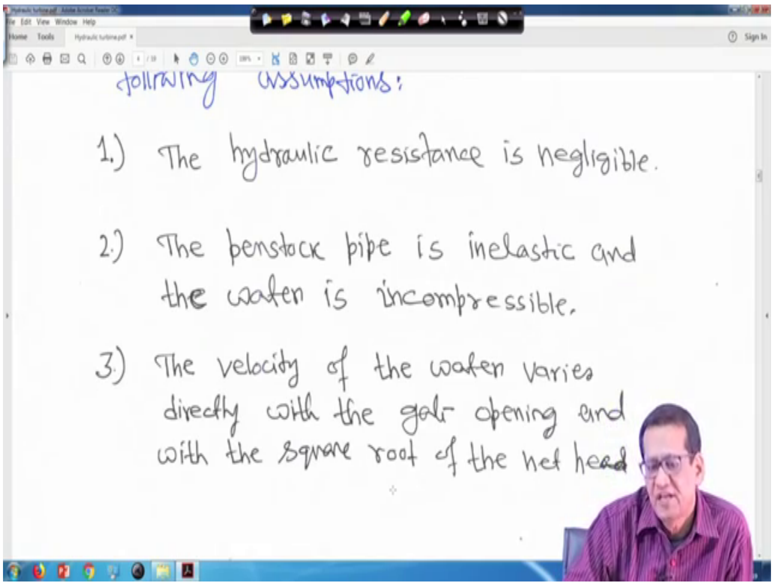

Number 1, the hydraulic resistance is negligible; this is first one. Second one, the penstock pipe is inelastic and the water is incompressible. So, this is another assumption, right. Now, number 3 is the velocity of the water varies directly with the gate opening and with the square root of the net head. This is important. So, it is actually velocity of the water varies directly with the gate opening and with the square root of the net head, right.

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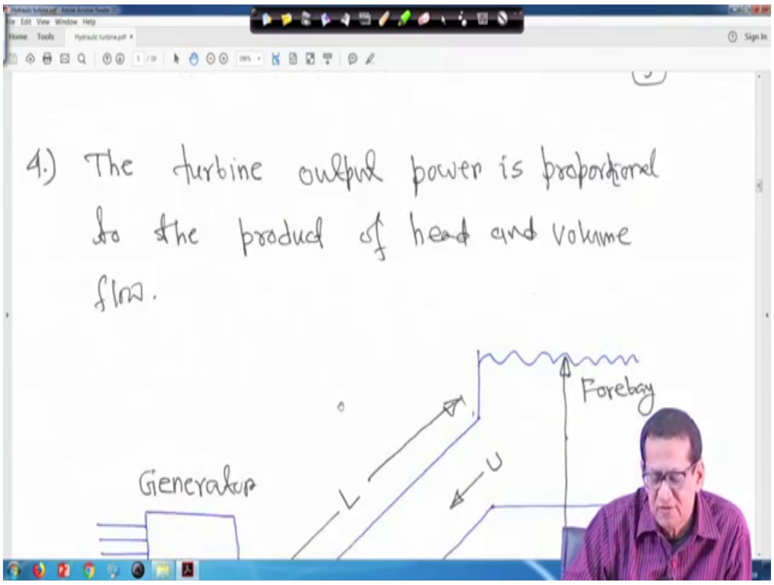

And number 4, the turbine output power is proportional to the product of head and volume flow. So, this things based on that only we will try to make it, assumptions.

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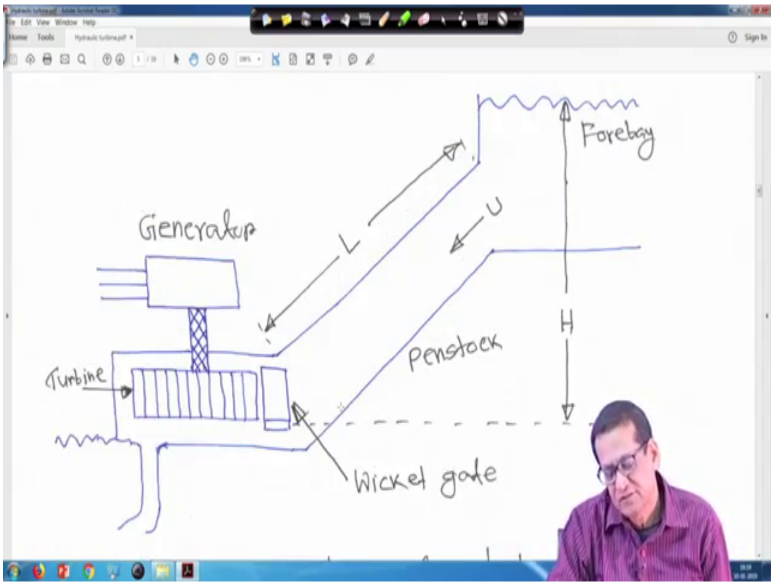

Now, if we assume a schematic diagram like this, right. So, this is your turbine, this is generator, this is wicked gate and this is the penstock this is that water flowing its speed of U and this is your length, right and this is forebay and this height is H, this length is L, and U is the velocity that is water, right and this is generator and this is the turbine. So, this is a simple diagram for the your what you call the turbine, right.

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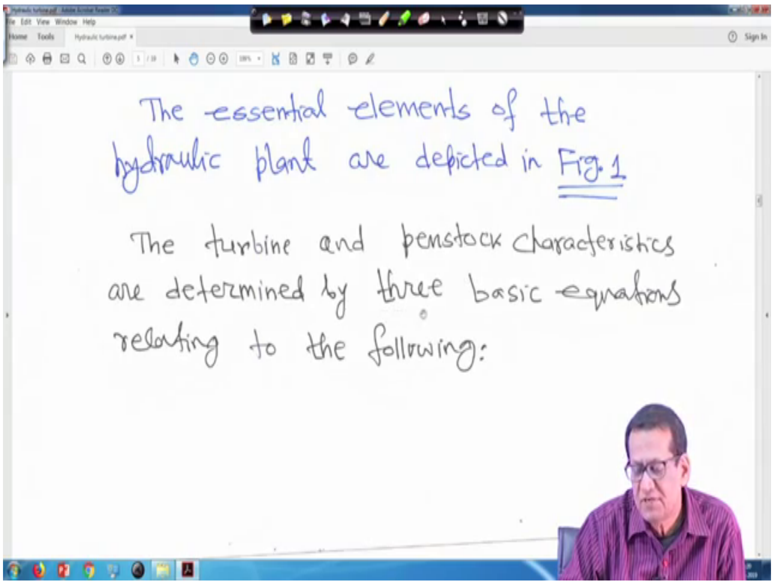

Now, schematic of a hydraulic plant; so, this is the simplest one. Now, the essential elements of the hydraulic plant are depicted in figure 1. So, this is figure 1, everything is given, right. So, turbine and penstock characteristics are determined by 3 basic equations relating to the following, right. So, basically 3 basic equations, that is velocity of water in the penstock, turbine mechanical power and acceleration of water column, right.

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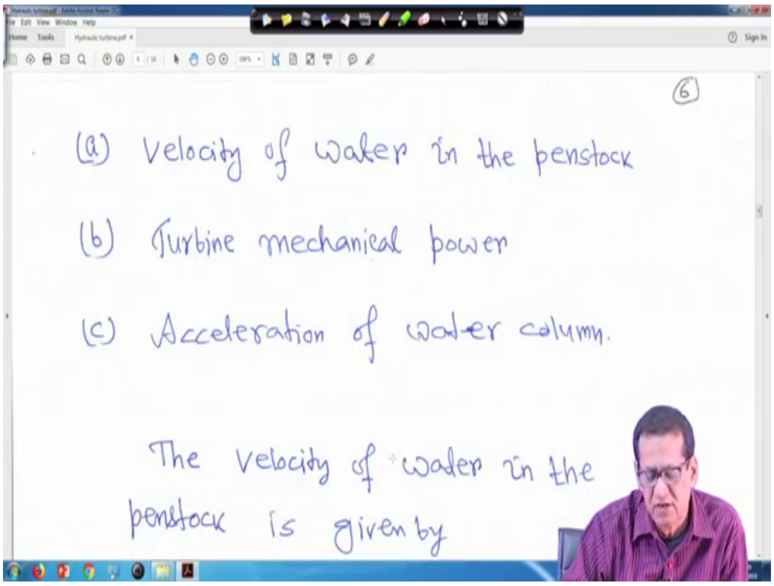

So, this 3 things, velocity of water in the penstock, turbine mechanical power, acceleration of water column.

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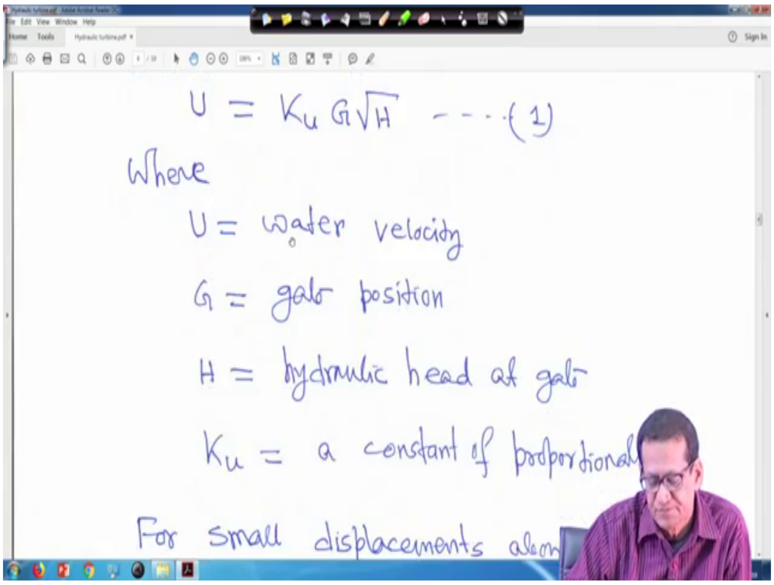

The velocity of water in the penstock is given by say U is equal to K u into G into root over H, right. So, this is actually it is given say the velocity of water, where U is equal to water velocity and G is equal to gate position and H is equal to hydraulic head at gate, right and K u a constant of proportionality. So, U is equal to K u into G root H, right. So, this is that your value, so water U is the water velocity.

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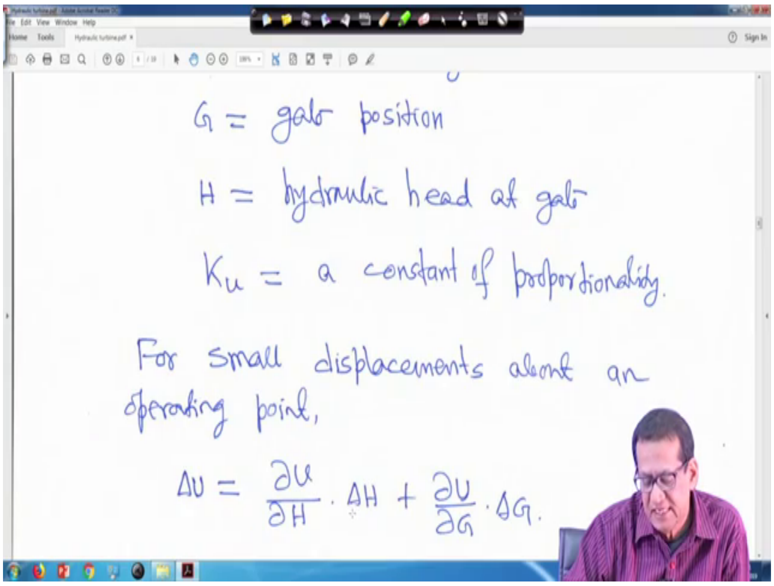

So, for small displacement about an operating point we can write this equation your what you call this for small displacement that delta U is equal to delta U upon delta H into delta H del U, del H into delta H plus del U del G into delta G, right. So, this equation for your what you call for small displacement this equation can be written like this, right. So, delta U is equal to del U del H into delta H plus del U del G into delta G, right.

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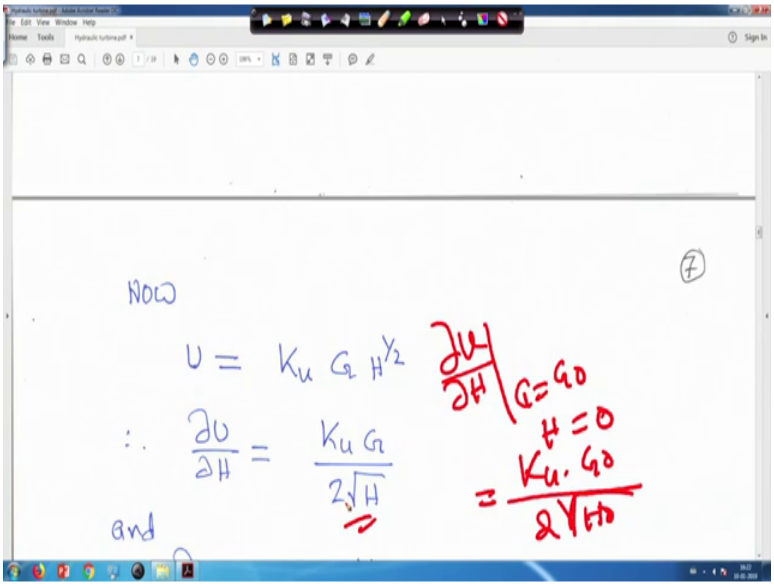

Now, we know that U is equal to here I have derived this thing, but U is equal to your what you call here I will write something that U is equal to K u G root H. So, H to the power half, right therefore, del U del H is equal to K u G to root over H. So, suppose initial value suppose you can write del U del H, right, if you have G is equal to G 0 and H is equal to H 0 then this equation; this equation it can be written as K u is a constant into G 0 divided by 2 root over H 0, right.

So, but directly I am I did not show it here, but this is we can write K u G 0 your divided by 2 root H 0. So, this way also we can write because we have to normalise the thing, right. So, that is your del U del H.

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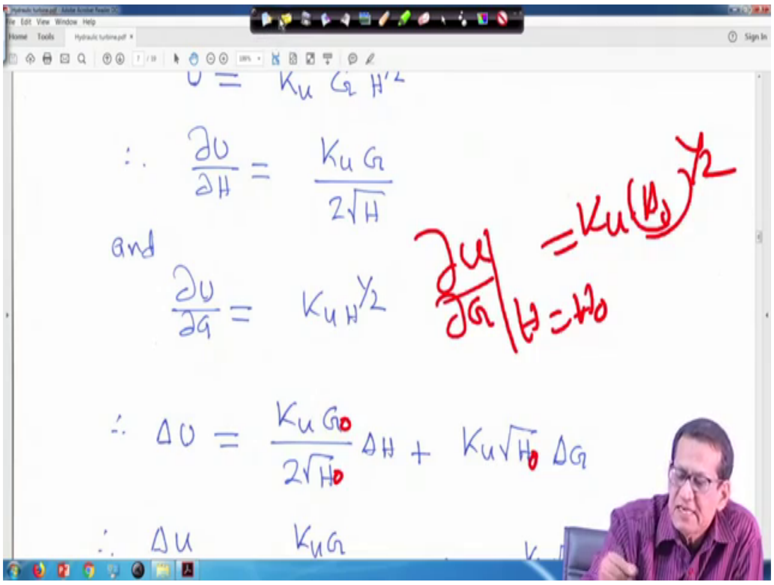

Similarly, del U del G is equal to K u into H to the power half, right. So, same thing here also del U del G or H is equal to H 0 is equal to we can write K u H 0 to the power half, right. But here I did not put it, but I am telling you this way we will do, therefore, this delta U is equal to directly I have written, right that K u G 2 root H delta H plus K u actually if you look into this then we can write that K u 0 this one 2 root over H 0 and this is also K u root over H 0 delta G, right because initial values where H 0 and G 0, right.

Therefore, so this is understandable. Here, it I actually directly wrote it, but here I thought I tell you this way. So, now, I am clearing it, right. Therefore, delta U, now delta U by, if you now this thing is your, come back to, just hold on.

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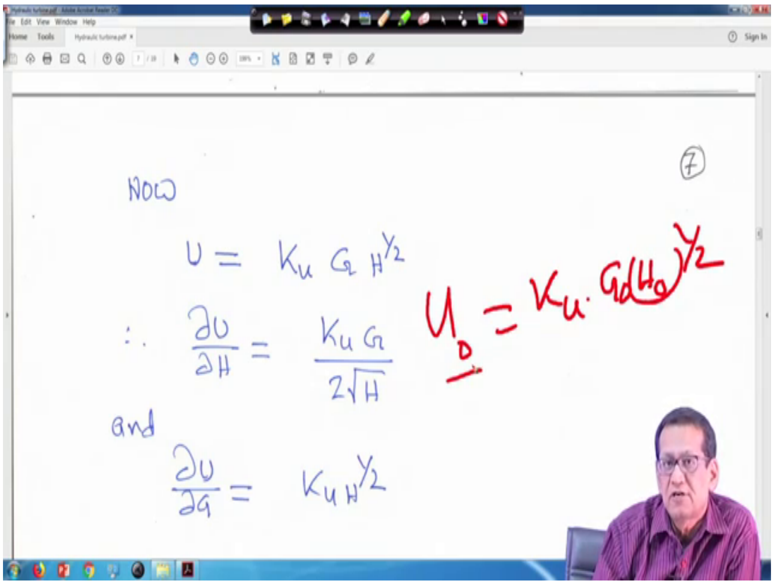

If you come back to this equation that U is equal to K u G H therefore, say U 0 it can be then K u this is a constant then it will be G 0 then H 0 to the power half, right. So, that is U 0 at when G is equal to G 0 and H is equal to H 0. So, that is U 0, right. Therefore this equation; therefore this equation that when you make it this equation that this is then H 0 and this is also H 0 and G 0, right.

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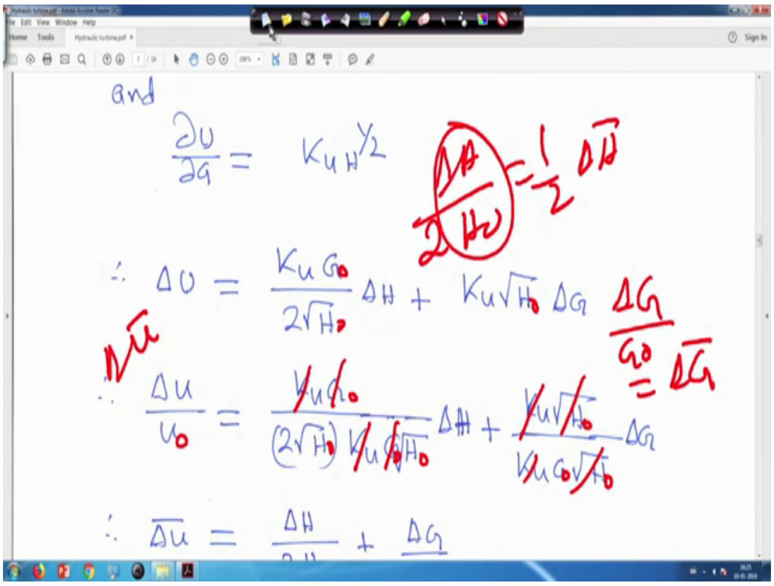

Therefore, both side if you divide by U 0 what I told U 0, it will be 2 H 0 G 0, then it will be G 0 H 0, it will be H 0, it will be G 0, it will be H 0, right. So, that means, if you normalise it the del U upon del U upon U 0 it will be say delta U bar, right. And here if you look into this the G 0 G 0 will be cancelled, right and K u K u will be cancelled, right and it will be root H root H 0 it will be your what you call that your this thing just hold it will be delta H upon 2 H.

This term actually G 0 G 0 will be cancelled, K u K u will be cancelled. So, it will be delta H upon your 2 H root over root. So, delta H divided by 2 H 0, right. So, is equal to it can be written as half it will be delta H bar; that means, this term is also normalized, right.

Similarly, here also if you look into these that this H 0 H 0 will be cancel K u K u will be cancelled, it is actually becoming your delta G upon G 0. So, that can be written as delta G bar, right it is normalised.

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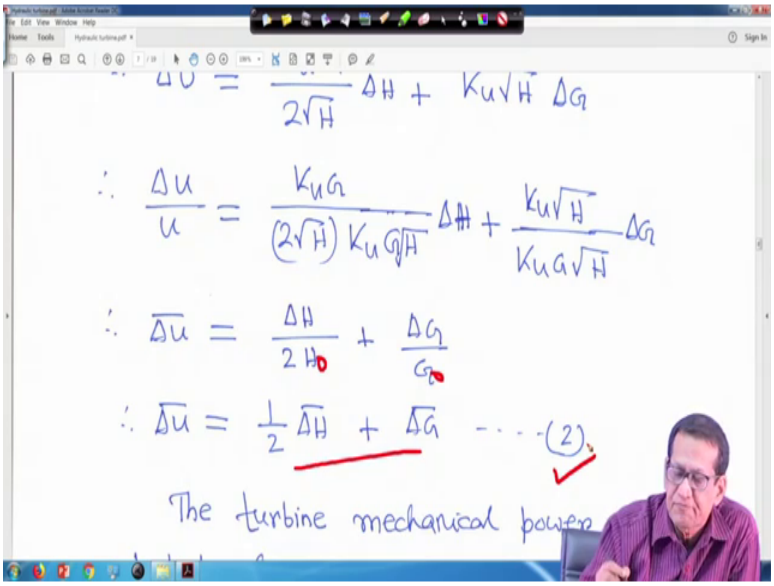

So, that is why this equation; that is why this equation delta U is equal to actually here based on that only it will be 2 H 0 it will be G 0, right. So, delta U bar is equal to half delta H bar plus delta G bar this is equation 2, right.

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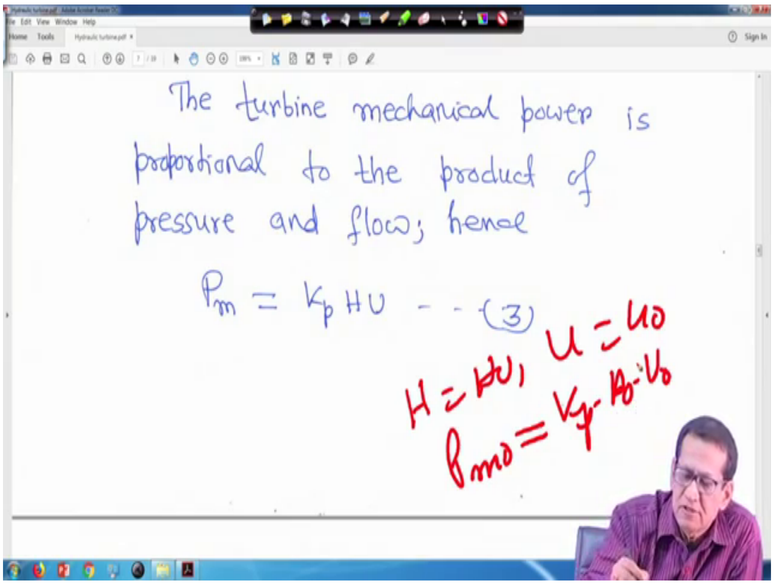

Therefore, the turbine mechanical power, next part is the turbine mechanical power is proportional to the product of the pressure and flow, right. So, here therefore, P m is equal to K p; K p is a constant of proportionality into H into U, right. So, same way when later we will see that, but I am writing here when H is equal to H 0 and U is equal to U 0 then it will be P m 0 is equal to K p then A 0 then U 0, right.

So, here also you take the your what you call delta P m, right. So, for linearizing you just linearize by considering a small displacement and again you have to normalise by dividing both sides by P m 0 is equal to K p, H 0, U 0. Just now I showed you, right. Therefore, if you what you call if you same as before, if you normalise that your this thing, right then what will happen you linearize and then normalise the delta P m upon P m 0 will be delta H upon a 0 plus delta U upon U 0.

So, this one also same as before you can write that delta P m is equal to K p into your what you call that your del a your what this thing that your del delta H your into U plus K p U into your del K p in to delta U into H, right. So, same way you can make it. So, and then you can normalise it.

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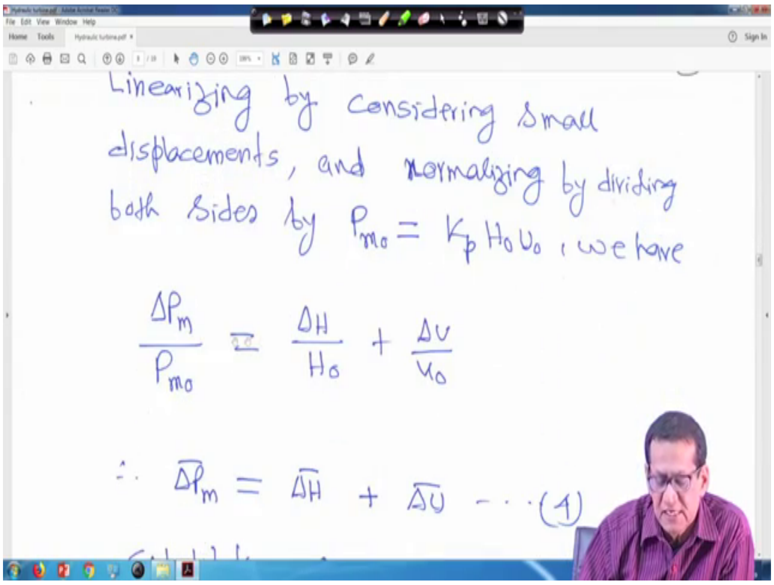

So, in this case also the delta P m upon P m 0 and P m 0 I told you this value, that your just now I told you P m 0 will be K p, A 0, U 0. So that means, delta P m upon P m 0 will be delta H upon H 0 plus delta U upon U 0, right. So, similar way you will get delta P m bar that is normalised, this is normalised is equal to delta H bar plus delta U bar, right. So, that is equation 4.

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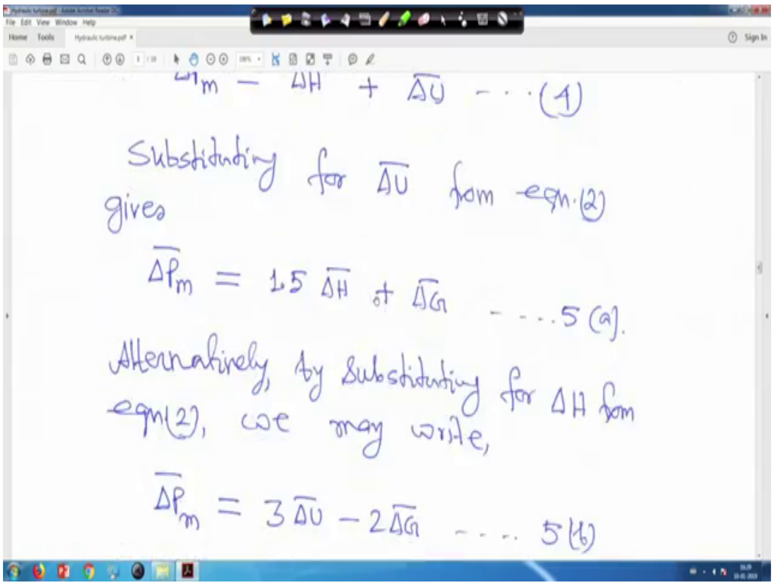

So, if you substitute for delta U bar from equation 2 it gives, right. So, from equation 2 from here; from equation 2 you substitute delta U bar is equal to half delta H bar plus delta G bar. If you do so, then it will be actually your delta P m bar is equal to 3 by 2 delta H bar plus delta G bar that is 1.5 delta H bar plus delta G bar. So, this is equation 5 a, right.

So, alternatively by substituting for delta H from equation 2, I mean from this equation 2 if you substitute that your delta H bar is equal to I mean you substitute this one that your just hold on. You substitute this one that delta H bar is equal to your 2 delta U bar minus delta G bar. This one you substitute in equation 4, right.

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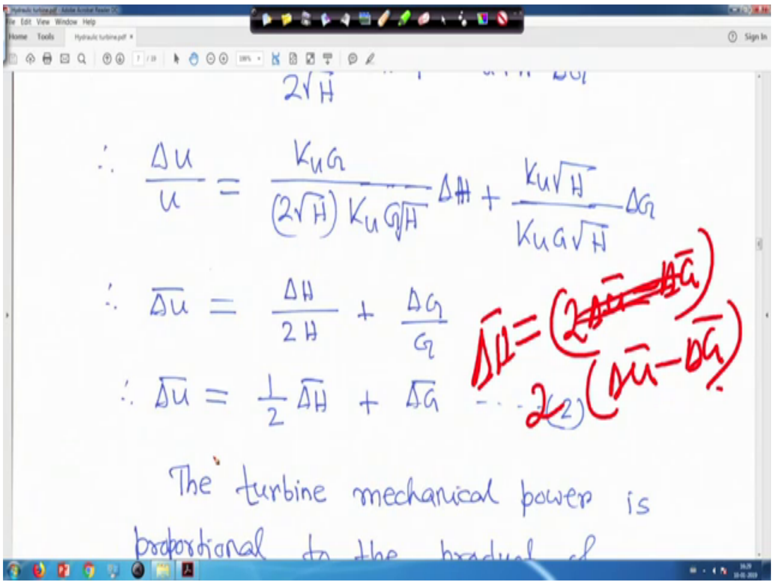

So, just hold on; just hold on not like that. It is your at your delta H bar is equal to 2 into your delta U bar minus delta G bar, right. So, that one the delta H bar is equal to 2 into delta U delta U bar minus delta G bar that you substitute in equation 4, right. If you do so; if you do so, then you will get this delta P m bar will be 3 delta U bar minus 2 delta G bar. This is equations say 5 b, right.

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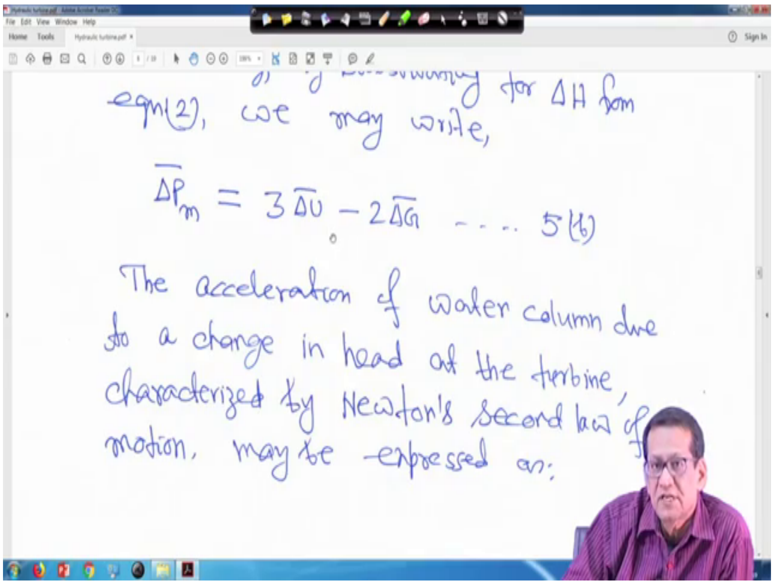

Now, the acceleration of water column due to a change in head at the turbine characterised by Newton’s second law of motion, right may be expressed as. So, now, we have to determine the modelling of hydro turbine, so we will make it like this after making all sort of small derivation.

The acceleration of water column due to change in head at the turbine characterised by Newton’s second law of motion may be expressed as it can be written as rho into L into A into d dt of delta U is equal to minus A into rho a g into delta H. Here is a question for you, right that why this minus sign? It is a small question for you, right. Why it is minus?

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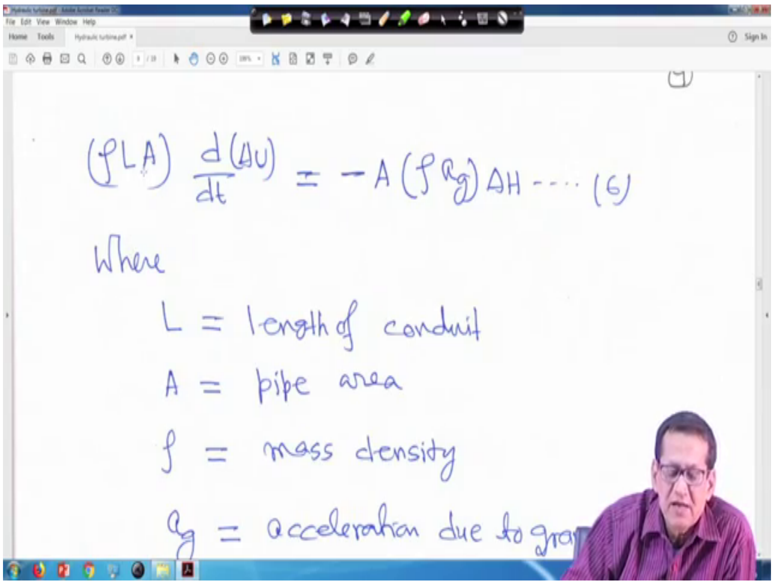

So, here if you look that means, this is the this equation you have written. Now, L is equal to length of conduit that was given in your what you call that was given in figure 1, right. So, that is I am going back to the figure 1 once again, just hold on. So, this is actually L, this is actually your L, right. So, just hold on. So, and A is the pipe area, right. So, and rho is mass density, right and a g is acceleration due to gravity.

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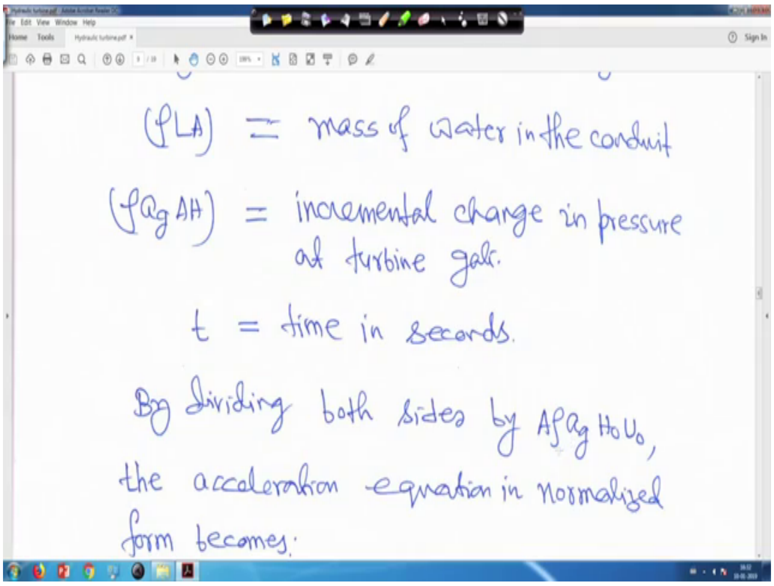

And therefore, this rho L A rho into L into A, right is equal to your mass of water in the conduit, right. And this side rho a g this side rho a g, right delta H so, that is incremental change in pressure at turbine gate that is rho into a g into delta H and t is equal to time in seconds, right.

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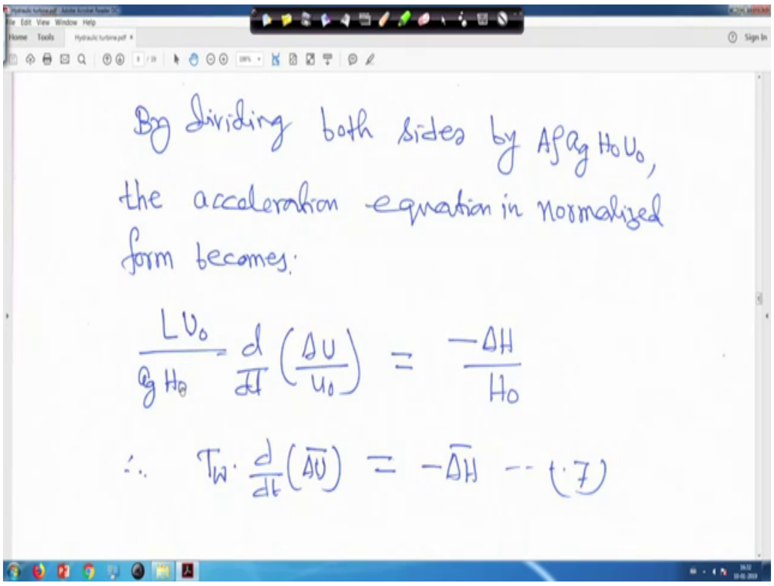

By dividing both sides by A rho a g H 0 U 0 you divide both side, then you will get the acceleration equation in normalised form it will become L U 0 upon a g H 0 d dt of delta U upon U 0 is equal to minus delta H upon H 0, right or we can write this equation we can write the T W into d dt of delta U bar is equal to minus delta H bar, right. Because this is normalise and this is your this is T W is defined and this is d dt of delta U bar is equal to minus delta H bar, right, where T W is equal to L U 0 upon a g H 0, right.

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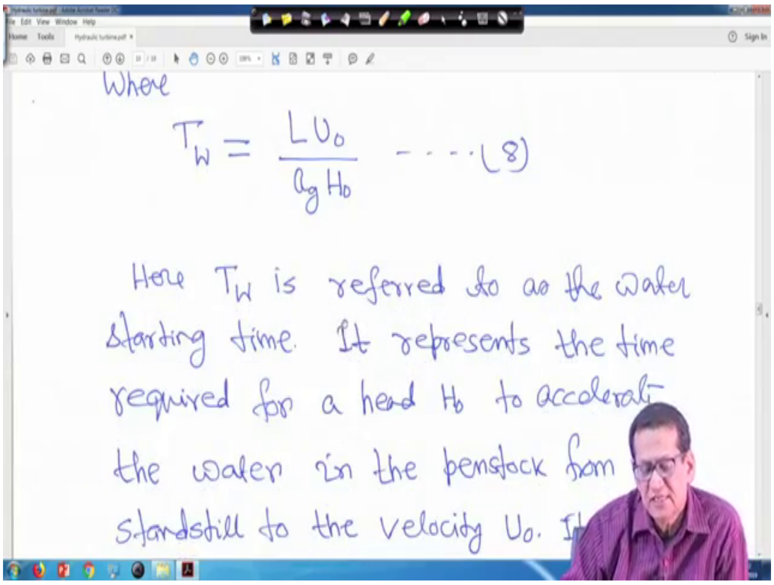

So, this is actually called that water starting time. So, this is actually T W. So, here T W is referred to as the water starting time, right. So, it represents the time required for a head H 0 to accelerate the water in the penstock from standstill to the velocity U 0, right.

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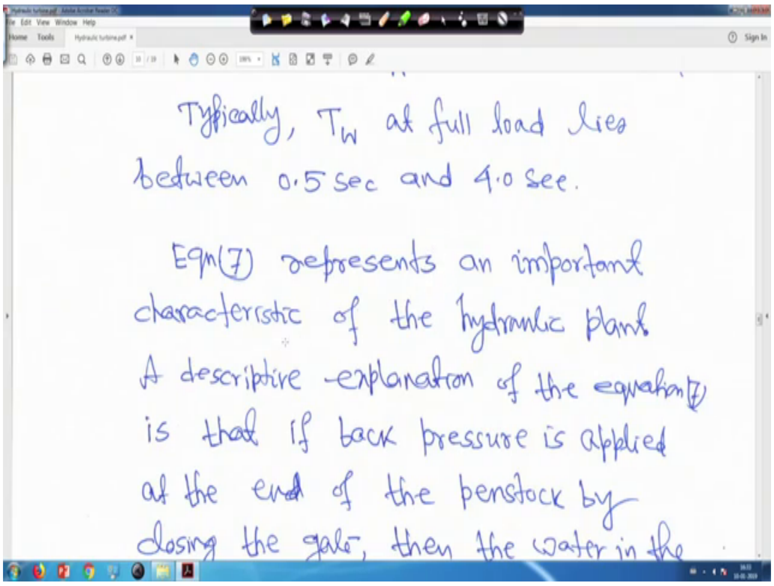

It should be noted that T W varies with load, right. Typically, T W at full load lies between 0.5 and your 4 second, right. Therefore, the T W we sometimes we call that is referred to water starting time actually, it represent the time required for a head H 0 to accelerate the water in the penstock from standstill so, the velocity U 0. So, it should be noted that T w varies with load. So, typically T W varies from 0.5 to 4 second, right. So, equation 7 actually represents an important characteristic of the hydraulic plant, right.

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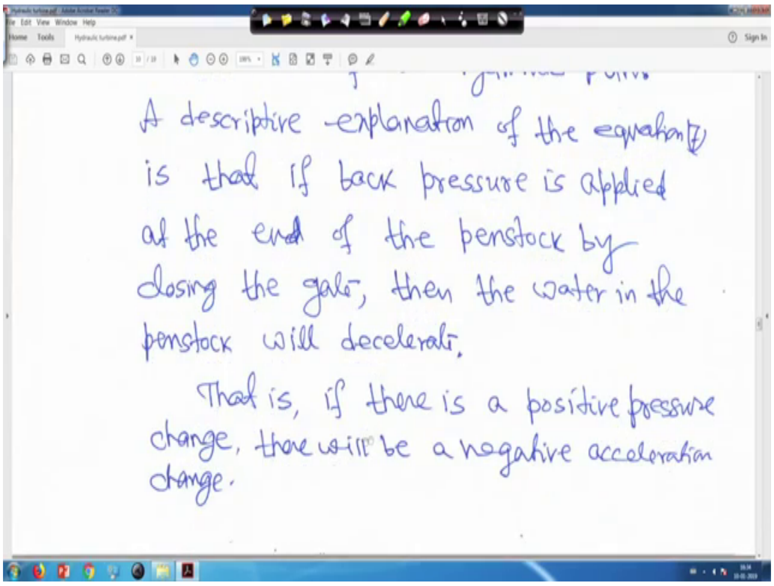

A descriptive explanation of the equation 7 is that if back pressure is applied at the end of the penstock by closing the gate, right then the water in the penstock will decelerate. Thus, that is if there is a positive pressure change there will be a negative acceleration change and vice versa, right.

Thank you very much, we will be back again.