Power System Dynamics, Control and Monitoring

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Lecture - 53
State estimation in power system (Contd.)

So we are, we are back again, so in the previous example we have seen that, the delta 1 and delta 1 and delta 2 the state variables for the three-bus system since knowing them allows all other quantities to be calculated right.

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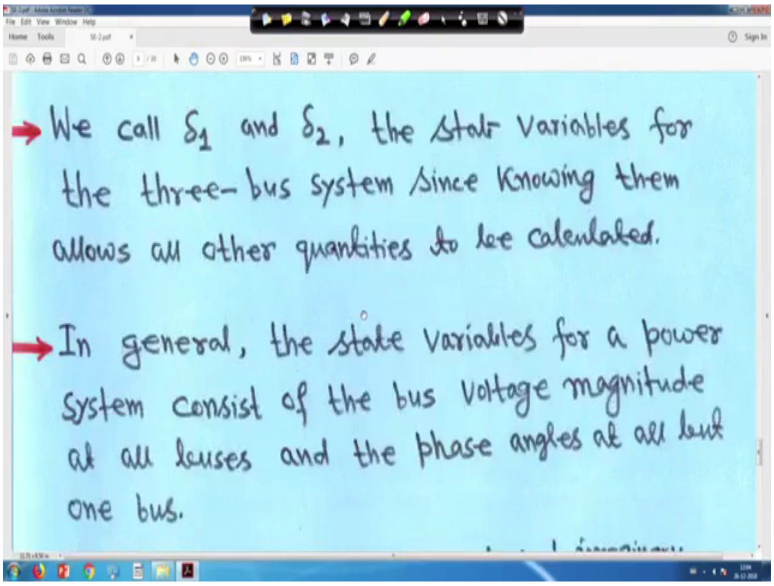

So, in general the state variables for a power system consists of the bus voltage magnitude at all buses and the phase angle at all but one bus that is slag bus right.

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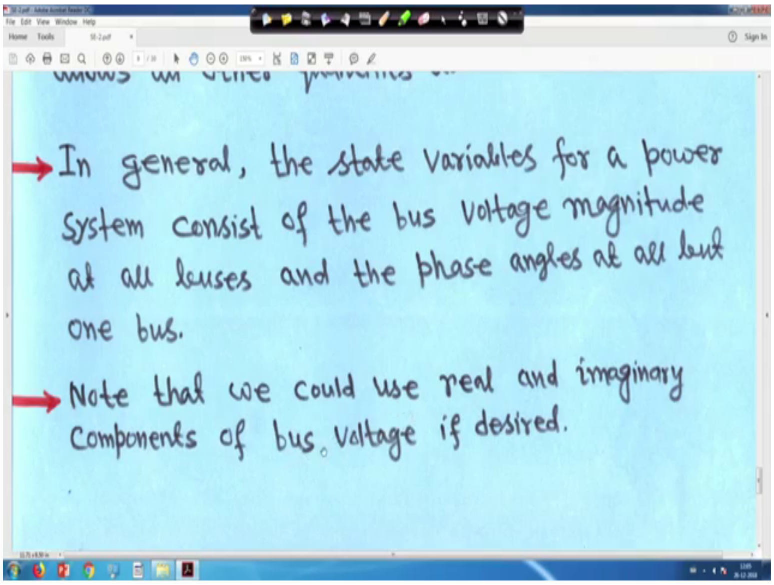

Note that we could use real and imaginary components of bus voltage if desired.

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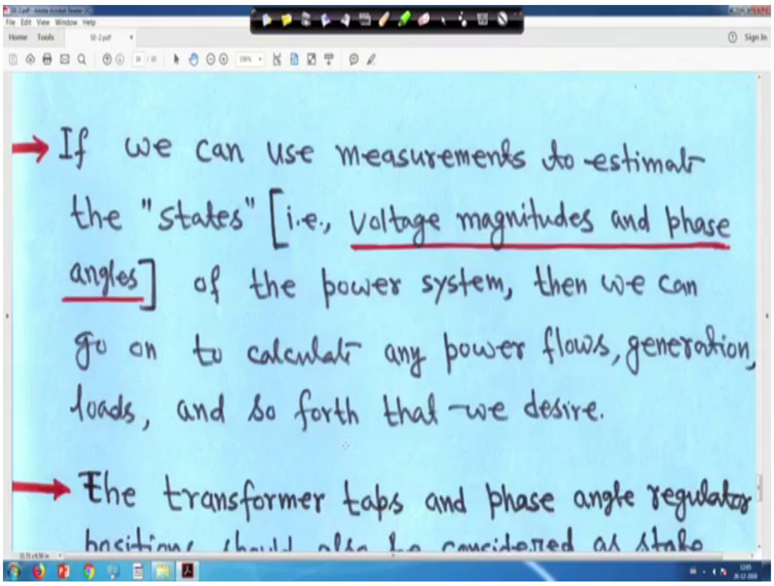

So, this all these things we have seen. If we, but just starting from the end of the previous lecture. If we can use measurements to estimate the “states” that is, voltage magnitude and phase angle in this case of the power system, then we can go on to calculate any power flows, generation loads, and so forth right; that we desire.

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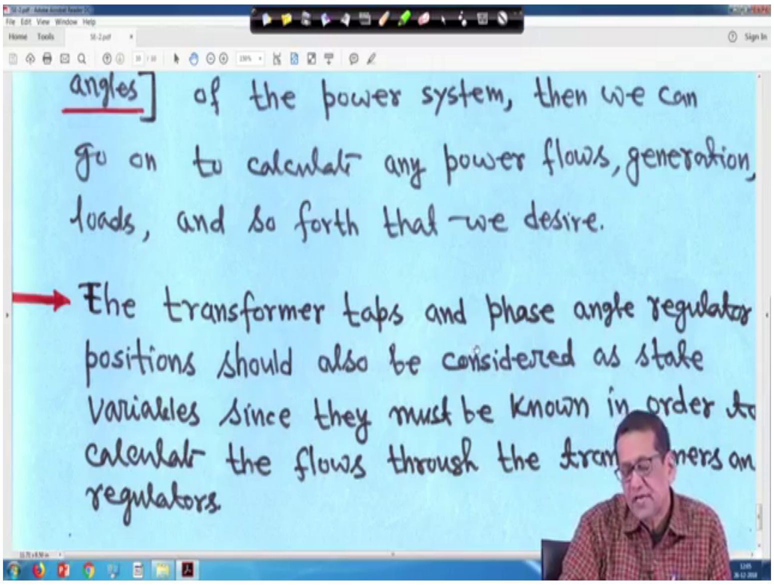

The transformer taps and phase angle regulator positions should also be considered as state variable since they must be known in order to calculate the flows through the transformers on and regulators right. So, that is what we have seen. Now just hold on. Let me increase the size.

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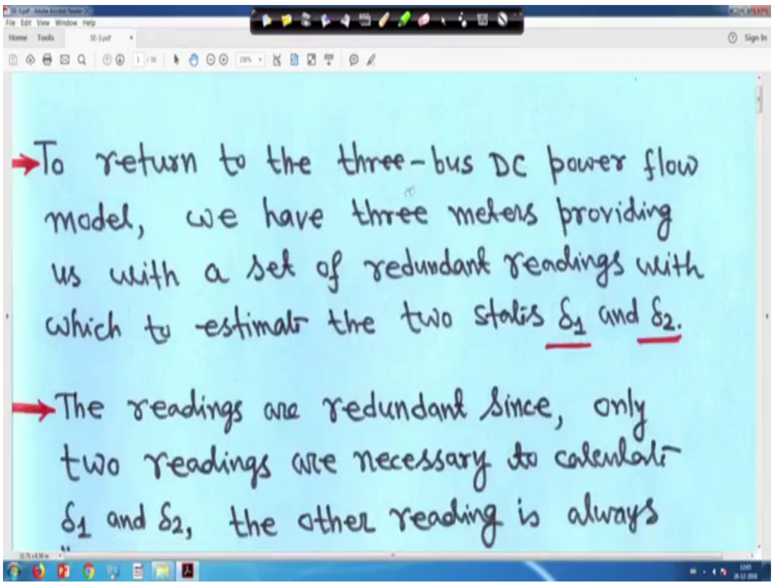

So, now to return to the three-bus DC power flow model, we have three meters providing us with a set of redundant readings with which to estimate the two states that is delta 1 and delta 2. There are two states, but we had three measurements right.

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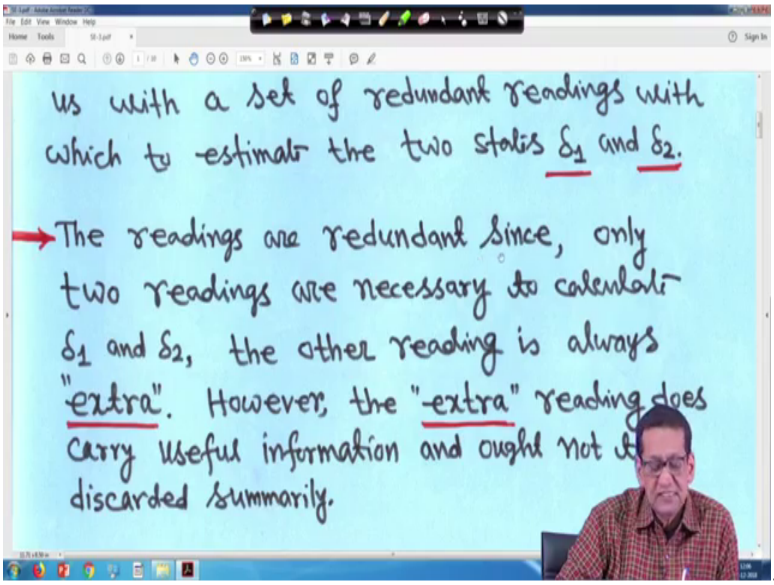

So, re readings are redundant since only two readings are necessary to calculate delta 1 and delta 2. T he other reading is always "extra". However, the "extra" reading does carry useful information and what not to be discarded summarily right.

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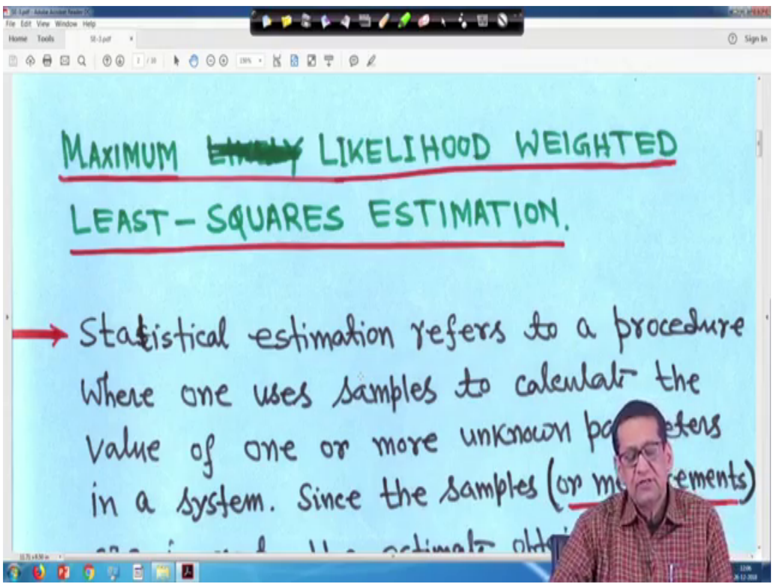

So, that is why if it is so, then that then we will go for a new concept right. That how we can accommodate all this thing. So, for state estimation that we will do static state estimation, but we will see three types of phases way of c. Mainly one is that when number of state variables are greater than your number of measurement. That is n s greater than n m. Another case number of state variable is equal to number of measurement that is n s is equal to n m. Another is number of state variables where n s less than n m that is number of state variables less than number of measurements right.

So, Maximum likelihood weighted least-square estimation. Likelihood means probability right. So, statistical estimation refers to a procedure which are you which one use a use a samples to calculate the value of one or more unknown parameter in a system. Since the samples that is measurements right. I have underlined this one are inexact; that is not true value not correct value right.

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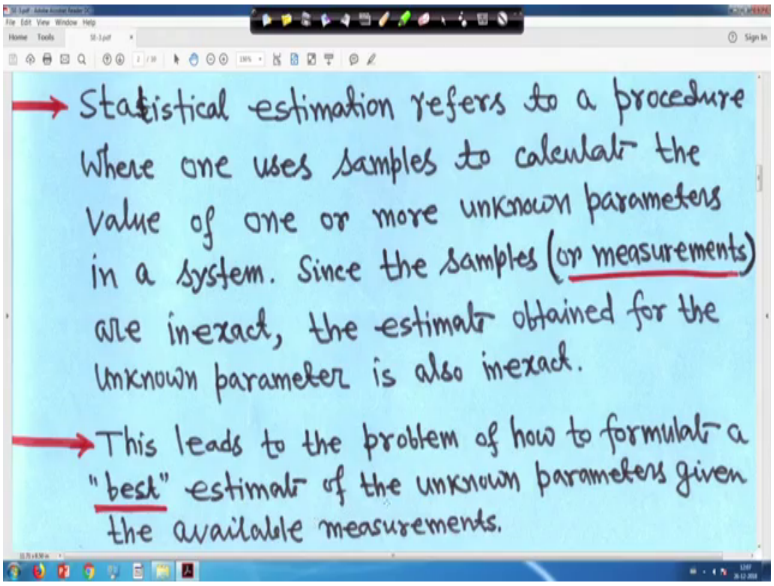

The estimate obtained for the unknown parameter is also in a inexact, but we have to see that how close it is to the true values right. This actually leads to the problem of how to formulate a “best” estimate of the unknown parameters given the available measurement right.

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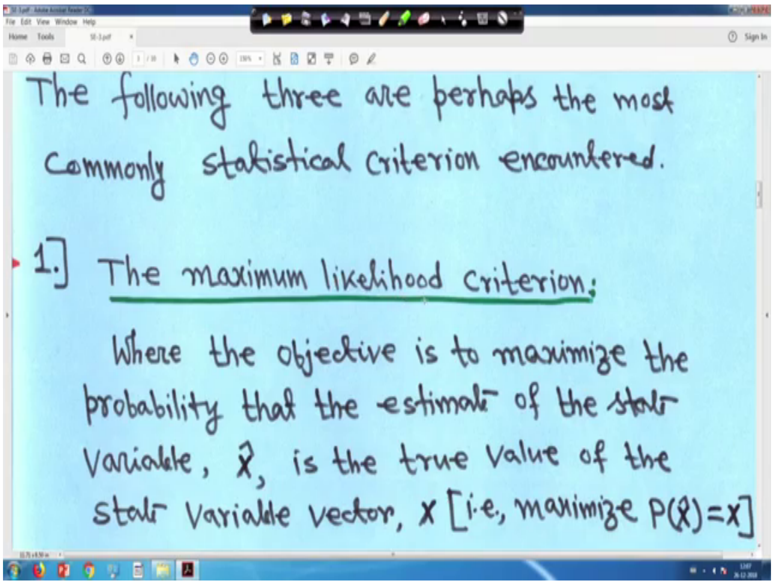

The following three are perhaps the most commonly statistical criteria encountered. Number 1 is: The maximum likelihood criteria. Here actually that is maximum likelihood means it is probability. Then you are maximize the probability right. Where the objective is that to maximize the probability that the estimate of the state variable X cap right is the true value of the state variable vector X. That is maximize P X cap is equal to X you have to maximize the probability this is maximum likelihood criteria right.

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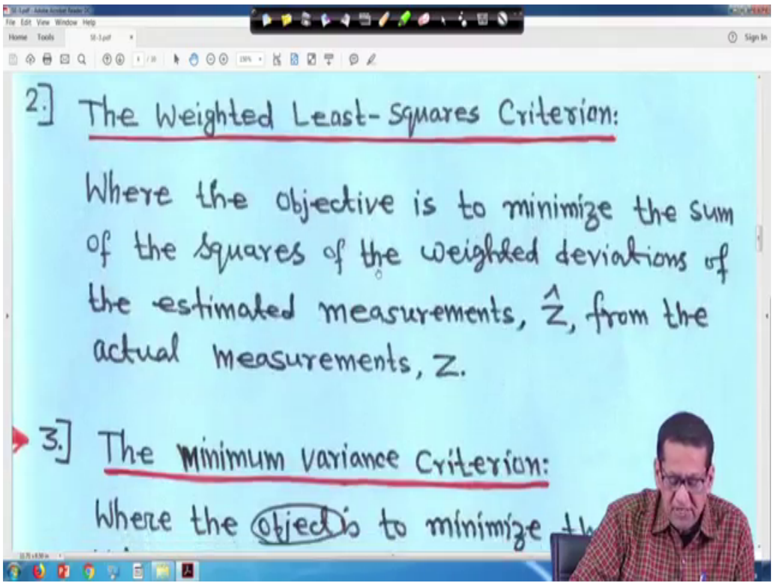

Next one is, that you are the weighted least-square criteria; here where the objective is to minimise the sum of the squares of the weighted deviations of the estimated measurements that is z cap from the actual measurement that is z right. sorry. So, this is weighted ;east-square criteria.

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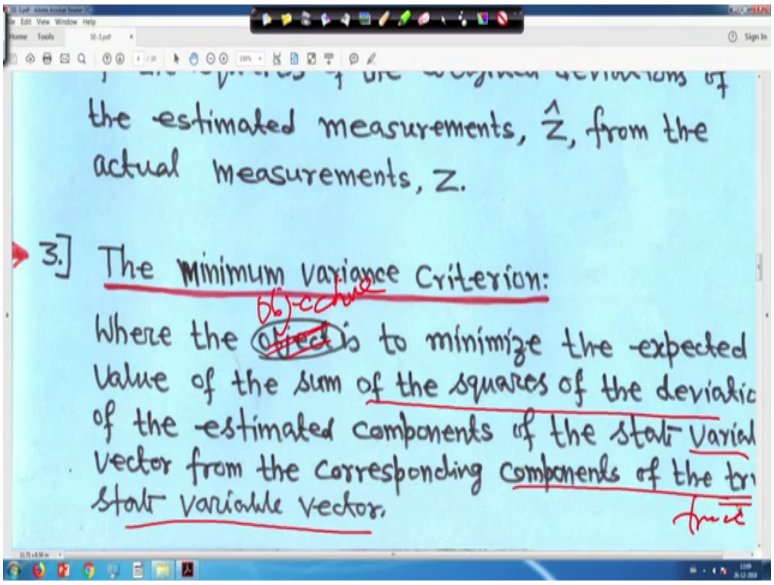

Third one is the minimum variance criteria right. In this case that the object is actually objective right, by it is written object it is objective. Where the objective is to minimise the expected value of the sum of the squares of the deviation of the estimated components of the state variable vector from the corresponding components of a true state variable vector right.

So, these are the actually it was your what you call it is variable and it is actually true right; true state variable vector. This is little bit cut but a just I am telling this one, this is actually true value right. So, that is actually this three different criterias you have and we have to follow certain things right; rhe certain mathematical procedure.

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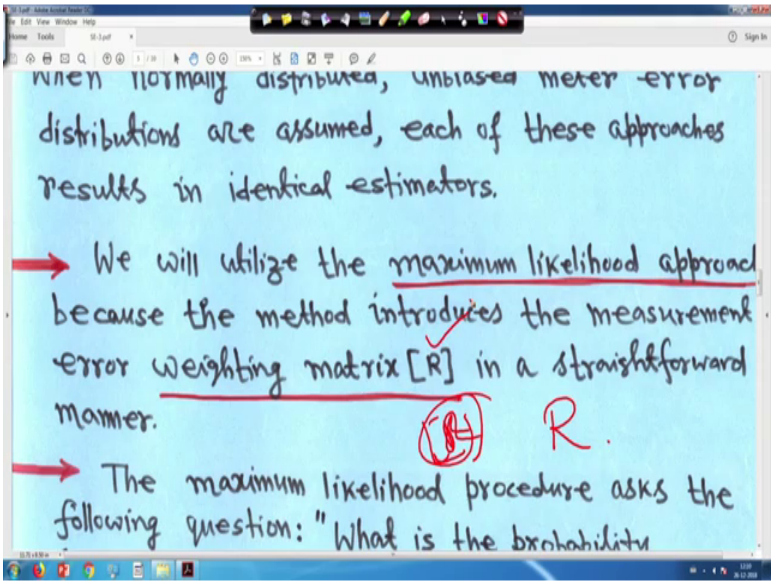

Now, when normally distributed, that is unbiased meter error distributions are assumed that each of these approaches results in identical estimators right. We will utilise the maximum likelihood approach because the method introduces the measurement error weighting metrics R. Sometimes what I will do, when I am writing it is actually R it is R matrix instead of that some later values R only; so understandable right. Just put in bracket that is the source that your matrix right. So, it is in a straight forward manner.

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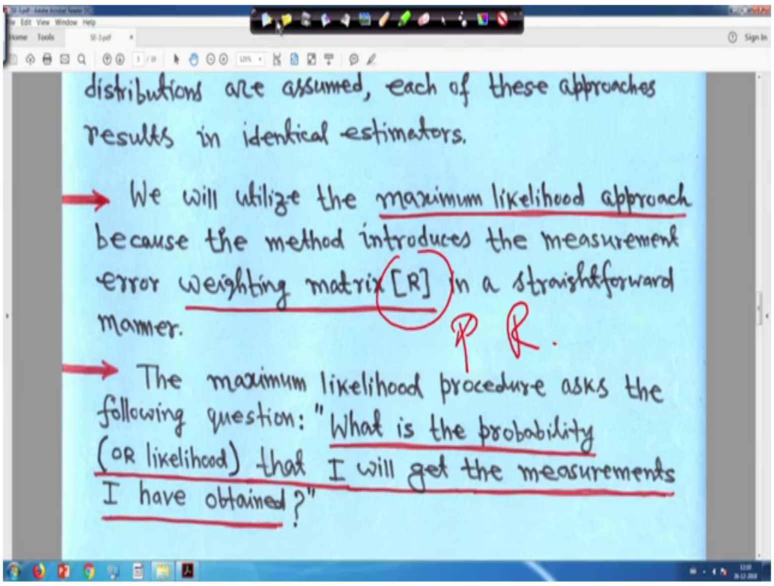

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We will utilise the maximum likelihood approach because the method introduces the measurement error waiting matrix R right in a straight forward manner. So, sometimes I will simply right that simply right r right. So, but here I am made it in bracket R to show that it is a matrix. So, the maximum likelihood procedure ask the following questions. That is what is the probability in bracket I have written or likelihood that I will get the measurement I have obtained right; so this is that criteria the maximum likely proceed your likelihood procedure right.

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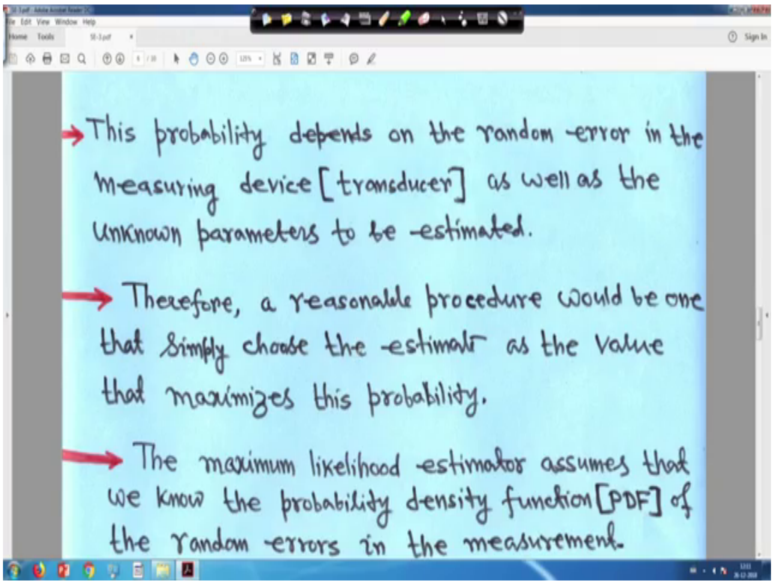

So, this probability actually depends on the random error in the measuring device that is transducer as well as the unknown parameters to be estimated. Therefore, a reasonable procedure would be one that simply choose the estimate as the value of maximizing, maximizes this that provide that maximizes the probability right.

The maximum likelihood estimator assumes that we know the probability density function that is equal PDF right, of the random errors in the measurement.

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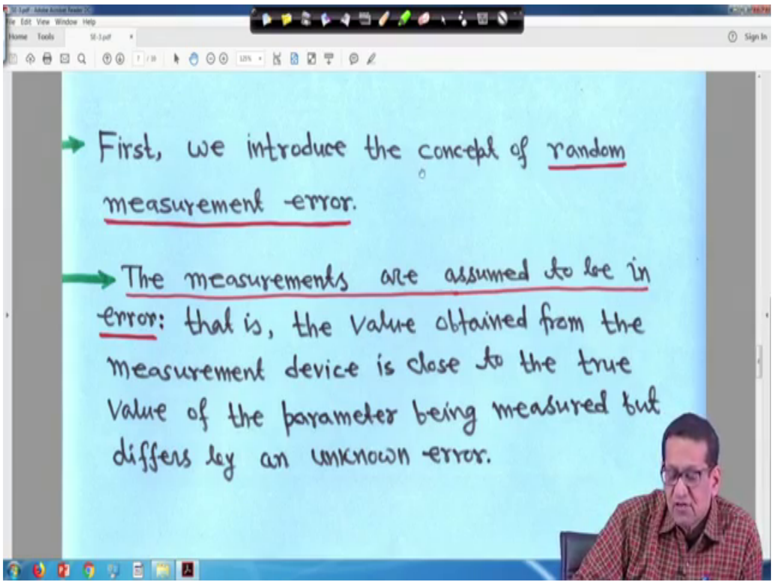

Now, first let us introduce the concept of random measurement error. The measurement errors are assumed to be in error, we are assuming that every measurement has some errors. For example, when you are doing your experiment in the laboratory you have ammeter, voltmeter, wattmeter and other type of meters when you take the reading you will see that as soon as you connect and try to take the reading you will find the point are actually oscillating around some point. When that pointer is settling to some point you are taking the reading, basically you are taking the mean reading right. Because it is or two reading is there, but both side you will find a point that is oscillating.

So, ultimately when it is settle down your taking the reading basically that is mean reading right. And in the ammeter or voltmeter just check it will be written somewhere are so, instruments that accuracy. Suppose if it is written that accuracy plus minus 3 percent right. That is if it is written like this, so that is actually 3 sigma right is equal to plus minus 3 percent the standard deviation right.

So, that the value obtained from the measurement device is close to the true value of the parameter being measured, but differs by an unknown error. Because it we are measuring, but measurement is not your exact right; it is in exact inexact. So, some error will be there.

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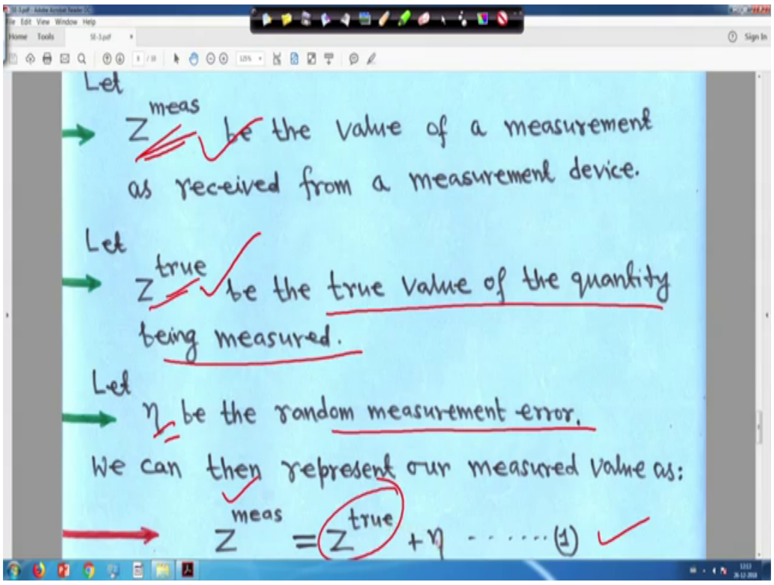

So, now let us let us make it like this, that you assume that Z measure right; let Z measure that is your this one be the value of a measurement or as received from a measurement device. It may be it may be volt, it may be current, ampere, it may be megawatt, it may be mega bar right like this.

So, that the hand and let Z true, this is Z true be the true value of the quantity being measured. This is the measured value and this is the true value right. Now let zeta be the random measurement error right. Therefore, we can right that Z measure is equal to Z true plus zeta; this is equation 1. So, this is our true value, but we are assuming that zeta be the random measurement error. So, Z measure actually g g Z true plus zeta right.

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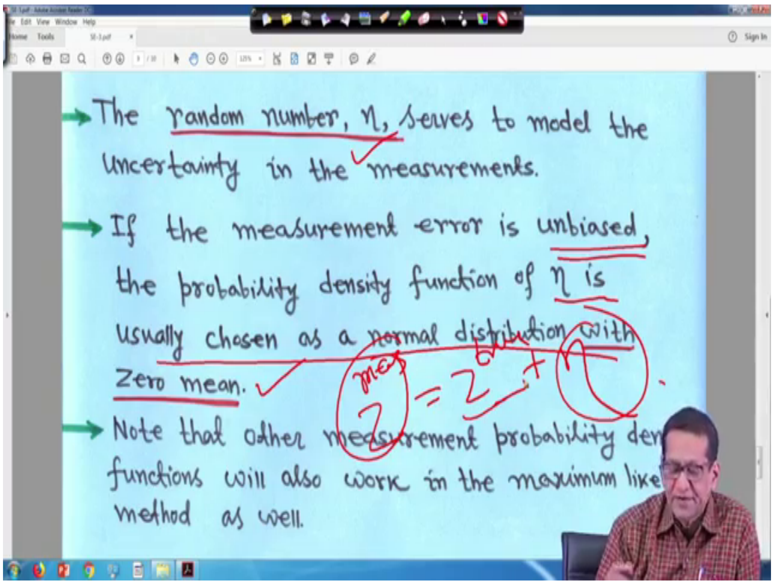

So, that way that way we will define that, than now the random number zeta actually subs to the model uncertainty in the measurement right. So, this is actually random number it subs to a model the uncertainty in the measurement.

If the measurement error is unbiased right the probability density function of zeta is usually chosen as a normal distribution with zero mean. So, that means, if the measurement error is unbiased the probability density function of zeta is usually chosen as normal distribution with zero mean I mean would not mean is zero. Therefore, if it is happens, so note that the other measurement probability density function will also work in the maximum likelihood method as well.

So, it mean value of I mean we are writing Z measure in the previous equation 1, Z true plus zeta right. If the mean value of the your what you call, that a your random number that zeta is 0. Therefore, mean value of Z a measure will be is equal to the mean value of Z I mean this thing Z true right.

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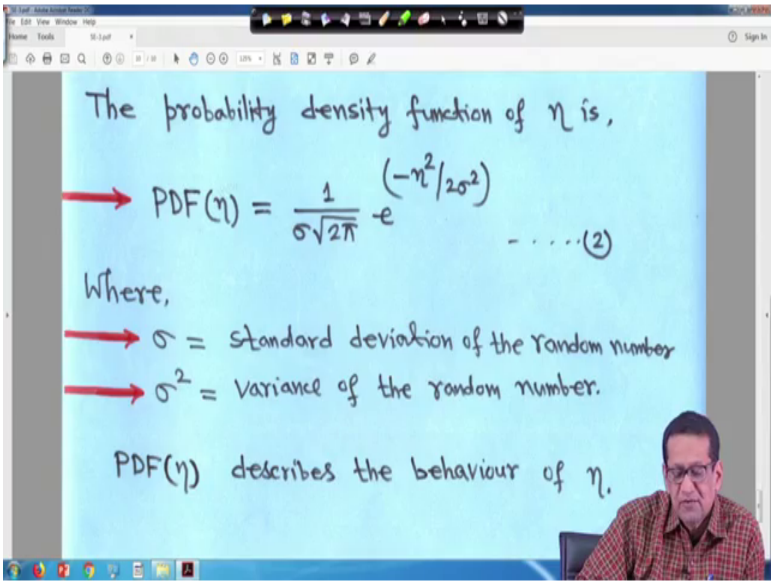

So, that is why the probability density function of zeta can be written as is a normal distribution your probability for probability expression that is PDF that is probability density function zeta is equal to one upon sigma root over 2 pi into e to the power minus your zeta square upon 2 sigma square; this is equation 2. This you know this expression from your mathematics probability chapter. You know this, this expression is a standard expression for normal distribution.

Where sigma is equal to you know the standard deviation of the random number and sigma square is the variance of the random number right. Now PDF zeta actually describes the behaviour of the behaviour of zeta right. So, this is a stand we define this one as a probability density function zeta is equal to this is a normal distribution sorry. So, now we will go to the next page just hold on.

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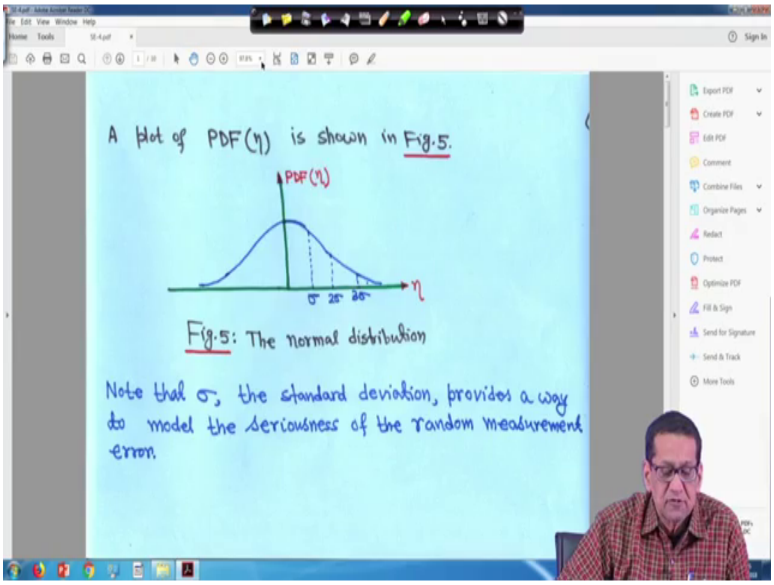

So, the plot, just hold on; so the plan your the plot or probability density function that is PDF zeta, versus zeta is given this is sigma, this is 2 sigma, this is 3 sigma and it is actually your what you call the normal distribution. The normal distribution, the PDF zeta versus zeta curve right. And note that sigma the standard deviation provides a way to model the seriousness of the random measurement error right.

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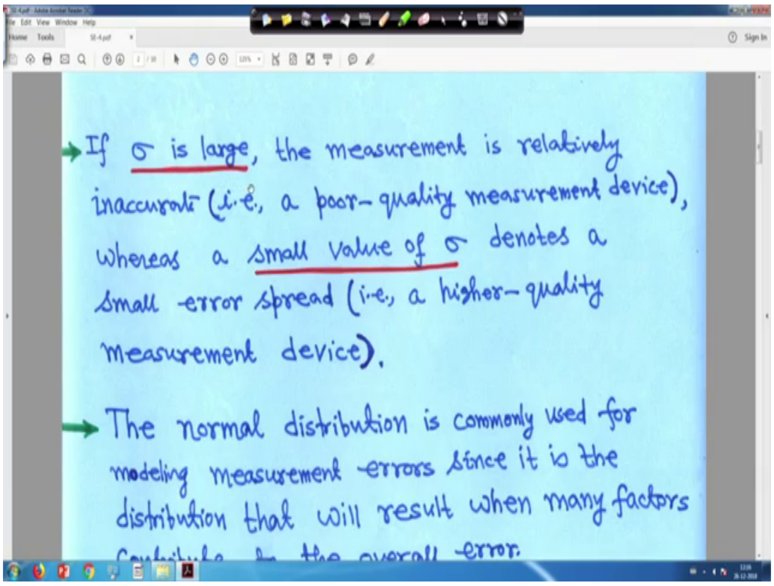

So, if sigma is large, the measurement is relatively inaccurate that is a poor quality measurement device. Because you are standard division is large; that means, the measurement device is not giving you the best answer right. Be best measurement. So, it is a poor quality measurement device. Where as a small value of sigma, when sigma is very low that is standard deviation is very low denotes a small error spread that is a higher quality of measurement, higher quality measurement device right. Now the normal distribution actually is commonly used for modelling measurement errors since it is the distribution that will result when many factors contribute to the overall error right.

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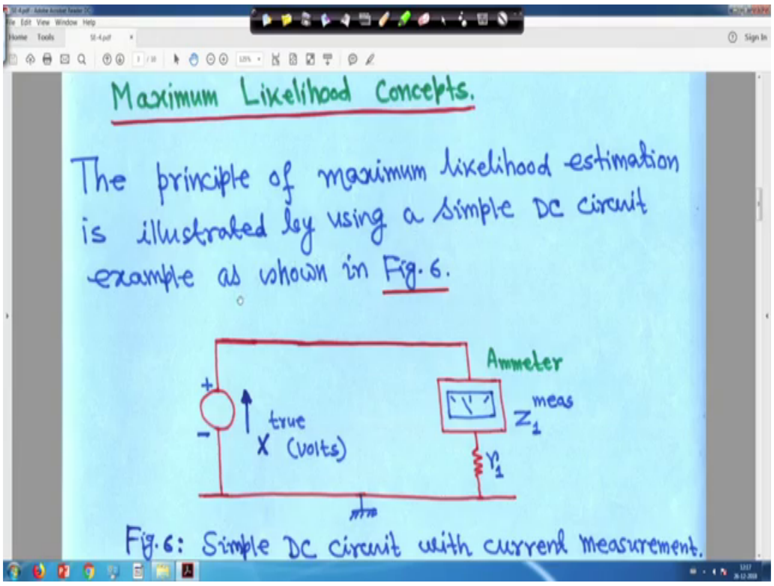

So, for example, for example, say you consider a maximum likelihood concept; that how things are. For example, you consider a simple DC circuit right. It has a voltmeter, it is given like this X true is actually it is a volt, and there is an ammeter also and it is a me measuring the current we get Z 1 measurement and r 1 is the resistance of the circuit right. So, the principles of maximum likelihood estimation is illustrated by using a simple DC circuit example and so as shown in figure 6 right. So, this is the voltage it is the true value we have to estimate this true value of the voltage another taken a simple circuit phase.

So, it is a voltage source and an ammeter is there when resistance is there in r that is r 1 right. And this is Z 1 measure; actually if this is Z 1 measure means it is the ammeter, amp ampere. So, Z 1 measure means it will be X true upon x upon r 1 in general right. Because X is the volt and Z is the current ampere right.

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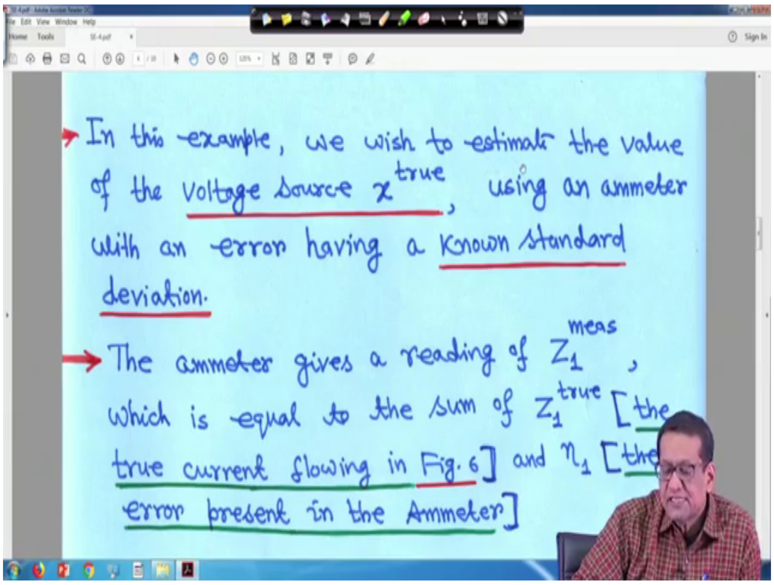

So, now in this example we used to estimate the value of the voltage source x true using an ammeter with a error having a known standard deviation. We know the ammeter and we know the known standard deviation right. Now ammeter gives a reading of Z 1 measure, this is actually giving editing that give Z 1 measure right. So, which is equal to the sum of the Z 1 true; that is the true current flowing in figure 6; there is a true current flowing in this figure 6 right. And zeta 1 the error present in Ammeter. This expression we have seen the Z measure is equal to Z true plus zeta. So, here it will be Z 1 measure will be is equal to Z 1 true plus zeta 1.

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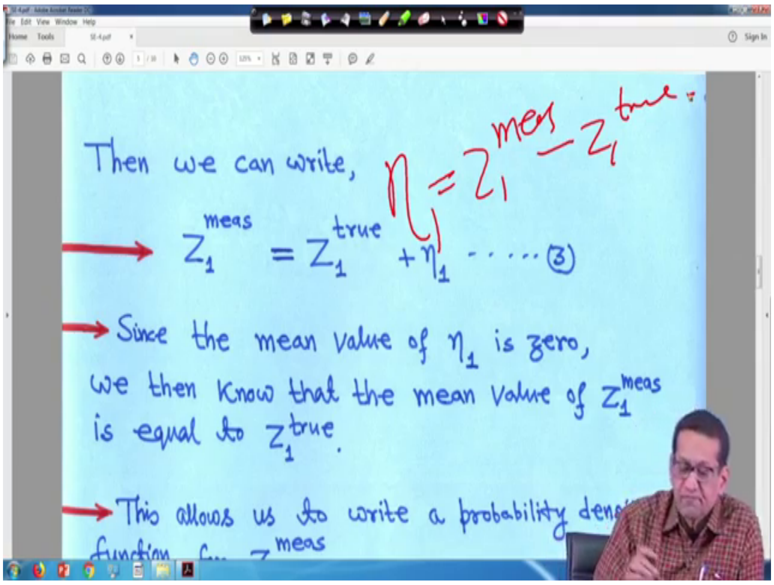

So, here now we can write that, Z 1 measure is equal to Z 1 true plus zeta 1 this is equation 3. Therefore, is zeta 1 therefore, zeta 1 is equal to your Z 1 measure minus Z 1 true right. So, because that zeta 1 expression we will use since the mean value of zeta 1 is zero I told you with a no that mean value of Z 1 me; that means, there is equal to zeta 1 true right because this zeta 1 mean value is zero. So, mean value of Z 1 measure is equal to Z 1 true Z true right.

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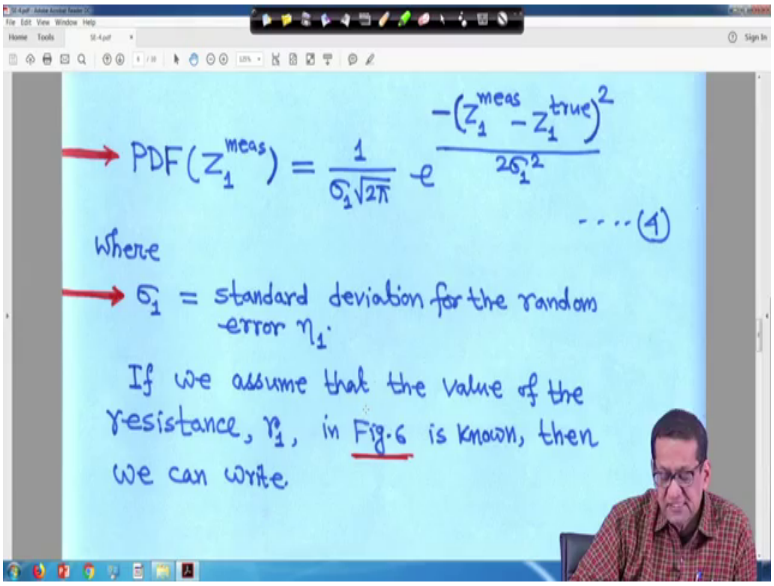

These actually allow us to write a probability density function for Z 1 measure as. So, in this case, we can write then PDF Z 1 measure is equal to 1 upon sigma 1 the standard deviation is sigma 1 to 2 pi right. It into e to the power actually it was minus your zeta 1 square upon 2 sigma 1 square. But zeta 1 is equal to Z 1 measure minus Z 1 true. So, that is why it is minus Z 1 measure minus Z 1 true whole square upon 2 sigma 1 square. This is equation four this is the probability density function for Z 1 measure right. Where sigma 1 is equal to standard deviation for the random error that is zeta 1.

If we assume that the value of the resistance, r 1, in figure 6 is known, then we can write right. We are assuming that that in the figure 6 the resistance of the DC circuit r 1 is known then we can write.

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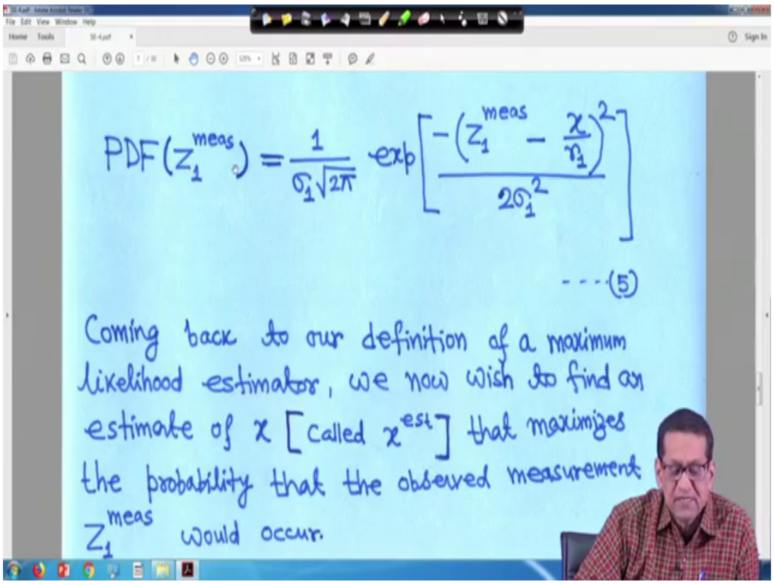

That your this term that Z 1, Z true that is your Z true it will be x upon r, because Z is the ampere your ammeter reading say, it is ampere and x is the volt, so volt by your resistance r so ampere. So, Z actually, Z true is actually x upon r 1, this term is replaced by x upon r 1. That means, by PDF Z 1 measure is equal to 1 upon sigma 1 root over 2 pi e to the power minus bracket Z 1 measure minus x upon r 1 whole square upon 2 sigma 1 square this is equation 5 right.

Now, coming back to our definition of a maximum likelihood estimator, we now wish to find an estimate of x called x x is equal to x est that is x estimator right. That maximizes the probability there that the observe measurement Z 1 measure would occur right. So, this is for a just a simple circuit we have taken and this is the probability density function for Z 1 measure.

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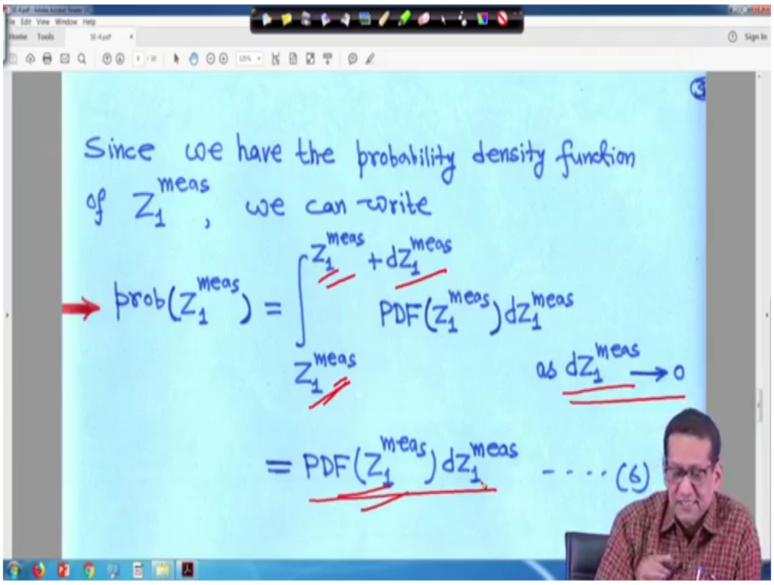

Since we have the probability density function of Z 1 measure, we can write like this the probability of Z 1 measure is equal to, it is actually Z 1 measure to Z 1 measure plus d Z 1 measure right. That is integration this is a limit.

Then PDF Z 1 measure into d Z 1 measure right. As d Z 1 measure actually tends to 0, you can write PDF Z 1 measure into d Z 1 measure right. Because d Z 1 measure is a very small thickness very small one, I suggest you go to the graph and just find out how things are happening, but from your probability studies you have studied this right. So, since we have the probability density function like this, then this one actually can be written as PDF Z 1 measure into d Z 1 measure this is equation 6 right.

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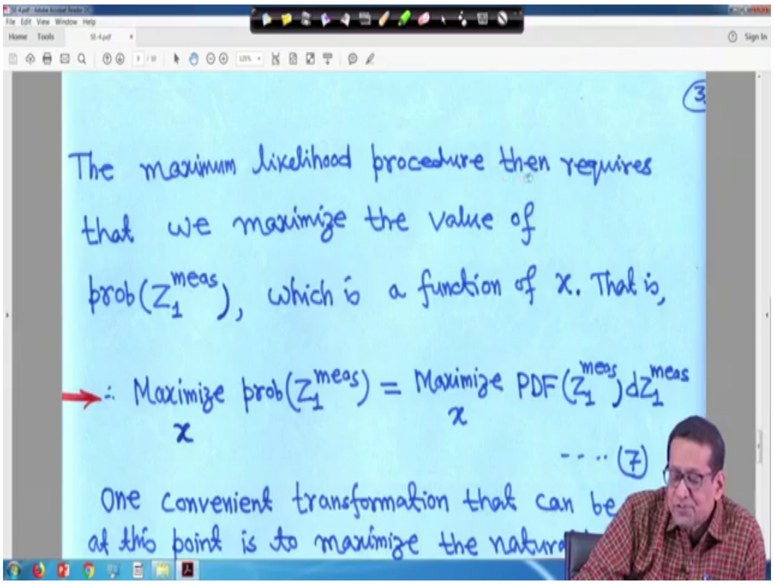

So, the next the maximum likelihood procedure then requires that we will maximize the value of probability Z 1 measure.

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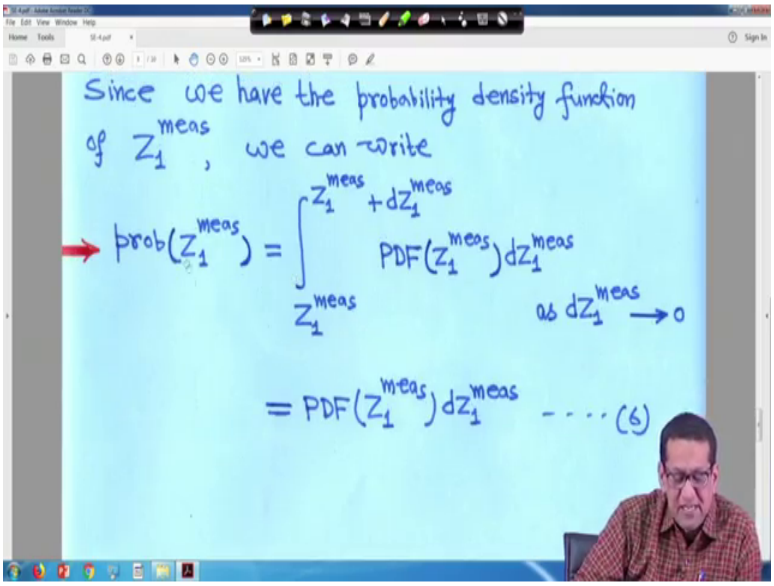

We have to actually maximize this one, that probability of Z 1 measure we have to maximize this one right. So, nothing but you have to maximize this one PDF Z 1 measure right.

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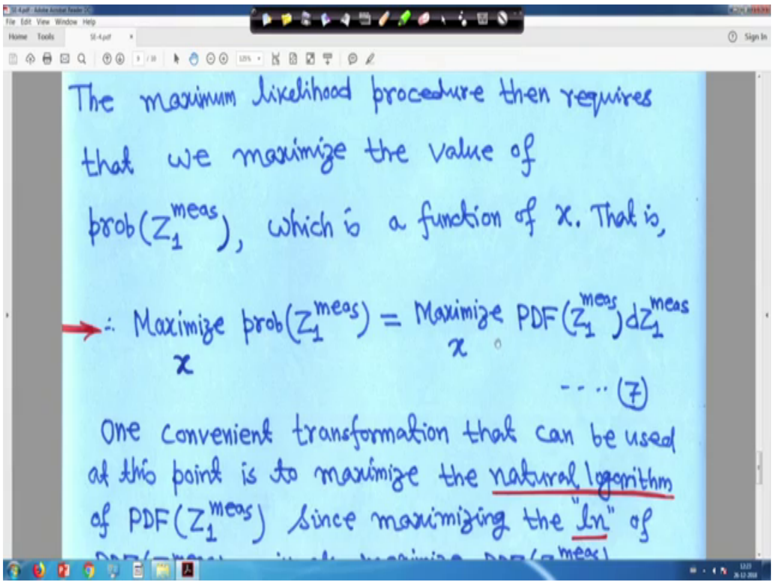

So, therefore, which is function of x; so the maximum likelihood procedure then requires that we maximize the value of probability of Z 1 measure, which is actually is a function of x. That is maximize probability of Z 1 measure and it is function of x is equal to maximize your PDF Z 1 measure into d Z 1 measure right. And that is your equation 7.

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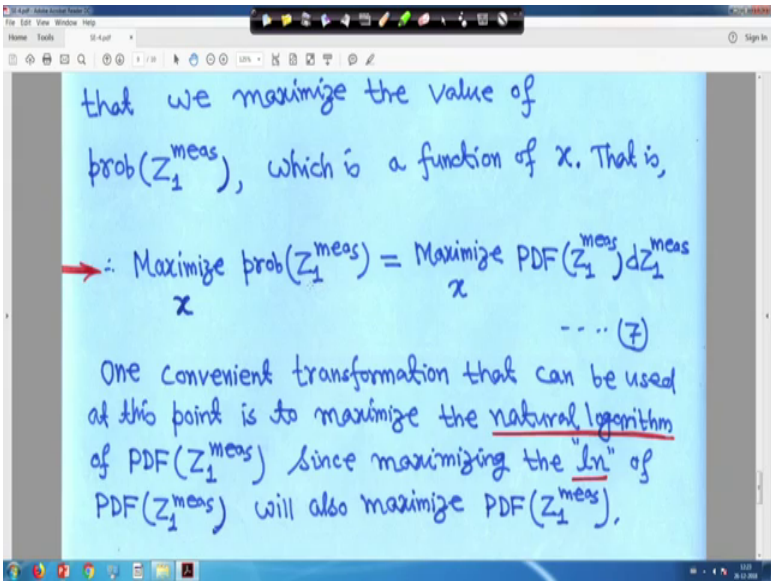

Now, how to maximize this? Things are very simple. One convenient transformation that can be use at this point is to maximize the natural logarithm of PDF Z 1 measure. Since maximize the your “ln” that is natural logarithm of PDF Z 1 measure will also maximize

PDF of your Z 1 your what you call measure right. So, when you are try to maximize PDF of Z 1 measure if you take is natural log logarithm it will be the same thing.

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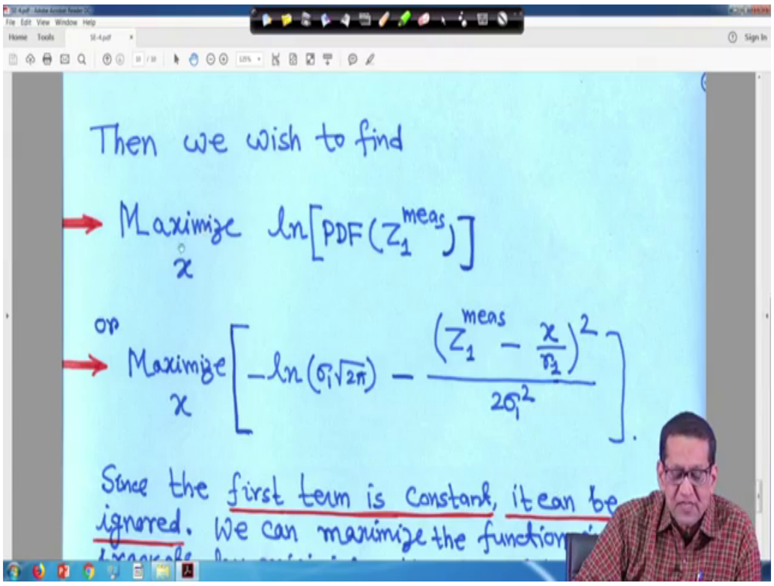

If it is so then we wish to maximize, wish to find out maximize the natural logarithm of PDF Z 1 measure right, for which we can find out the true value.

So, if you do so, if you do s o, you take natural log of this equation, of this equation. This equation you take the natural log right. Then what will happen? That it will be minus l n sigma 1 root 2 pi my then, minus you are what you call 2 in to l n Z 1 measure minus x y, x upon r 1 divided by your what you call divided by 2 sigma 1 square right. So, here also we have written that one, that you are maximize; that means, maximizing PDF function Z 1 is same thing that maximizing natural log of PDF of Z 1 measure. So, this will be maximize minus l n sigma 1 root 2 pi and this is minus your Z 1 measure minus x upon r 1 whole square upon 2 sigma 1 square, because we are taken the natural log right.

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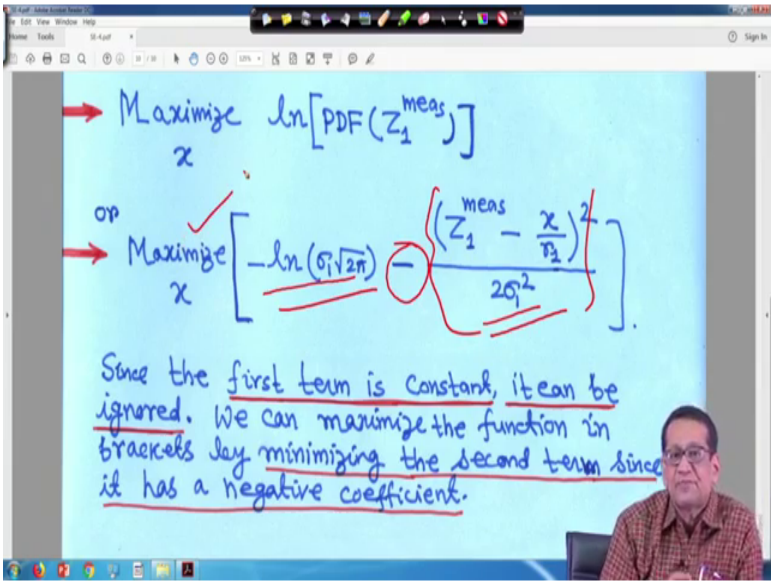

Now, since the first term is constant, because this is a constant; if first term is a constant right. It is a first time is a constant. So, generally maximize means we have to take d d x, but anyway of 0, d d x of that term is first term is your what you call, it is a constant and second term a minus sign is there before this before this term a minus sign is there, that mean maximize this term means minimization of this term actually we will maximize this term. Because first term is a constant right and a minus sign is there before that. Therefore, we can maximize the function in brackets by minimising the second term since it has a negative coefficient this is negative right.

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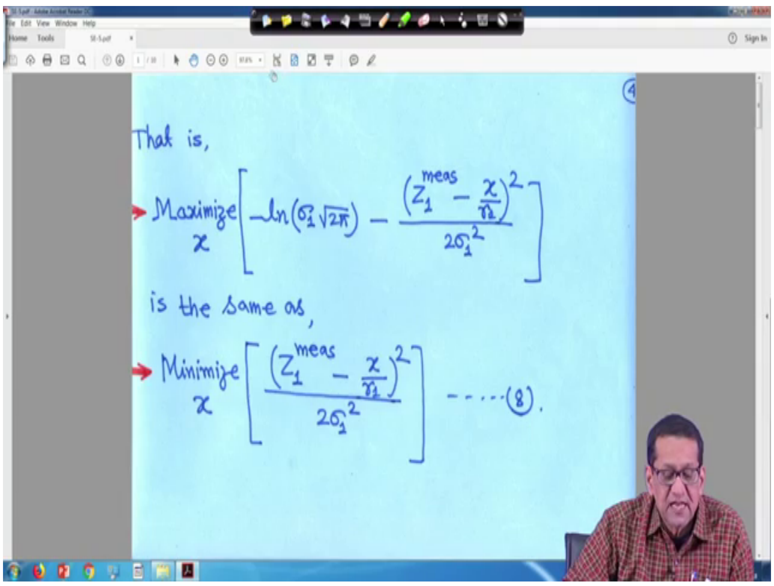

Therefore, just hold on. That is your maximize your just hold on let me increase its size. That is your maximize x minus l n sigma 1 root 2 pi minus Z 1 measure minus x upon r 1 your what you call whole square upon 2 sigma whole square. It is simply actually minimise x that is Z 1 measure minus x upon r 1 whole square divided by 2 sigma 1 square this is equation 8. So, minimise x means you will take d d x of this one is equal to 0 right.

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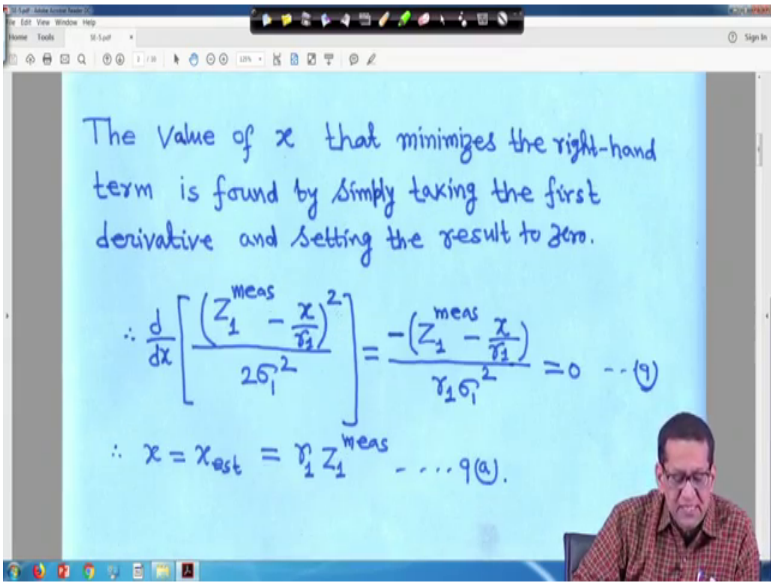

That means the value of x sorry that minimises the right hand term is found by simply taking the first derivative and settling the your setting the result to zero. That means, d d x of this one is equal to 2 will come, but 0, 0 both side are right hand side is 0. So, in generally it will come minus Z 1 measure minus x upon r 1 that is in bracket divided by r 1 sigma 1 square is equal to 0. This is equation 9. Solving this at for if you make that x is equal to x est right estimated value.

Therefore x is equal to x estimated value will simply r 1 into Z 1 measure, this equation I have marked as a 9 a. So, for a simple voltmeter or ammeter you are getting the sa same result like b is equal to i r. Because Z 1 measure is nothing but the i and this is the r 1 and this is x is nothing but the b right. So here, actually 1 me only 1 meter is there and you are just trying to find out the true value of the voltage, so it is quiet straight forward right. But which is simply it is b is equal to i r; so this is you simple one.

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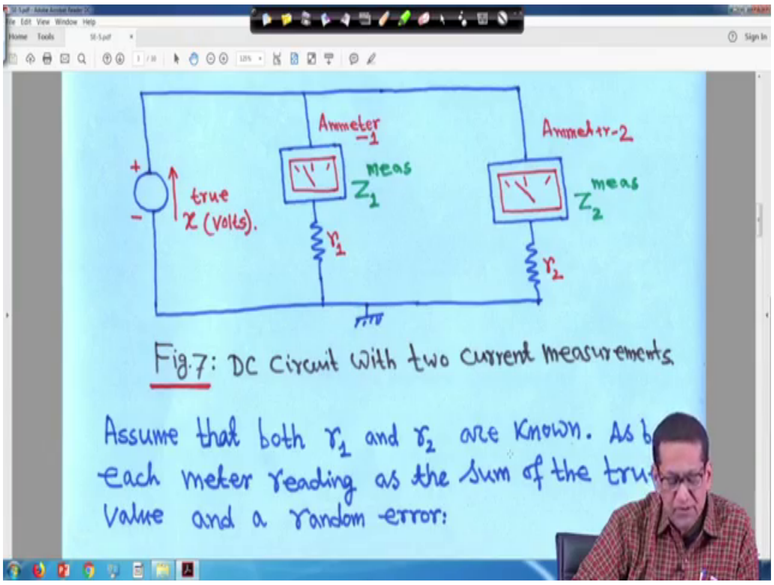

Now, suppose now I have now we have say two ammeters, pay in parallel and one resistance is r 1 here one resistance is r 2 here, current measure by ammeter-1 is Z 1 measure and current measured by your ammeter-2 it is your Z 2 measure right. And this is the true value of the voltage that is v. We have to estimate that true value, and this ammeter has standard deviation sigma 1 and this ammeter has standard deviation sigma 2 right.

And that that error for this ammeter it is zeta 1 and for this one it is zeta 2. So, it is simple parallel circuit two registers are connected in parallel right. And it an across that a voltage source is there that mean all three quantities are in parallel and these two ammeters measuring Z 1 measure and Z 2 measure the current flowing through the ammeter right.

So, it is a simple DC circuit, now assume that both r 1 and r 2 are known as before each meter reading as the sum of the true value and a random error right.

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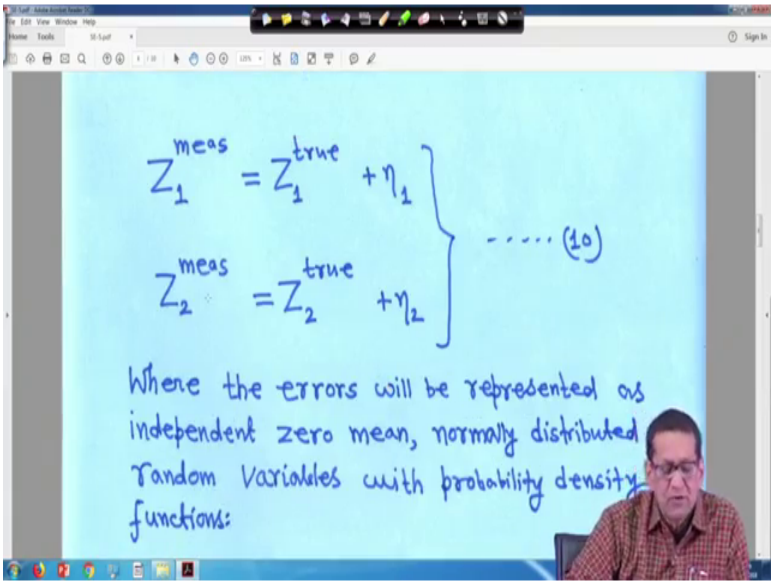

Now, we know from the same pa procedure Z 1 measure will be is equal to Z 1 true plus zeta 1 and Z 2 measure will be Z 2 true plus zeta 2. So, this two equation combine we mark as a equation 10 right where the errors will be represented as independent zero mean, normally distributed random variables with probability density function.

So, it is I mean there not a your what you call influencing one metre reading is not influencing the other metre reading, that completely independent right.

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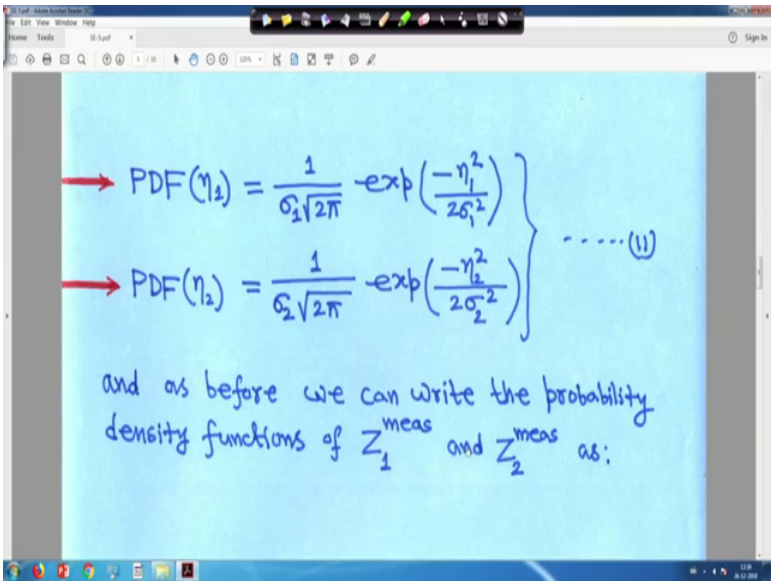

So, therefore, PDF zeta 1 we can write, 1 upon sigma 1 root over 2 pi x e to the power minus sigma 1 square upon 2 sigma 1 square zeta 1 square upon 2 sigma 1 square. So, zeta 1 is Z 1 measured minus your Z 1 true value.

Similarly PDF zeta 2 is equal to 1 upon sigma 2 root over 2 pi e to the power minus zeta 2 square upon 2 sigma 2 square. So, zeta 2 is nothing, but z measure to minus your what you call that your Z true value divided by your 2 what that will zeta 2 and divided by 2 sigma 2 square right. This is this equation combining equation 11 right and as before we can write the probability density function of Z 1 measure and Z 2 measure.

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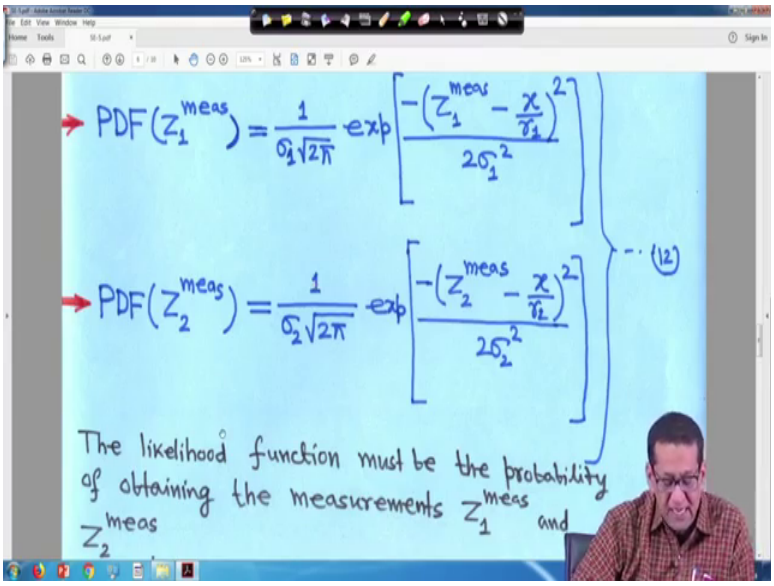

So, in this case, if we do so, so PDF Z, PDF Z 1 measure is equal to 1 upon sigma 1 root 2 pi e to the power minus of Z 1 measure minus x upon r 1 whole square divided by 2 sigma whole square, is same as before that your Z 1 measure minus Z true and Z 2 is equal to it is a parallel circuit. So, Z 1 true will be x upon r 1 similarity it is a parallel circuit that Z 2 will be x upon r 2 right.

That is why, this PDF Z 1 measure is equal to 1 upon sigma 1 root over 2 pi e to the power minus of Z 1 measure minus x upon r 1 whole square upon 2 sigma whole square right. Similarly PDF Z 2 measure will be 1 upon sigma 2 root over 2 pi e to the power minus in bracket Z 2 measure minus x upon r 2 whole square upon 2 sigma 2 square. This is equation this two equation is combined equation 12 right. Now we have to maximize the probability of Z 1 measure and Z 2 measure right.

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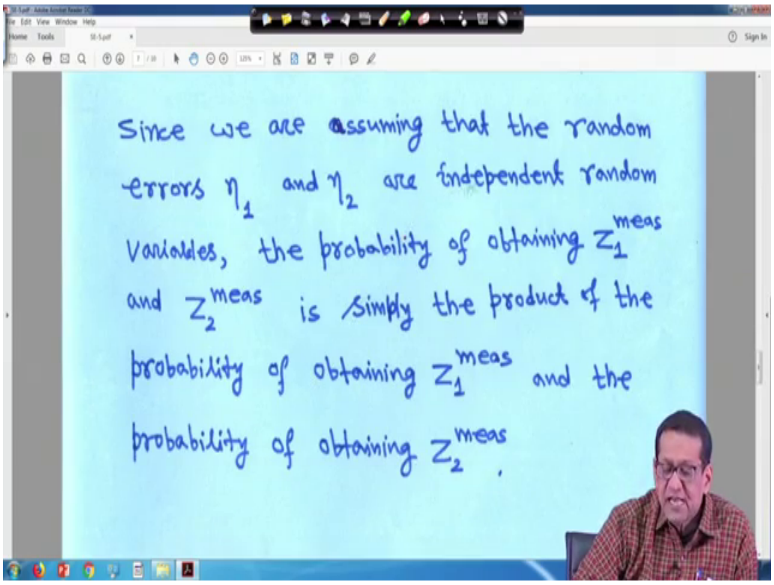

Now, the likelihood function must be the probability of obtaining the measurements Z 1 measure and Z 2 measure right. Therefore, since we are assuming that the random errors zeta 1 and zeta 2 are independent random variables the probability of obtaining Z 1 measure and Z 2 measure is simply the product of the probability obtaining probability of obtaining Z 1 measure and the probability of obtaining Z 2 measure right. That means, probability of Z 1 measure and Z 2 measure is equal to probability of Z 1 measure into probability of Z 2 measure.

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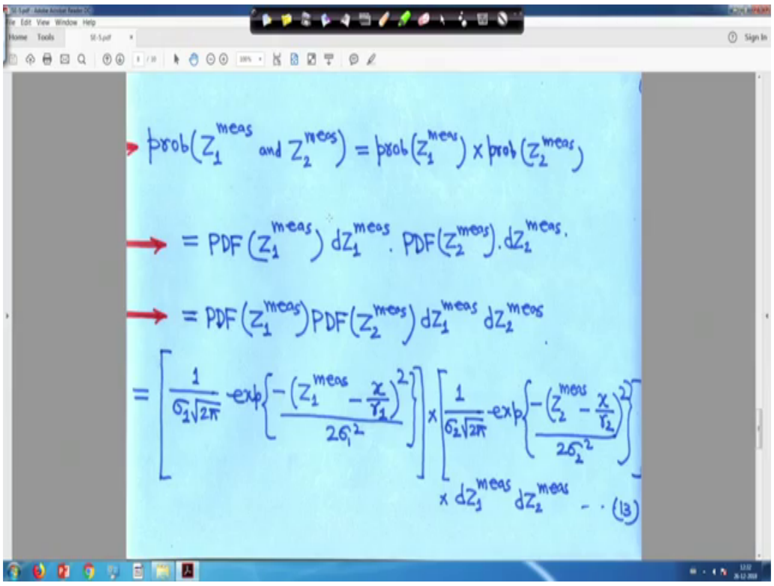

Because these two are independent, is equal to we can write that earlier we have seen probability of z measure is equal to in general PDF Z measure into d Z 1 measure. So, this one you can write PDF Z 1 measure d Z 1 measure into PDF Z 2 measure into d Z 2 measure right. Or we can right PDF Z 1 measure into PDF Z 2 measure then, d Z 1 measure into d Z 2 measure right. So, just let me reduce this volume little bit right. now this ok; sorry area.

So, just hold on. So, this one; that means, if you write if you write like this, then these two we can write that if you substitute the PDF Z 1 measure and PDF Z 2 measure expense your what you call expression. Then it will you will get 1 upon sigma root 2 pi e to the power minus Z 1 measure minus x upon r 1 whole square upon 2 sigma 1 square into 1 upon sigma 2 root 2 pi e to the power minus Z 2 measure minus x upon r 2 whole square upon 2 sigma 2 square into d Z 1 measure d Z 2 measure that is equation 13.

Actually here also what will do, that we got this expression right. So, we have to maximize the probability then ultimately what we have to do is we have to maximize the actually a your what you call PDF your Z 1 measure into PDF Z 2 measure. These two we have to maximize again we have to take the natural logarithm of this. I told you that maximizing the PDF function here is nothing but the maximizing of the logarithm your natural logarithm will take and that same function the l n function right.

Thank you very much we will be back again.