Power System Dynamics, Control and Monitoring
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Lecture – 25
Power System stability, Eigen properties of the state matrix (Contd.)

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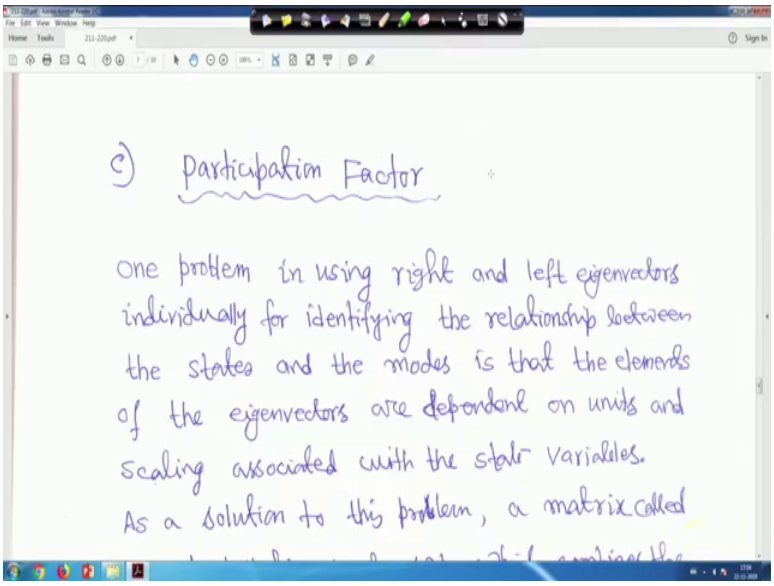

Ok, we are back again. In the previous lecture, we have seen that participation factor, but just starting again one or two pages before that such that our continuity will be maintained right. So, one problem in using your right and left eigenvectors right individually for identifying the relationship between the states and the modes is that the elements of the eigenvectors are dependent on units and scaling associated with the state variables.

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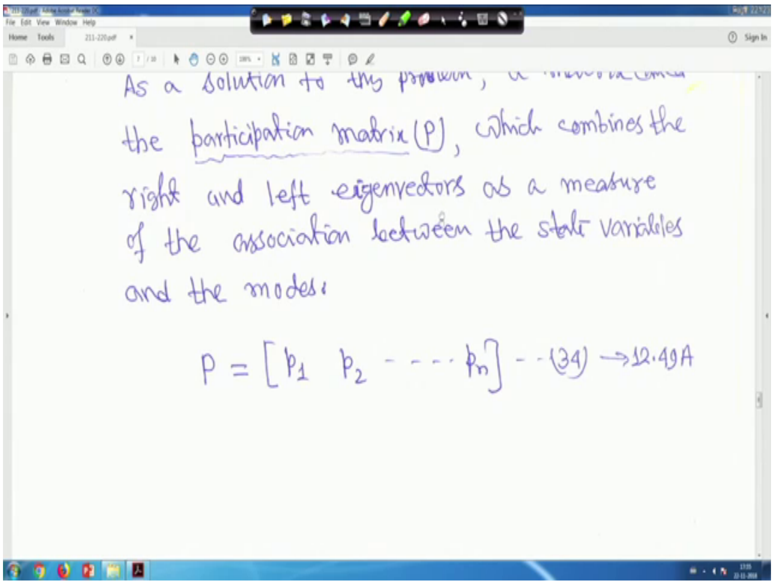

As a solution to this problem, this we have seen in the previous lecture a matrix called the participation matrix P, which combines the right and left eigenvectors as a measure of the association between the state variables and the modes. For example, suppose p 1, p 2 basically it is a column vector, and your what you call p 1, p 2 up to this p n, this is equation-34. This is nothing actually this is for my own reference.

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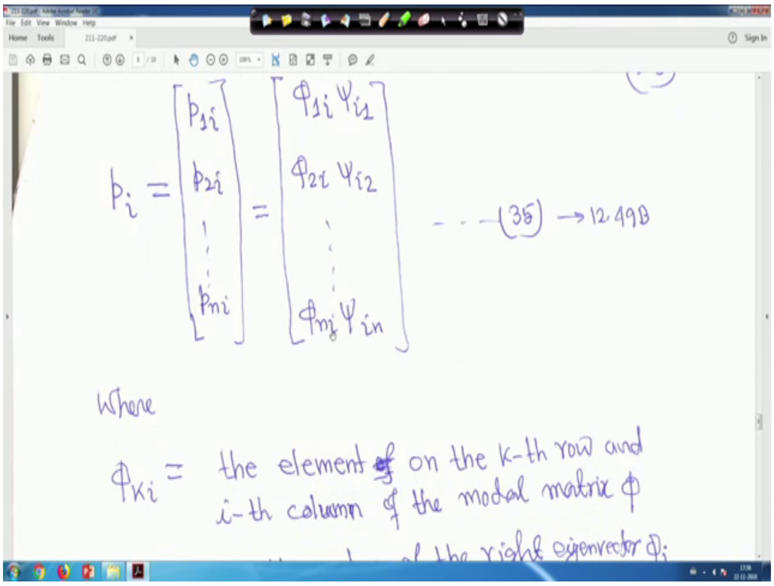

So, that means just hold on that means your p i, it can be given as p 1 i, p 2 i, up to p n i is equal to phi 1 i psi i 1, phi 2 i psi i 2 up to phi n i psi i n that means, basically it is a product of your right eigenvector and left eigenvector, this is equation-35 a this one nothing right, this is for my own reference this is nothing. Now, where in general here phi 1 i, phi 2 i, up to phi n i for psi i 1, psi i 2, up to psi i n.

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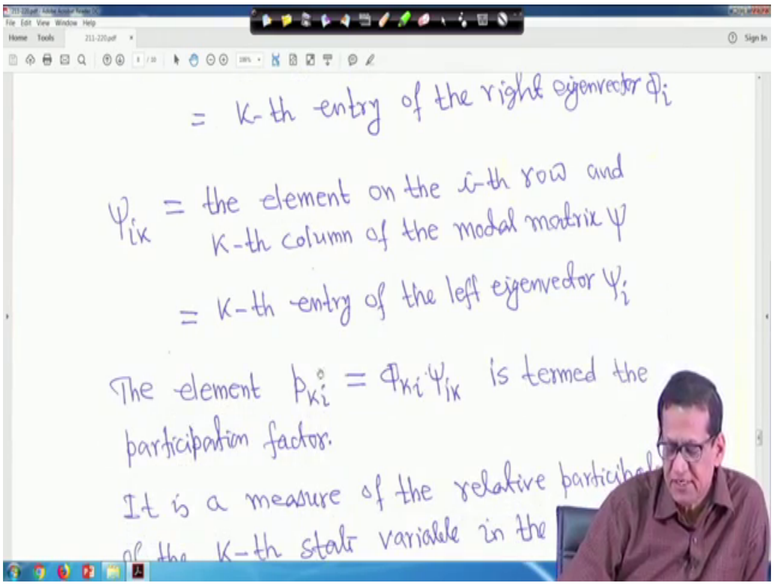

So, for k-ths row that phi k i is equal to the element on the k-th row and i-th column of the modal matrix phi right. So, from that suppose you have a phi matrix, so you can find out suppose K is equal to 2, and i is equal to 3, so that is phi 2, 3 something like this, is equal to the k-th entry of the right eigenvector phi i.

Similarly, psi i K the left eigenvector. The element on the i-th row and k-th column of the modal matrix psi is equal to k-th entry of the left eigenvector psi i. Suppose, here it is k is equal to 2 i is equal to 3, it will be phi 2 3. And here it will become psi 3 2 right. Now, the element p k i that is is equal to phi k i into psi i k is termed the participation factor in general.

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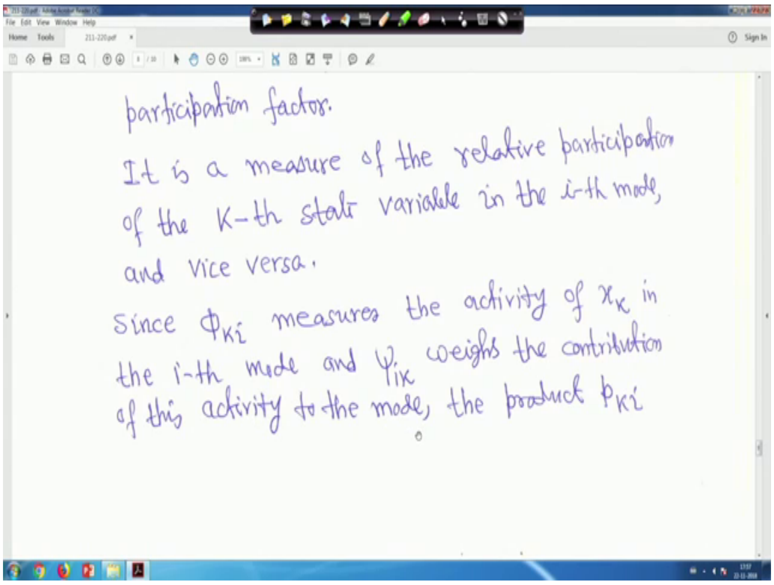

Basically, it will come in the matrix form. So, it is a measure of the relative participation of the k-th state variable in the i-th mode and vice versa. Since, phi k i measures the activity of x k your in the i-th mode and psi i k weights the contribution of this activity to the mode the product p k i measures the net participation.

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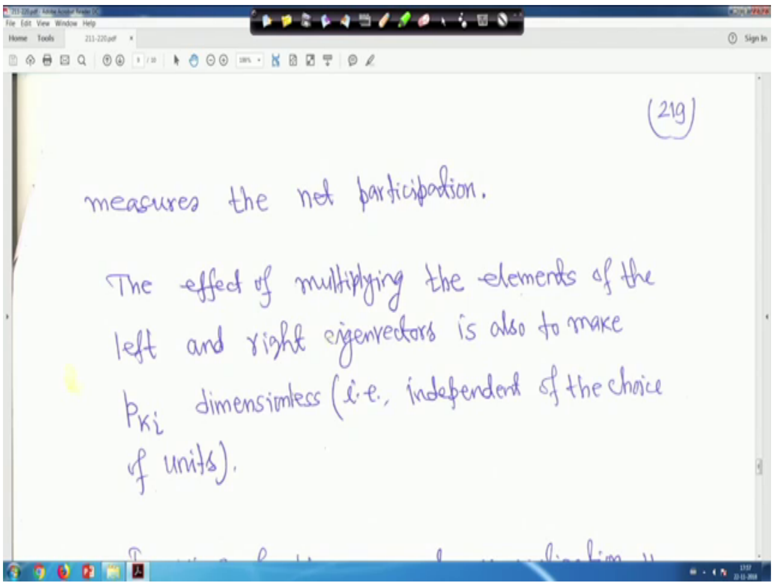

The effect of multiplying the elements of the left and right eigenvectors is also to make p k i dimensionless that is independent of the choice of unit.

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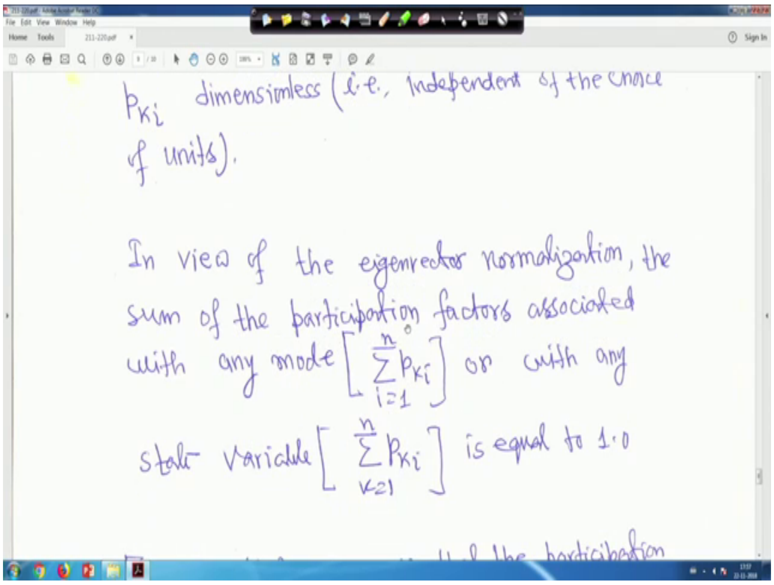

So, in view of the eigenvector normalization, the sum of the participation factors associated with any mode that is sigma i is equal to 1 to n p k i or with any state variable sigma k is equal to 1 to n p k i is equal to 1 that is after normalization, later also much more we will see after your this thing that what is your some different derivation but meaning is same.

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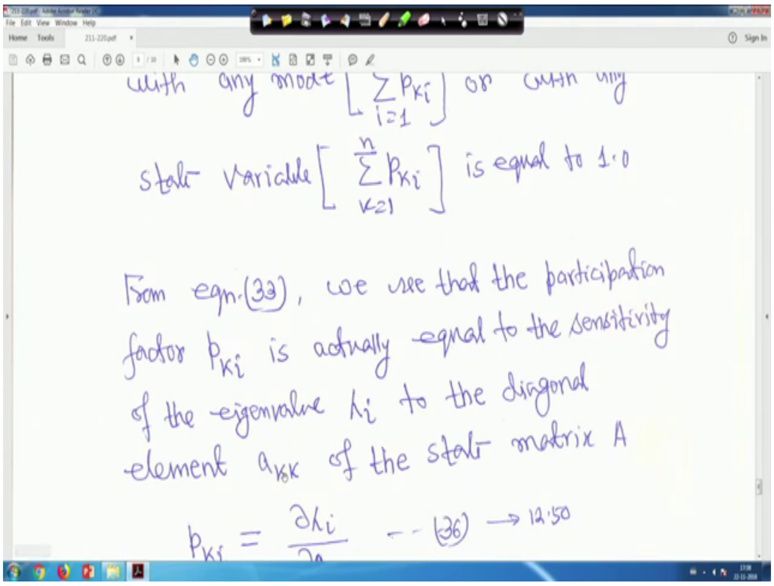

So, from equation-33, we see that the participation factor p k p k i is actually equal to the sensitivity of the eigenvalue lambda i to the diagonal element a k k of the state variable.

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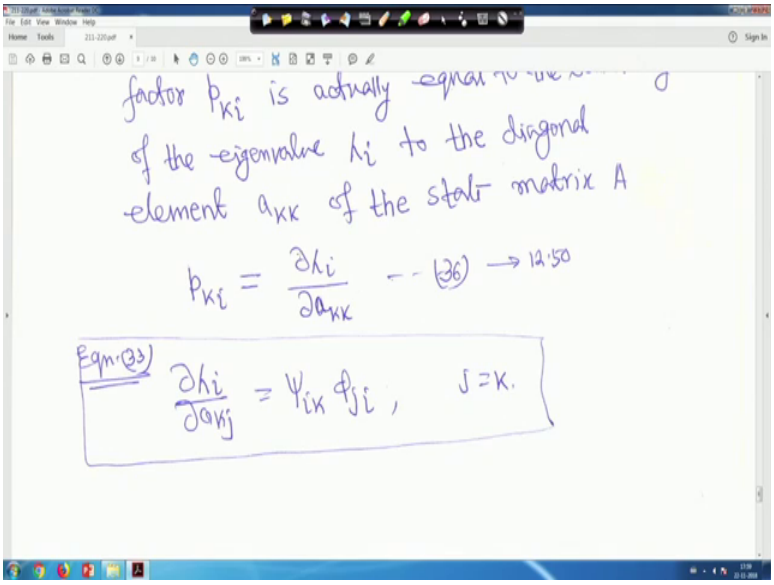

This we have seen that means p k i is equal to delta lambda i upon delta a k k, this is equation-36 right. And from equation 33 that delta lambda i upon delta a k j is equal to psi i k into phi j i, but for j is equal to k right, this is nothing and this is for my own reference. So, p k i will be delta lambda i upon delta a k k. So, just hold on.

So, after this that participation your what you call participation matrix pi for that example, which you took earlier that for (Refer Time: 04:55) state variable delta omega r and delta delta we will see, but before that just one page just for small thing, after that we will find out that thing.

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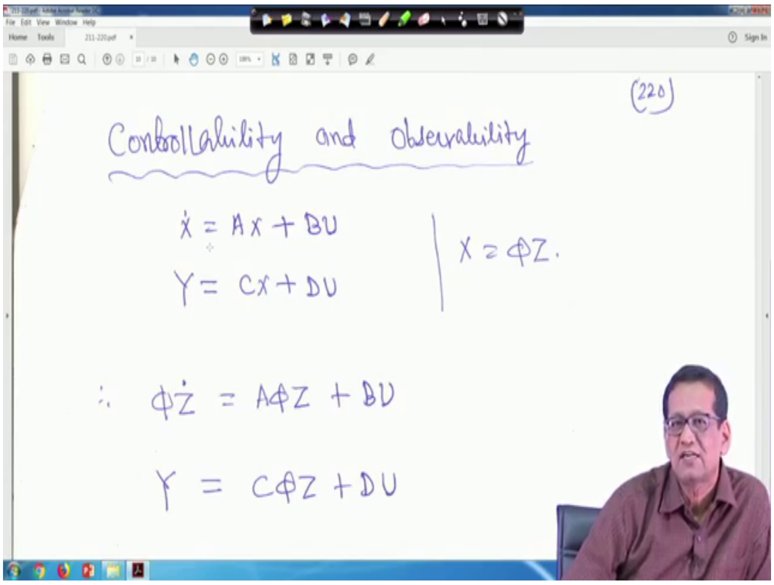

That controllability and observability. You know this from your undergraduate control system studies suppose X dot is equal to A X plus B U, and Y is equal to C X plus D U, where X is equal to phi Z right, this we have defined before.

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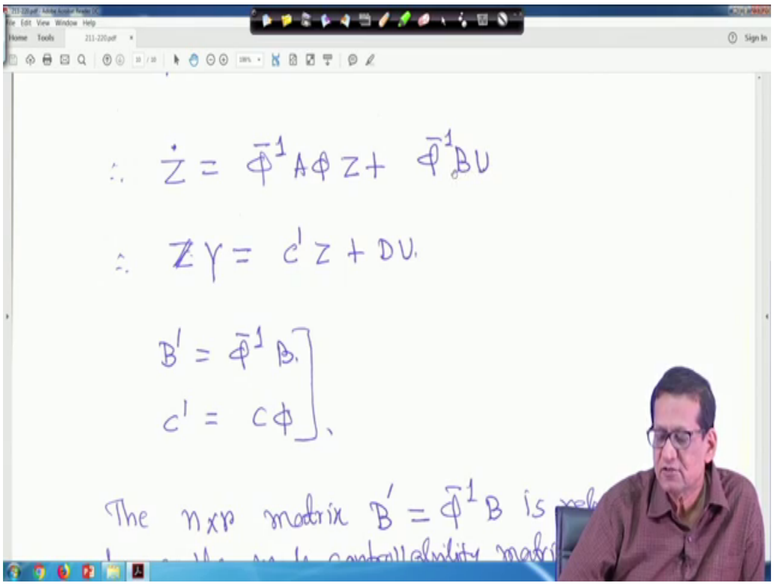

Therefore, if you put here X is equal to phi Z, it will be phi Z dot is equal to A phi Z plus BU or and this one this Y should be is equal to this Y this Y should be is equal to X is equal to phi Z, you are putting here X is equal to phi Z. So, it will be C phi Z plus D U.

Therefore, this equation both sides you multiply by phi inverse if you multiply by phi inverse both side, phi inverse phi inverse both side, so phi inverse phi is identity matrix. So, this side will be this side will be Z dot right is equal to phi inverse A phi Z plus phi inverse B U right or this output also thus your this thing Y is equal to C phi Z plus your D U that means, this one this is nothing this is just hold on this is actually nothing. It is Y is equal to we can write C dash Z plus D U. Therefore, C dash is is equal to basically your C phi from here C dash is equal to C phi right plus D U.

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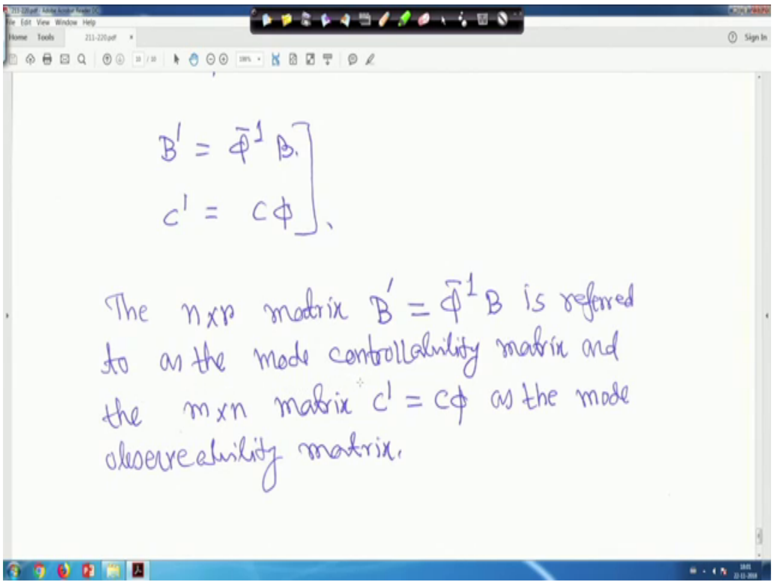

So, therefore B dash is equal to phi inverse B, because here also B dash is equal to phi inverse B, and C dash is equal to C phi. Therefore, the n into r matrix B dash is equal to phi inverse B is referred to as the mode controllability matrix, and the m into n matrix C dash is equal to C phi as the mode observability matrix right. So, this one your B dash is equal to phi inverse B, it is a mode controllability matrix and C dash is equal to C phi as the mode observability matrix, only this much only this much for this course not much in detail. Now, participation factor I have taken in the next page just hold on right, just hold on next page I have taken.

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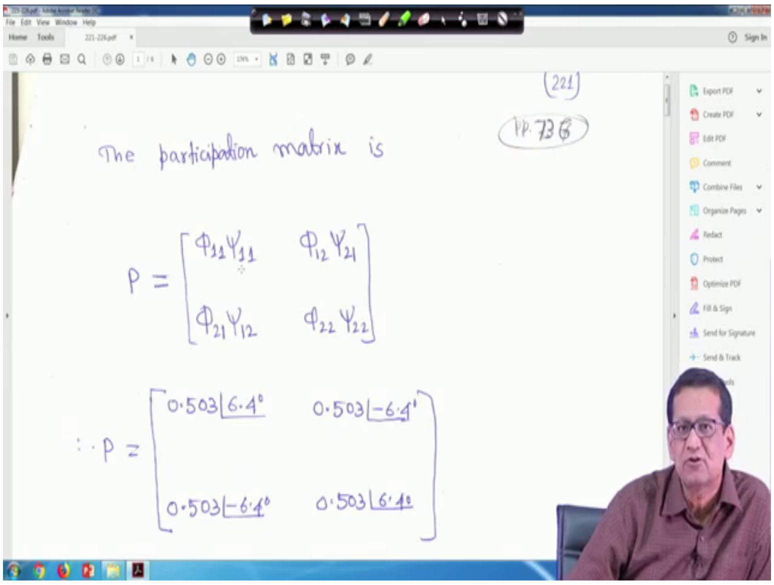

So, the participation matrix whatever we have got for 2 into 2 matrix phi matrix is known psi matrix is known. Now, it is actually phi 11 psi 11, then phi 12 psi 21 is easy to remember, if it is a 3 into 3 matrix that will be phi 13 psi 31 like this right. And here it will be phi 21 psi 12 and phi 22 psi 22.

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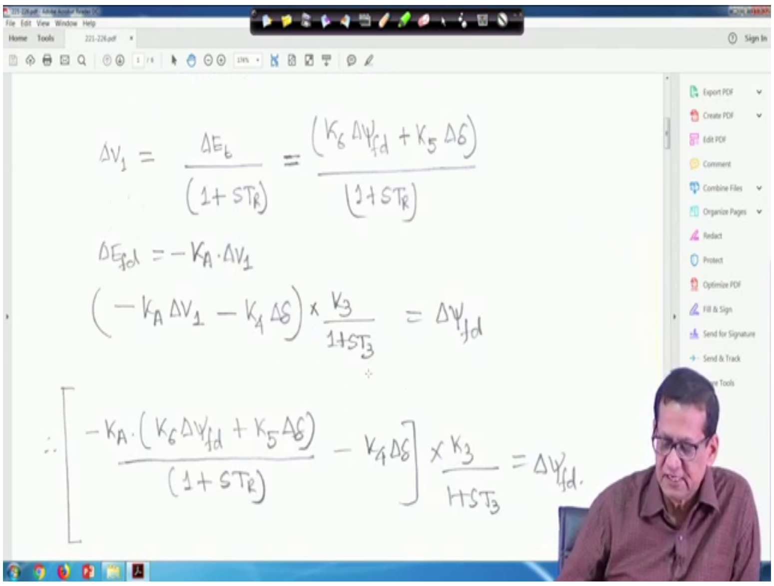

So, phi and psi matrix elements are known just multiply, your this one your P will get 0.503 angle 6.4, 0.503 angle minus 6.4, 0.503 angle minus 6.4, here also 0.503 angle 6.4. This is a simple 2 into 2 your matrix, later we will take a 3 into 3 that is why your what you call that is why, we looking at this delta omega r and delta delta they have equal participation in terms of this we take is a all are 0.503 right.

Now, after this, after this so up to your this thing participation factor, we will come to the participation factor again. But, before that one interesting thing we will try to see that is that is your what will be the steady state value of your delta omega r right, what is the steady state value. I mean it is steady state value of delta omega r that is your delta omega r ss right. So, for this we will we will go to your I think it will go to figure-39, before coming to this just hold on before coming to this we will go to figure your this thing just hold on this figure.

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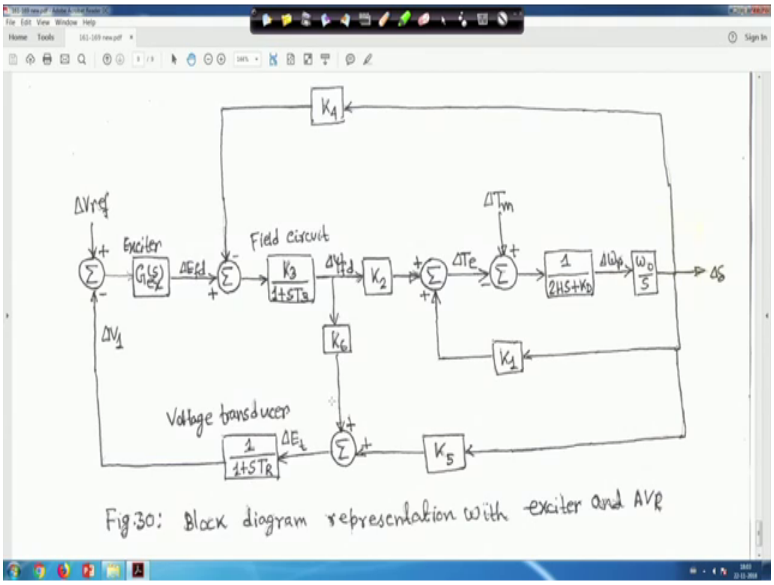

So, this is actually your figure-30. So, in this case what you have to do is for delta V reference you take as a 0, delta V reference we will take 0 that means, these value we will take as a 0 delta V reference right. And then this delta T m, we will take a step input. If we take a step input that means, when you take the Laplace transform step input, it will be delta T m by S.

Then we have to find out what is the value of delta omega r steady state value right, these are the thing other parameters are given. For example, these value K is given right. And for simplicity we may neglect this one. If we want we may neglect T R, because T R is very small. So, whenever I am going to this only from this figure actually, we will derive this that what is your what you call that delta omega R in terms of delta T m considering all other parameters K 1, K 2, K 3, K 4 right K 5, K 6 all were given before we will use the same parameter.

So, this is figure-30 and page number 169. So, if it is so just hold on your this thing, this is actually if you go to this thing actually forget about this thing, so if you go to this that delta V 1 that is your figure-30 you go to figure-30 right, and page number is 169. So, what you do is that your just hold on hold on. So, first you find out look at this figure your 30 delta V 1 will be delta E t upon 1 plus S T R that is your K 6 delta psi f d plus K 5 delta delta upon 1 plus S T R.

Now, from that block diagram delta V reference is equal to 0, I told you that figure-30 you will when you will listen to this you open figure 30 that we have consider say delta V reference is equal to 0. If it is so, then delta E f d will be minus K into delta V 1 right is equal to your that means, and delta V 1 is equal to this thing, then you substitute here. So, it will become your minus your K A delta V 1, and your what you call this delta V 1 will be K 6 delta phi psi f d plus K 5 delta delta. And delta V 1 is K 6 delta phi f d plus K 5 delta delta plus 1 plus your S T R right.

Then this one delta psi f d, it can be this is delta E f d and delta psi f d let me go to the figure once again right just hold on. Let me go to the figure once again, then I will come back. But, when we will listen to this, you will just open this one.

For example, this figure the delta when delta E f d, this delta E f d delta V reference is 0, this is delta V 1, and this is your K A right. So, it is minus feedback. So, delta E f d is equal to your minus K A into delta V 1 this is delta V 1. And delta V 1 is equal to that K 6 delta psi f d plus K 5 delta delta into 1 upon 1 plus S T R.

Now, delta psi f d if you write, I am writing over it delta psi f d right delta psi f d will be that this side it will come K 4 your what you call this side delta E f d, this is delta E f d and this side minus K 4 delta delta into your K 3 upon this one 1 plus S T 3 right. So, same thing we are writing there same thing we are writing, this is the block diagram just step by step you move.

Therefore, this equation we are writing that delta psi f d is equal to minus K A delta V 1 minus K 4 delta delta into your K 3 upon 1 plus S T, because delta E f d is equal to this one minus K A delta V 1 that is right. So, now if you now say delta V 1 just hold on, let me think now delta V 1 is given here this delta V 1 is given here right K 6 delta psi f d plus K 5 delta delta upon 1 plus S T R right.

So, you substitute this delta V 1, this whole thing this whole thing delta V 1 you substitute here. This is what has been written here minus K A into K 6 delta psi f d plus K 5 delta delta upon 1 plus S T R minus K 4 delta delta into K 3 upon 1 plus S T 3 is equal to delta psi f d right, next is you simplify.

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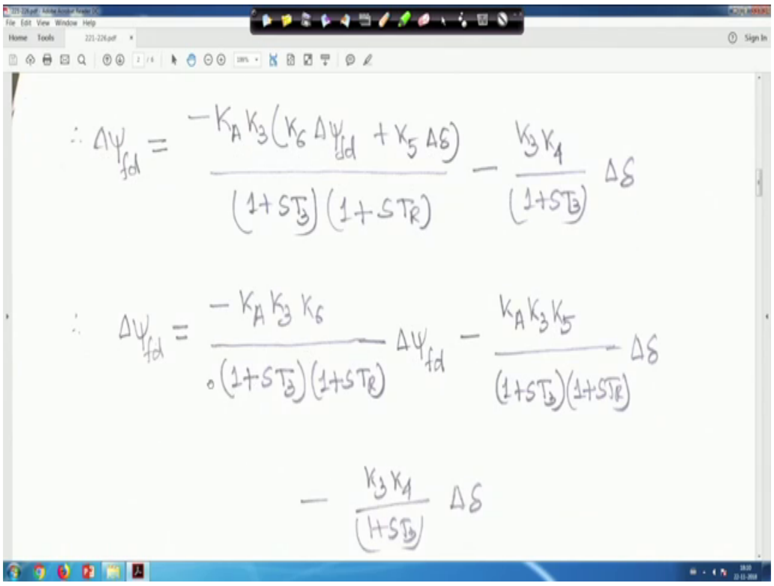

Now, therefore delta psi f d can be written as minus K A K 3 after simplification into K 6 delta psi f d plus K 5 delta delta divided by 1 plus S T 3 into 1 plus S T R minus K 3 K 4 upon 1 plus S T 3 delta delta right or you can further simplification delta psi f d will be minus K A K 3 K 6 upon 1 plus S T 3 into 1 plus S T R delta psi f d minus K A K 3 K 5 upon 1 plus S T 3 into 1 plus S T R delta delta minus K 3 K 4 upon 1 plus S T 3 delta delta right.

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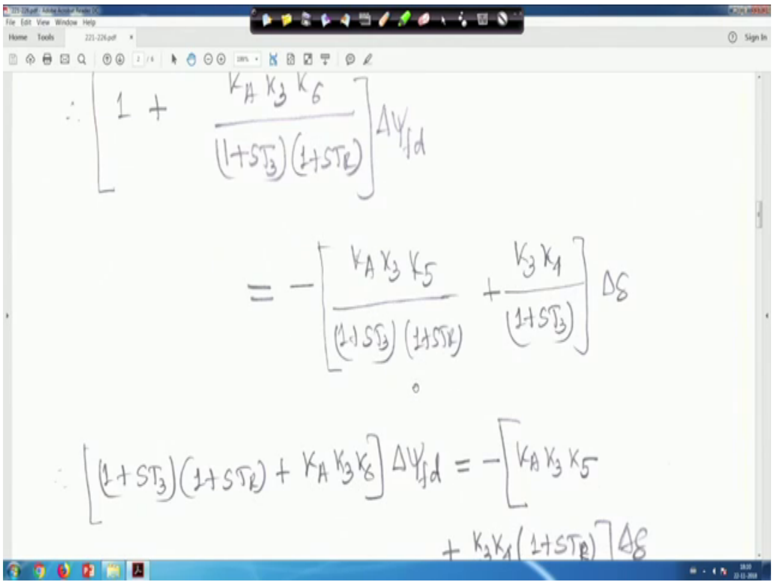

So, once this is done, then after simplification you will get that all the delta psi f d term moving to the left hand side. So, 1 plus K A K 3 K 6 upon 1 plus S T 3 upon 1 plus S T R delta psi f d is equal to minus in bracket K A K 3 K 5 upon 1 plus S T 3 into 1 plus S T R plus K 3 K 4 upon 1 plus S T 3 bracket close delta delta.

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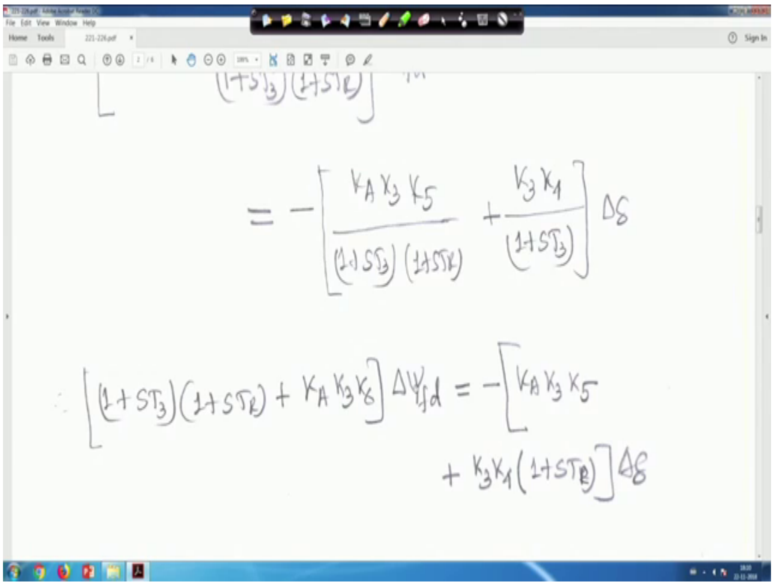

Now, further simplification will give you this 1 plus S T 3 into 1 plus S T R plus K A K 3 K 6 into delta psi f d is equal to minus in bracket K A K 3 K 5 plus K 3 K 4 1 plus S T R into delta delta.

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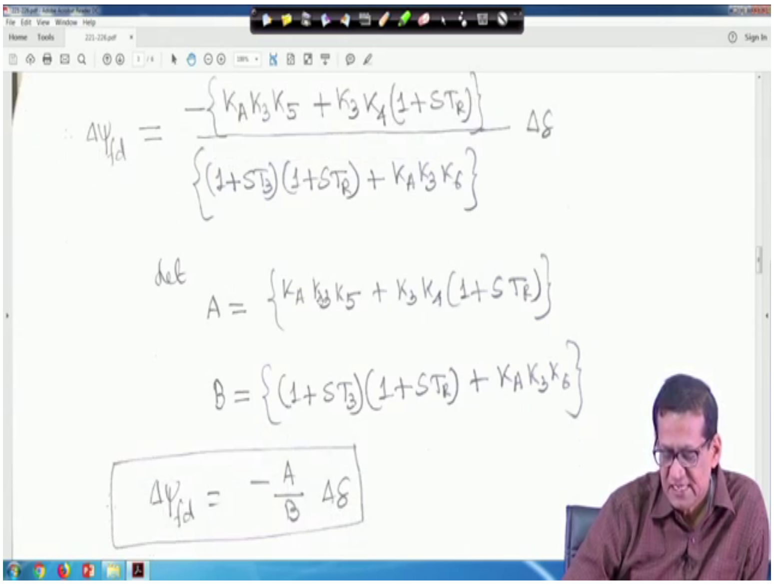

Now, therefore delta psi f d, we can write that minus K A K 3 K 5 in bracket plus K 3 K 4 into 1 plus S T R divided by bracket close divided by 1 plus S T 3 into 1 plus S T R plus K A K 3 K 6 into delta delta. Now, you assume A is equal to whatever is there in the denominator right. And B is equal to we assume that your sorry A is equal to numerator that is your K A K 3 K 5 plus K 3 K 4 1 plus S T R that is in numerator. And denominator B is equal to 1 plus S T 3 1 plus S T R plus K A K 3 K 5, K A K 3 K 6.

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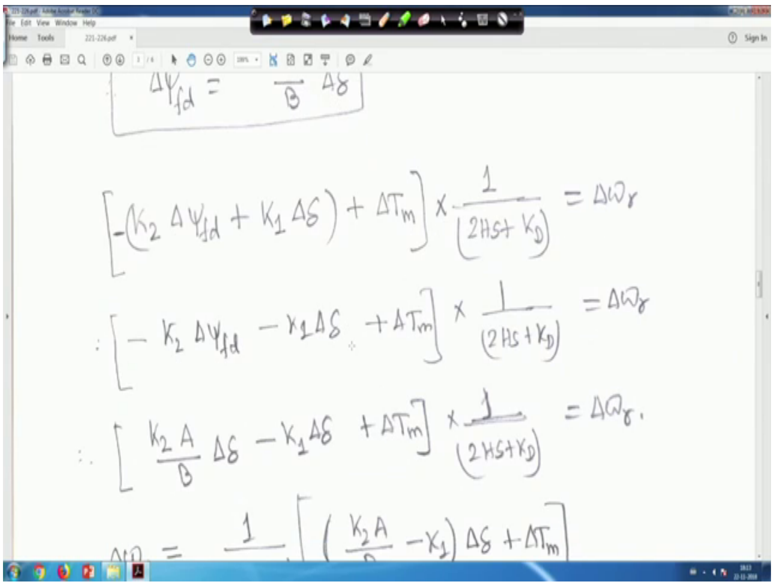

Therefore, delta psi f d can be written as minus A by B delta delta. Now, this one that your what you call if you go back to the your again you go back to that your what you call that figure-30, page number 169 from there you can write minus K 2 delta psi f d plus K 1 delta delta plus delta T m into 1 upon 2 H S plus K D just let me go through that right let me go through that page your 169 here.

Now, here from here you can easily make it that K 2 delta psi f d right K 2 delta psi f d, then here it is coming your what you call K 1 delta delta, so it is minus here right. So, minus K 2 delta psi f d minus K 1 delta delta plus delta T m into 1 upon 2 H S plus K D is equal to delta omega r. So, so from this from that only we are writing that equation.

So, so from this we are writing this equation that your this one minus K 2 delta psi f d plus K 1 delta delta bracket close plus delta T m into 1 upon 2 H S plus K D is equal to delta. But, delta T m, so far it is actually step input later we will replace for easy of analysis later delta T m has to be replaced delta T m upon S, because it is your step input I have not put yet delta T m by S at the end I will put it.

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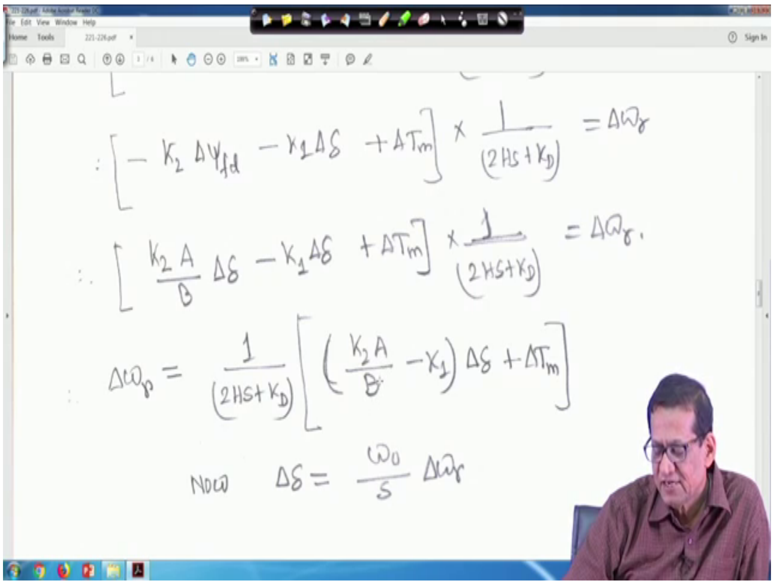

So, in this case what you call that now upon simplification all set of simplification that delta omega r will become 1 upon 2 H S plus K D in bracket that K 2 upon A K 2 A upon B minus K 1 bracket close delta delta plus delta T m. And from the figure-30 again we know that delta delta is equal to omega 0 S by S into delta omega r right. So, what we will do it that we have to substitute or replace this delta delta by omega 0 upon S delta omega r, and simplify.

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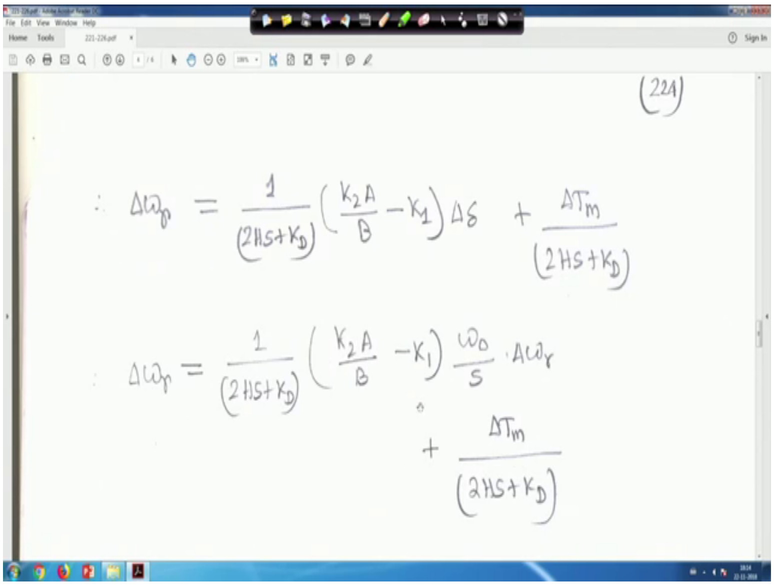

So, if you do so if you do so that delta omega r will become like this 1 upon 2 H S plus K D K 2 upon A B minus K 1 delta delta plus delta T m upon 2 H S plus K D right. So, now delta T m that delta omega r, it is 1 upon 2 H S plus K D, this is K 2 A upon B minus K 1 and delta delta we replace upon omega 0 upon S delta omega r, this I told you this delta omega S should be replaced by this one plus delta T m upon 2 H S plus K D.

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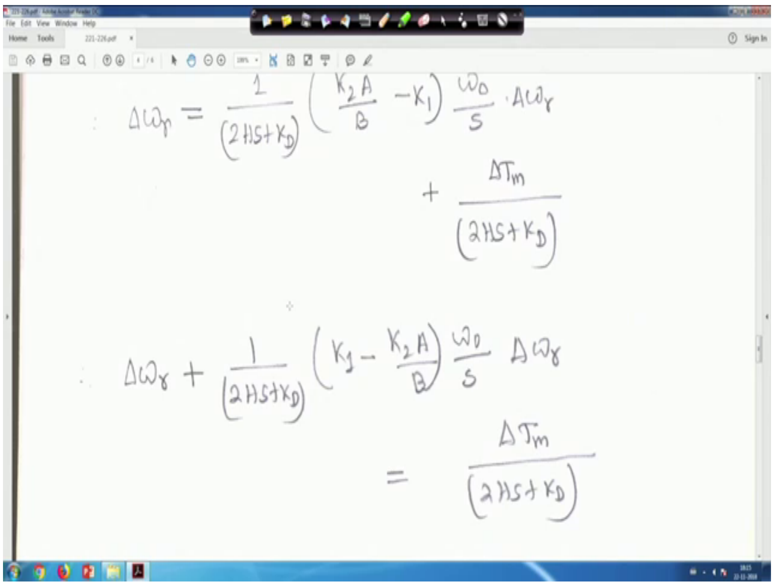

That means, that if you simplify this I mean bring this delta omega r term to the left hand side and simplify right, then what will happen that equation can be written like this right hand side will be delta T m upon 2 H S plus K D.

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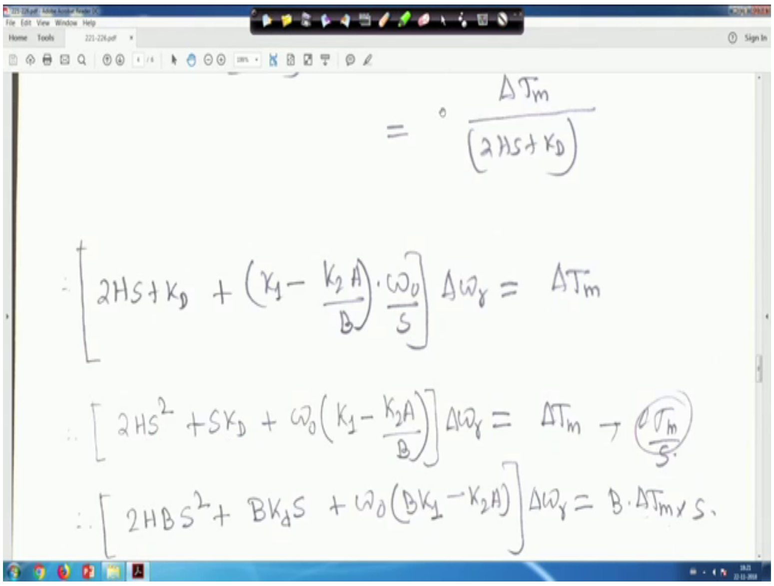

So, this one actually first what you can do is that delta T m delta T m by S that will replace later first you see this. This S is there, so both side you multiply your by B S right. So, it will be your what you call if you multiply both side by B into S first we have multiplied both side by S right, so if you multiplied both side by S, so it will be 2 H S square plus S K D plus omega 0 your K 1 minus K 2 A upon B S will not be there right omega 0 is there. So, both side if you merge this and this will be S into delta T m, this is we will see later. Now, your what you call now both side you multiply by B. If you do so, it will be 2 H B S square plus B K d S plus omega 0 B K 1 minus K 2 A delta omega r is equal to B into delta T m into S.

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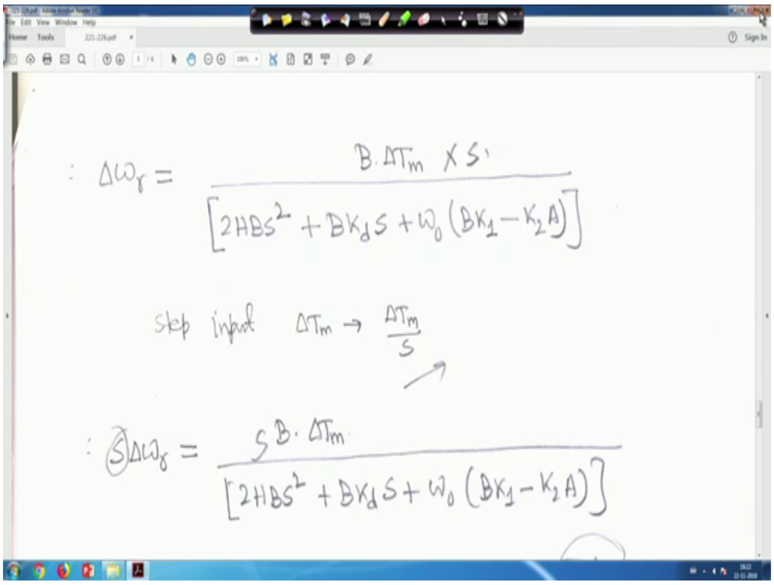

So, that means your this delta omega r actually B delta T m into S. Now, step input delta T m actually now we will substitute delta T m by S right. So, if you do so, you substitute here if you that numerator that is your B term is there B term is there and delta T m by S and another thing is S is there.

So, basically S S will be cancel. And numerator it will be your simply B delta T m that means, let me clear it that means, your numerator will simply become your B delta T m right numerator will simply become B delta T m right, and multiply both side by now S. Therefore, your delta omega r steady state is equal to limit S tends to 0 your what you call S delta omega r. So, this is your nothing but delta omega r ss right, and this side limit S tends to 0.

So, this numerator S tends to 0 means, it will be 0 by something. So, steady state error of delta omega r S s will be 0 right that means this value will be 0 for delta v reference right. So, this is actually I tried of my own actually book it is not given just check one thing that your delta omega r S s will be 0 right.

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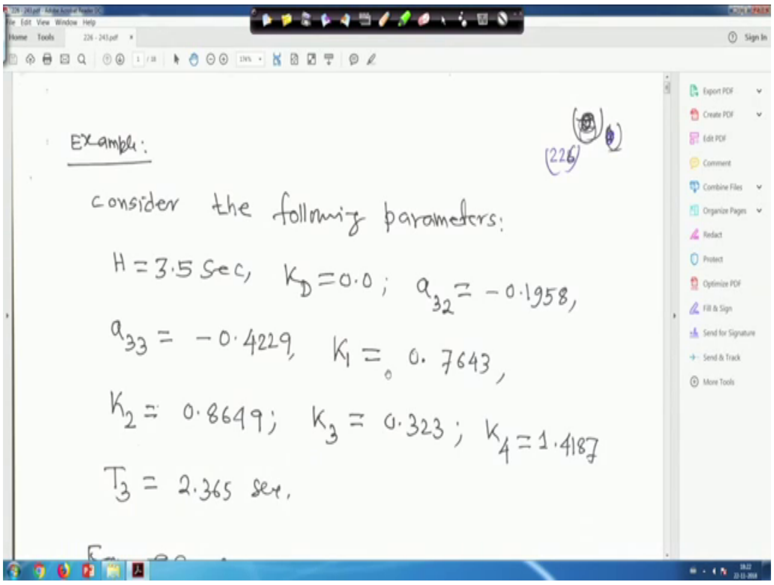

So, next will be again we will come to this that your participation example one example, then we will take the participation factor now example. So, consider the following parameters. Now H is equal to say 3.5 second, K D damping coefficient is taken 0.0, and suppose a 32 is given I am made things simplified for you a 32 is given minus 0.1958, a 33 is equal to minus 0.4229, K 1 is 0.7643, K 2 is 0.8649, K 3 is 0.323, and K 4 is 1.4187, and T 3 is 2.365 second with all this parameter, you please go back to equation 267, and please form the a matrix right.

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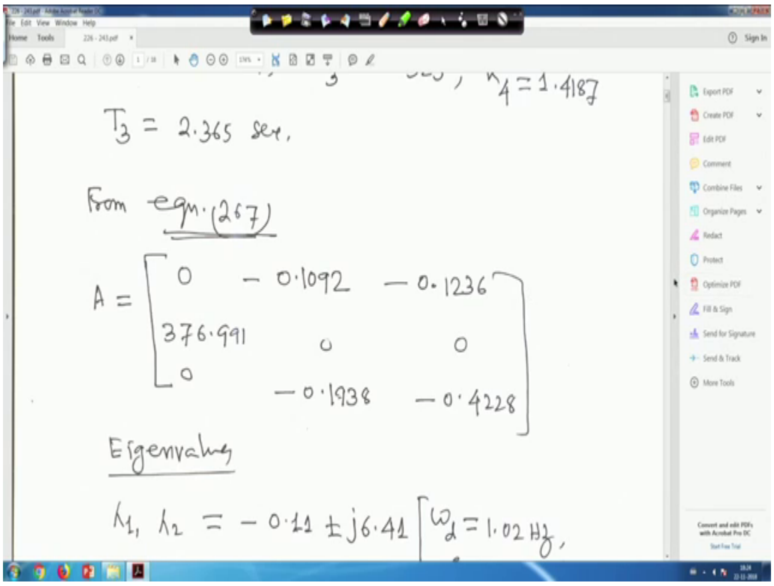

So, if you form the A matrix, so A will be 0, this will be minus 0.1092 minus 0.1236. This is 60 hertz frequency we have taken, so this is 376.991, this is 0, this is 0. And here it is 0 minus 0.1938, and this is your minus 0.4228.

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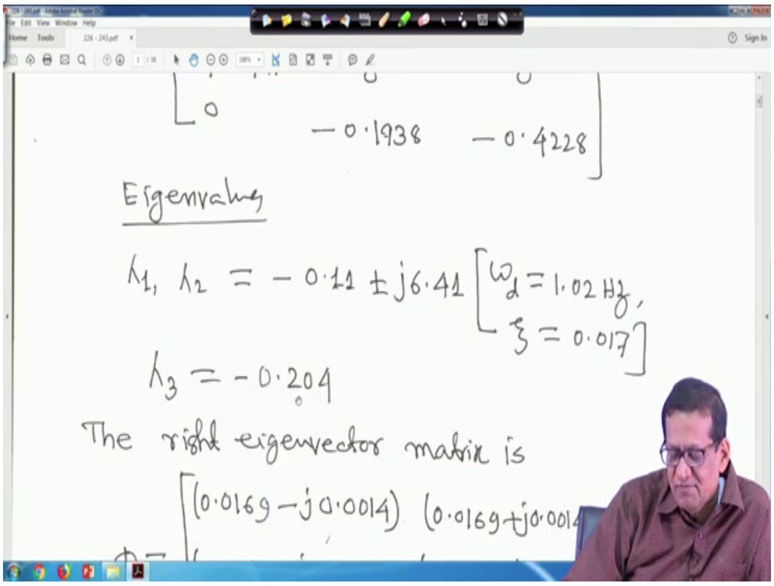

Now, if you try to find out the eigenvalues of this matrix right, if you try to find out eigenvalues of this matrix, so it is lambda 1 2 Eigen three eigenvalues are there, so one is complex conjugate pair that is minus 0.11 plus minus j 6.41. From here we see that omega d will come 1.0 hertz, you know omega d is equal to omega n root over 1 minus psi square. And psi will be damping ratio 0.017 based on that you can easily calculate, I did not give the calculation, but previously we have done it right.

So, and the another eigenvalue is the real eigenvalue lambda 3 is minus 0.204 right I do not know maybe you will be knowing one thing that regarding matrix. If all the eigenvalues of a matrix right, your what you call I mean if the matrix is a real and symmetric, then you will find eigenvalues are real right. But, if it is not, then it maybe it may be real, it maybe some complex conjugate pair right.

Another thing of there interesting property is there for matrix, before proceeding further. Suppose, you have a matrix right and your you suppose your you are getting a suppose you have a 8 into 8 matrix some matrix you have now you are getting eigenvalues something that is symmetric real symmetric matrix right, it is a real symmetric matrix.

And suppose you are getting eigenvalues of this supposing if one is minus 3, another is plus 3 right. If one is plus 2, another is minus 2 right. If one suppose 8 such eigenvalues are there, if one is plus 1, another you are getting minus 1. If one is say 2.2, another you are getting minus 2.2, this may happen for some special characteristic of the matrix right.

So, if you get this kind of eigenvalue if I mean plus minus plus minus pair this if you get the eigenvalues right, then whenever you are trying to find the eigenvalues for control system that is that is basically you are trying to find out the characteristic equation right. And in that case the characteristic equation is an even function that is why, Eigen value may come like this right. I do not know book may be given, I do not know, but this is for some to improve our general knowledge, I am telling you this right.

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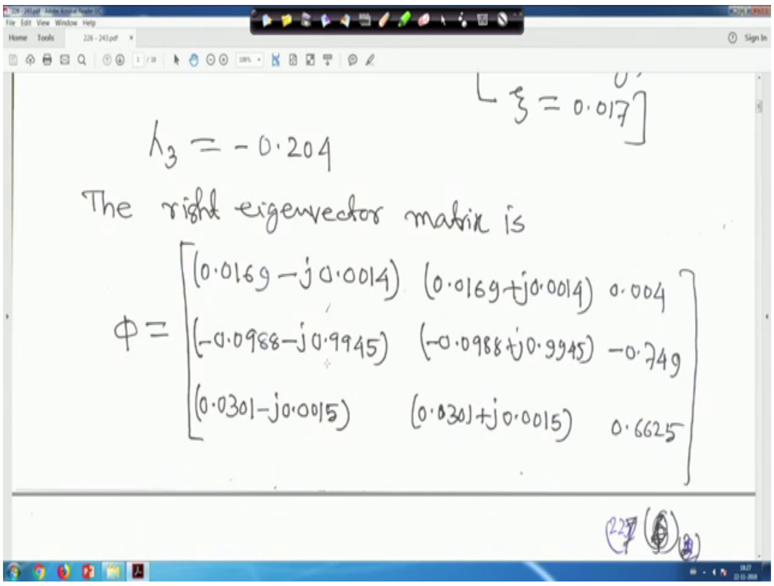

So, and another real value that lambda 3 is equal to minus 2.204. Now, that we have already we have already shown how to find out right and left eigenvector. So, so right Eigen eigenvector matrix is you will get phi is equal to that is 0.0169 minus j 0.0014, then 0.0169 plus j 0.0014, and this is 0.0014. Similarly, phi 2 1 minus 0.0988 88 minus j 0.9945 minus 0.0988 plus j 0.9945, because two eigenvalue complex conjugate pair is there.

So, if it is a minus here, if it is a plus; if it is a minus here, if it is a plus. And last one is one real eigenvalue is there, so it is minus 0.749. And similarly, the last one 3 1, 3 2, and 3 3 is a 0.0301 minus j 0.0015. And here it is also 0.0301 plus j 0.00 these two are complex conjugate actually this one and this one right. And last one is 0.6625 this is your right Eigen vector matrix.

Thank you very much. We will be back again.