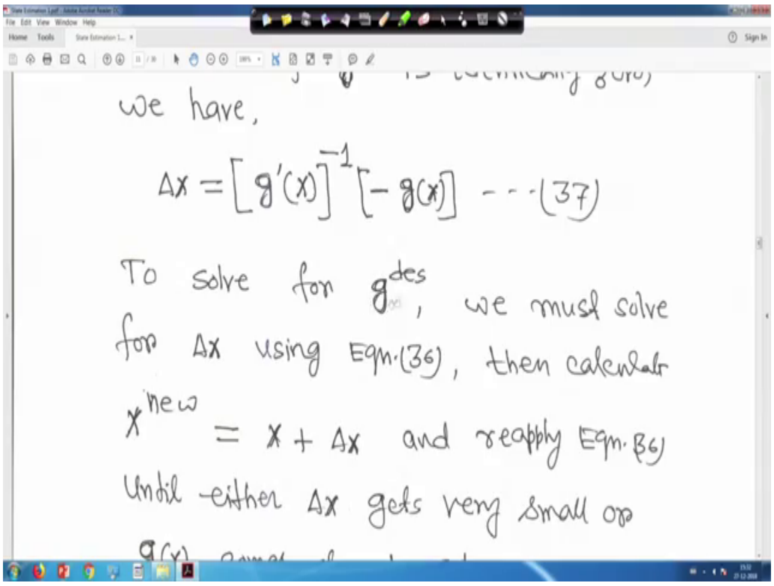
Power System Dynamics, Control and Monitoring
Prof. Debapriya Das
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 56
State estimation in power system (Contd.)

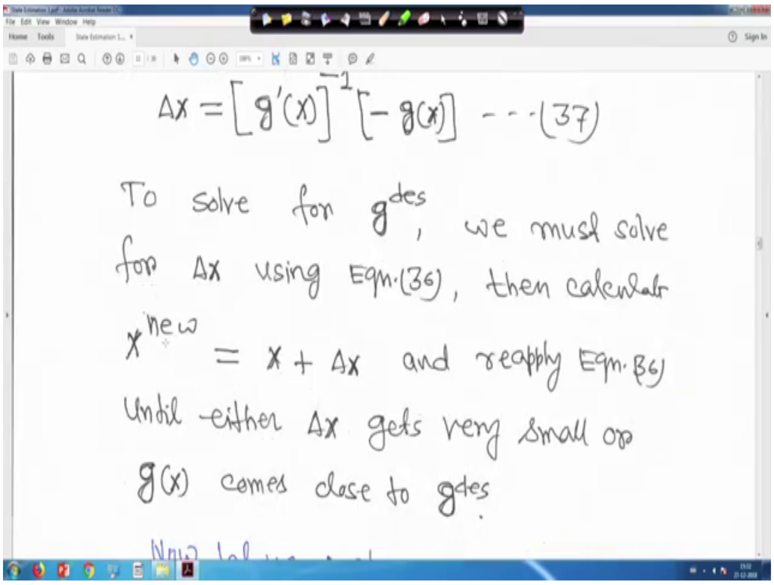
We are back again to this equation right.

(Refer Slide Time: 00:27)



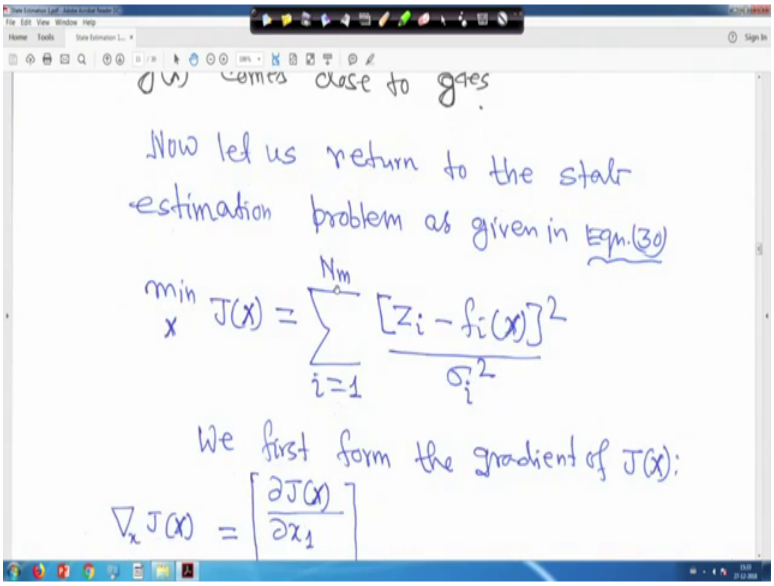
Now, to solve for g desire, we must solve for delta x using equation 36 right earlier 36 I have shown.

(Refer Slide Time: 00:29)



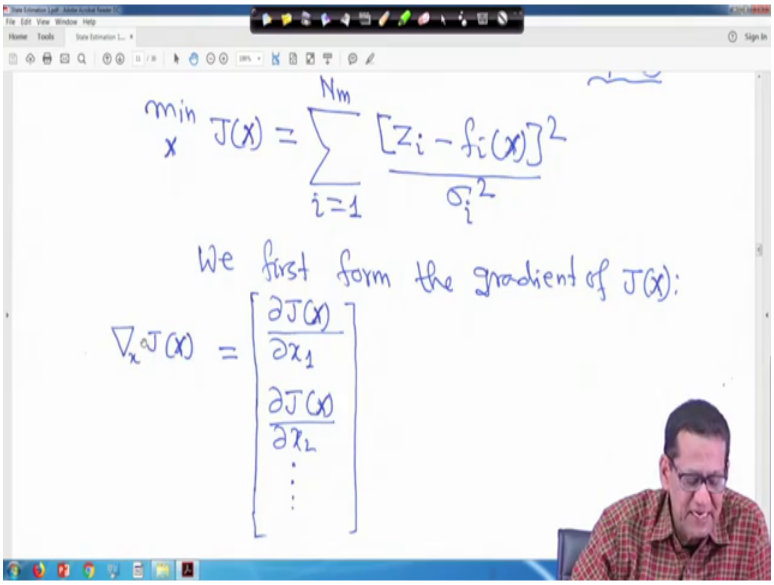
Then calculate x new the new values of all x right is equal to x plus delta x and reapply equation 36 until either delta x get very small or gx come close to g desire right.

(Refer Slide Time: 00:45)



Now, let us return to the state estimation problem as given in equation 30.

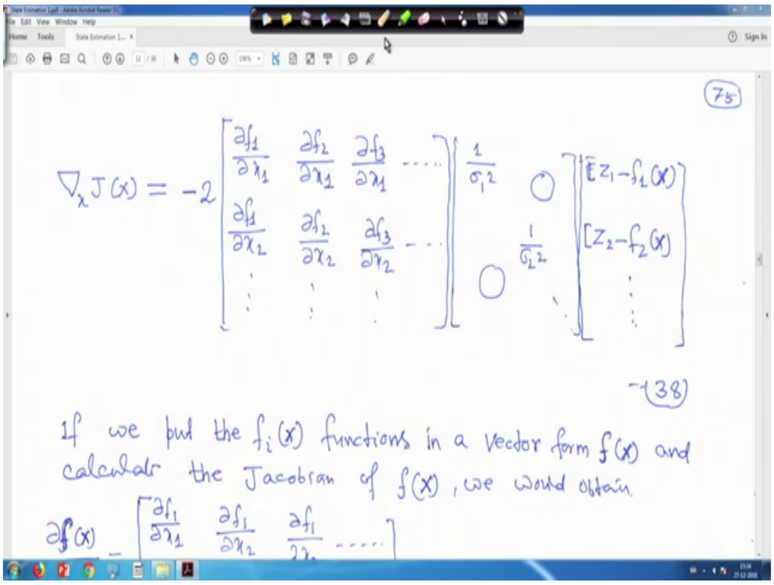
(Refer Slide Time: 00:49)



So, earlier we have seen that your minimize your J X i is equal to 1 to N M that is number of measurement then in bracket Z i minus fi x whole square divided by sigma i square. I told you once again that why you have divided it by sigma i square its a small question to you right. So, you will you apply you will answer it in forum.

First what we have to do is, we have to first find out the gradient of JX. So, gradient of J X it can be written as delta J X delta x 1 delta J X delta x 2 and so, on you have n ns number of state variables right. So, this way you have to get the gradient now and this is the function J X. So, you have to take derivative with respect to x 1 x 2 x right all partial derivatives you have to take.

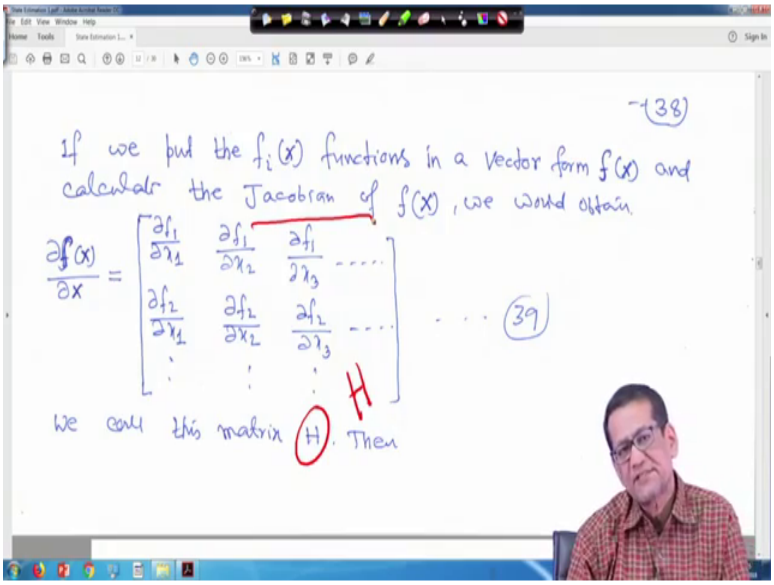
(Refer Slide Time: 01:39)



Now, if you do so, then gradient of J X it can be written as that minus 2 will come because of your this thing because square is here, square is here right. If you do so, then it will be minus 2 into delta f 1 upon delta x 1, delta f 2 upon delta x 1, delta f 3 upon delta x 1 and so on right you have if you look into that you have i is equal to 1 to N m. So, N M number of measurement, so, it will go up to N M. So, then delta f 1 upon delta x 2 delta f 2 upon delta x 2 delta f 3 upon delta x 2 and so on right.

So, actually it is continuing it is also continuing, it will also continue understandable. Then this term automatic will come 1 upon sigma diagonal matrix 1 upon sigma 1 square, 1 upon sigma 2 square it will continue all other elements are 0 that is why these are shown 0 right into Z 1 minus f 1 x Z 2 minus f 2 x and it will continue because you have nm number of your measurements right.

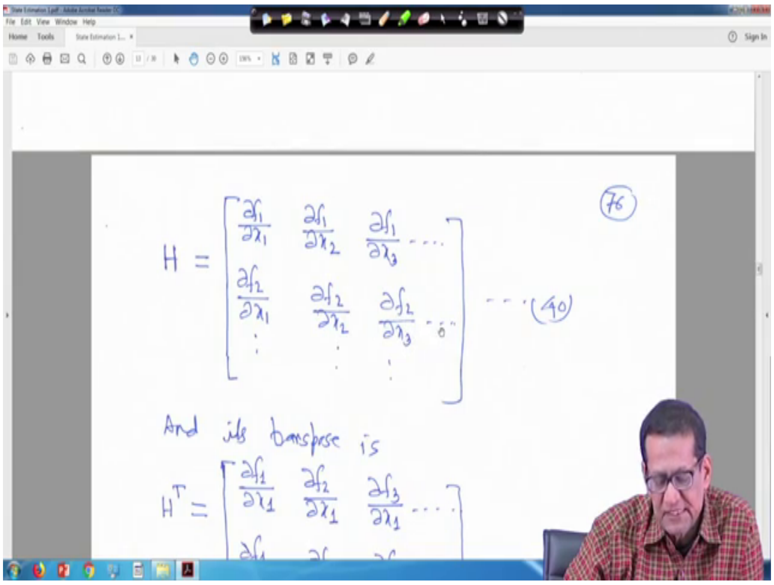
(Refer Slide Time: 02:47)



Now, if we put that fi x function in a vector form your fx and calculate the Jacobean of fx we would obtain. Now if we write delta fx upon delta x in general right. So, it is actually it will like a if you do it. So, it will be delta f 1 upon delta x 1 delta f 1 upon delta x 2 delta f 1 upon delta x 3 and so on.

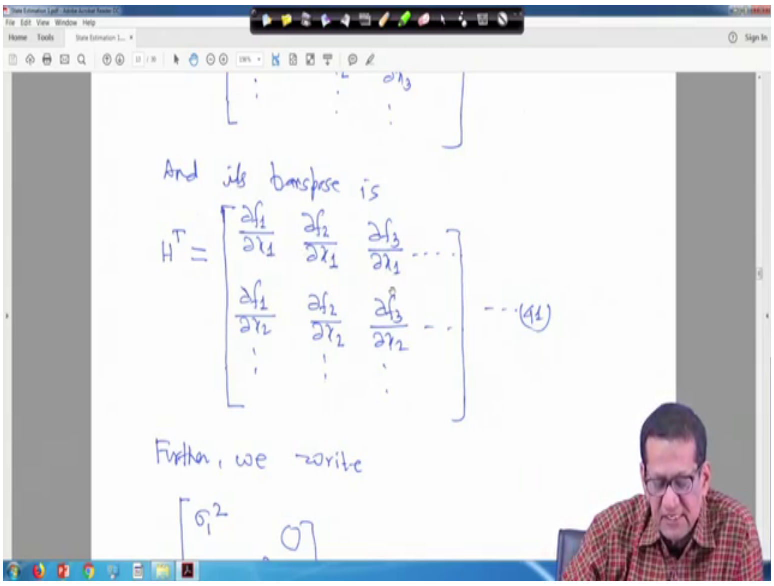
Similarly, delta f 2 upon delta x 1 delta f 2 upon delta x 2 delta f 3 upon delta f 2 upon delta x 3 and so, on. So, it is basically Jacobian matrix its basically Jacobian matrix. If you look into this matrix and if you look into this matrix you will find it is transpose of another one right. Therefore, we call this matrix H; that means, this matrix this Jacobian matrix this one we are calling is as a matrix H right. So, basically this is a Jacobian matrix Jacobian of f x right.

(Refer Slide Time: 03:41)



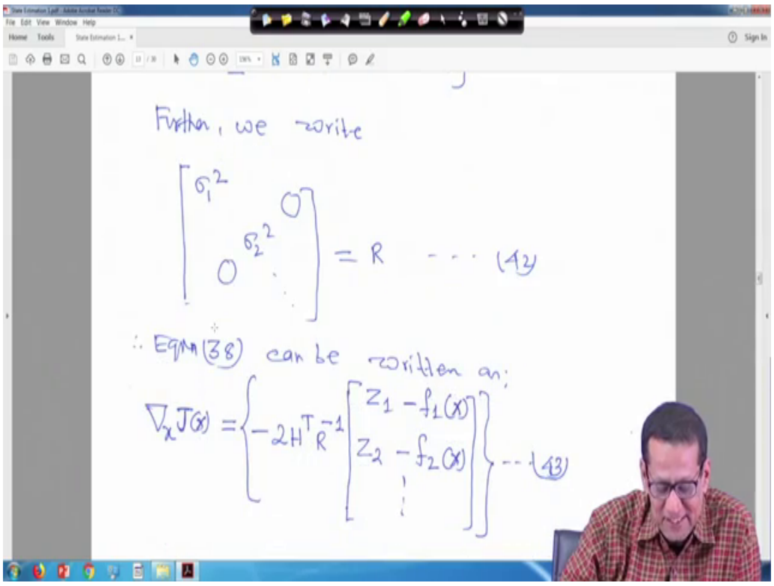
That means; that means, this H can be written as delta f 1 delta x 1, delta f 1 delta x 2 delta f 1 delta x 3 and so on up to nm number of your what you call measurement you have right. Similarly, delta f 2 upon delta x 1, delta f 2 upon delta x 2, delta f 2 upon delta x 3 and so on right.

(Refer Slide Time: 03:56)



Therefore it transposes if you take the transpose of this matrix, it H transpose it will be delta f 1 upon delta x 1 delta f 2 upon delta x 1 delta f 3 upon delta x 1 and so on. So, delta f 1 upon delta x 2, delta f 2 upon delta x 2 and delta f 3 upon delta x 2 and so on right; that means, this equation this actually this will be H transpose right.

(Refer Slide Time: 04:24)

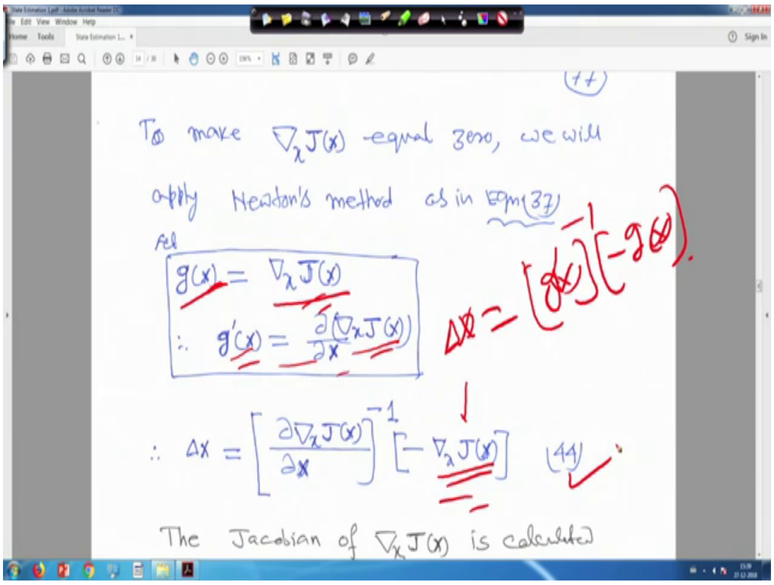


And then that is your then this one that your R is equal to sigma 1 square sigma 2 square initially at the beginning of this state estimation R, somehow I overlooked that 1 I mentioned that it is R inverse it is not R inverse it is R right because it is 1 upon sigma 1 square right. So, when you take R inverse at that time it its basically R matrix right. So, we are defining like this. So, everything is actually everything is.

So, now equation 38 now can be written as that it is if we take this one is R it is 1 upon sigma 1 square 1 upon sigma 2 square will come because it is a diagonal matrix that is why this R inverse right. So, equation 38 can be written as the gradient of J X actually is equal to minus 2 into then I told you it will be H transpose then R inverse into Z 1 minus f 1 x, Z 2 minus f 2 x and so on right.

That means, this equation; that means, this equation minus 2 H transpose then R inverse because R is equal to sigma 1 square sigma 2 square like this and these are all 1 upon 1 upon other elements are 0 that is why R inverse and this is Z 2 1 minus f 1 x, Z 2 minus f 2 x right. So, that is this is the equation 43 right.

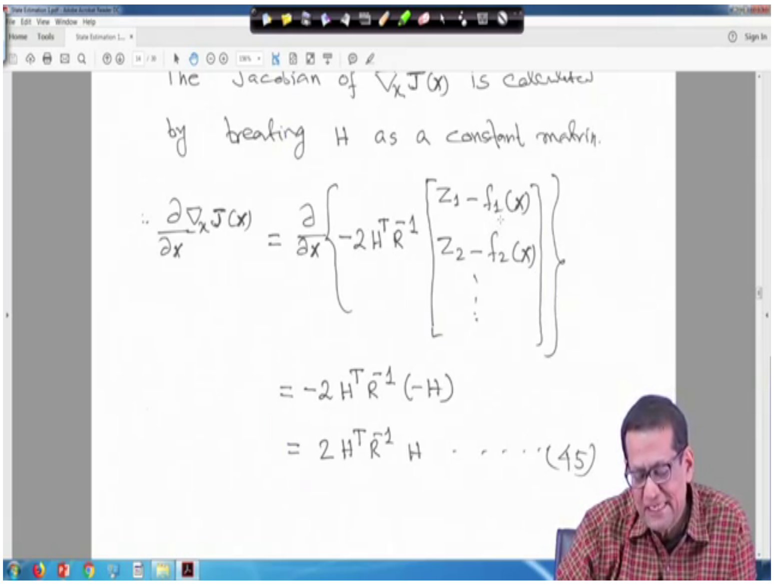
(Refer Slide Time: 05:37)



Now, to make this gradient 0 right we will apply Newton method as given in equation 37 right. So, in equation 37 actually this is actually assume that your gx is equal to; gx is equal to gradient of J x right therefore, g; that means, g dash x will be del x of gradient of J x, because J because you have to; you have to solve it iteratively right.

So, in that case you have to first sorry first you assume gx is equal to the gradient of J X, then g dash x will be del del x of gradient of J X. So, you have to first the gradient to 0 iteratively. So, we know we knew that formula that delta x is equal to just now we have done it in equation 37 right it is actually g dash x inverse into minus g x right, so, g dash x inverse minus. So, this is your this is your g your gx. So, that is why it is minus gradient of J X and this is your g dash x that is your del del x of gradients of Z derivative of this one. So, this is actually delta x. So, this is equation 44 I hope this is understandable to you right.

(Refer Slide Time: 06:57)

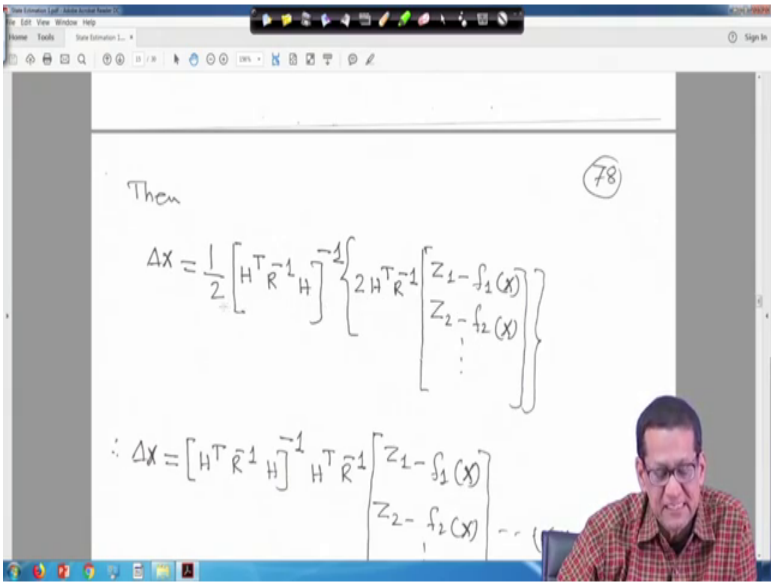


So, the Jacobian of that gradient J X is calculated by treating H as a constant matrix. Now assume H is a constant matrix then what we will do that, this del del x of gradient of J X you have to compute. That means this is my this is my your gradient of J X, we are assuming H is a constant matrix R of course, is a constant matrix H is a constant matrix, then you have to take the derivative with respect to x of this gradient right. So, if you do so, that equation 43 if you take if you do so, then del del x of gradient J X is equal to del del x of minus 2 H transpose R inverse into that Z 1 minus f 1 x Z 2 minus f 2 x and so on right.

So, if you take this thing, then it will become actually minus 2 H transpose R inverse and if you take the derivative of again, it will come your in general it will come del f upon your what you call del f upon del x in general. So, basically it will be nothing, but your Jacobian matrix. Because here we have shown that here we have shown that if you take del fx upon del x it is nothing, but your Jacobian matrix that is your H right here it is H. So, that is equation 39.

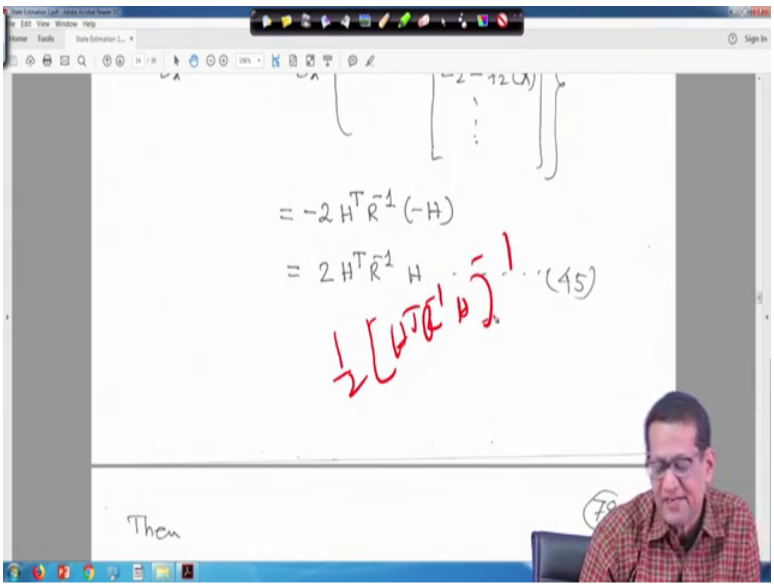
So; that means, if you take with respect to this one also. So, basically it is minus sign is there that is minus will be there and in general del f upon del x is nothing, but the H the Jacobian matrix right; that means, the gradient of your derivative of this gradient is nothing, but 2 H transpose R inverse H right this equation 45. So, this is after doing this you have got the gradient right. So, assuming that H and your all H matrix is a constant one.

(Refer Slide Time: 08:47)



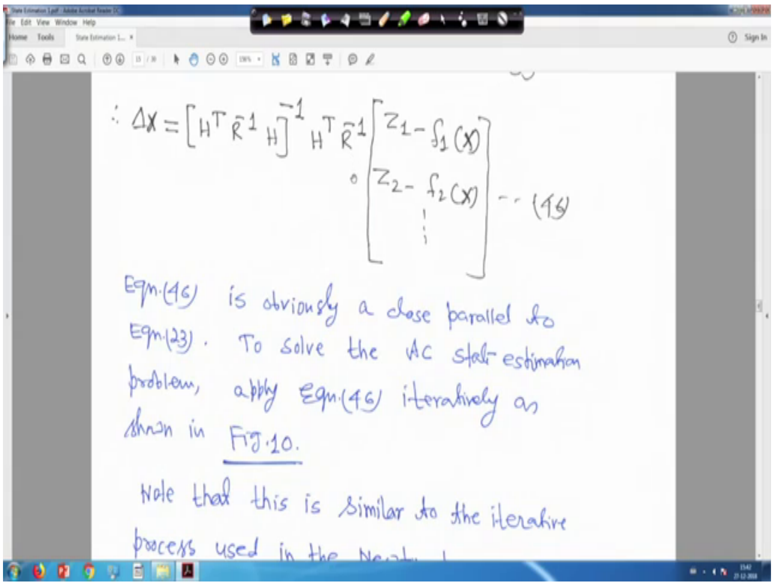
So, next is, then we know that delta x is equal to your what we have seen? Delta x is equal to g dash x inverse right g dash x nothing, but the gradients derivative. So, this is our this is our g dash x that is del del x of gradient of J X. So, that is H transpose R inverse H inverse right and was 2 was there. So, because of these 2 we have made it 2 inverse that is half, so, it is half right.

(Refer Slide Time: 09:13)



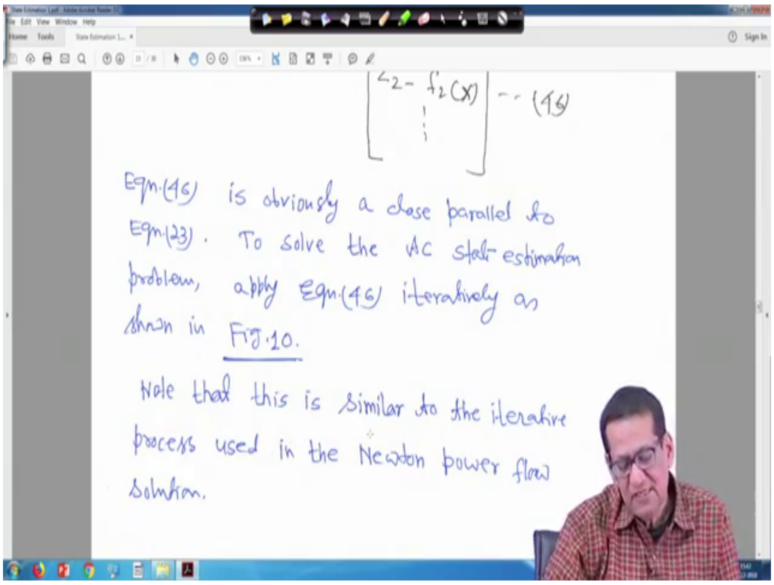
Because here, so, here it was 2 was there. So, when you take inverse of this one, when you take inverse of this one as you are taking inverse because of 2 it will be half, and then H transpose R inverse H inverse right. So, that is what we are writing there that is what we have written here right half H transpose R inverse H whole inverse into the 2 H transpose R inverse Z 1 minus fx 1 Z 2 minus f 2 x, f f 2 x and so on right. So, 2 2 will be cancel this is 2 and this is 2 will be cancel.

(Refer Slide Time: 09:48)



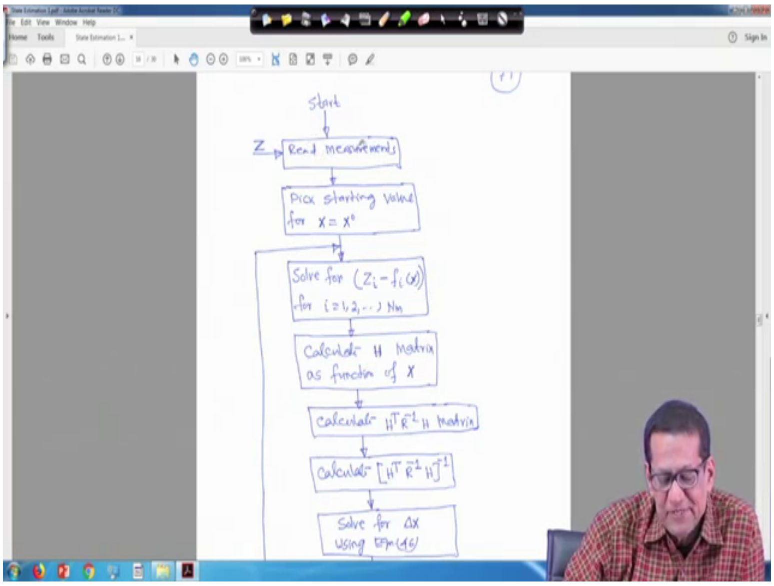
So, ultimately it will become delta x is equal to H transpose R inverse H whole inverse H transpose R inverse, Z 1 minus f 1 x Z 2 minus f 2 x this is equation 46 right. So, equation 46 is; obviously, close parallel to equation 23, because if you look equation 23 you will find very close to that right. To solve the ac state estimation problem you apply equation 46 iteratively as shown in figure 10 that means, a flow chart is given right.

(Refer Slide Time: 10:22)



So, note that this is similar to the iterative process used in the Newton's power flow method. Its basically Newton's method that your what you call that your iterative method right.

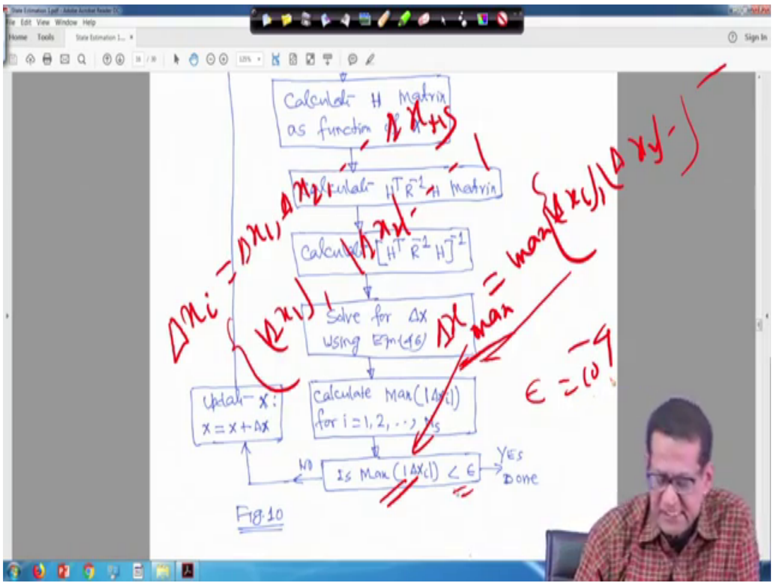
(Refer Slide Time: 10:35)



Now, if we come back to if we come back to this your what just let me reduce the size. So, if we come back to this that first you have to start, then read measurement; that means, all Z measured values all the measurement data you read. Then pick starting value for x is equal to x naught right superscript is given x is equal to x naught. Then you solve for you are what you call Z i minus f ix for i is equal to 1 2 up to n M number of your measurement right

So, once you made it, then calculate H matrix as function of x right. So, that also you have the H matrix I told you that is a Jacobian matrix of f actually right and with the initial values you have to calculate H also right. If it is not this thing if it is not function of f then directly you will get it right.

(Refer Slide Time: 11:22)



So, calculate now H transpose R inverse H matrix right so, that you can calculate. Now calculate H transpose R inverse H inverse now you have to calculate this one right step by step. Then you solve for delta x using equation 46, that this equation that is your this using this equation you solve for delta that is delta x 1, delta x 2 like this it is in general is what you call is a vector right.

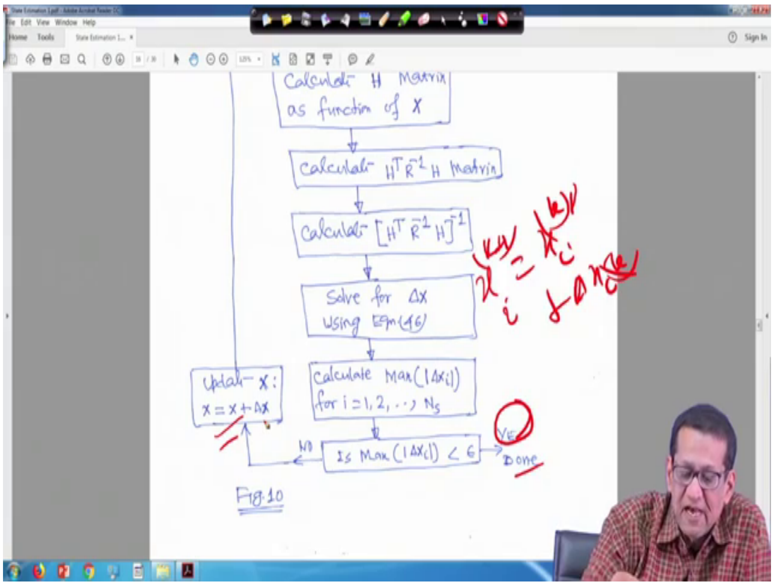
So, once you have done this right then calculate maximum of delta x i I mean you have you have so, many your. So, you are what you call you are getting I am just let me increase the size right just hold on just hold on. So, let me increase the size then I will tell you this one.

So, suppose what you are doing is, suppose you are you are you have so, many variables that delta x i right is equal to delta x 1, delta x 2 like this you have ns number of state variable right. So, every time you are trying to find out the mismatch right, then you are taking absolute that what you take? That you take absolute of your delta x 1 I am putting in my way then absolute of delta x 2 all you are taking for n number of state variables right.

Out of which what to do? You find the max of that; that means, here I am making like this, but for your understanding, suppose delta x max is equal to your max of right your delta x 1, then delta x 2 like this right you find out maximum I take all the your what you call delta x values and take out of which the maximum or absolute of course, the absolute value will take right.

And if that max value if that delta x that is max delta x i is nothing, but the your delta x max. If that max value is your less than epsilon you specified epsilon for example, say 10 to the power minus 4, the specified epsilon if it is less than epsilon your solution has converged right.

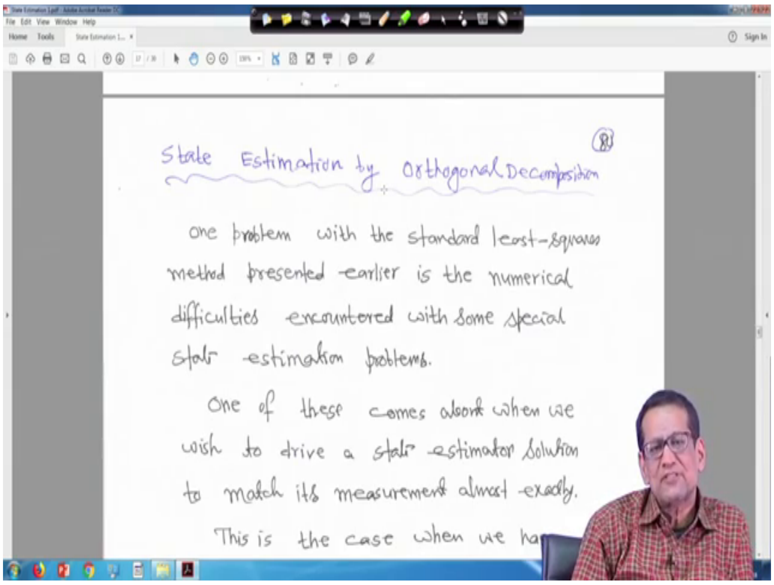
(Refer Slide Time: 13:30)



Otherwise; that means, if it is less than epsilon, yes then your what you call then your it is solution as converge that is yes right that is done. If it is not, then you update x is equal to your x plus delta x. That means, x 1 is equal to in general x 1 I mean in general iterative method when you write that is in general, it will be x i k plus 1 is equal to x i k right plus delta x i k that is in general. So, that is why I mean in the flowchart form we have written x is equal to x is a vector actually right.

So, many state values are there. So, x is equal to x plus delta x update and after updating that you will come back to here this thing again you start right you got the new value of x, your what you call x then again solve for this 1 and repeat this process till the solution as converge right.

(Refer Slide Time: 14:17)



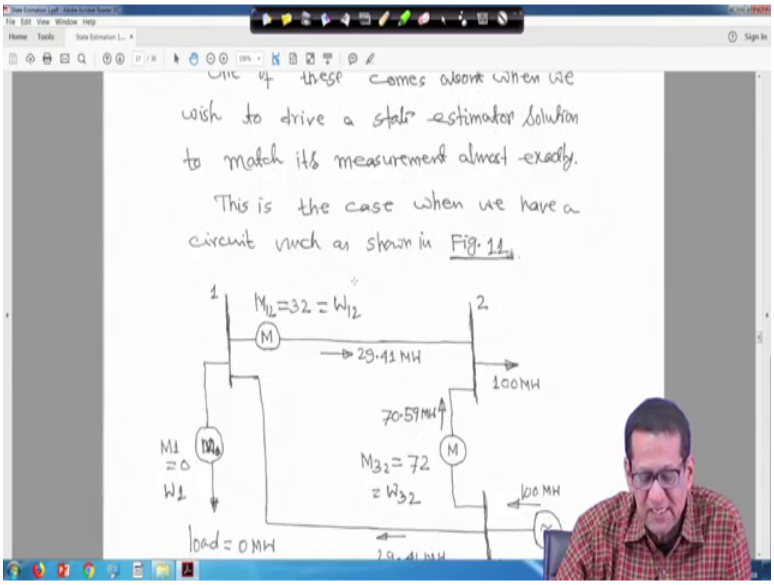
So, this is actually AC, but only thing is that, that in AC network right we cannot solve problem in the classroom without computer. So, whatever little bit whatever you get from the steady state point of view and theory point of view right. So, we otherwise it is not possible.

So, next is your what you call that your state estimation by orthogonal decomposition. So, in this case that what happened why we will go for orthogonal decomposition? So, basically in this method we will use one thing any orthogonal decomposition, you can see any control system book or any article also any control system book or any optimization thing, that orthogonal decomposition we will get. And here in this course it is not possible to explain in detail, that givens rotation method will be used right.

So, directly I will tell you right and if you have any problem then you put the question in the forum we will see that, but givens rotational method will be used. So, those givens rotational method cannot be explained here, but directly I will write assuming that you will study of your only this part only this small part right.

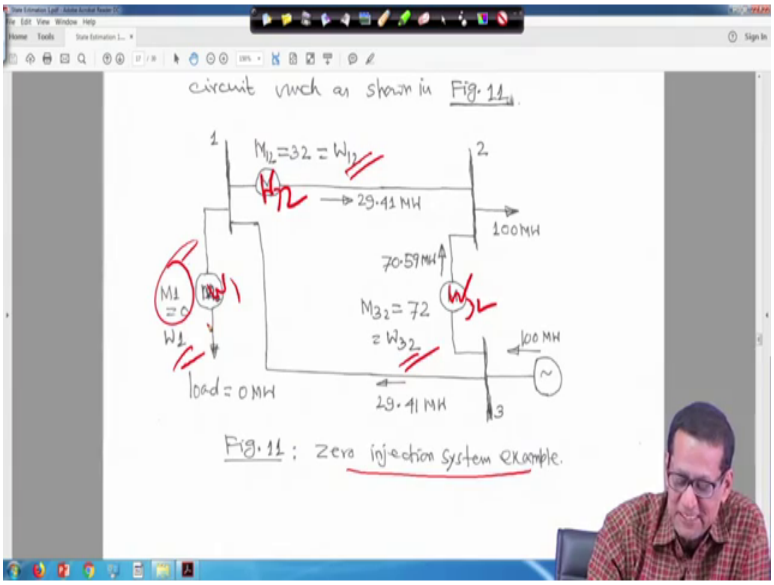
So, now state estimation by orthogonal decomposition that why we will go for orthogonal decomposition. Definitely there must be some reason or that right. So, one problem with the standard least square method presented earlier is the numerical difficulties, encountered with some special state estimation problems right.

(Refer Slide Time: 15:43)



Sometimes that you know matrix may become singular or something like that. So, you may not get any result any result right you have to overcome that one of these; one of these comes about your when wish to give a state estimation solution, to match its measurement almost exactly right. So, this is the case when we have a circuit which are shown in figure 11 right.

(Refer Slide Time: 16:08)

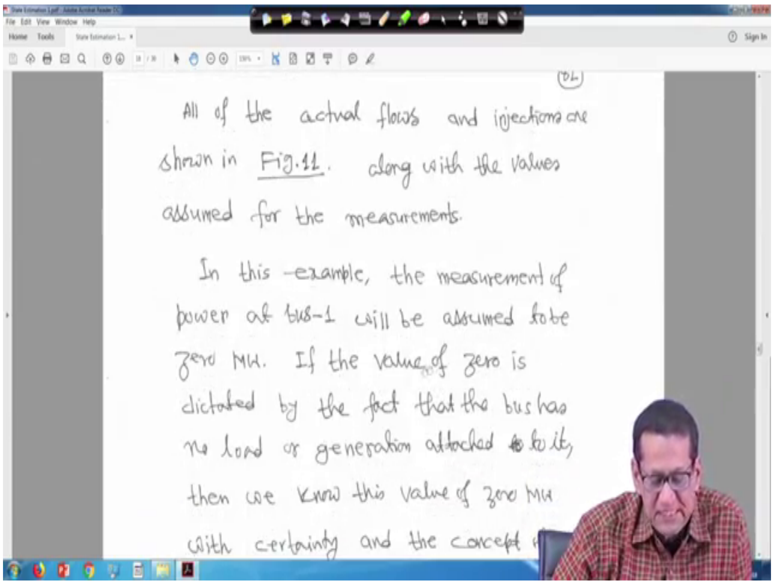


So, this is actually W 12 measurement is 32 megawatts right and this is your load here your load is there or generation whatever you will take it does not matter that, it is meter 1. So, its reading is 0 it is 0 right. And another meter is there that is your W 32 it is 72 megawatt all are megawatt right and this is your what you call that your load is going that 100 megawatt and here it is 100 megawatt is entering because if you just go for power summation thing then you will get this result right. So, these are meter.

So, meter means same thing same thing it is actually M 1 2 is nothing, but W 1 2 right it is M is nothing, but your W 3 2 right and here it is M 1, so, it is basically W 1. So, same thing right by when i write meter. So, I put meter here W and M that is why I made it here W, here it is W, and here it is W such that there should not be any confusion right.

So, 0 injection system example, so, here injection should be 0 right because load is 0, so, no power injection. So, these are the things suppose this is meter reading and these are the thing. Now, some data we have taken and given this thing right.

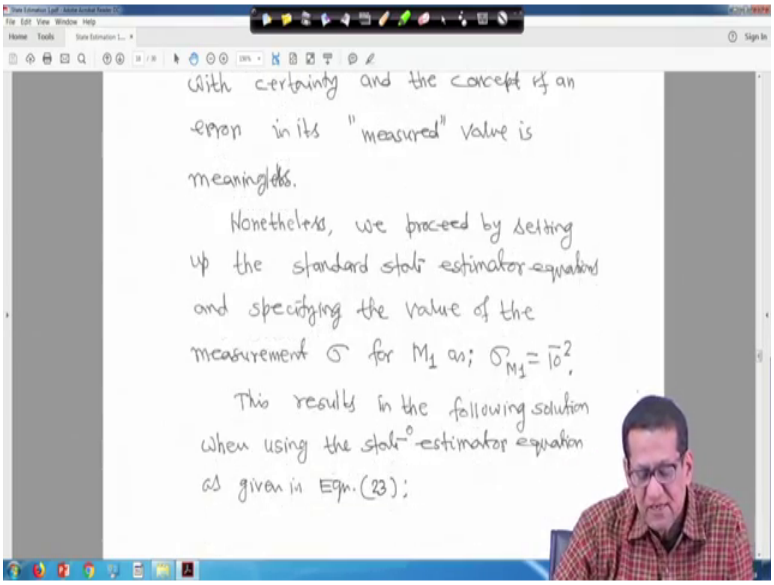
(Refer Slide Time: 17:20)



So, now all of the actual flows and injections are shown in figure 11. So, this is meter reading and based on that some line flows and other things are given, some data are given these are the given data right.

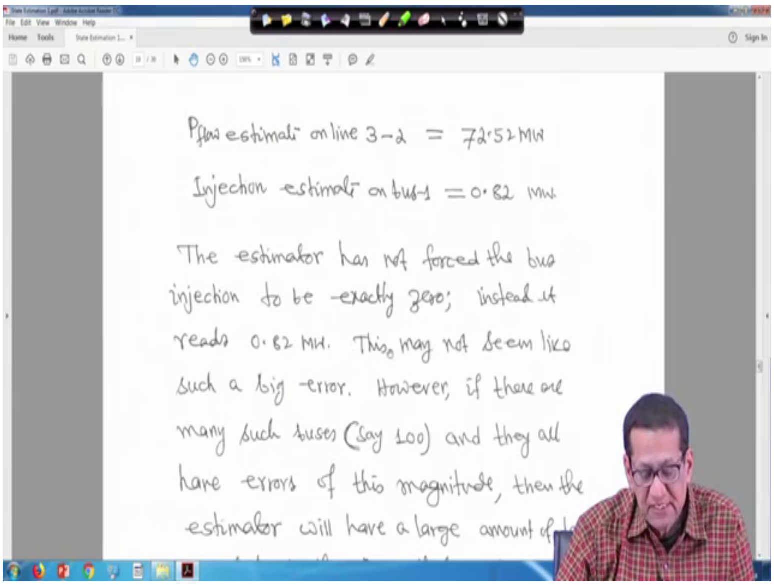
Now, in this example the measurement power at bus 1 will be assumed to be 0 megawatt. That is here at bus 1 that measurement power will be 0 megawatt right. Suppose your if the value of 0 is dictated by the fact that the bus has no load or generation attached to it, then you know this value of 0 megawatt with certain with certainty and the concept of an error in this measure value is meaningless right.

(Refer Slide Time: 18:04)



Now, nonetheless we proceed by setting of the standard state estimator right equations and specifying the value of the measurement sigma for M 1. Suppose for M 1 that sigma M 1 that is a standard deviation is 10 to the power minus 2 per meter 1 which was corrected near bus 1 right this result in the following solution when using the state estimated equation 33.

(Refer Slide Time: 18:28)

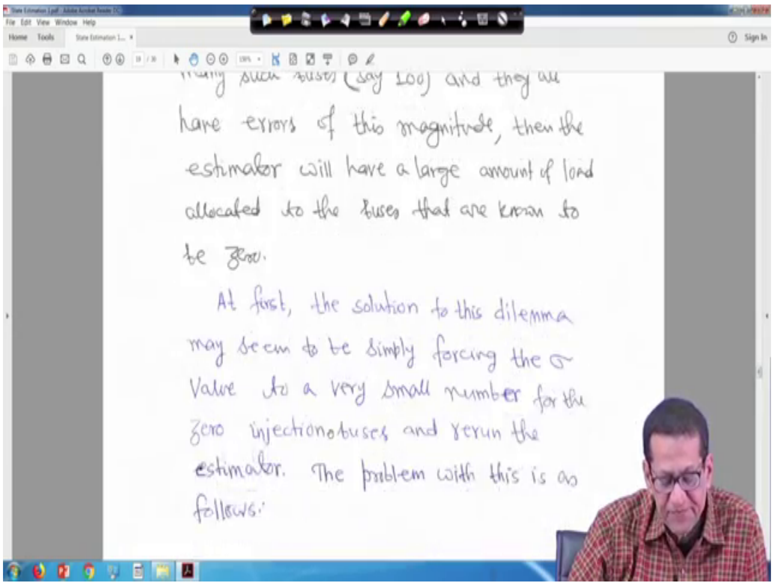


Suppose if we do so, suppose power flow estimate on line suppose the data is given 1 to 2 30.76 megawatt using the your same data. Now power flow estimate on line 3 2 72.52

megawatt and injection estimate on bus 1 is 0.82 megawatt; that means, this is not 0 because although we expect it should be 0, but it is showing some 0.82 megawatt.

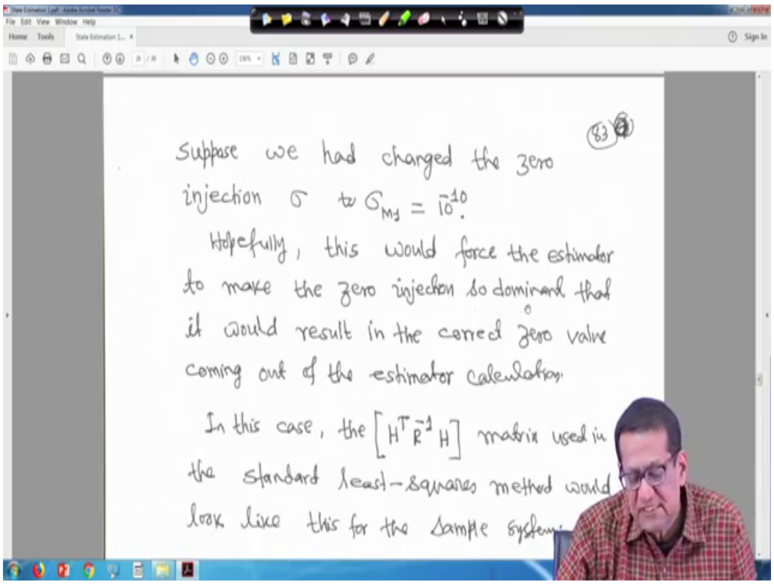
And in a power system you have a huge many numbers of measurement things are there so, gross error will be quite large right. So, the estimator has not force the bus injection to be exactly 0, instead it reached its 0.82 megawatt, this may not seem like such a big error right.

(Refer Slide Time: 19:09)



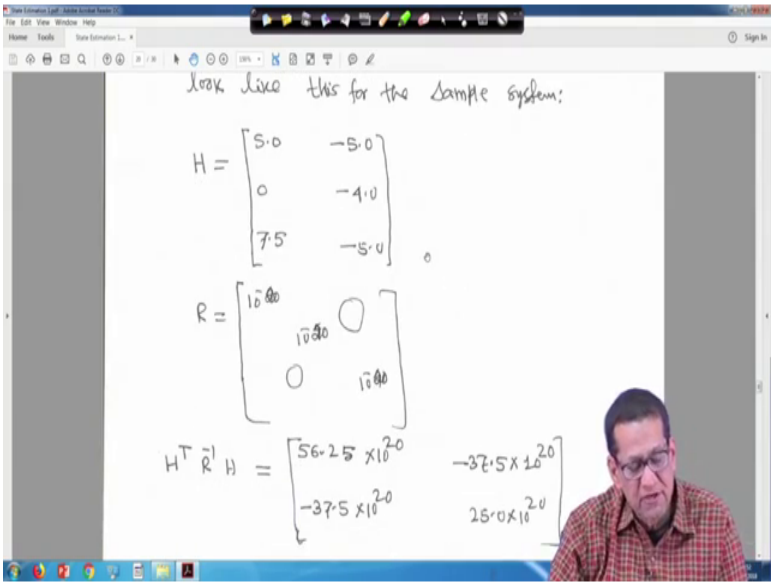
However if there are many such buses say 100 buses in a network right and they all have errors of this magnitude, then the estimator will have a large amount of load allocated to the buses that are known to be 0 right. At first the solution to this dilemma may seem to be simply forcing the sigma value to a very small number for the 0 injection buses and rerun the estimator the problem with this is as follows right.

(Refer Slide Time: 19:35)



Suppose we had a change the 0 injection sigma for that is sigma M 1 is 10 to the power minus 10; that means, the meter is actually very accurate 10 to the power minus 10 sigma means, it is almost 0 error right suppose you have made it. Hopefully this would force the estimator to make the 0 injection. So, dominant that it would result in the your correct 0 value, coming out of the estimator calculator right.

(Refer Slide Time: 20:03)

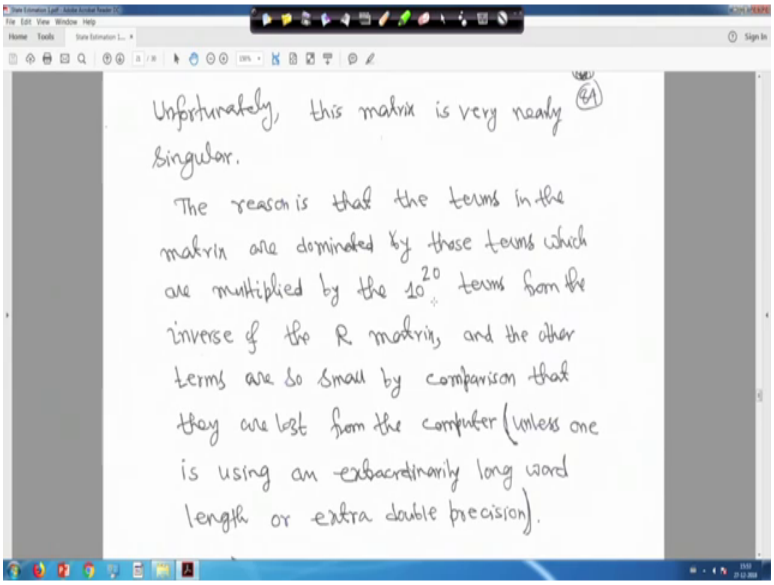


In this case the H transpose R inverse H matrix using the standard least square method would look like this for the sample system sorry. So, H was given the same H was taken previously. So, in this case of your what you call your R only your meter 1 it is not all minus 20 this thing right only for sigma 1 that is sigma M 1 we have taken 10 to the power minus 10.

So, its square will be 10 to the power your minus 20 right. So, this R is given 10 to the power your this is actually other things remain same previously. So, it will be 10 to the power minus 4, 10 to the power this is actually 10 to the power minus 20 right.

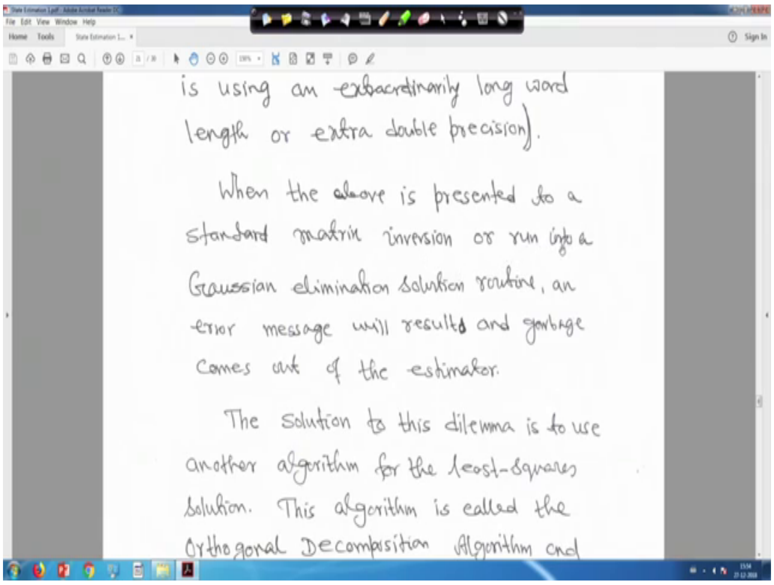
So, if we do so; if we do so, then H transpose R inverse H actually it will be something like this. 56.25 into 10 to the power 20 minus 37.5 into 10 to the power 20 minus 37.5 into 10 to the power 20 and 25 into 10 to the power 20 these values are very large rise.

(Refer Slide Time: 21:07)



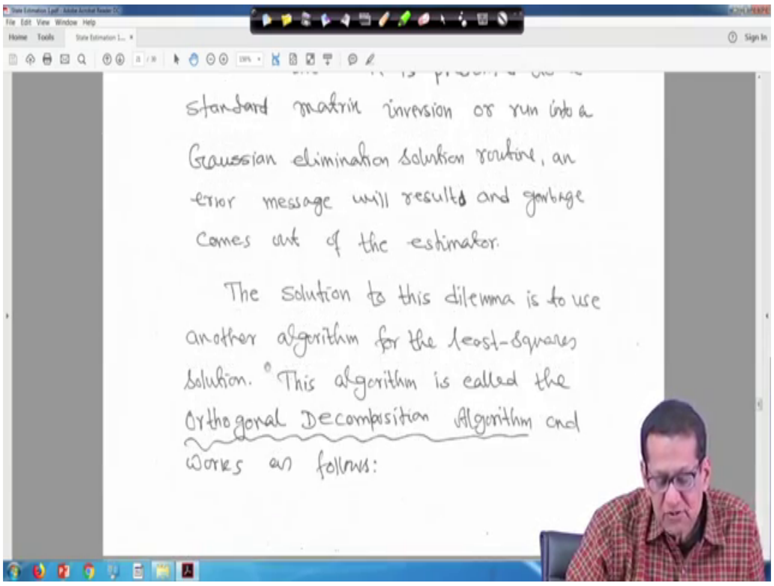
Unfortunately this matrix is very nearly singular right. Singular means it inverse may not exist because you have to ultimately you have to calculate this inverse right. So, in this case what will happen? The reason is that the terms in the matrix are dominated by these terms which are multiplied by the 10 to the power 20 terms right from the inverse of the R matrix. And the other terms are so, small by comparison that they are lost from the computer; that means, unless one is using an extraordinary long word length or extra double precision right.

(Refer Slide Time: 21:42)



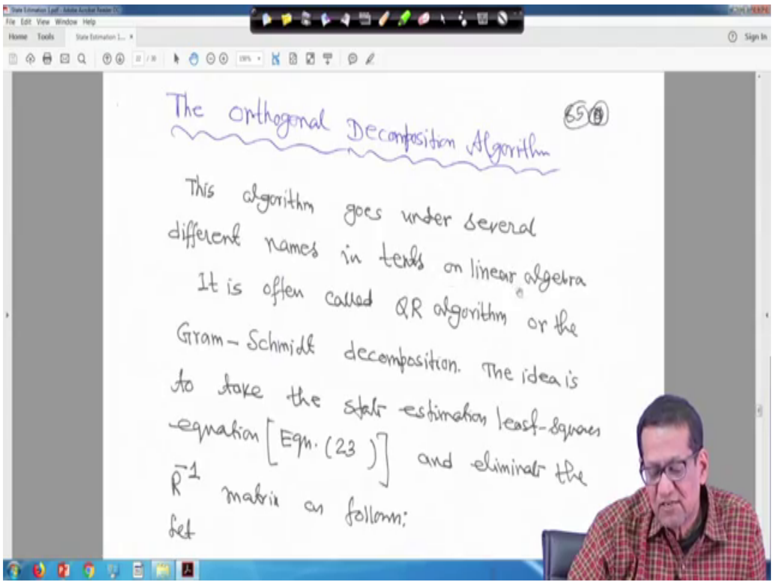
So, when the above is presented to a standard matrix inversion or run into a Gaussian elimination solution routine right and error message will result and garbage comes out of the estimator, because you will not get anything because matrix becoming singular.

(Refer Slide Time: 21:57)



So, the solution to this dilemma is to use another algorithm of the least square solution and this algorithm is called the orthogonal decomposition algorithm and works as follows right. So, that is why we follow orthogonal decomposition such that you will get a reasonable good solution.

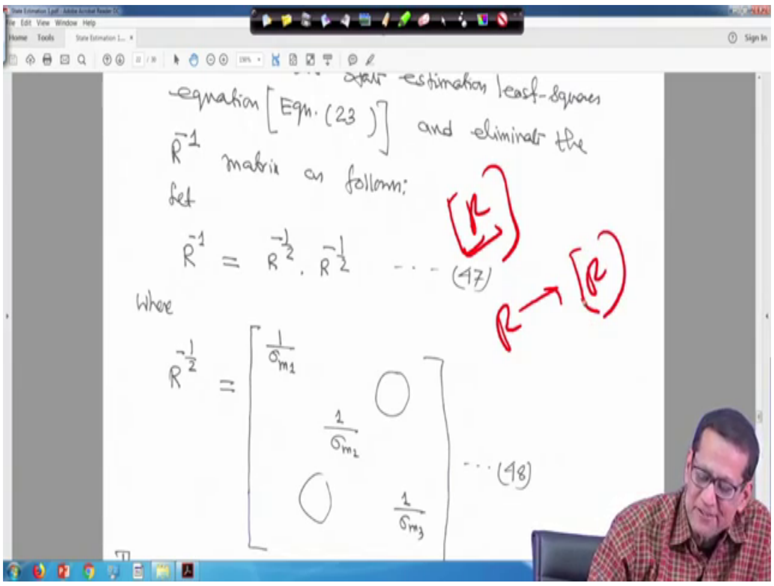
(Refer Slide Time: 22:15)



So, the orthogonal decomposition algorithm. This algorithm actually goes under several different names in texts on non-linear, your what linear algebra right. It is often called QR algorithm or the Gram Schmidt decomposition technique right because, but here we are not using this QR algorithm or this thing because we are using R matrix something right. So, we will name this as a QU rather than QR algorithm right.

The idea is to take the state estimator you are what you call least square equation that is equation 23 we have seen and eliminate the R inverse matrix as follows.

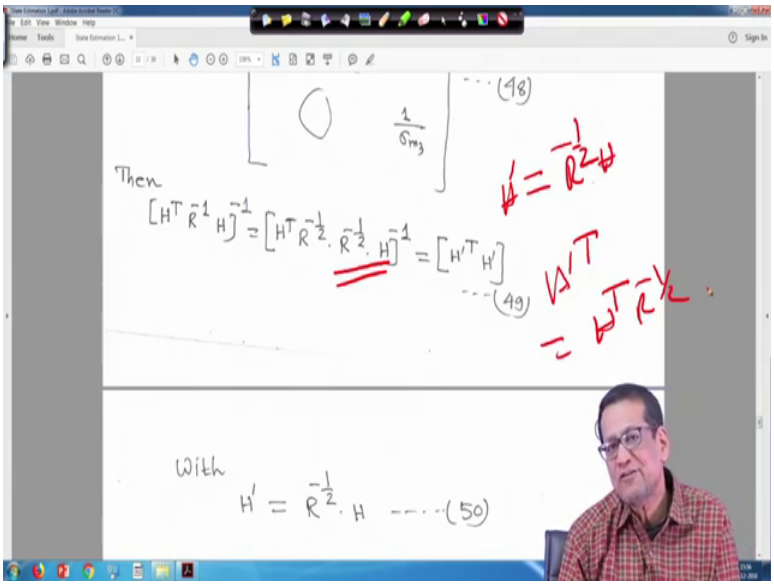
(Refer Slide Time: 22:52)



So, we have to eliminate R inverse, because this actually creates problem that is why you go for orthogonal decomposition. So, R inverse you can write R to the power it is actually earlier I wrote that it actually your what you call in bracket R to mention R matrix the same thing, these R actually these R this is same thing just write in bracket again and again I did not put it right I did not write it. So, it is R matrix actually right. So, these R inverse it can be written R to the power minus half into R to the power minus half right.

That means, your; that means, your where R to the power minus half will be actually earlier it was R to the power your what you call earlier R 2 inverse it was 1 upon sigma M 1 square 1 upon sigma M 2 square 1 upon sigma M 3 square like this. So, in this only you have taken that your 3 measurement case right. So, therefore, R to the power minus half will be 1 upon sigma M 1 1 upon sigma M 2 1 upon sigma M 3 like this is equation 48 right.

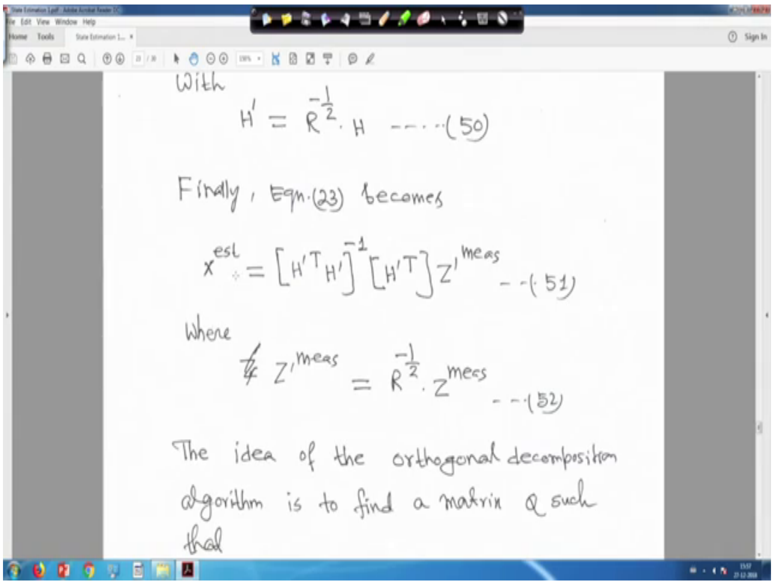
(Refer Slide Time: 23:53)



Therefore this equation H transpose R inverse H inverse which whole inverse is equal to you can write H transpose R to the power minus half into R to the power minus half into H to the power minus 1 right that can be written as H dash transpose H dash. Because this one because this one this one if you assume that this is my H dash, this is my H dash is equal to R to the power minus half then your H right.

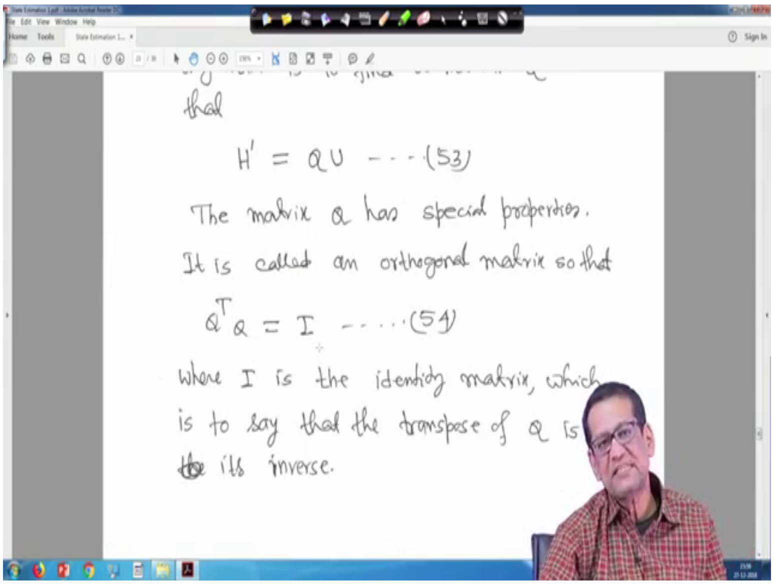
This you have then H dash transpose will become H transpose right then you are what you call that you are R to the power minus half because it is a diagonal matrix. So, no question of putting transpose right. So, that is why this 1 we can write that H dash transpose into H dash this is equation 49 right.

(Refer Slide Time: 24:51)



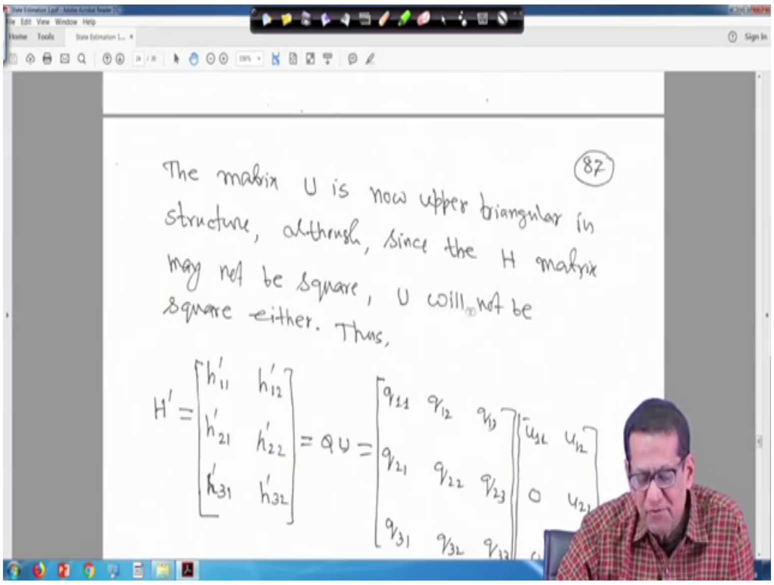
So, with H dash is equal to just hold on with H dash is equal to R to the power your minus half into H right that is the equation 50. Finally equation 23 will become that x estimated is equal to H transpose H dash transpose H dash to the power inverse into H dash transpose into j dash measure, this is equation 51 right. Now let us assume what we have done is that Z dash Z dash measure is equal to R to the power minus half into Z measure right. So, this is equation 52. So, just assuming this right, so, that is why it is Z dash measure.

(Refer Slide Time: 25:25)



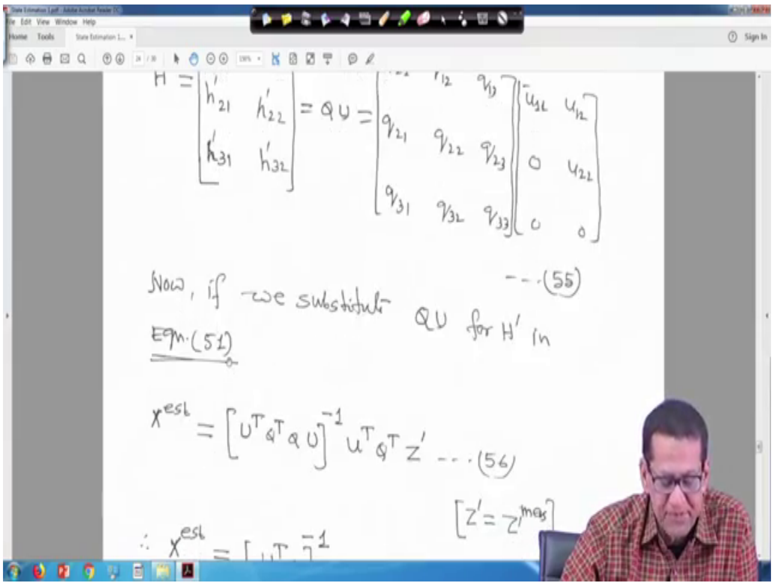
Now, the idea of the orthogonal decomposition algorithm is to find a matrix Q such that H transpose is equal to QU right this is equation 53. The matrix Q has a special properties it is called an orthogonal matrix. So, thus Q transpose Q will be identity matrix i right; that means, Q transpose will be is equal to basically Q inverse right. So, this is Q transpose Q is equal to say I, where I is the identity matrix which is to say that the transpose of a Q is its inverse right.

(Refer Slide Time: 25:59)



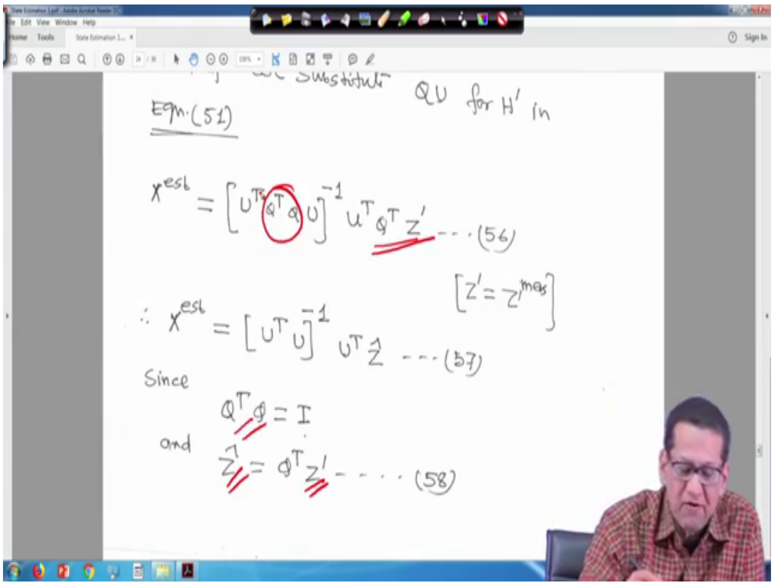
The matrix U is now a upper triangular in structure although since the H matrix may not be square U will not be square either right. Therefore, we can write H dash is equal to h dash 1 1, h dash 1 2, h dash 2 1, h dash 2 2, h dash 3 1, h dash 3 2 is equal to Q U is equal to Q is a 3 into 3 matrix it will be q 1 1, q 1 2, q 1 3, q 2 1, q 2 2, q 2 3, q 3 1, q 3 2, q 3 3 and u is a upper triangular. So, u 1 1, u 1 2 it is 0 u 2 2 and it is 0 0 right.

(Refer Slide Time: 26:35)



Now, if we substitute Q U for H dash in equation 51. In equation 51 you just substitute that you are this thing right, you substitute here.

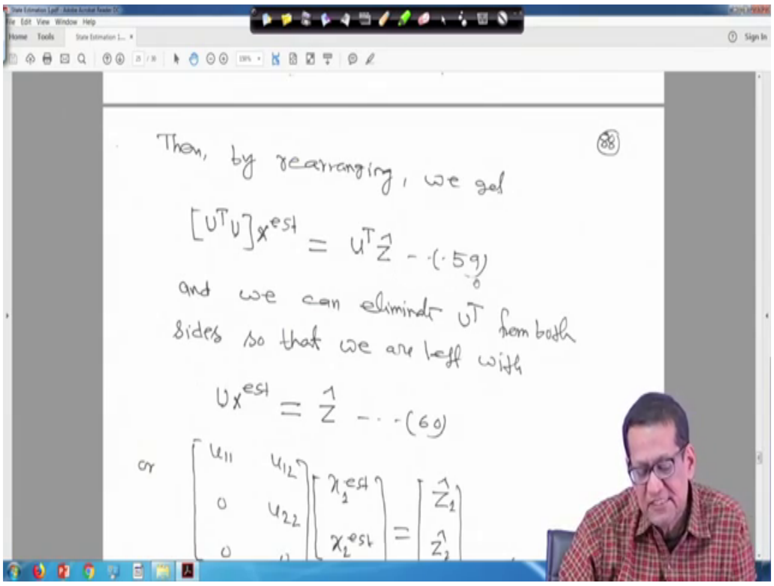
(Refer Slide Time: 26:50)



If you substitute then you will get X estimated will be U transpose Q transpose Q U whole inverse, then U transpose Q transpose Z right Z dash this is equation 56 where Z dash is equal to Z dash measure right. Therefore, X estimated will be U transpose U inverse because Q transpose Q is equal to identity matrix I right because this Q transpose Q is equal to identity matrix I right therefore, your this x estimated will be U transpose U inverse into U transport Z cap right.

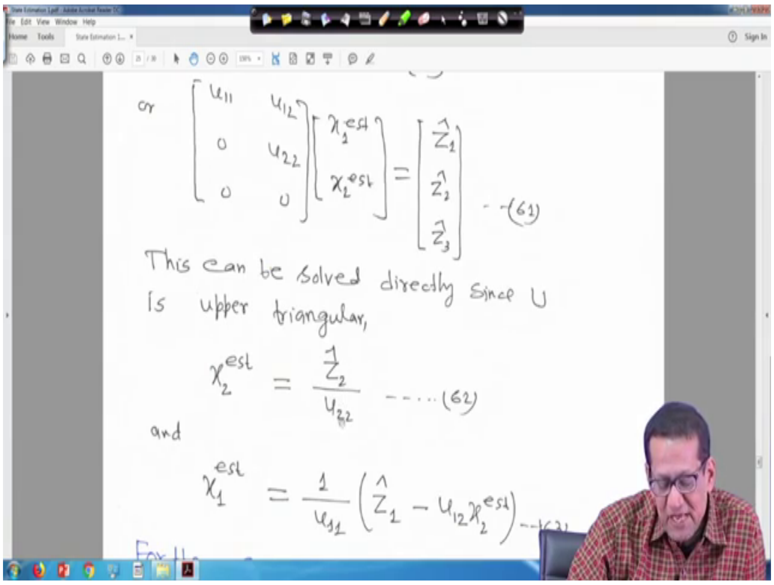
So, Z that is your Z cap is equal to Q transpose your what you call Z dash that is Q transpose Z dash is equal to Z cap and Q transpose Q is equal to I. So, this is actually I right. So, our objective was to eliminate that R inverse that is R matrix from that mathematical equation right.

(Refer Slide Time: 27:48)



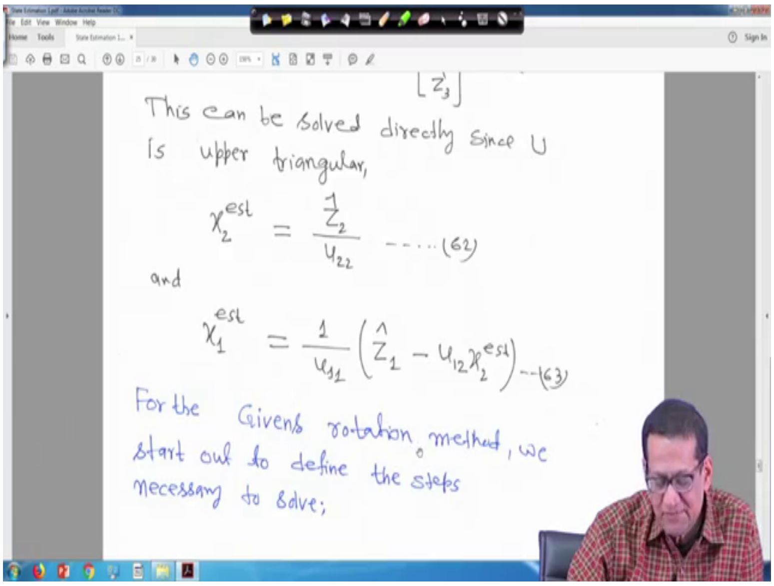
So, then by rearranging, you will get U transpose U X estimated is equal to u transpose Z cap this is equation 59 right. And we can eliminate U transpose from both sides so, that we are left with. So, both side U transpose U transpose you can eliminate. So, you are left with UX estimated is equal to Z cap look how simple it has come now right. The expression finally, had become in the simple form by orthogonal decomposition right or u is equal to you know u 1 1, u 1 2 it is upper triangular. So, 0 u 2 2, 0 0 x 1 estimated x 2 estimated this is Z 1 cap Z 2 cap and Z 3 cap.

(Refer Slide Time: 28:26)



So, if you solve this one then X 2 your estimated is equal to Z 2 cap upon u 2 two this is equation 62 and x 1 estimated is equal to 1 upon u 1 1 Z 1 cap minus u 1 2 x 2 estimated right.

(Refer Slide Time: 28:39)



For the givens rotation method we start out to define the step necessary to solve right. So, this givens rotation method actually little bit we will see not much little bit we will see and then your what you call then we will try to see few examples at the end right.

So, we will make few examples for you and throughout the course its actually its course is mostly your mathematical. So, mathematical oriented and mathematics are there. So, and all the problems are not small problems, but we will see how things can be done right and all these analysis we have tried to see it.

So, thank you very much we will be back again.