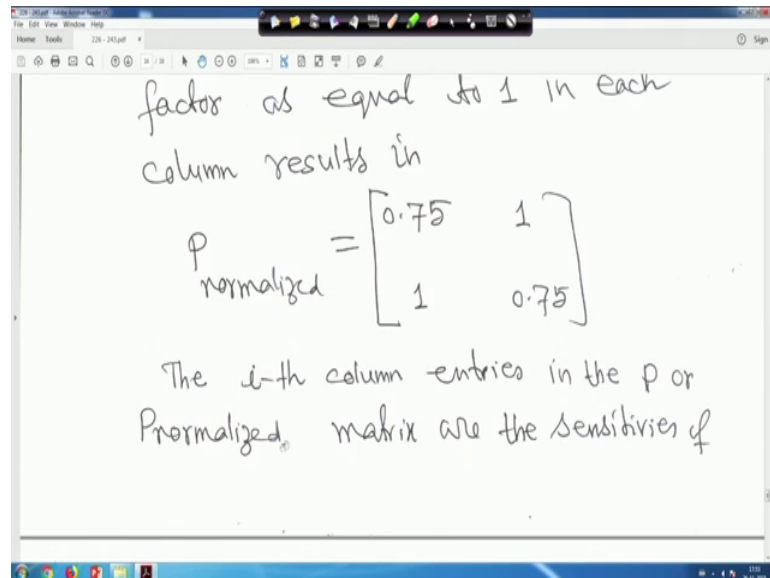


Power System Dynamics, Control and Monitoring
Prof. Debapriya Das
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 27

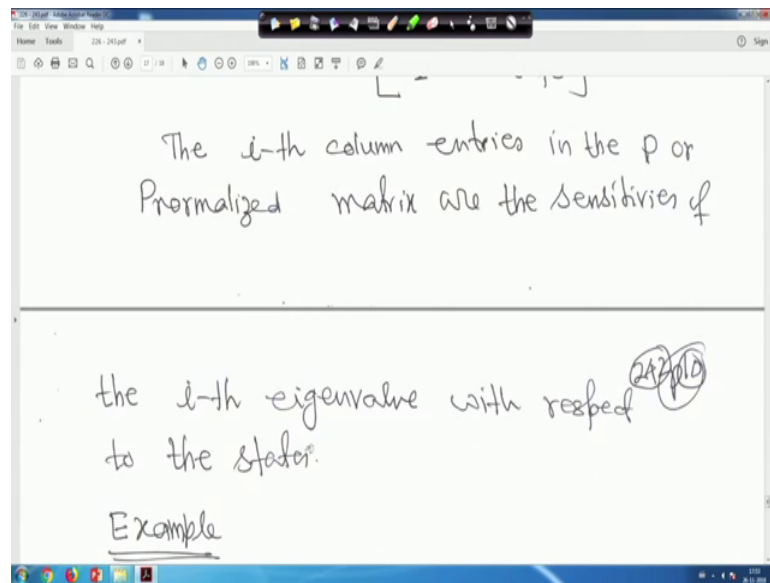
Power System stability, Eigen properties of the state matrix, Transient stability

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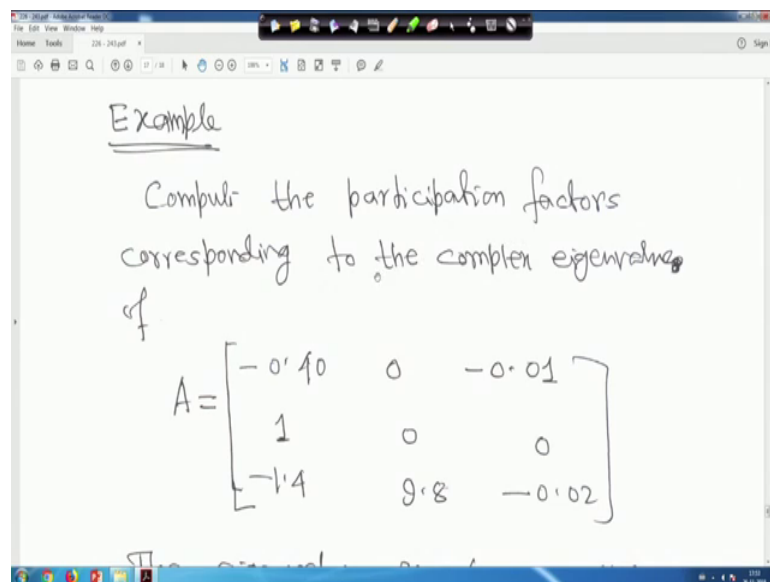
Ok. So, in the previous lecture, we have seen that your normalized one that is just a 2 into 2 matrix right, and this is your what you call that normalize your participation factor right. So, the i -th column entries in the P or $P_{\text{normalized}}$ matrix are the sensitivities of the i -th eigen value with respect to the states.

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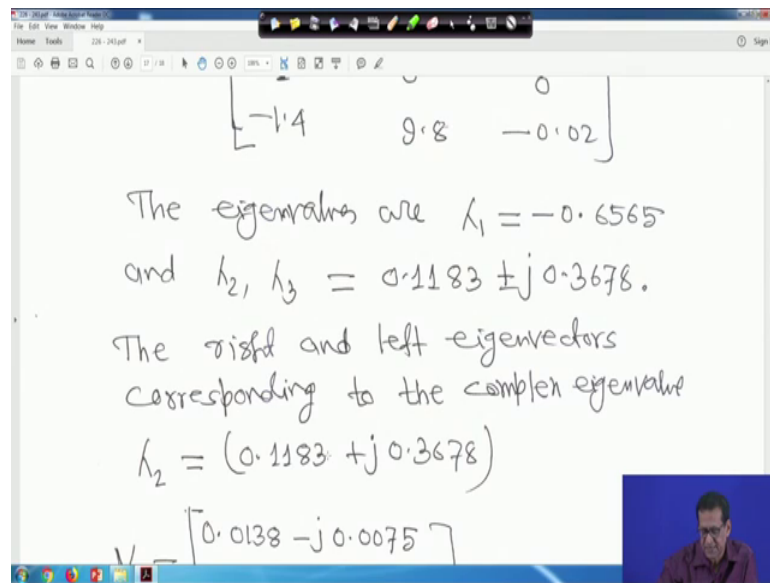
This is the previously we have seen some example, but I again took couple of example just to demonstrate certain things right.

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Now, next another example is taken, so compute the participation factors corresponding to the complex eigen values of right. A is equal to this is my a matrix it is a 3 into 3 matrix.

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$$\begin{bmatrix} -1.4 & 9.8 & -0.02 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

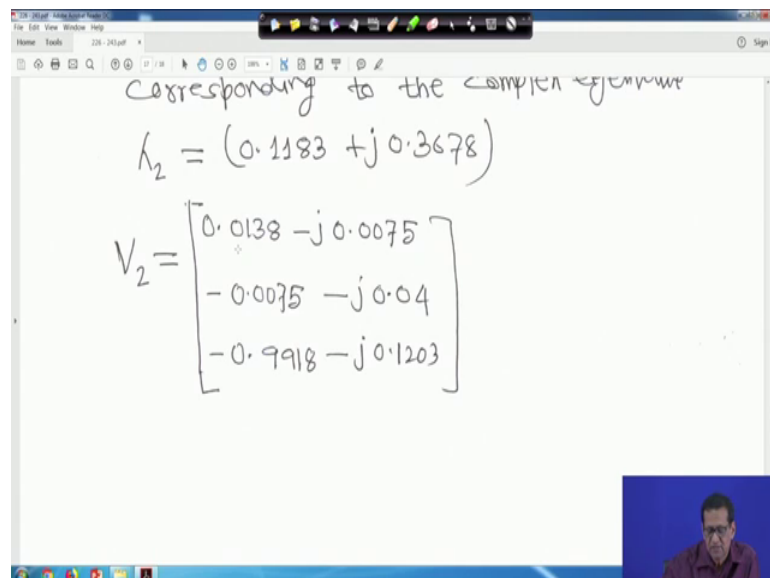
The eigenvalues are $\lambda_1 = -0.6565$
and $\lambda_2, \lambda_3 = 0.1183 \pm j0.3678$.

The right and left eigenvectors
corresponding to the complex eigenvalue
 $\lambda_2 = (0.1183 + j0.3678)$

$$V_2 = \begin{bmatrix} 0.0138 - j0.0075 \\ -0.0075 - j0.04 \\ -0.9918 - j0.1203 \end{bmatrix}$$

So, minus 0.40, this is 0, this is minus 0.01. This is 1 0 0, and this is minus 1.4, 9.8 and minus 0.02 right. When you find the eigen values of these matrix, so one eigen value will be real that is minus 0.6565. And another two are complex conjugate pairs that is lambda 2, and lambda 3 are 0.1183 plus minus j 0.3678. So, these three are the eigen values right. Here the right and left eigenvectors corresponding to the complex eigen values, you find out right.

(Refer Slide Time: 01:35)



The screenshot shows a whiteboard with the following content:

Corresponding to the complex eigenvalue
 $\lambda_2 = (0.1183 + j0.3678)$

$$V_2 = \begin{bmatrix} 0.0138 - j0.0075 \\ -0.0075 - j0.04 \\ -0.9918 - j0.1203 \end{bmatrix}$$

So, only corresponding to these you find out the right and left eigenvectors right. Earlier how to find out right and left eigenvector that everything had been shown. Similar, way you proceed right. Here I am just giving the answer. Similar way you proceed, you will get this is my right eigenvector.

So, it is your this two stands for this is eigen value lambda 2 that is 0.183 plus j 0.3678 that is why, it is written V 2 right eigenvector, this is for lambda 2. It is point 0.138 minus j 0.0075. This is minus 0.0075 minus j 0.04. And this is minus 0.9918 minus j 0.12 zee 3 right 03 right. So, this is right eigenvector.

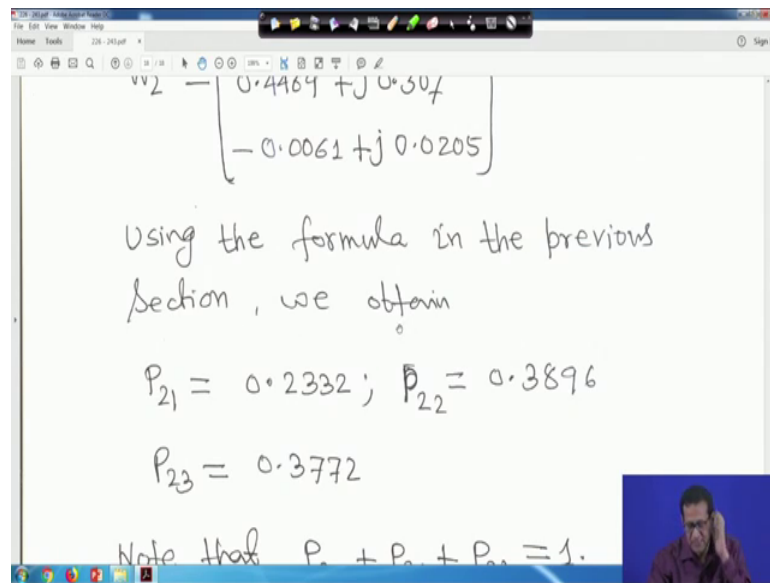
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$$w_2 = \begin{bmatrix} 0.838 - j0.0577 \\ 0.4469 + j0.307 \\ -0.0061 + j0.0205 \end{bmatrix}$$

Using the formula in the previous section, we obtain

Similarly, left eigenvector corresponding to same eigen value lambda 2, it is 0.835 minus j 0.0577. This is 0.4469 plus j 0.307, and this is minus 0.0061 plus j 0.0205, this you know how to how to find out right. So, just you find out at your basically your what you call you can find out that B matrix, phi matrix, and psi matrix here also B matrix and W matrix. Now, how you have obtained previously same way you follow right.

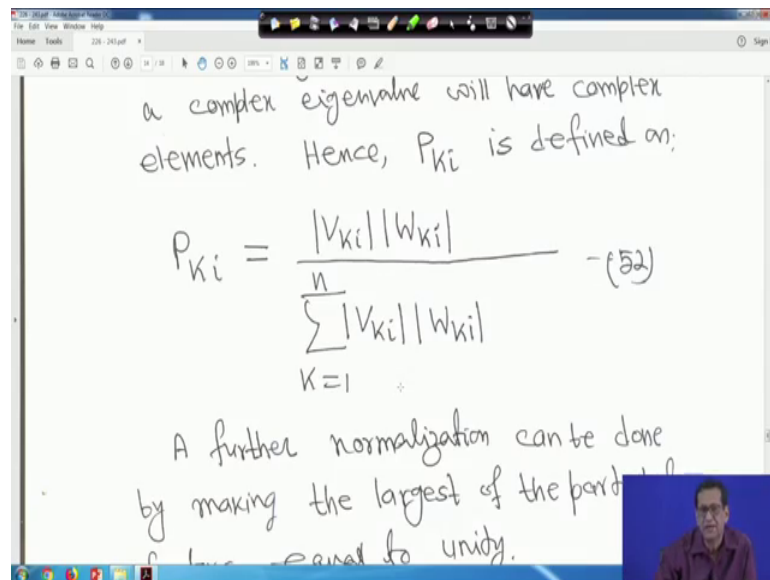
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A screenshot of a whiteboard with a blue border. At the top, a matrix is written:
$$V_2 = \begin{bmatrix} 0.4464 + j0.307 \\ -0.0061 + j0.0205 \end{bmatrix}$$
 Below the matrix, the text reads: "Using the formula in the previous section, we obtain". Then, three values are listed:
$$P_{21} = 0.2332; P_{22} = 0.3896$$
$$P_{23} = 0.3772$$
 At the bottom, it says: "Note that $P_{21} + P_{22} + P_{23} = 1$." A small video inset of a man is in the bottom right corner.

So, using the formula in the previous section, you can obtain that P using the previous formula means this one right, I will go back to this right this one.

(Refer Slide Time: 03:19)



A screenshot of a whiteboard with a blue border. The text reads: "a complex eigenvalue will have complex elements. Hence, P_{ki} is defined as:". Below this, equation (52) is written:
$$P_{ki} = \frac{|V_{ki}| |W_{ki}|}{\sum_{k=1}^n |V_{ki}| |W_{ki}|} \quad \text{--- (52)}$$
 At the bottom, it says: "A further normalization can be done by making the largest of the parts of P_{ki} equal to unity." A small video inset of a man is in the bottom right corner.

This formula that is equation-52 right. So, follow the same thing equation-52 right, and then you come back to this. So, if you follow this that equation-52, you will get the P_{21} 0.2332, P_{22} 0.3869, and P_{23} 0.3772 right.

(Refer Slide Time: 03:43)

$P_{23} = 0.3772$

Note that $P_{21} + P_{22} + P_{23} = 1$.

We can normalize with respect to P_{22} by making it unity, in which case

$P_{21}(\text{norm}) = 0.598$

$P_{22}(\text{norm}) = 1$

$P_{23}(\text{norm}) = 0.968$

So, note that P_{21} because it is normalized, so P_{21} plus P_{22} plus P_{23} is equal to 1. So, we can normalize with respect to P_{22} right making it unity in which it can be I mean highest is your what to call that your P_{22} P_{22} is 0.3896. So, divide everything by 0.3896.

(Refer Slide Time: 04:04)

to P_{22} by making it unity, in which case

$P_{21}(\text{norm}) = 0.598$

$P_{22}(\text{norm}) = 1$

$P_{23}(\text{norm}) = 0.968$

So, in that case P_{22} normalise will be 1. And if you divide by P_{22} values along P_{21} and P_{23} , it will be P_{21} normalize will be 0.598, and P_{23} normalize will be 0.968 right, this way one can find out. But, in this case one thing you have noticed that you are it is there

is no what to call as it is, what to call that is magnitude is considered right. And it is normalised and you will see that no angle is appearing here right only magnitude.

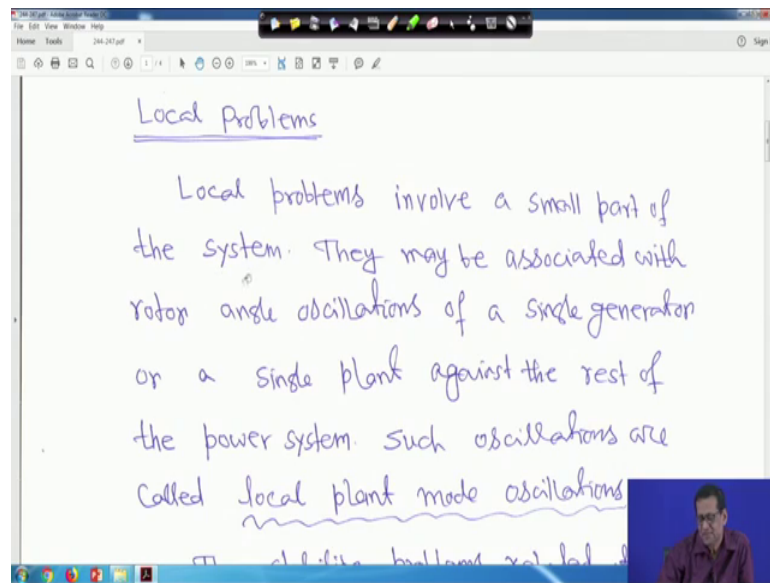
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The image shows a digital whiteboard interface with handwritten text in blue ink. At the top right, the number '244' is circled. The main title is 'Characteristics of Small-Signal Stability Problems', with 'Small-Signal' and 'Stability Problems' underlined. Below the title, the text reads: 'In large power systems, small-signal stability problems may be either local or global in nature.' Underneath this, the words 'Local Problems' are underlined. At the bottom, it says 'Local problems involve a small part of'. On the right side of the whiteboard, there is a vertical toolbar with various icons for editing and exporting. In the bottom right corner, there is a small video inset showing a person's face.

So, after this after this, we will go to some general thing. Now, whatever we have seen so far that regarding your dynamics, basically it dynamics control of what you call small signal stability analysis for synchronize machine. So, only we have restricted to that which is possible for your classroom class room exercise right, so that is why stimulus and other things.

Generally, I have tried my best to avoid those things right. So, now characteristic of small-signal stability problems Now, in large power systems, small-signal stability problems may be either local or global in nature some general ideas right.

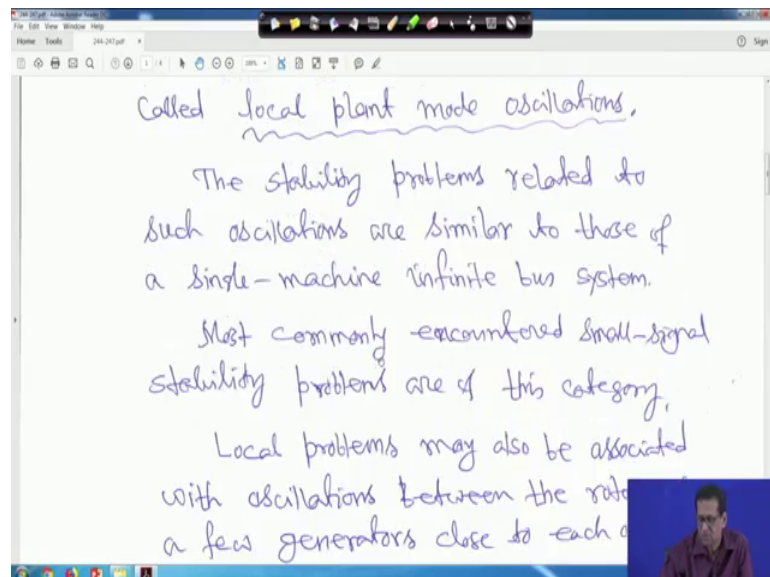
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The screenshot shows a digital whiteboard with the title "Local Problems". The text on the board reads: "Local problems involve a small part of the system. They may be associated with rotor angle oscillations of a single generator or a single plant against the rest of the power system. Such oscillations are called local plant mode oscillations." A small video inset of a man is visible in the bottom right corner of the whiteboard window.

Now, local problems. Now, local problem involves small part of the system right. They may be associated with rotor angle oscillations of a single generator or a single plant against the rest of the power system right, such oscillations are called local plant mode oscillations.

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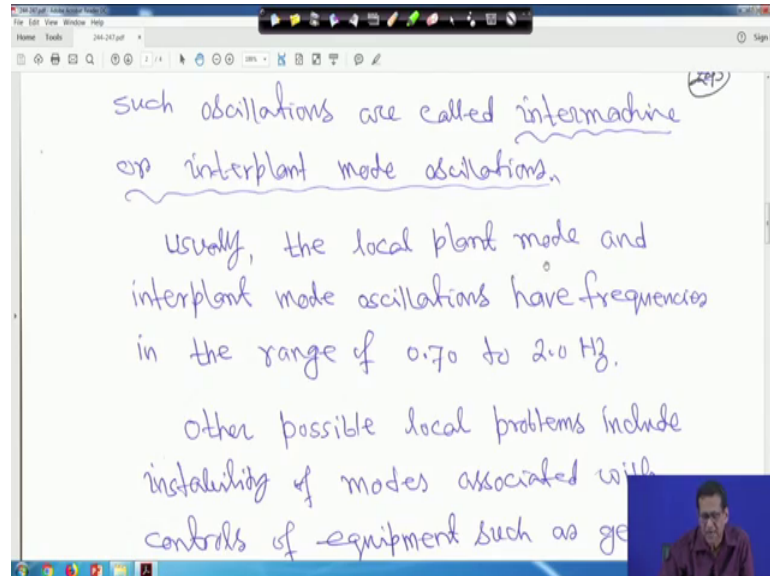


The screenshot shows a digital whiteboard with handwritten text. The text reads: "Called local plant mode oscillations. The stability problems related to such oscillations are similar to those of a single-machine infinite bus system. Most commonly encountered small-signal stability problems are of this category. Local problems may also be associated with oscillations between the rotor of a few generators close to each other." A small video inset of a man is visible in the bottom right corner of the whiteboard window.

So, certain terminology you should keep it in your mind, so that your this is called local plant mode oscillations right. Now, next is the stability problems related to such

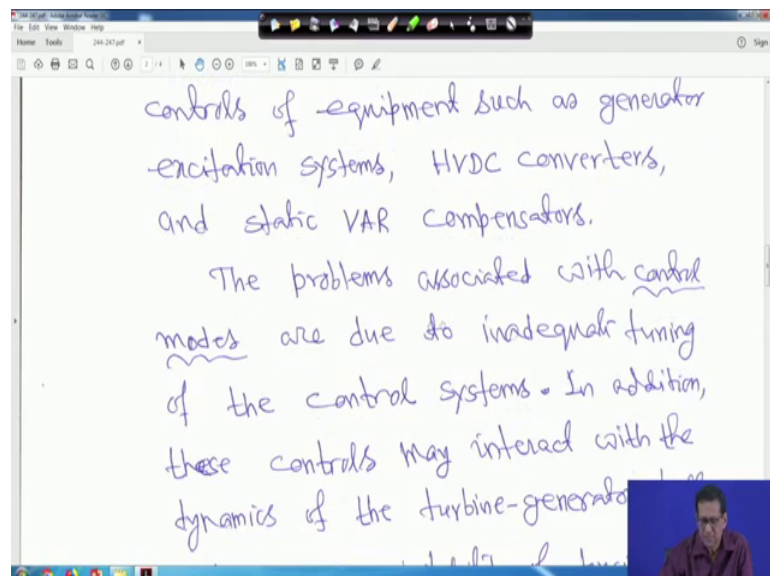
oscillations are similar to those of a single machine infinite bus system. Most commonly encountered your small-signal stability problems are of this category right.

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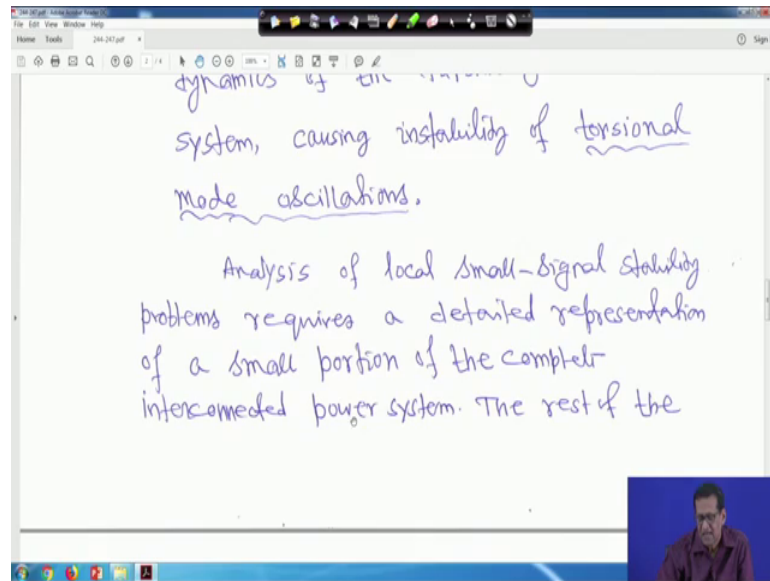
Local problems may also be associated with oscillations between the rotors of few generators close to each other; such oscillations are called inter-machine or inter plant mode oscillations. This terminology you should keep it in your mind, because this is underlined also right. Usually, the local plant mode and inter plant mode oscillations have frequencies in the range of 0.7 to 2.0 hertz right.

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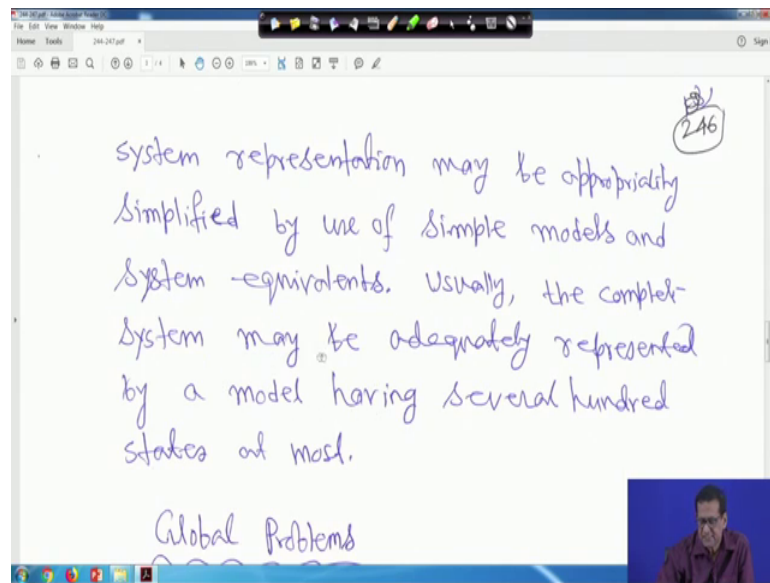
Other possible local problems include instability of modes associated with controls of equipment such as generator excitation systems, HVDC converters, and static VAR compensators right that is the SVC. The problem associated with control modes are due to inadequate tuning of the control system that the parameters are optimized or proper tuning is necessary that control parameters.

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In addition, right these controls may interact with the dynamics of the turbine-generator shaft system, causing instability of torsional mode oscillations. This one this torsional mode oscillations I thought I will cover little bit, but later I decided I should skip it. By chance if I get some time at the end, I will see that right, it will take few equations and how it is done. So, this is actual instability of torsional mode oscillations right. Analysis of local small-signal stability power systems require a detailed representation of a small portion of the complete interconnected power system right.

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system representation may be appropriately simplified by use of simple models and system equivalents. Usually, the complete system may be adequately represented by a model having several hundred states at most.

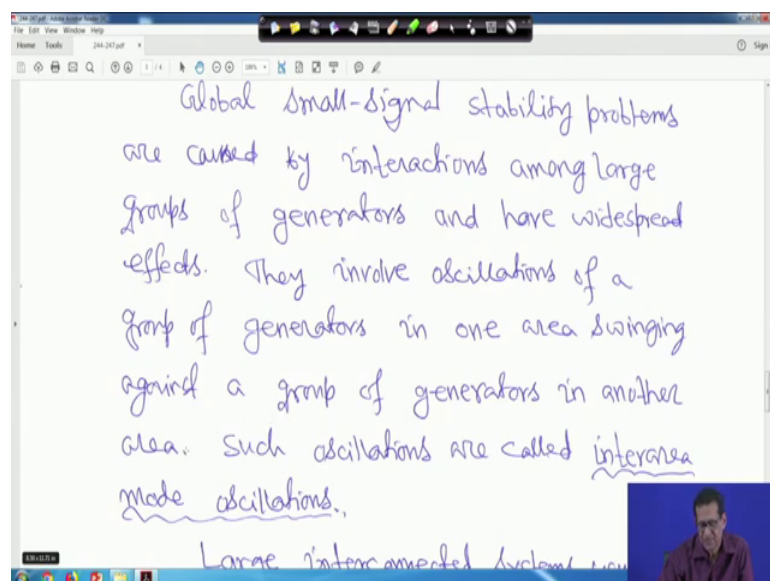
Global Problems

246

The screenshot shows a whiteboard with handwritten text in blue ink. The text discusses simplifying system representation using simple models and system equivalents. A small video inset in the bottom right corner shows a man speaking. The number '246' is circled in the top right corner. The title 'Global Problems' is written at the bottom.

The rest of the system representation may be appropriately simplified by use of simple modes and system equivalents, simple models and your system equivalents right. Usually, the complete system may be adequately represented by a model having several hundred states and states that is (Refer Time: 08:01) If you consider that interconnected system so many machines are there, so you will have several hundreds of states variable right.

(Refer Slide Time: 08:08)



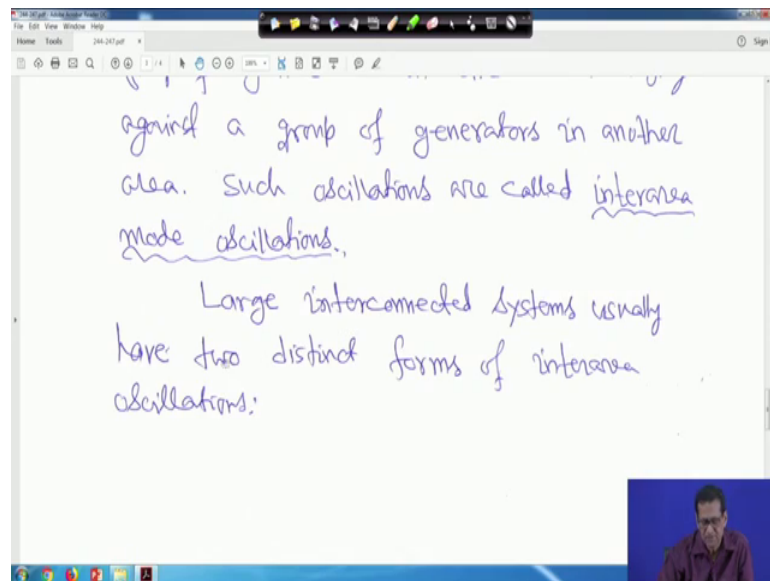
Global small-signal stability problems are caused by interactions among large groups of generators and have widespread effects. They involve oscillations of a group of generators in one area swinging against a group of generators in another area. Such oscillations are called interarea mode oscillations.

Large interconnected systems can

The screenshot shows a whiteboard with handwritten text in blue ink. The text describes global small-signal stability problems caused by interactions among large groups of generators. It mentions interarea mode oscillations. A small video inset in the bottom right corner shows a man speaking. The title 'Large interconnected systems can' is partially visible at the bottom.

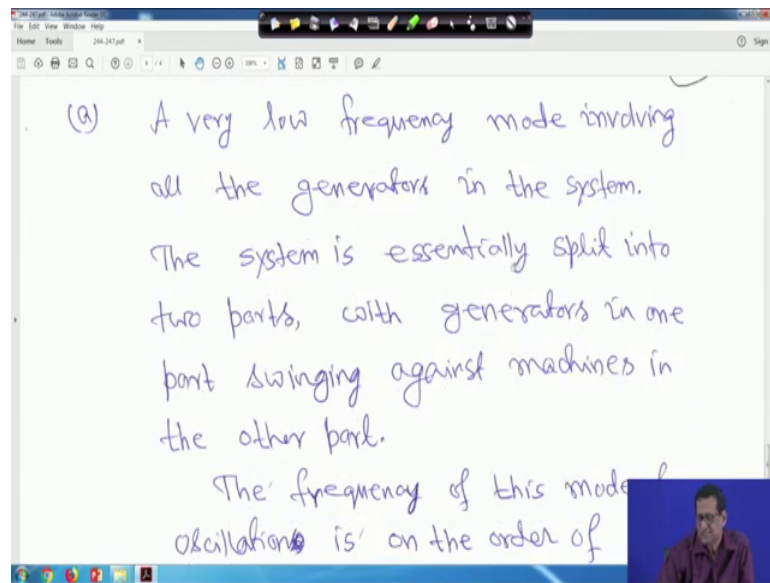
Another thing is that global problems. Global small-signal stability problems are caused by interaction among large groups of generators, and have wide spread effects right. They involve oscillations of a group of generators in one area swinging against a group of generators in another area right. This is actually this kind of stability problem that is I mean multi machine system right. So, it is beyond the scope of this core (Refer Time: 08:37) as per as classroom exercise is concerned right.

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Such oscillations are called inter area mode oscillations. Large interconnected systems usually have two distinct forms of Inter area oscillations.

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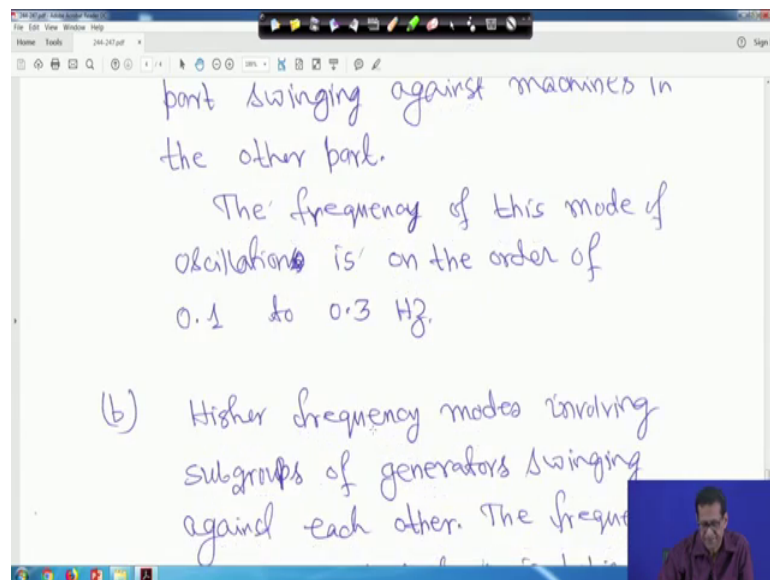


(a) A very low frequency mode involving all the generators in the system. The system is essentially split into two parts, with generators in one part swinging against machines in the other part. The frequency of this mode of oscillations is on the order of

The screenshot shows a presentation window with a whiteboard background. The text is handwritten in blue ink. A small video inset of a man is visible in the bottom right corner of the whiteboard area.

First one is a very very low frequency mode involving all the generators in the system right. So, system is essentially split into two parts with generators in one part swinging against machines in the other part right.

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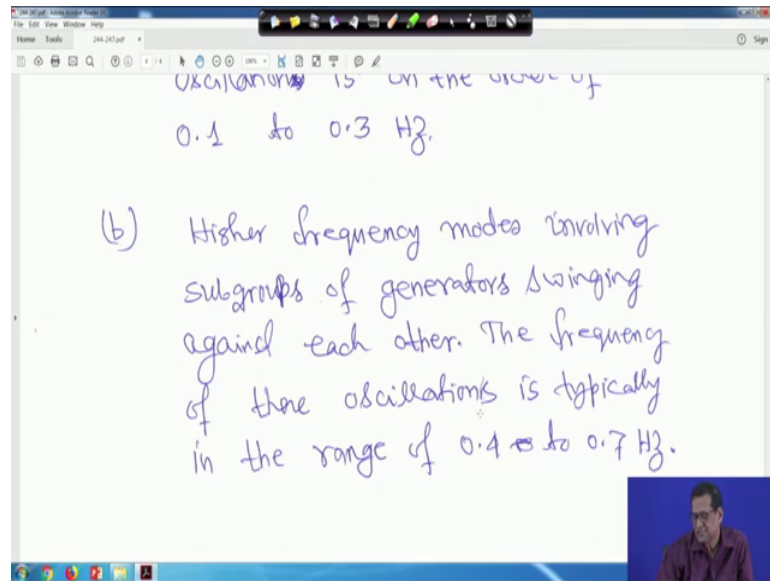
part swinging against machines in the other part. The frequency of this mode of oscillations is on the order of 0.1 to 0.3 Hz.

(b) Higher frequency modes involving subgroups of generators swinging against each other. The frequency

The screenshot shows a presentation window with a whiteboard background. The text is handwritten in blue ink. A small video inset of a man is visible in the bottom right corner of the whiteboard area.

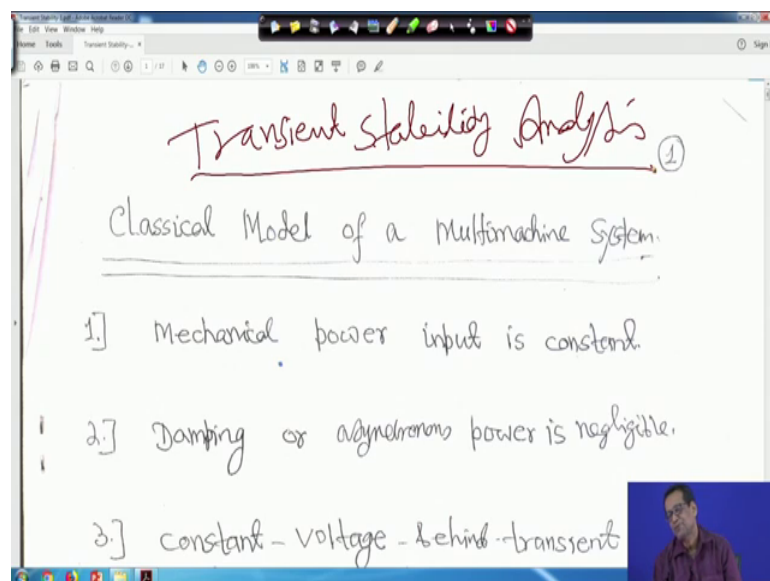
Then we group of generators right swinging together swinging against that your what to call group of machines in another area something like this right. The frequency of this mode of oscillation is on the order of 0.1 to 0.3 hertz right.

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Number second one is at higher frequency modes involving subgroups of generators swinging against each other. The frequency of the your frequency of this oscillations is typically in the range of 0.4 to 0.7 hertz right. So, with this that your synchronized machine part we call this it is almost over. Only thing is that if time permits, I will see of distributary exiting system of synchronized machine that is if time permits right. Now, next what we will do after this, we will move to the transient stability analysis particularly for multi machine system right.

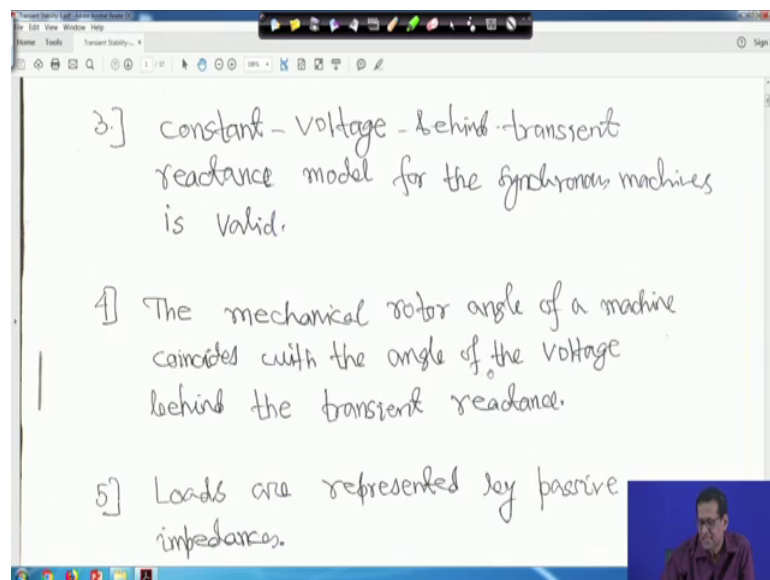
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Next one will be that your transient stability analysis for multi machine system. So, here I forgot to write that headline, it is transient stability analysis transient stability analysis right. So, basically your in power system analysis codes that single machine infinite was we have seen it.

So, here we will see that multi machine system, but let me tell you one thing that the what you call that your stimulus and other thing right. And those techniques we will skip, because in the classroom we cannot do that right. So, only I will think from the point of your classroom exercise. So, in the case of this transient stability analysis this first we have to see the classical model of a multi machine system.

(Refer Slide Time: 11:07)



So, number-1 mechanical power input we assume that it is a constant; we have to make some assumption. Number-2 damping or asynchronous power is negligible right, it is based on certain assumptions we will move. Number-3 constant voltage behind transient reactance model for the synchronous machines is valid right. Number-4 the mechanical rotor angle of a machine coincides with the angle of the voltage behind the transient reactance. Number-5 loads are represented by passive impedances or admittances right. Here admittances not written either passive impedances or admittances.

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concern with the swing of the system behind the transient reactance.

5] Loads are represented by passive impedances.

This model is useful for stability analysis but is limited to the study of transients for only the "first swing" or for periods on the order of one second.

This model is useful for stability analysis, but is limited to the study of transients for only the first swing or for periods on the order of one second right maximum.

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(2)

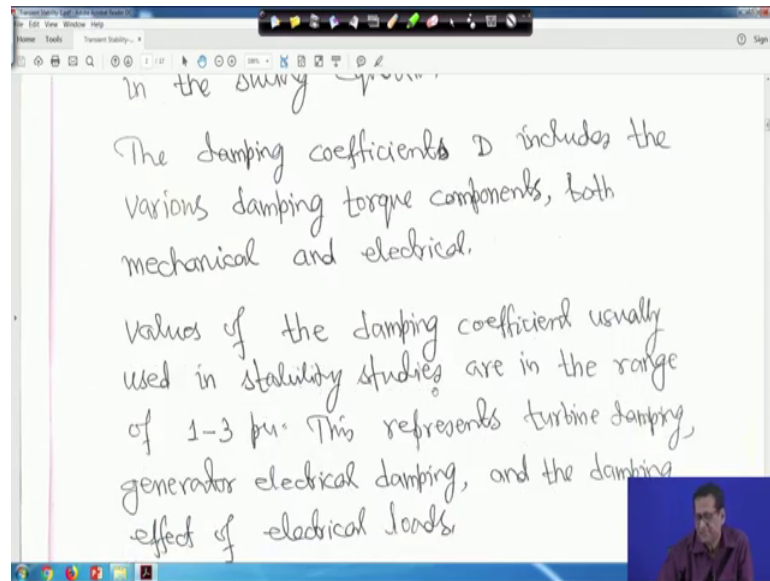
Assumption-2 is improved upon somewhat by assuming a linear damping characteristic.

A damping torque (or power) $D\omega$ is frequently added to the inertial torque (or power) in the swing equation.

The damping coefficient D includes the various damping torque components, both

Now assumption-2 is this one that damping or asynchronous power is negligible right. So, assumption-2 is improved upon somewhat by assuming a linear damping characteristic. A damping torque or power the $D\omega$ that we have seen previously right is frequently added to the inertial torque or power in the swing equation.

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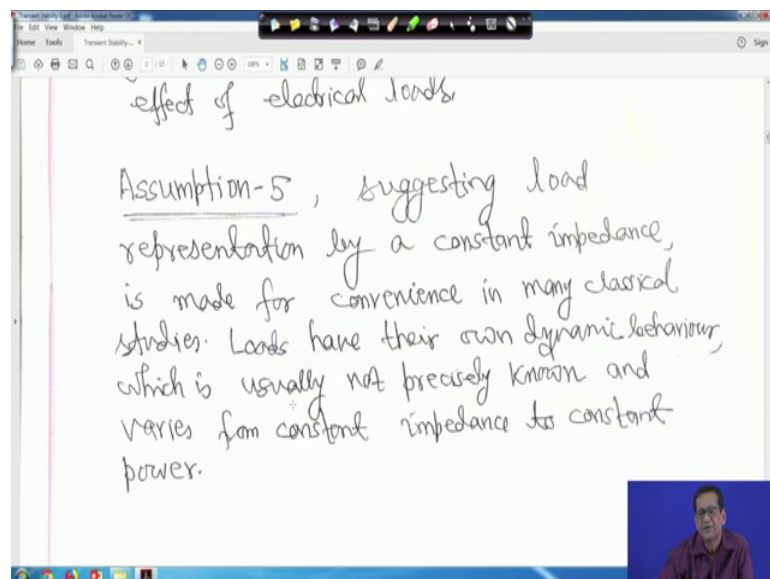
In the swing system,

The damping coefficient D includes the various damping torque components, both mechanical and electrical.

Values of the damping coefficient usually used in stability studies are in the range of 1-3 pu. This represents turbine damping, generator electrical damping, and the damping effect of electrical loads.

The damping coefficient D includes the various damping torque components, both mechanical and electrical. Values of the damping coefficient usually used in stability studies are in the range of 1 to 3 per unit. This represents turbine damping, generator electrical damping, and the damping effect of electrical loads. So, this is the range of the value of the damping coefficient in between 1 to 3 per unit.

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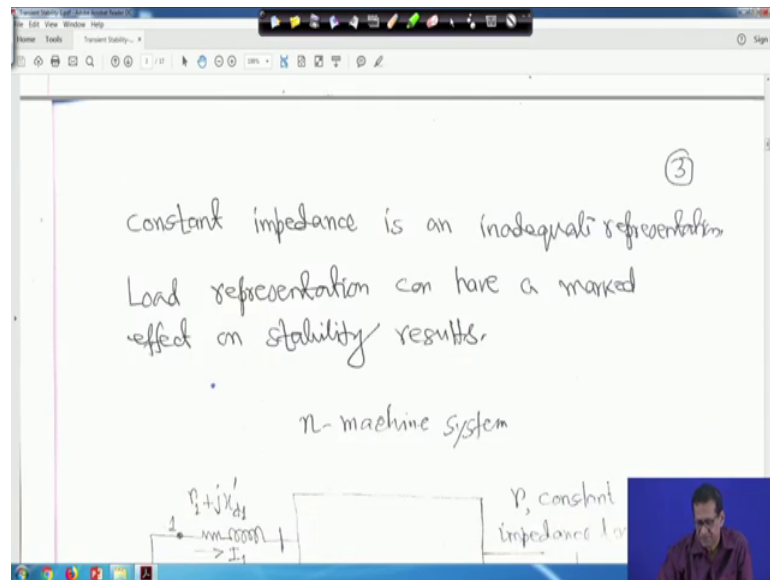
effect of electrical loads

Assumption-5, suggesting load representation by a constant impedance, is made for convenience in many classical studies. Loads have their own dynamic behaviour, which is usually not precisely known and varies from constant impedance to constant power.

Now, assumption-5 is this one that loads are represented by passive impedances or admittances. So, assumption-5 suggesting load representation by a

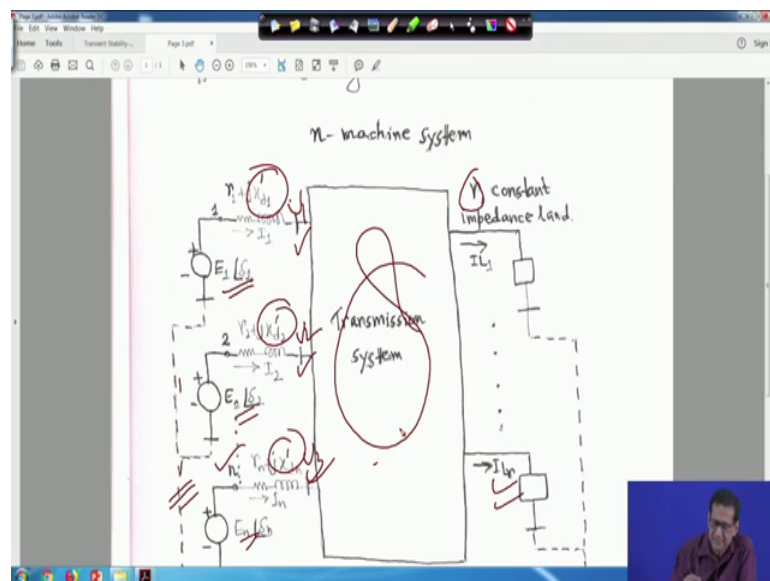
constant impedance is made for convenience in many classical studies right. Loads have their own dynamic behaviour, which is usually not precisely known and varies from constant impedance to constant power right. So, we will take here constant impedance right.

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So, now constant impedance is an inadequate representation. Load representation can have a marked effect on stability results right.

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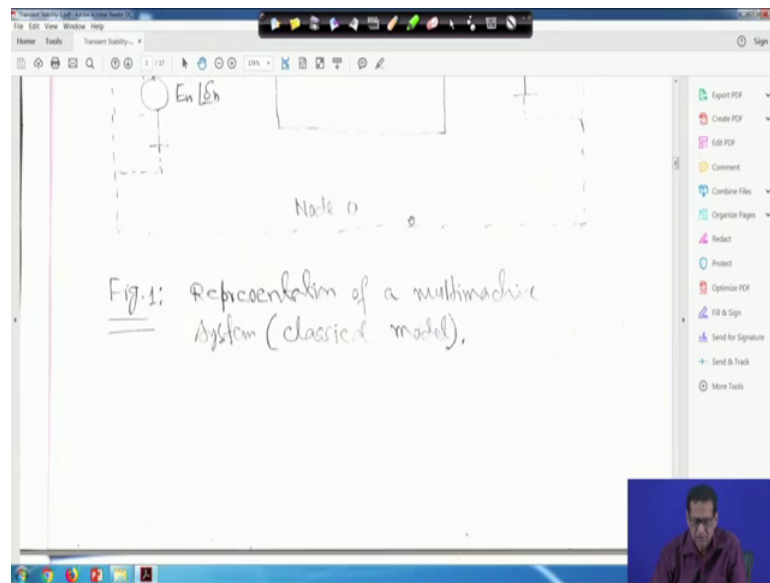
So, this figure it will be faint right, so I will be bring a good figure just hold on right. So, same page I mean just I have redrawn this. So, constant impedance is an inadequate representation load representation can have a marked your what to call an stability results.

Now, suppose the you are a machine system right. And this one this one this is my this is my transmission network right. So, generators are there suppose you have n number of generators right. So, voltage behind this transient reactance it is given by $E_1 \angle \delta_1$, then $E_2 \angle \delta_2$ upto n right upto n $n \angle \delta_n$. Then the impedance set is given $r_1 + jx_{d1}$, $r_2 + jx_{d2}$ upto $r_n + jx_{dn}$, but usually that r_1 r_2 upto r_n right this is negligible.

So, in our studies we will only consider this x_{d1} in general right x_{d1} , x_{d2} , x_{dn} . And this E_1 , E_2 your what you call this (Refer Time: 14:44) up to E_n . So, these are generator voltage behind your transient reactance that is $E_1 \angle \delta_1$ $E_2 \angle \delta_2$ like this. And these are the bus bars.

Say for example, say this is bus voltage V_1 V_2 like this or may be any bus numbers, these are the buses these are the buses right. So, this and this dash line this is basically a reference node or common node that will that I will move later. So, and this side that you have a load you have constant impedance loads, we have r number of buses right. So, it is say current here is I_{L1} , here next one will be I_{L2} , ultimate it will come to I_{Lr} right. And this is my transmission network, and this is my generators n number of generators, and there you have r number of your load buses right.

(Refer Slide Time: 15:38)



And this is node 0; actually you call as reference node right. And this actually figure-1 representation of a multi machine system classical model. So, this way we can represent your what you call that in machine system right. So, this is shown actually this figure was faint that is why I showed the other one.

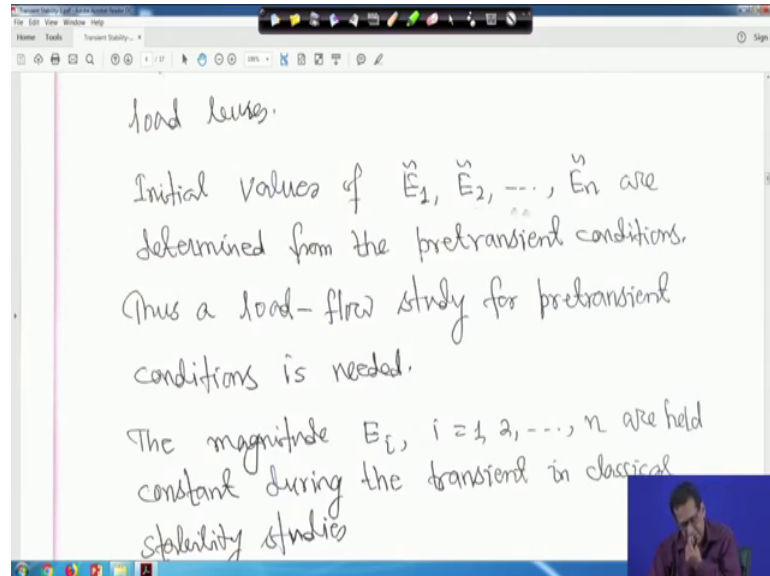
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Node '0' is the reference node (neutral).
Nodes 1, 2, ..., n are the internal machine buses, or the buses to which the voltages behind transient reactances are applied.
Passive impedances connect the various nodes and connect the nodes to the reference or load buses.
Initial values of $\tilde{E}_1, \tilde{E}_2, \dots, \tilde{E}_n$ are determined from the pretransient conditions.

Now, node 0 actually is the reference node that is neutral right. So, nodes 1, 2, n I told you are the internal machine buses right or the buses to which the voltage behind

transient reactances are applied right. So, passive impedances connect the various nodes and connect the nodes to the reference at load buses right.

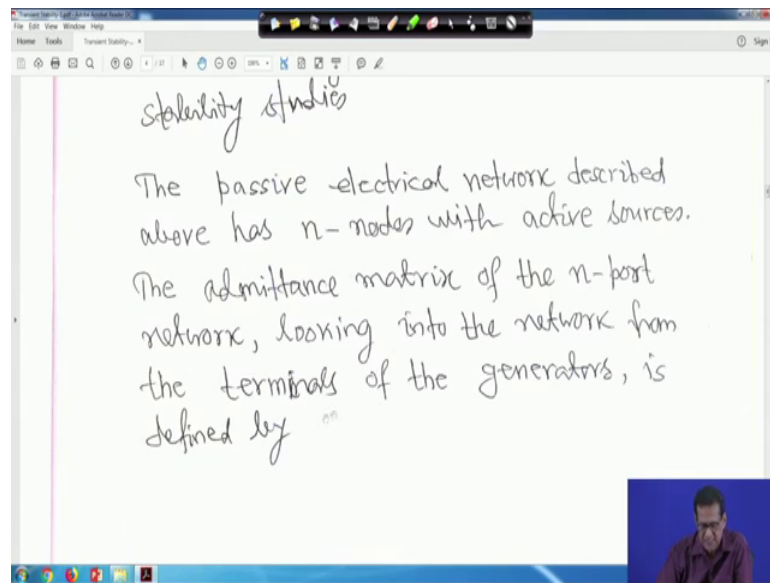
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So, initial values of \tilde{E}_1, \tilde{E}_2 up to \tilde{E}_n are determined from the pretransient conditions right. Thus a load-flow study for pretransient condition is needed right. So, this figure is actually better figure I shown, so I have shown you, this voltage $E_1 \delta_1, E_2 \delta_2$ right. So, once load-flow is known, then E_1, E_2, E_n and $\delta_1, \delta_2, \dots, \delta_n$ all can be computed that we will see later right.

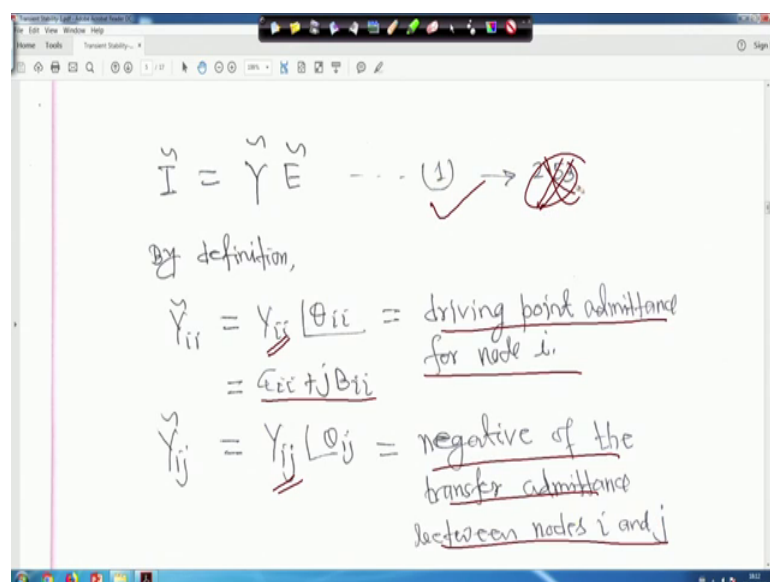
And the current flowing here I_1, I_2 up to I_n right and these are the internal your what you call this is marked actually this is internal 1, this is 2 internal machine buses up to this n right. So, let me clear it. So, initial value of \tilde{E}_1, \tilde{E}_2 up to \tilde{E}_n these are quantities that is why tilde are used are determined from the pre transient, what you call conditions thus a load-flow study for pre transient condition is needed.

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Now, the magnitude of E_i for i is equal to 1 to n are held constant during the transient in classical stability studies. Once that E_1, E_2 upto E_n 's are computed, then it will remain constant right during transient this is an assumption. The passive electrical network described above has n -nodes with active sources right. The admittance matrix of the n port network, looking into the network from the terminals of the generator is defined by.

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I mean we know that \tilde{I} is equal to $\tilde{Y} \tilde{E}$. And here it is \tilde{E} , so generate terminal voltage. So, \tilde{I} is equal to $\tilde{Y} \tilde{E}$ right. This is not for you this is for my own reference

right. So, by definition that Y_{ii} from a load flows studies you know it is Y_{ii} angled θ_{ii} . Y_{ii} means this is the magnitude right, and θ_{ii} is the angle and this is called driving point admittance for node i . This already you know from the load-flow studies is equal to we can write G_{ii} plus $j B_{ii}$.

And Y_{ij} is equal to Y_{ij} angled θ_{ij} . So, Y_{ij} is the magnitude and θ_{ij} is its angle, it is negative of the transfer admittance between nodes i and j , this also you know from your load flows studies right. So, this is the equation-1, and this is nothing this is for my own reference.

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The power into the network at node i , which is the electrical power output of machine i , is given by

$$P_{ei} = \text{Real} \left(\sum_{j=1}^n E_i E_j Y_{ij} \cos(\theta_{ij} - \delta_i + \delta_j) \right)$$

$$P_{ei} = \underline{E_i^2 G_{ii}} + \sum_{\substack{j=1 \\ j \neq i}}^n E_i E_j Y_{ij} \cos(\theta_{ij} - \delta_i + \delta_j),$$

$j = 1, 2, 3, \dots, n$

So, next one is that so Y_{ij} also we can write that G_{ij} plus $j B_{ij}$, this is equation-2 right. Now, the power into the network at node i , which is the electrical power output of the machine i is given by that P_{ei} we can write real of E_i tilde into I conjugate tilde, this is the real part of this one.

If you make real from your load-flows studies, you know how to do this right. So, we can write p_{ei} is equal to E_i square G_{ii} plus sigma j is equal to 1 to n , j not is equal to i $E_i E_j Y_{ij} \cos(\theta_{ij} - \delta_i + \delta_j)$ that that from load-flow studies you know when you have taken i -th term out from this 1 one, it is E_i square G_{ii} that's i why here it is written j not is equal to here. Load-flow studies also you know p_i is equal to j is equal to 1 to n your $b_{ij} y_{ij} \cos(\theta_{ij} - \delta_i + \delta_j)$ this way right now.

So, this equation when you write like this say j is equal to 1 to n right, then you can write E i, then you can write E j, then you can write Y i j, then you can write cosine theta i j minus delta i plus delta j. This is you know valued from load flow studies.

Now, from this expression when you are taking your i-th term out, now from this expression when you are taking j your i-th term out right, then when I mean when j is equal to say 1, 2 this way 2, 3 this way when I will come it will come to n right.

So, when i-th term is out from this sigma, so it will be when j is equal to i, so it will be basically your what will happen that it will be your E i into E i e i square it is Y i i. And this i is equal to they mean this term will vanish, it will be theta Y i i cos theta i i. So, basically E i square G i i that is why it is written like this right.

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The power into the network at node i , which is the electrical power output of machine i , is given by

$$P_i = \text{Real} (E_i I_i^*)$$

$$P_{ei} = E_i^2 G_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^n E_i E_j Y_{ij} \cos(\theta_{ij} - \delta_i + \delta_j),$$

for $i = 1, 2, \dots, n$

So, that is why the P i is equal to E square G i i j is equal to 1 to n, j not is equal to i right your what you call E i E j Y is equal to X cosine theta i j minus delta i plus delta j for i is equal to 1 to n right.

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machine i , is given by

$$P_i = \text{Real}(\tilde{E}_i \tilde{I}_i^*)$$

$$P_{ei} = E_i^2 G_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^n E_i E_j Y_{ij} \cos(\theta_{ij} - \delta_i + \delta_j), \quad i = 1, 2, \dots, n$$

$$\cos\{\theta_{ij} - (\delta_i - \delta_j)\} = \cos \delta_{ij} \cos(\delta_i - \delta_j) + \sin \delta_{ij} \sin(\delta_i - \delta_j)$$

Next is that P_i mean this term this term you can I mean before going to that right this term this cosine your θ_{ij} minus δ_i minus δ_j right, this one you can write now. Cosine θ_{ij} $\cos a \cos b$ plus $\sin a \sin b$, so $\cos \delta_i$ minus δ_j right plus your $\sin \theta_{ij}$, then $\sin \delta_i$ minus δ_j right this term, so that is what we have done here.

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$$P_{ei} = E_i^2 G_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^n E_i E_j [B_{ij} \sin(\delta_i - \delta_j) + G_{ij} \cos(\delta_i - \delta_j)]$$

$i = 1, 2, \dots, n$ -- (3) \rightarrow 2.55

The equations of motion are then given

This one we can write $E_i E_j B_{ij} \sin \delta_i$ minus δ_j plus $G_{ij} \cos \delta_i$ minus δ_j right.

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The image shows a whiteboard with handwritten mathematical equations. The first equation is $P_{ei} = \text{Real}(\vec{E}_i \vec{I}_i^*)$. The second equation is $P_{ei} = E_i^2 G_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^n E_i E_j Y_{ij} \cos(\theta_{ij} - \delta_i + \delta_j)$, with a note $i = 1, 2, \dots, n$. Below this, a term Y_{ij} is expanded into a bracketed expression: $Y_{ij} \{ G_{ij} \cos(\delta_i - \delta_j) + B_{ij} \sin(\delta_i - \delta_j) \}$. This is then simplified to $G_{ij} \cos(\delta_i - \delta_j) + B_{ij} \sin(\delta_i - \delta_j)$, which is further rearranged to $B_{ij} \sin(\delta_i - \delta_j) + G_{ij} \cos(\delta_i - \delta_j)$.

I mean if you when you are expressing this one, when you are expressing this one, so this is say Y_{ij} and this term just I wrote now cosine theta i, j cosine delta i minus delta j plus sin theta i, j , then sin delta i minus delta j right, so that $Y_{ij} \cos \theta_{ij}$, it will be your G_{ij} into cos delta i minus delta j .

And this one Y_{ij} into sin theta i, j , so $B_{ij} \sin \delta_i - \delta_j$ or this time we have written plus. So, $B_{ij} \sin \delta_i - \delta_j$ plus $G_{ij} \cos \delta_i - \delta_j$ right that is this term we have written there, so that is why this we are writing that $E_i^2 G_{ii} + \sum_{j=1}^n E_i E_j$, it is $B_{ij} \sin \delta_i - \delta_j$ plus $G_{ij} \cos \delta_i - \delta_j$ showed you that this is the thing. And this is also for i is equal to 1 to n , this is equation-3 right.

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The equations of motion are then given by

$$\frac{2H_i}{\omega_R} \frac{d\omega_i}{dt} + D_i \omega_i = P_{mi} - P_{ei}$$

$$\therefore \frac{2H_i}{\omega_R} \frac{d\omega_i}{dt} + D_i \omega_i = P_{mi} - \left[E_i^2 G_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^n E_i E_j Y_{ij} \cos(\theta_j - \delta_i + \delta_j) \right]$$

$$\frac{d\delta_i}{dt} = \omega_i - \omega_R, \quad i=1, 2, \dots, n$$

Now, the equation of motion are then given by this swing equations, so that should differ for synchronisation analysis this same thing we can write $2 H_i$ upon ω_r right over some other ω_r over the reference speed right into $D \omega_i$ dt plus D_i the damping term damping co efficient ω_i is equal to in general P_m minus P_i , this is for i -th machine this is for i -th machine right. And that means, that $2 H_i$ upon ω_r into $D \omega_i$ upon dt plus $D_i \omega_i$ is equal to P_m minus P_i . And this P_i minus P_e this is the expression for P_e you substitute here you substitute here, that is minus in bracket $E_i^2 G_{ii}$ plus j is equal to 1 to n , j not is equal to i $E_i E_j Y_{ij} \cos(\theta_j - \delta_i + \delta_j)$ right this is your equation for ω_i .

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$$\frac{2m_i}{\omega_R} \frac{d\omega_i}{dt} + D_i \omega_i = P_{mi} - P_{ei}$$

$$\therefore \left\{ \frac{2m_i}{\omega_R} \frac{d\omega_i}{dt} + D_i \omega_i = P_{mi} - \left[E_i^2 G_{ii} + \sum_{\substack{j=1 \\ j \neq i}}^n E_i E_j Y_{ij} \cos(\theta_{ij} - \delta_i + \delta_j) \right] \right\}$$

$$\textcircled{A} \frac{d\delta_i}{dt} = \omega_i - \omega_R, \quad i=1, 2, \dots, n \quad \text{--- (4) \rightarrow 2.56}$$

It should be noted that prior to the disturbance
 $(t=0^-) P_{mi0} = P_{ei0}$

Another thing is $d\delta_i$ upon dt delta upon dt , this also we have seen that ω_i minus ω_R is the reference speed and ω_i is the speed for the i -th motion ω_i minus ω_R . This is for i is equal to 1 to n ; actually, these two equations are equation-4 right, this is nothing. These two equations actually equation-4 together right I written here is 4 is a basically these two equation together, it is equation-4 right.

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$$\frac{d\delta_i}{dt} = \omega_i - \omega_R, \quad i=1, 2, \dots, n \quad \text{--- (4) \rightarrow 2.56}$$

It should be noted that prior to the disturbance
 $(t=0^-) P_{mi0} = P_{ei0}$

$$P_{mi0} = E_i^2 G_{ii0} + \sum_{\substack{j=1 \\ j \neq i}}^n E_i E_j Y_{ij0} \cos(\theta_{ij0} - \delta_{i0} + \delta_{j0}) \quad \text{--- (5) \rightarrow 2.57}$$

So, it should be noted that prior to the disturbance at t is equal to at 0 minus I mean just before disturbance P_{mi0} actually equal to p_i . Actually, it had balanced right, P_{mi0}

mechanical power input is equal to electrical power output P_i^0 so right, so that means just before disturbance P_m^0 is equal to then we can write same equation, but using one suffix 0 that $E_i^2 G_{ii}^0 + \sum_{j=1}^n, j \neq i E_i E_j Y_{ij}^0 \cos(\theta_{ij}^0 - \delta_i^0 + \delta_j^0)$, this is equation-5 right.

So, this is nothing this is actually before disturbance right, before disturbance these thing we will get it because I what you call load-flow studies are carried out. Later we will see on, I will take the example that from that load-flow studies your voltages. The internal machine bus voltages E_1, E_2, E_3 up to E_n are all will be computed all the angles will be known. So, at that time you can easily find out what is your P_{ij}^0 , and P_i^0 is equal to nothing but the that your P_m^0 right.

So, thank you, we will be back again.