

Power System Dynamics, Control and Monitoring
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Lecture – 26
Power System stability, Eigen properties of the state matrix (Contd.)

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$s = 0.017$

$= -0.204$

right eigenvector matrix is

$$\begin{bmatrix} \underbrace{(0.0169 - j0.0014)}_{\phi_{11}} & \underbrace{(0.0169 + j0.0014)}_{\phi_{12}} & \underbrace{0.204}_{\phi_{13}} \\ \underbrace{(-0.0988 - j0.9945)}_{\phi_{21}} & \underbrace{(-0.0988 + j0.9945)}_{\phi_{22}} & \underbrace{-0.249}_{\phi_{23}} \\ \underbrace{(0.0301 - j0.0015)}_{\phi_{31}} & \underbrace{(0.0301 + j0.0015)}_{\phi_{32}} & \underbrace{0.6625}_{\phi_{33}} \end{bmatrix}$$

So, with this therefore, the right eigen vector for further clarification actually this is your phi 11, this is phi 12 and this is phi 13. So, if you look into this phi 1 and phi 22, they are basically 12 rather, they are basically complex conjugate because two eigen values are there so it is complex conjugate pair, and one real eigen values. This third, your what you call that your third column it is all are real. Similarly, this is your phi 21, this is your phi 22, and this is your phi 23, and this one is phi 31, this is phi 32 and this is phi 33, right. So, here also these two are complex conjugate, these two are complex conjugate, right.

Now, if you for to get the left eigen vector psi will be is equal to phi inverse, but we have seen because your what you call that your phi into psi is equal to identity matrix, right. So, psi is equal to phi inverse.

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for eigenvector matrix normalized
that $\Psi\Phi = I$

$(1.3884 + j1.9097)$ Ψ_{11}	$(-0.041 + j0.499)$ Ψ_{12}	$(-0.0643 + j0.563)$ Ψ_{13}
$(9.3884 - j1.9097)$ Ψ_{21}	$(-0.041 - j0.499)$ Ψ_{22}	$(-0.0643 - j0.563)$ Ψ_{23}
-2.682 Ψ_{31}	0.0014 Ψ_{32}	1.5126 Ψ_{33}

So, if you invert it just let me tell you, if you invert it that you will get that your psi phi is equal to I, right therefore, your psi is equal to your what your call that your phi inverse. So, in that case what you will get that if you take inversion you will get this one. In this case also this is your this is your psi 11 and psi 12, psi 13, right, this is psi 21, psi 22, psi 23. You know it, but still I am writing, psi 31, psi 32, psi 33, right. So, if you look into this column wise these two eigen values are complex conjugate pair.

Similarly, these two also complex conjugate pair, these two also and third your row is basically real because one eigen value is real. But only one thing I would like to tell that suppose some questions are given in exam or assignment or we understand that 3 into 3 inversion particularly complex one is a time consuming process and it is difficult also, it consume lot of time. So, in that case sufficient data will be provided such that you write that phi psi all will be provided such that you can easily compute the rest of the thing, right.

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participation matrix is

$$\begin{array}{ccc|l}
 \left. \begin{array}{l} \frac{1}{\lambda_1} \begin{array}{l} -1^\circ \\ \psi_{11} \end{array} \\ \frac{1}{\lambda_2} \begin{array}{l} -1^\circ \\ \psi_{12} \end{array} \\ \frac{1}{\lambda_3} \begin{array}{l} 94^\circ \\ \psi_{13} \end{array} \end{array} \right\} \Delta \omega_r \\
 \left. \begin{array}{l} 0.501 \begin{array}{l} 1^\circ \\ \psi_{21} \end{array} \\ 0.501 \begin{array}{l} 1^\circ \\ \psi_{22} \end{array} \\ 0.017 \begin{array}{l} -94^\circ \\ \psi_{23} \end{array} \end{array} \right\} \Delta \delta \\
 \left. \begin{array}{l} 0.001 \begin{array}{l} 180^\circ \\ \psi_{31} \end{array} \\ 0.001 \begin{array}{l} 180^\circ \\ \psi_{32} \end{array} \\ 1.002 \begin{array}{l} 0^\circ \\ \psi_{33} \end{array} \end{array} \right\} \Delta \psi_{fd}
 \end{array}$$

So, now in our participation matrix if you try to find out that P, right it is directly computed that P actually is equal to this one basically if you multiply like this it will be basically phi 11, psi 11, this term, right. This will be phi 12 then just see whether I am not missing any term that psi 21. If it is phi 13, it is psi 31, right. Similarly, this one if it is phi 21 then it will be psi 12, right and if it is your phi 22 then it will be psi 22; if it is phi 23 it will be psi 32, right. Just you have to multiply all the elements are there for left and, right eigen vector. And if it is phi 31 then it will be psi 13, this product of this it is coming like this if it is phi 32 then it is psi 23 and if it is phi 33 then it is psi 33.

So, all these phi and psi elements are there in the right and left eigen vector. So, if you multiply it will be like this. If you look into this, this first one is for delta omega r then delta delta and then delta psi fd, right. So, everywhere magnitude is 0.501, 0.501, everything here also, but this one 0.017, 0.017, this is 1.002 and here it is 0.001, 0.001. So, what does it mean?

So, because 3 variables are there delta omega r, delta psi and delta delta and delta psi fd because it is 3 into 3 you have taken, right. So, this is for lambda 1 this is for lambda 2 and this is for lambda 3, right. So, based on that this participation factor is your participation factor is computed. So, I have marked this is for lambda 1, lambda 2, and lambda 3.

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the participation matrix, it is seen $\Delta\omega_r$ and $\Delta\delta$ have a high participation in the oscillatory mode (corresponding eigenvalues λ_1 and λ_2); the field linkage has a high participation in the

From the participation matrix it is seen that $\Delta\omega_r$ and $\Delta\delta$ have a high participation in the oscillatory, oscillatory mode corresponding to λ_1 , λ_2 . I mean this is $\Delta\omega_r$, this is $\Delta\delta$. So, here also 0.501, here also 0.501, here also 0.501, here also 0.501, right; therefore, the $\Delta\omega_r$ and $\Delta\delta$ have high participation in the oscillatory mode, corresponding to eigen values λ_1 and λ_2 .

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latory mode, represented by the mode λ_3 .
steady-state synchronizing torque end due to $\Delta\psi\delta$ is
 $K_f = - 0.8649 \times 0.323 \times 1.4187$

The field plus linkage has high participation in the non-oscillatory mode represented by the eigen values lambda 3. If you see the lambda 3 here it is 1.002. It is high participation, whereas in that participation of delta omega r, delta delta, is 0.017, 0.017 very negligible, right. Similarly, for psi fd here in this mode 0.001, 0.001 almost negligible, right.

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end due to $\Delta\psi_{fd}$ is

$$K_3 K_4 = -0.8649 \times 0.323 \times 1.4187$$

$$K_3 K_4 = \underline{-0.3963}$$

$$K_2 K_3 K_4 = \underline{0.3963}$$

total steady-state synchronizing
coefficient is .

So, the steady state synchronising torque coefficient due to delta psi fd this we know, this from the block diagram we pick up, right. So, this we have seen earlier, the steady state synchronising torque coefficient due to delta psi fd minus K 2, K 3, K 4. I mean K 2 K 2, K 3, K 4 minus symbol that is we are taking from the block diagram. That is minus K 2, K 3, K 4 all the data were given initially just multiply, it will be minus K 2, K 3, K 4 will be minus 0.3963; that means, K 2, K 3, K 4 is equal to this is minus K 2, K 3, K 4 we have taken. So, minus 0.3963 that is K 2, K 3, K 4 is equal to 0.3963, right. The total steady state synchronising torque coefficient. So, it is K 1, your minus K 2, K 3, K 4, right.

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$$= K_1 - K_2 K_3 K_4$$
$$= (0.7643 - 0.3963)$$
$$s = 0.3679 \text{ pu torque/rad.}$$

the block diagram of Fig.31

So, it is K_1 minus $K_2 K_3 K_4$. K_1 is given 0.7643 given in the data, and K_2, K_3, K_4 ; here K_2, K_3, K_4 , right this value you substitute that K_1 minus $K_2 K_3 K_4$. So, whatever it come that is your K synchronising torque coefficient, right. Therefore, K_s is equal to 0.3679 per unit torque per radian, right.

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$$s = 0.3679 \text{ pu torque/rad.}$$

the block diagram of Fig.31

↓ Fig.30

Now, from the block diagram of figure 31, right; figure 31, just check figure 31 or figure 30. Please check this one, both block diagram are same you can take 30 also just check, right. Only that pss term may be there, but this figure 30 or figure 31, probably figure 30

just check, right. So, if it is figure 31 also just see similar diagram, only one pss term may be there, right.

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$$\frac{\Delta \psi_{fd}}{\Delta S} \Big|_{\text{due to } \Delta \psi_{fd}} = \frac{-K_2 K_3 K_4}{(1 + sT_3)}$$

$$\frac{\Delta \psi_{fd}}{\Delta S} \Big|_{\text{just due to } \Delta \psi_{fd}} = \frac{-K_2 K_3 K_4 (1 - sT_3)}{1 - s^2 T_3^2}$$

So, delta T S upon delta delta S due to delta psi fd we know this minus K 2, K 3, K 4 upon 1 plus S T 3, right or delta T upon delta delta instead of due to delta psi fd just due to delta psi fd we are writing minus K 2, K 3, K 4, numerator and denominator you multiply by 1 minus S T 3. So, which will become, this one multiplied by 1 minus S T 3 upon 1 minus S square T 3 square, right.

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The due to $\Delta \psi_{fd}$ is

$$\Delta \psi_{fd} = \frac{-K_2 K_3 K_4}{(1 - s^2 T_3^2)} \Delta S + \frac{K_2 K_3 K_4 T_3}{(1 - s^2 T_3^2)} (s \Delta S)$$

the Fig. 31,

$$\Delta S = \frac{\omega_0}{s} \Delta \omega_r.$$

Therefore, delta T due to delta psi fd; that means, multiply both side by delta delta therefore, delta T due to delta psi fd will be minus K 2, K 3, K 4 upon 1 minus S square T 3 square delta delta plus K 2, K 3, K 4 T 3 upon 1 minus S square T 3 square S delta delta, these two term we are separating, right. So, one is delta delta, another is S delta delta, this is from figure 31. That pss term is there, that your that thing, right. So, which is correct one figure 31.

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$$\psi_{fd} = \frac{-k_2 k_3 k_4}{(1 - s^2 T_3^2)} \Delta s + \frac{k_2 k_3 k_4 T_3}{(1 - s^2 T_3^2)} (S \Delta s)$$

the Fig. 31,

$$\Delta s = \frac{\omega_0}{s} \Delta \omega_r.$$

$$\therefore (S \Delta s) = \omega_0 \Delta \omega_r.$$

So, go back to figure 31, I am not being figure 30 also you will get the same thing, right. It is just development step by step. Therefore, delta delta is omega 0 upon S delta omega r, therefore S delta delta can be written as omega 0 delta omega r, right. Here it is S delta delta. So, replace S delta delta by this omega 0 delta omega r, right.

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the Fig. 31,

$$\Delta S = \frac{\omega_0}{s} \Delta \omega_r.$$
$$\therefore (S \Delta S) = \omega_0 \cdot \Delta \omega_r.$$
$$\Delta \psi_{fd} = \frac{-k_2 k_3 k_4}{(1-s^2 T_3^2)} \Delta S + \frac{k_2 k_3 k_4 T_3}{(1-s^2 T_3^2)} \Delta \omega_r \cdot \omega_0$$

Therefore, delta T due to delta psi fd is minus K 2, K 3, K 4 upon 1 minus S square T 3 square delta delta plus K 2, K 3, K 4 T 3 upon 1 minus S square T 3 square then into delta omega r into omega 0, right. So, this is delta T in your what you call in terms of delta delta and delta omega r, due to delta psi f d, right. Now, this is simple thing just go back to your figure 31, right, again and again I am not going just when you listen to this just keep figure 31 in front you, right.

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$$\Delta \psi_{fd} = K_s (\Delta \psi_{fd}) \Delta S + K_D (\Delta \psi_{fd}) \Delta \omega_r \omega_0.$$

the eigenvalues, the complex
frequency of rotor oscillation is
(-1.1 + j6.41). Since the real

Therefore, this can be written as this thing can be written as the delta T due to delta psi fd
 $K_S \Delta \psi_{fd}$; that means, K_S due to $\Delta \psi_{fd}$ into $\Delta \delta$ plus K_D that
damping part, $\Delta \psi_{fd}$ it is synchronising torque coefficient this is damping torque
coefficient K_D into; this is due to $\Delta \psi_{fd}$ into ω_r into ω_0 . ω_0 is
your $2\pi f_0$.

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From the eigenvalues, the complex
frequency of rotor oscillation is
 $-0.11 + j6.41$. Since the real
component is much smaller than the
imaginary component, we can compute
 K_S and K_D at the oscillation frequency
setting $s = j6.41$ without loss of

From the eigen values, the complex frequency of rotor oscillation we have seen it is
minus 0.11 plus $j 6.41$, right, but this real part is very small compared to the imaginary
part.

So, for our approximate analysis more or less it is true who will ignore the real part,
right. Since the real component is much smaller than the imaginary component we can
compute K_S and K_D at the oscillation frequency by setting S is equal to $j 6.41$ without
loss of much accuracy because this one will ignore, right, because it is very small. So,
only we will take that is if you substitute in this equation in this equation S is equal to
your $j 6.41$ here you will substitute, right.

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ch accuracy.

$$\Delta\psi_{fd} = \frac{-k_2 k_3 k_4}{(1 - s^2 T_3^2)} = \frac{-0.3963}{1 - (j6.41 \times 2.365)^2}$$

$$\Delta\psi_{fd} = -0.00172 \text{ pu torque/rad}$$

$$\Delta\psi_{fd} = \frac{k_2 k_3 k_4 T_3 \omega_0}{(1 - s^2 T_3^2)}$$

That means, that K; that means, K S delta psi fd has been defined as one that is minus K 2, K 3, K 4 upon 1 minus S square T 3 square this is actually K S due to delta psi fd into delta delta. So, this is my K S delta psi fd is equal to this term, right.

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$$\Delta\psi_{fd} = \frac{-k_2 k_3 k_4}{(1 - s^2 T_3^2)} \Delta s + \frac{k_2 k_3 k_4 T_3}{(1 - s^2 T_3^2)} \Delta\omega_r \cdot \omega_0$$

$k_2 k_3 k_4$ $k_2 k_3 k_4$

That means, this one this one actually K S delta psi fd, right and this one whatever it is this one is given, right this term. Later we will see it is K D your delta psi fd, right omega 0 term is also there that is constant. So, if you do so, if you do so, right then put K 2, K 3, K 4 all you have got it, it is 0.3963 and divided by 1 minus j 6.41 and T 3 value as given

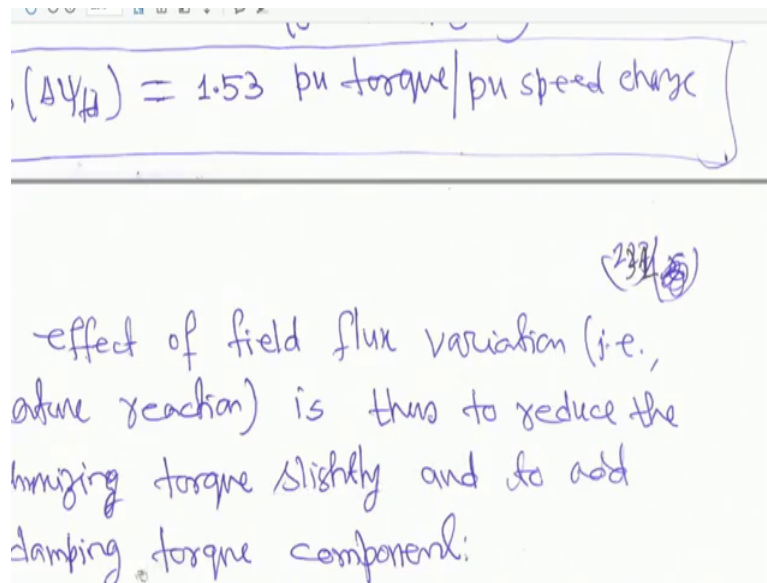
in the data 2.365 whole square. If you simply K S due to delta psi fd becoming minus 0.00172 per unit torque per radian, this is dimensionless quantity, right.

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The image shows a whiteboard with handwritten mathematical equations. The first equation is $(\Delta\psi_{fd}) = -0.00172 \text{ pu torque/rad}$. The second equation is $\Delta\psi_{fd} = \frac{k_2 k_3 k_4 T_3 \omega_0}{(1-s^2 T_3^2)}$. The third equation is $(\Delta\psi_{fd}) = \frac{0.3963 \times 2.365 \times 377}{1 - (j6.41 \times 2.365)^2}$. The final result is $(\Delta\psi_{fd}) = 1.53 \text{ pu torque/ pu speed change}$.

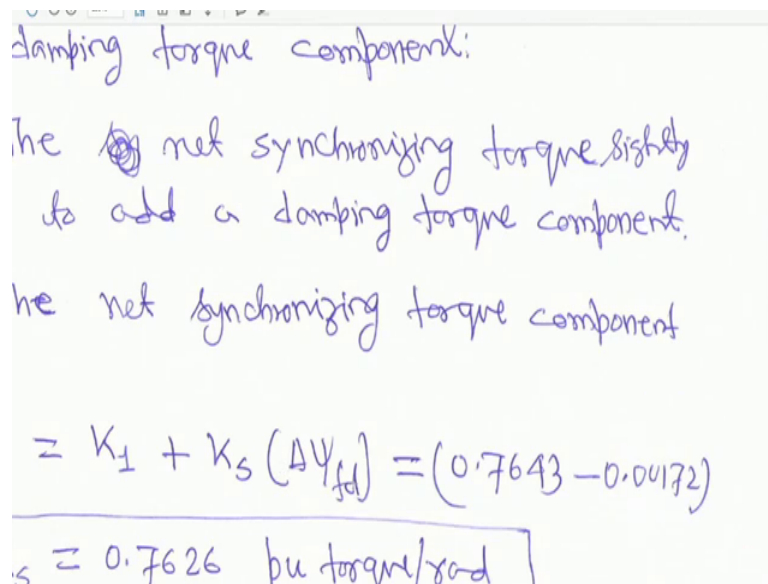
Similarly, K D delta psi fd is K 2, K 3, K 4 T 3 into omega 0 upon 1 minus S square T 3 square, right. So, K 2, K 3, K 4 you have computed before 0.3963 into 2.365 that is T 3 into omega 0 it is 60 hours, so 377, right 2 pi f 0 we have taken f 0 is equal to 60 hours divided by 1 minus j 6.41 into 2.365 square. If you simplify it is coming actually K D delta psi fd is 1.53 per unit torque per unit speed change that is basically dimensionless quantity, right.

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Therefore, the effect of field flux variation that is the armature reaction, right is thus to reduce the synchronising torque slightly and to add a damping torque component because this is actually K_S is minus 0.00172 negative means that its effectively slightly it will reduce, right because it will be K_1 minus K_S , right.

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Therefore, the net synchronising torque component in general we write K_1 plus K_S $\Delta\psi_{fd}$ is equal to K_1 is given 0.7643 and K_S due to $\Delta\psi_{fd}$ we got minus 0.00172. So, more or less unchanged, right. So, it is 0.7626, right.

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$$s = 0.7626 \text{ pu torque/rad}$$

The only source of damping is due to flux variation. Hence, the net damping torque coefficient is

$$K_D = K_D(A\psi_{fd}) = 1.53 \text{ pu torque / pu speed change}$$

Similarly, the only source of damping is due to the field flux variation is the net damping torque is K_D because other K_D in the data K_D was given 0 there was no K_D it will be simply as K_D is equal to $K_D \Delta \psi_{fd}$ that is 1.53 it is a dimensionless quantity, but we are writing per unit torque by per unit speed change, right.

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$$K_D = K_D(A\psi_{fd}) = 1.53 \text{ pu torque / pu speed change}$$

Now, the undamped natural frequency is

$$\omega_n = \sqrt{\frac{K_S \omega_0}{2H}} = \sqrt{\frac{0.7626 \times 377}{2 \times 3.5}}$$

$$\omega_n = 6.41 \text{ rad/sec.}$$

Now, we know the undamped natural frequency is this expression we have made it earlier ω_n is equal to root over $K_S \omega_0$ upon $2H$. So, K_S we got 0.7626

$\omega_0 = 2\pi f_0$, f_0 is 60 Hz. So, 377 and 2 into H, H in the data given 3.5. So, ω_n is actually 6.41 radian per second, right.

Now, here also if you look into this, if you look into this that here we, here also if you look into this that it is 6.41, right. So, I mean just it is your what you call just telling that according to our data it is coming like this, but nothing to relate with that, right.

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The whiteboard shows the following handwritten equations:

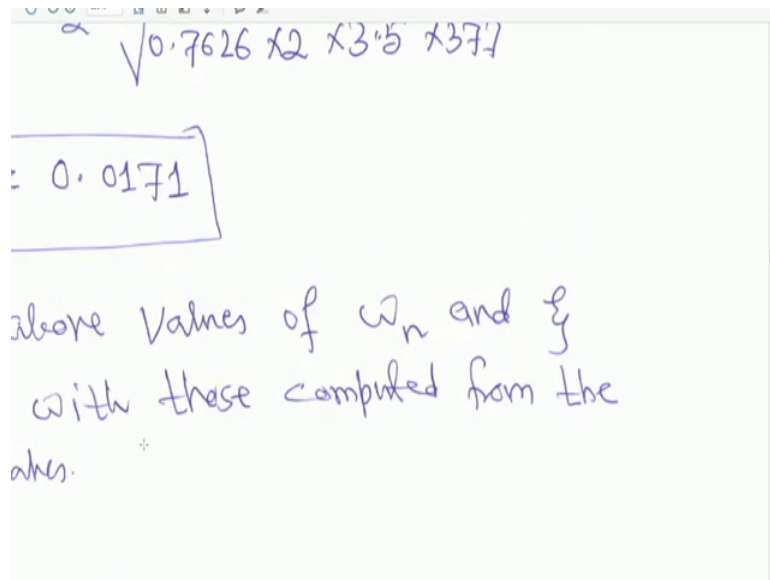
$$\frac{1}{2} \frac{K_D}{\sqrt{2H\omega_0 K_S}}$$

$$\frac{1}{2} \frac{1.53}{\sqrt{0.7626 \times 2 \times 3.5 \times 377}}$$

$$= 0.0171$$

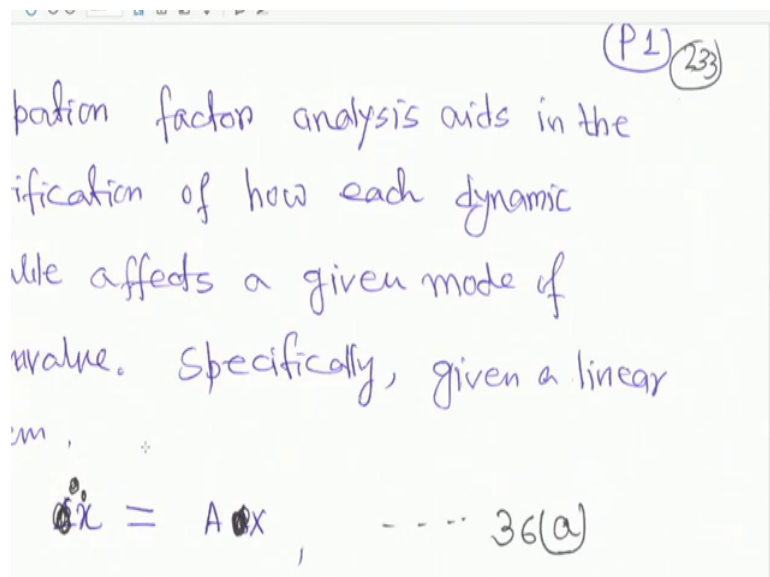
And similarly your ψ that is your damping ratio that also we have derived before that is half into K_D upon root over $2H\omega_0 K_S$. If you substitute that your H is your ah what you call $2H$, right $\omega_0 K_S$, K_S is actually 0.7626 then $2H$, 2 into 3.5 into $\omega_0 = 2\pi f_0$, 377. So, $\psi = 0.0171$, right, this is the answer.

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So, the above value of omega and psi agree with these computed from the eigen values, right. So, if you see the eigen values and see this one you will find that omega n also we got 6.41 and here also we got this thing. So, it agrees with that (Refer Time: 15:03) that means, the approximation is ok and just have a look, right. So, this is the your what you call that another example, right.

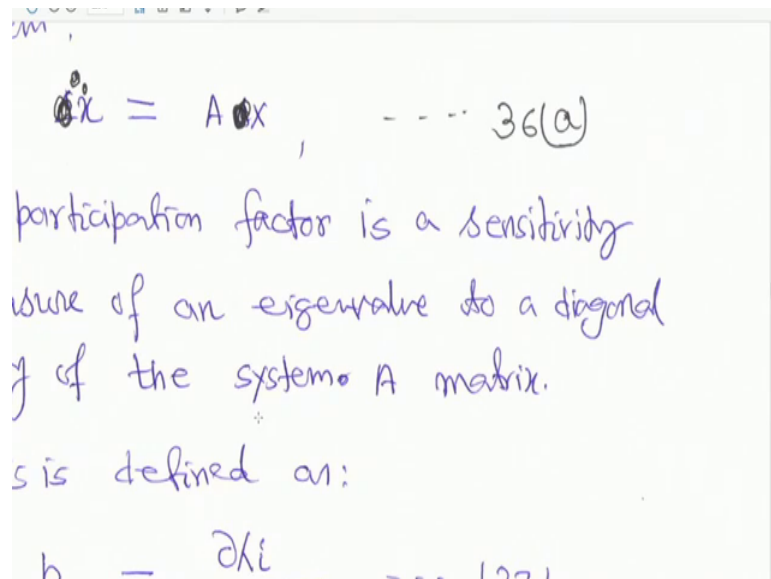
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So, now question is that the participation factor we have studied already. And we have seen that P is equal to in general phi into psi in matrix form, right that also I told you how to take it. Now, see little bit more.

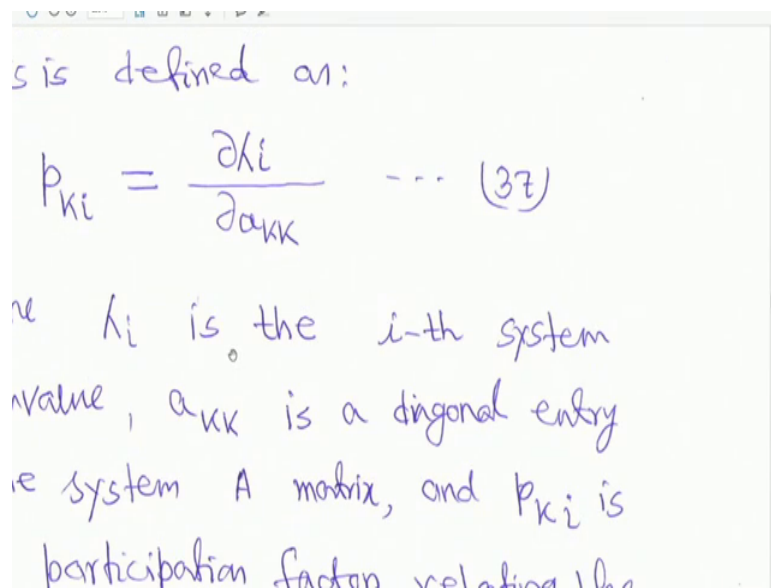
So, participation factor analysis again aids in the identification of how each dynamic variable affects a given mode of eigen value specifically given a linear system say \dot{x} is equal to $A x$, right. Actually, this is I have written small then you take small x , right. This equation we have made it 36 a, right. Just we are ah trying to put it in another way, right.

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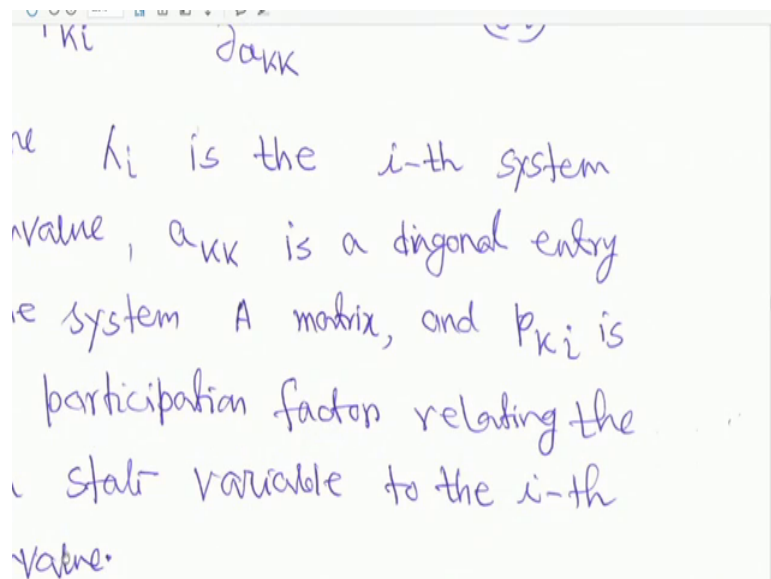
So, a participation factor is a sensitivity measure of an eigen value to a diagonal entry of the system A matrix. This we have seen earlier, but why I had taken this again just we will see.

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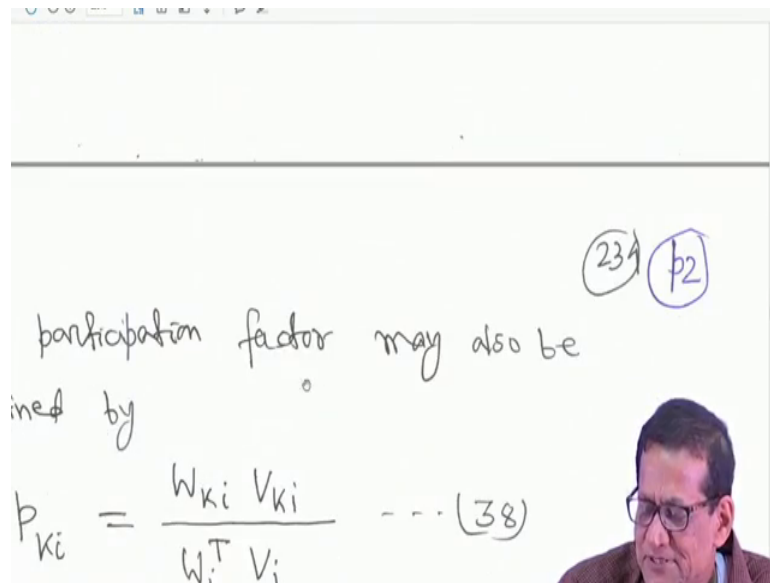
So, this is also here defined that p_{ki} is equal to $\frac{\partial \lambda_i}{\partial a_{kk}}$ this is equation say 37, right. Well, λ_i is the i th system eigen value, a_{kk} is the diagonal in the system A matrix and p_{ki} is the participation factor relating the K th state variable to the i th eigen value, right.

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So, this is your what you call you have seen this before, right.

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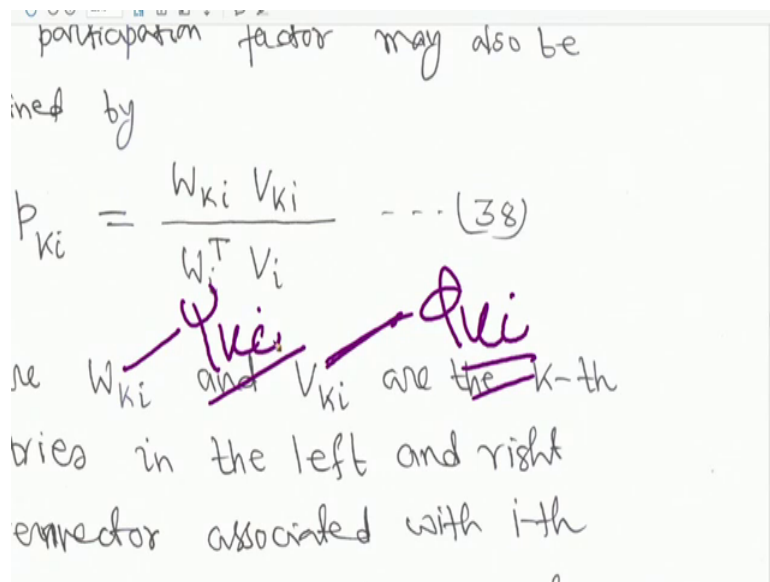
participation factor may also be defined by

$$p_{ki} = \frac{w_{ki} v_{ki}}{w_i^T v_i} \dots (38)$$

(23) (p2)

Now, the participation factor may also be defined by p_{ki} is equal to $w_{ki} v_{ki}$ upon $w_i^T v_i$.

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participation factor may also be defined by

$$p_{ki} = \frac{w_{ki} v_{ki}}{w_i^T v_i} \dots (38)$$

where w_{ki} and v_{ki} are the k -th entries in the left and right eigenvector associated with i -th

Annotations: w_{ki} is crossed out and replaced with ψ_{ki} ; v_{ki} is crossed out and replaced with ϕ_{ki} .

Actually v_{ki} is the your what you call w_{ki} and v_{ki} are the k th entries in the left and your right eigen vector. Actually, v_{ki} is nothing, but your ϕ_{ki} previously you have taken, the right eigen vector and w_{ki} is nothing but ψ_{ki} that is your left eigen vector meaning is same, right.

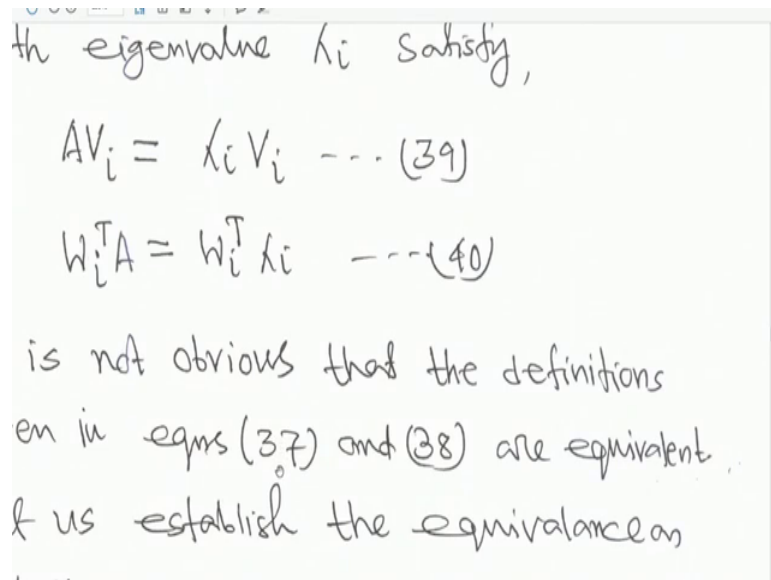
As I have taken different example that is why this terminology I have changed it, right. So, it is $W_{ki} V_{ki}$ upon $W_i^T V_i$, this is equation 30. So, basically your this is actually V_{ki} is V_i is actually nothing, but ϕ_i that is right eigen vector and W is nothing but ψ_i that is your left eigen vector, right. So, where W_{ki} and V_{ki} are the k th just hold on, are the k th entries right in the left and right eigen vector associated with i th eigen value, right.

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W_{ki} and V_{ki} are the k -th entries in the left and right eigenvector associated with i -th eigenvalue. The right eigenvector V_i and the left eigenvector W_i [$\phi_i = V_i$ and $\psi_i = W_i$] associated with the i th eigenvalue λ_i satisfy,

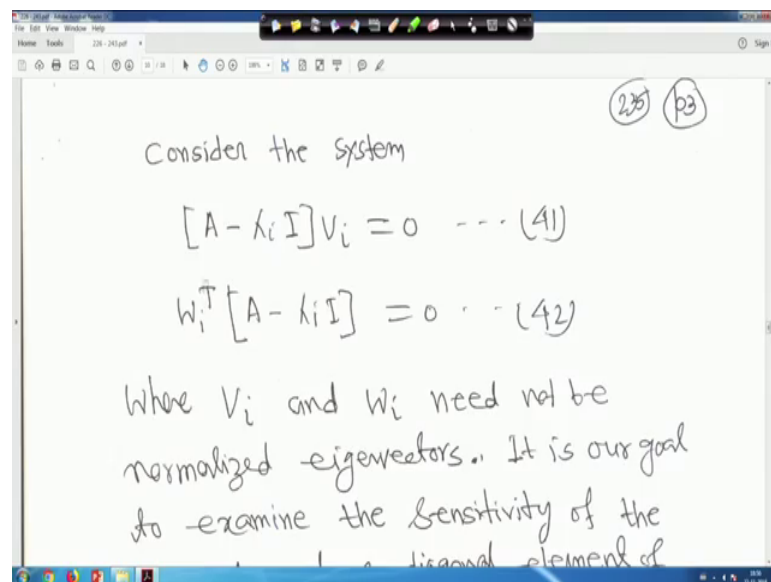
The right eigen vector V_i and the left eigen vector W_i , I have written here ϕ_i actually V_i and ψ_i is equal to W_i hence there is no confusion, right. Associated with the i th eigen value λ_i satisfying this we have seen before, there it was a ϕ_i is equal to $\lambda_i \phi_i$, but I am putting here $A V_i$ is equal to $\lambda_i V_i$ and here your, there it will be your $\psi_i^T A$ is equal to $\psi_i^T \lambda_i$ that is $W_i^T A$ into is equal to $W_i^T \lambda_i$. This is equation 40, right.

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Now, it is not obvious that the definition given in the equation 37 and 38 are equivalent, right. So, let us establish the equivalents as follows, right.

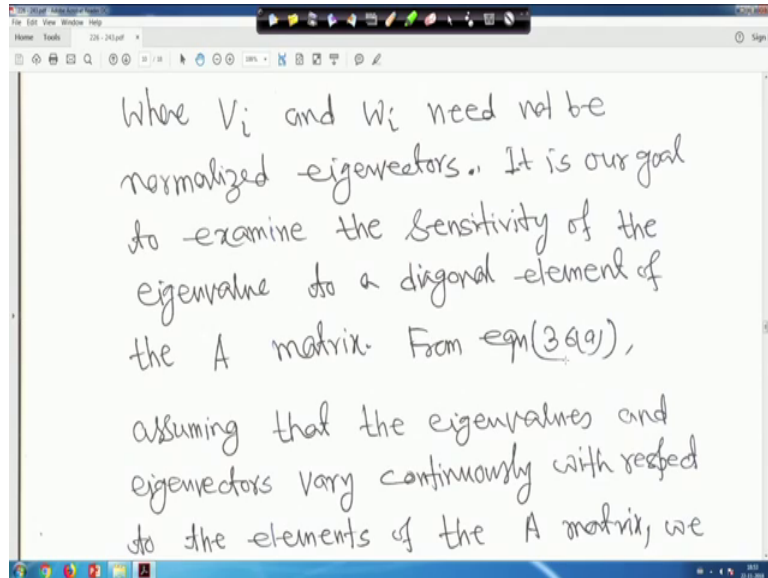
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So, for example, consider the system, this we have seen earlier that your A minus λ_i into I into V_i is equal to 0 to find out your right eigen vector this we have seen before also, right. And another one left eigen vector W_i transpose we are making A minus λ_i into I is equal to 0 this is 41, this is 42, right. Now, where V_i and W_i need not be normalised eigen values. So, V_i and W_i , it is need not be normalised eigen values, right,

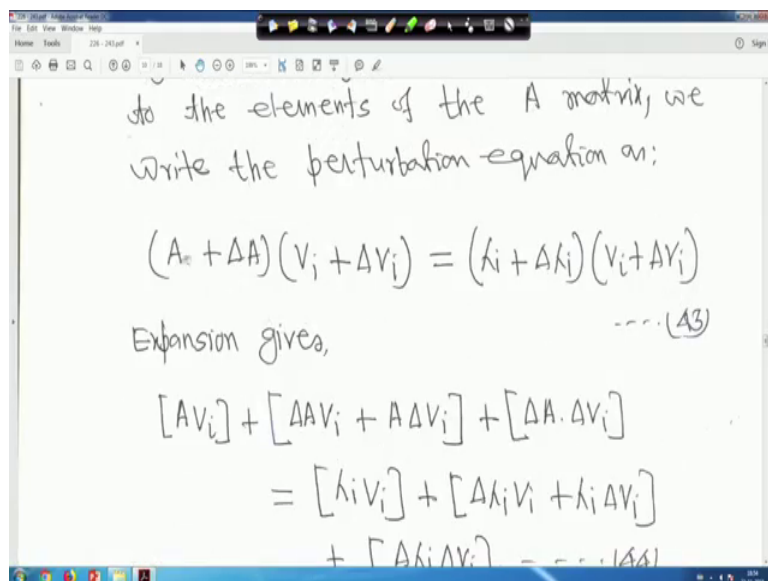
but we have seen before. It is our goal to your what you call to examine the sensitivity of the eigenvalue to a diagonal element, right, diagonal element of the A matrix.

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So, from the equation 36 a, that is \dot{x} is equal to your $A x$, right I believe equation 36 that is \dot{x} is equal to $A x$, right. So, from equation 36 a, that assuming that the eigenvalues and eigenvectors vary continuously with respect to the elements of the A matrix, say we assumed this one, right.

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We write the perturbation equation as that means, these equation these equation can be written as this equation 39 it can be written as, right equation 39 it can be written as A plus ΔA into V_i plus ΔV_i is equal to λ_i plus $\Delta \lambda_i$ into V_i plus ΔV_i , right if it is like this. Like your, like your small perturbation thing, right.

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$$(A + \Delta A)(V_i + \Delta V_i) = (\lambda_i + \Delta \lambda_i)(V_i + \Delta V_i) \quad \dots (43)$$

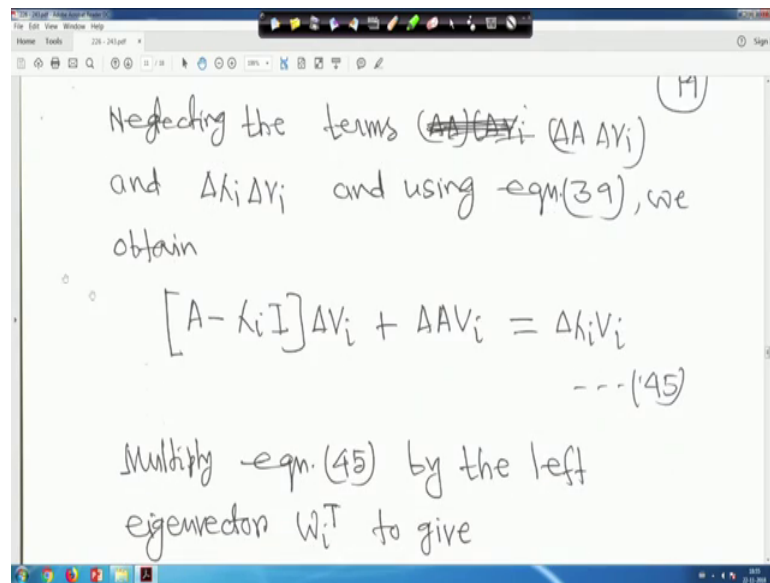
Expansion gives,

$$[AV_i] + [\Delta AV_i + A\Delta V_i] + [\Delta A \cdot \Delta V_i]$$

$$= [\lambda_i V_i] + [\Delta \lambda_i V_i + \lambda_i \Delta V_i] + [\Delta \lambda_i \Delta V_i] \quad \dots (44)$$

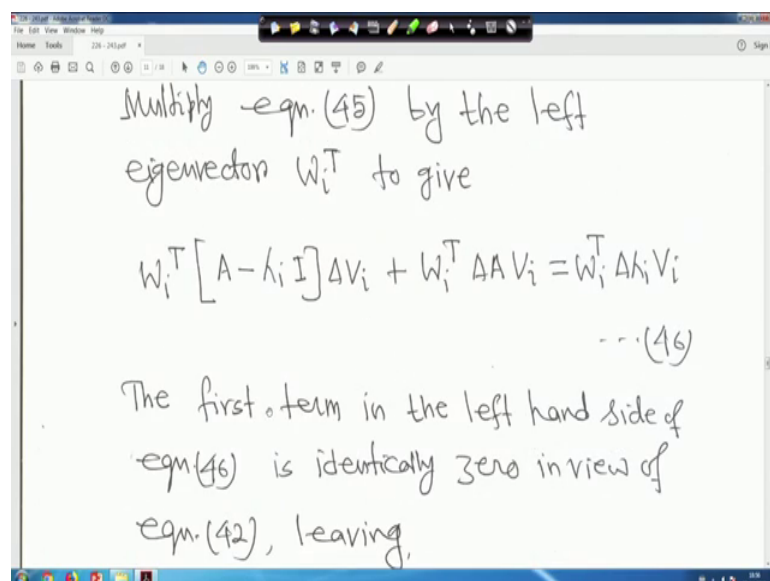
So, this expansion gives if you multiply it will be $A V_i$ plus in bracket ΔA into V_i plus A into ΔV_i plus ΔA into ΔV_i , is equal to $\lambda_i V_i$ plus bracket $\Delta \lambda_i V_i$ plus $\lambda_i \Delta V_i$ plus $\Delta \lambda_i \Delta V_i$. Plus ΔA into ΔV_i , this term this quadratic type of term so, we will neglect. This term and these two terms so, we will neglect; we assume that its contribution is negligible and this is equation 44, these two terms we will neglect, right.

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So, if you do so, if you do so, neglecting the terms that delta a delta V i and delta lambda i delta V i and using equation 39 we have obtained, right; that means, this is your just you have to go back to equation 39, right. So, basically you neglect these two terms and equation from equation 44 and 39 you will get a minus lambda i into i delta V i plus delta a V i is equal to delta lambda i V i, right. But this term A minus lambda i I into delta V i is 0 that we have seen, right Sorry this A minus lambda i I into delta V i plus delta A V i is equal to delta lambda A V i now multiply this is not 0 we will come to that, sorry, sorry.

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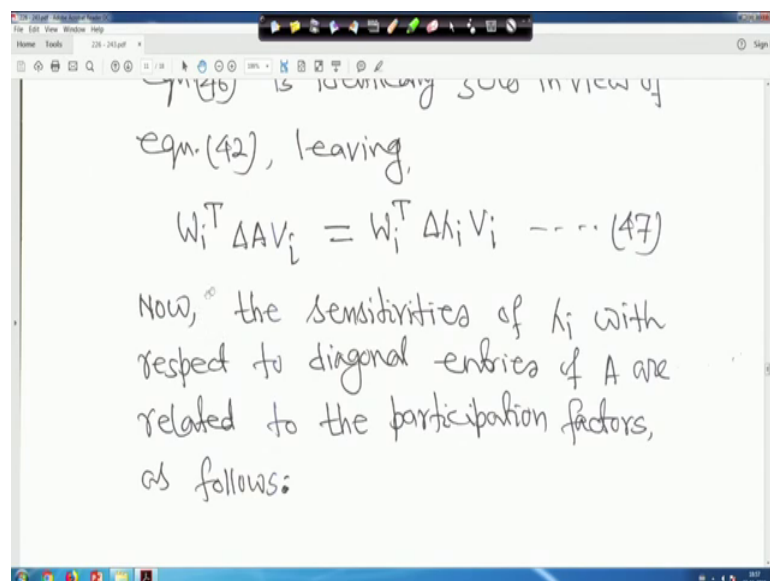


Now, multiplying equation 45 by the left eigenvector. So, please multiply this equation by your left eigenvector. So, say W_i^T transpose, right. So, the $A - \lambda_i I$ into ΔV_i plus $W_i^T \Delta A$ into V_i is equal to $W_i^T \Delta \lambda_i V_i$, right. The first term in the left-hand side of equation 46 is identically 0 in view of equation 42. So, come to equation 42, right.

If you see equation 42 that $(A - \lambda_i I) V_i$ that was Δ multiplying both side W_i^T into $(A - \lambda_i I)$ is equal to 0, right. So, if you come to this equation 45, right, here if you come to this that is 46, so $W_i^T (A - \lambda_i I) V_i$ this term effectively is 0. So, this term is 0.

So, if it is so, then the first term if it is so, then this equation can be written as $W_i^T \Delta A V_i$ is equal to $W_i^T \Delta \lambda_i V_i$, right.

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So, that means your, this equation now from this equation we will get this expression same as before. Now, the sensitivities of λ_i with respect to diagonal entries of the A are related to the participation factors as follows, right only diagonal elements, right a KK_{jj} like this, right.

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Assume that, only the k-th diagonal entry of A is perturbed so that

$$\Delta A = \begin{bmatrix} 0 & \dots & 0 & \dots & 0 \\ 0 & \dots & \Delta a_{kk} & \dots & 0 \\ 0 & \dots & 0 & \dots & 0 \end{bmatrix} \quad \text{--- (48)}$$

Then, in eqn (17), the left-hand side

So, in that case assume that only the Kth diagonal entry of A is perturbed, right only Kth element so that means, delta A all other elements remain fixed. So, participation is 0 only Kth element that is a KK is perturbed. So, it is delta a KK, right, this is equation 48. So, previously whatever we have done, so now, concept will be clear, right.

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$[0 \dots 0 \dots 0]$

Then, in eqn (17), the left-hand side can be simplified resulting in

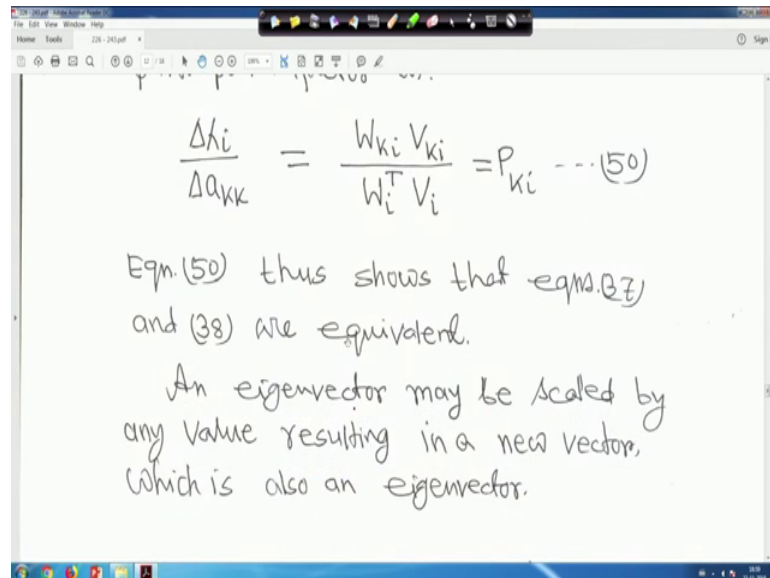
$$\underline{W_i^T} \underline{\Delta A} \underline{V_i} = \underline{W_{ki}} \underline{\Delta a_{kk}} \underline{V_{ki}} = \underline{W_i^T} \underline{\Delta \lambda_i} \underline{V_i}$$

Solving for the sensitivity gives the participation factor as:

So, then in equation 47 the left-hand side can be simplified as it is W i transpose delta A V i is equal to W Ki a KK, V Ki is equal to W i transpose delta lambda i V i. Then delta A, this delta A actually, this delta only Kth element, right only diagonal element that Kth

row and Kth column the diagonal are perturbed. So, ΔA is equal to Δa_{kk} , so this is in general we can write them because now it is only single element Δa_{kk} . So, we can write $W_{ki} \Delta a_{kk} V_{ki}$, right is equal to your $W_i^T \Delta \lambda_i V_i$ the right-hand side, right. Therefore, solving for the sensitivity it gives your participation factor, right.

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$$\frac{\Delta \lambda_i}{\Delta a_{kk}} = \frac{W_{ki} V_{ki}}{W_i^T V_i} = P_{ki} \quad \dots (50)$$

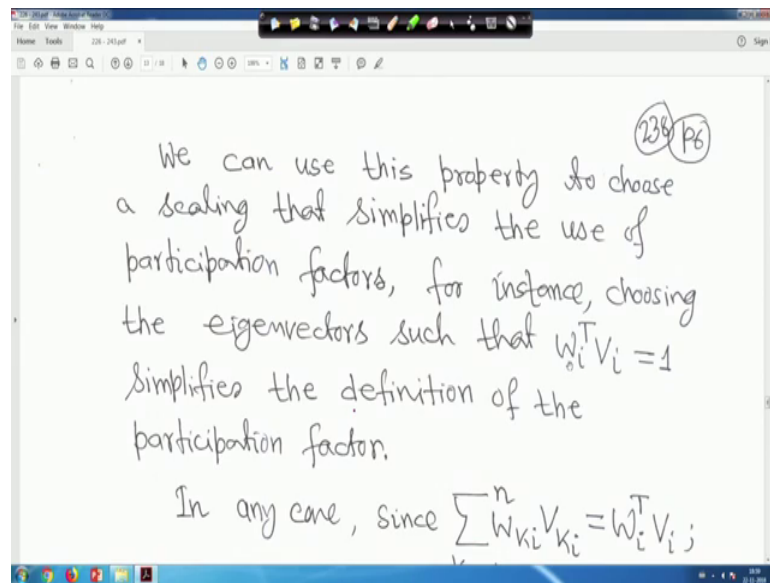
Eqn. (50) thus shows that eqns. (37) and (38) are equivalent.

An eigenvector may be scaled by any value resulting in a new vector, which is also an eigenvector.

As therefore, $\Delta \lambda_i$ upon Δa_{kk} is equal to $W_{ki} V_{ki}$ upon $W_i^T V_i$ is equal to P_{ki} this is equation 50. So, we see another form of participation factor.

Now, equation 50 thus shows that equation 37 and 38 are equivalent, right. Now, an eigenvector may be scaled by any value resulting in a new vector which is also an eigenvector, right.

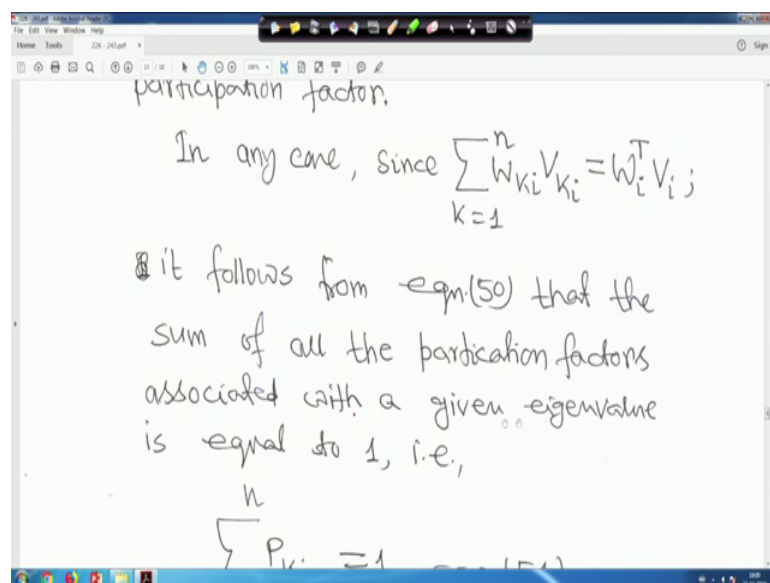
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So, we can use this property to choose the scaling that simplifies the use of participation factor, right. For instance, choosing the your what you call eigenvector such that $W^T V$ is equal to say 1, simplifies the definition of the participation factor that we have seen also that K is equal to 1 to n is $W_{ki} V_{ki}$ is equal to $W^T V$.

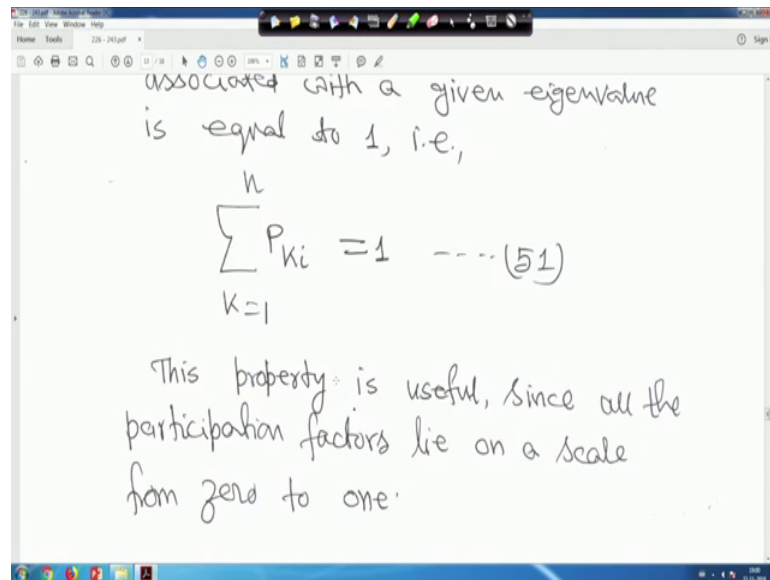
Similarly, this thing we have seen also that K is equal to 1 to n , this is $\psi K \phi$ is equal to it can be written as $\psi^T \phi$, right.

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So, it follows from equation 50; that means, from this equation, that means, from this equation, right. It follows from equation 50 that the sum of all the participation factor associated with a given eigenvalue is equal to unity that is one. That means, $\sum_{k=1}^n P_{ki}$ is equal to 1 this is 51; that means, it is normalised, right.

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associated with a given eigenvalue
is equal to 1, i.e.,

$$\sum_{k=1}^n P_{ki} = 1 \quad \dots (51)$$

This property is useful, since all the participation factors lie on a scale from zero to one.

This property is useful, since all the participation factor lie on a scale from 0 to 1, right. So, previously when we started participation factor everything is to has been mentioned, but once again I thought I should take it in little bit in different way such that that idea will be clear, right.

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To handle participation factors corresponding to complex eigenvalues, we introduce some modifications as follows.

The eigenvectors corresponding to a complex eigenvalue will have complex elements. Hence, P_{ki} is defined as

$$P_{ki} = \frac{|V_{ki}|}{|W_{ki}|}$$

So, now to handle participation factors corresponding to complex eigenvalues we introduce some modification as follows, right. Because we have, earlier we have seen participation factor and complex values, right. Now, the eigen this is another form the eigenvectors corresponding to a complex eigenvalue will have complex element, right.

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$$P_{ki} = \frac{|V_{ki}| \dots |n_i|}{\sum_{k=1}^n |V_{ki}| |W_{ki}|} \quad \text{--- (52)}$$

A further normalization can be done by making the largest of the participation factors equal to unity.

Example-

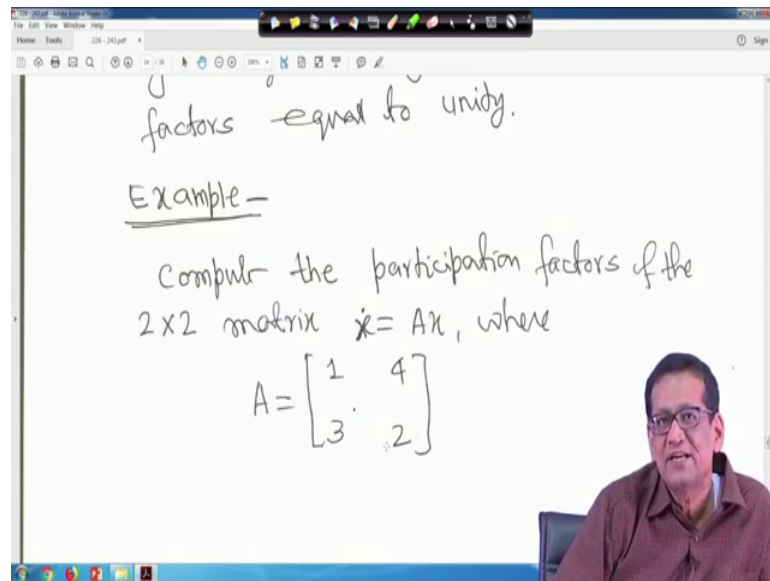
combine the participation

Hence, P_{ki} is defined as P_{ki} can be taken as mod of V_{ki} that is the absolute value, then absolute of W_{ki} sigma K is equal to 1 to n , absolute of V_{ki} into absolute of W_{ki} this is equation 52. When I will take couple of example, you will find things have been

very transparent, right. Only my suggestion to all of you, just try to understand in little bit try yourself derivation and little bit you have to keep it in your memory from the point of view of your examination, you have to keep this in mind, right.

So, a further normalization can be done by making the largest of the participation factor equal to unity, right.

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factors equal to unity.

Example -

Compute the participation factors of the 2x2 matrix $\dot{x} = Ax$, where

$$A = \begin{bmatrix} 1 & 4 \\ 3 & 2 \end{bmatrix}$$

So, example, compute the participation factor of the 2 into 2 matrix \dot{x} is equal to Ax , where A is equal to 1 4 3 2, right. So, this is your what you call that your matrix is given, right.

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The eigenvalues are $\lambda_1 = 5$, $\lambda_2 = -2$

The right eigenvectors are

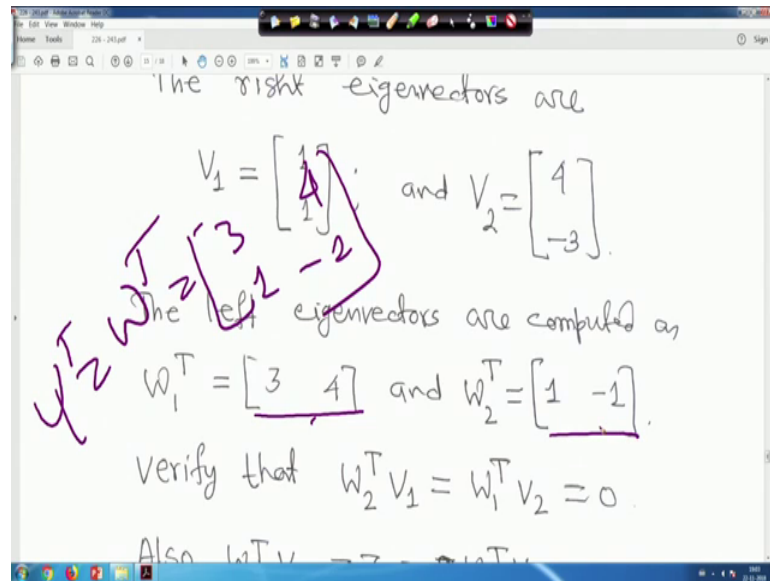
$\Phi = V$ $V_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$; and $V_2 = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$.

The left eigenvectors are computed as

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Now, the eigenvalues of this matrix you can easily find it out. So, eigenvalues are lambda 1 is equal to 5, where lambda 2 is equal to minus 2, right. So, in this case your eigen values are real. Now, you following the same procedure you find out the, right eigenvectors, right it is V 1 the first column of the matrix 1 1 and V 2, 4 minus 3; that means, your basically your, basically your phi is equal to nothing but V the right eigenvector. So, first column that is 1 1 and that is your 4 minus 3 this is actually my right eigenvector, right. So, this way we write like this V 1 means first column V 2 means for second column, right. So, if you find out the right eigenvector following the same procedure you will get like this, right.

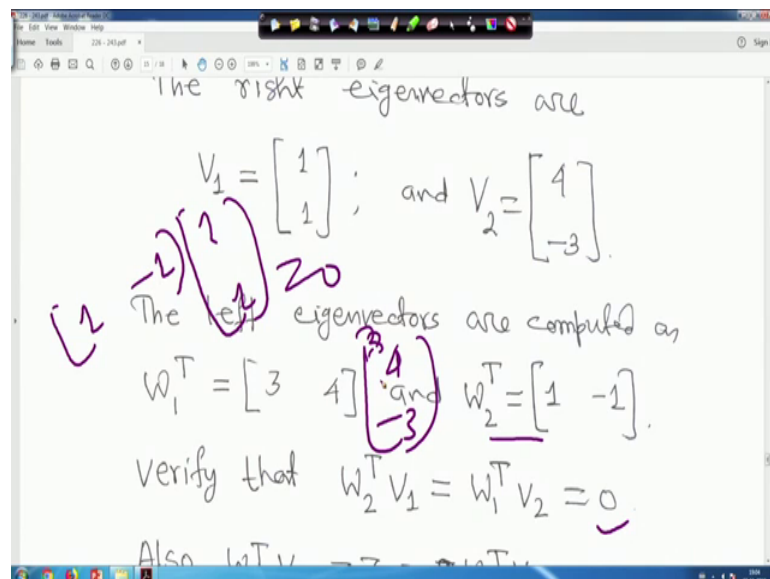
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So, the left eigenvector similarly you can compute, so it is given W_1 your transpose that has 3 4 and W_2 transpose 1 minus 1; that means, your basically ψ is equal to W the left eigenvector this can be written as actually 3 4 and it is 1 minus 1. This way it can be written, right.

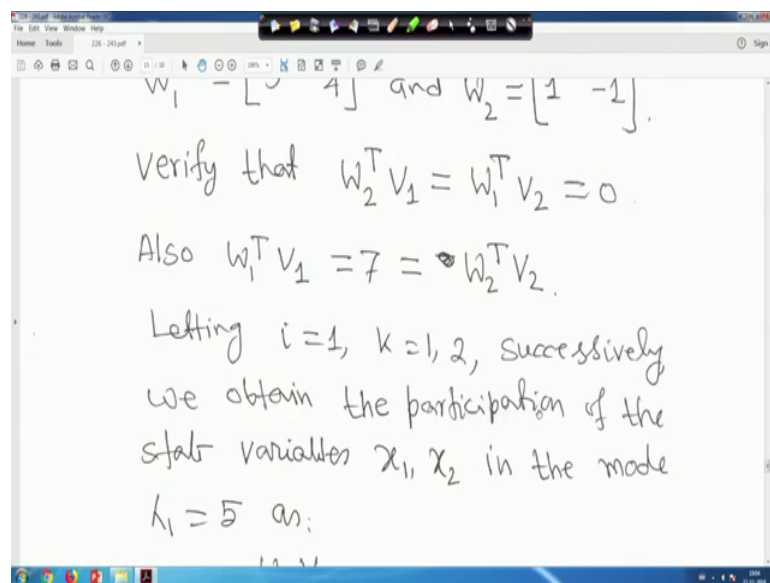
So, your basically it is your what you call that your ψ transpose W transpose this way you can write, right. So, that is why it is written as that was column wise, this is row wise, right.

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So, now to verify that you will see the W just hold on. W_2 transpose V_1 is equal to W_1 transpose V_2 is equal to 0. For example, W_2 your transpose; that means this one, this one actually 1 minus 1, right and then your V_1 that is 1 1. If you multiply 1 into 1 minus 1 into 1 it is basically 0, right. So, it is 0 similarly if you make W_1 transpose, right this one 3 4 into your 4 minus 3 that is 3 4 I am making it here for you 4 minus 3, 12 minus 12, so it is 0. So, W_2 transpose V_1 , W_1 transpose V_2 is equal to 0. Please you do it yourself, I have given the final thing you find out right eigenvector and left eigenvector. Of course, you know this, right. So, now to verify this, right.

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And also, W_1 transpose V_1 is equal to 7 is equal to W_2 transpose V_2 you can do it you will get 7, right. Now, letting i is equal to 1 and K is equal to 1 2, your successively we obtain the participation of the state variable x_1 and x_2 in the mode $\lambda_1 = 5$.

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The image shows a whiteboard with handwritten mathematical expressions. At the top, it says λ_1, λ_2 in the middle. Below that, it shows $\lambda_1 = 5$ as:
$$P_{11} = \frac{W_{11} V_{11}}{W_1^T V_1} = \frac{(3)(1)}{7} = \frac{3}{7}$$

$$P_{21} = \frac{W_{21} V_{21}}{W_2^T V_2} = \frac{(4)(1)}{7} = \frac{4}{7}$$

You take that your what you call that expression is there here know, expression is there your what you call P Ki expression is there, right. So, in that case your this thing, your what you call you take one by one, right, one by one this is your what you call the totally per unit is one and here it is, here it is, here it is, right.

So, if you look into this expression that you put your in this case i is equal to 1 and K is equal to 1 2; that means, in this equation you put your i is equal to 1 and K is equal to 1 to 2; that means, n is equal to 2, right. And you do this, you will find, if you do so, and simplify you do this and you simplify, you will get P 11 will become W 11 V 11 upon W 1 transpose V 1 because another term will become 0. So, it will be 3 into 1 upon 7, 3 by 7 and P 21, W 21, V 21 upon W 2 transpose V 2 it is 4 into 1 upon 7, right. So, basically it will be 4 by 7; 3 by 7, 4 by 7 if you add it, it will be unity P 11 plus P 21 will become unity, right.

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Letting $i=2$, and $k=1, 2$, we obtain the participation ~~factor~~ of the state variables x_1, x_2 in the mode $\lambda_2 = -2.01$.

$$P_{12} = \frac{W_{12} V_{12}}{W_2^T V_2} = \frac{4}{7}$$

Similarly, for i is equal to 2 and K is equal to 1 2, you will get $P_{12} W_{12} V_{12}$ upon W_2^T transpose V_2 that will get 4 by 7.

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Letting $i=2$, and $k=1, 2$, we obtain the participation ~~factor~~ of the state variables x_1, x_2 in the mode $\lambda_2 = -2.01$.

$$P_{12} = \frac{W_{12} V_{12}}{W_2^T V_2} = \frac{4}{7}$$
$$P_{22} = \frac{W_{22} V_{22}}{W_2^T V_2} = \frac{3}{7}$$

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$W_2^T V_2$ 7

The participation matrix is therefore

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} 3/7 & 4/7 \\ 4/7 & 3/7 \end{bmatrix}$$

Normalizing the largest participation factor as equal to 1 in each column results in

And, P_{22} will get $W_{22} V_{22}$ upon W_2 transpose V_2 that is 3 by 7, right. So, participation matrix P_{11} P_{12} , P_{21} P_{22} ; it is 3 by 7, 4 by 7, 4 by 7, 3 by 7. There we did ϕ into ψ , ϕ_{11} ψ_{11} , ϕ_{12} ψ_{21} , ϕ_{21} ψ_{12} and ϕ_{22} ψ_{22} , here directly we are getting it, right using that matrix because everything has been taken care of, right. So, please go through this, right.

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P_{21} P_{22} $\begin{bmatrix} 4/7 & 3/7 \end{bmatrix}$

Normalizing the largest participation factor as equal to 1 in each column results in

$$P_{\text{normalized}} = \begin{bmatrix} 0.75 & 1 \\ 1 & 0.75 \end{bmatrix}$$

The i -th column entries

Normalizing the largest participation factor as equal to 1 in each column; that means, largest is your 4 by 7 divide this all the element by 4 by 7. If you do so, it will be P normalised 0.75, 1, 1, 0.75, right.

So, thank you very much. We will be back again.