Power System Dynamics, Control and Monitoring
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Lecture – 24
Power System stability, Eigen properties of the state matrix (Contd.)

So, we are again we are back, right.

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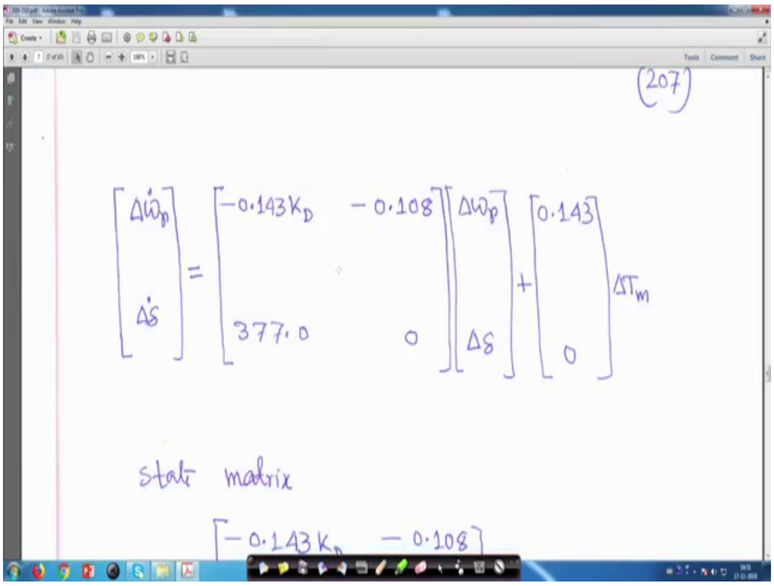

So, this is actually second order system.

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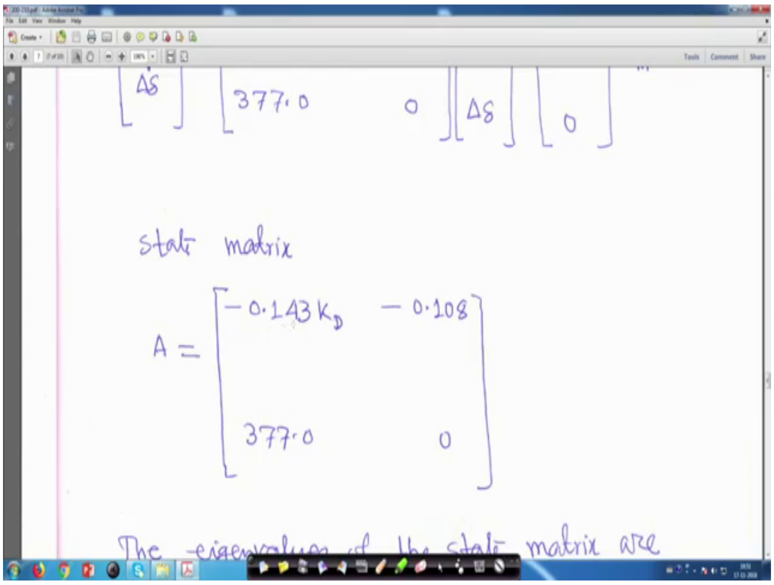

Now, the state matrix A is equal to this much whatever we have got here this is my state matrix.

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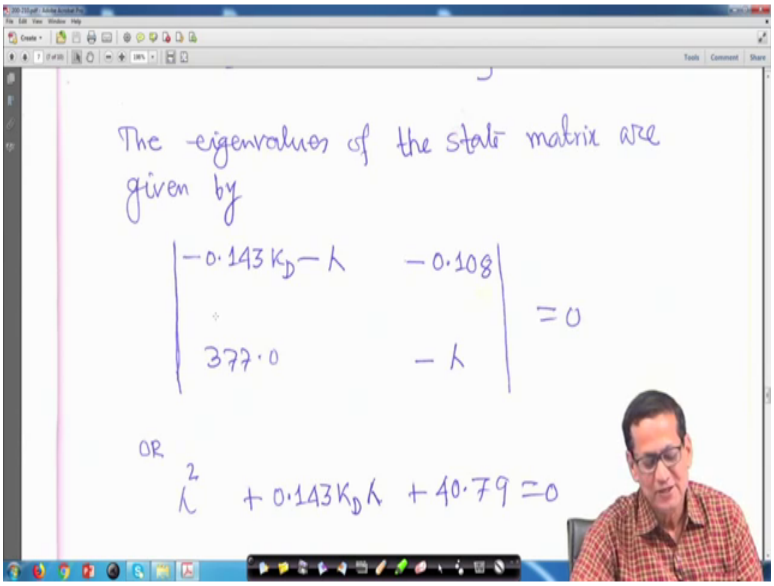

So, the Eigenvalues of the state matrix can be written as I mean you have to say determinant 0. So, minus 0.143 K D minus lambda minus 0.108 then 377 minus lambda the determinant of this one is equal to 0, right. I mean your A minus determinant of A minus lambda is equal to 0, right.

So, if you simplify this equation it will be lambda square plus 0.143 K D lambda plus 40.79 is equal to 0. If you simplify it will come like this. It is simply quadratic equation, right.

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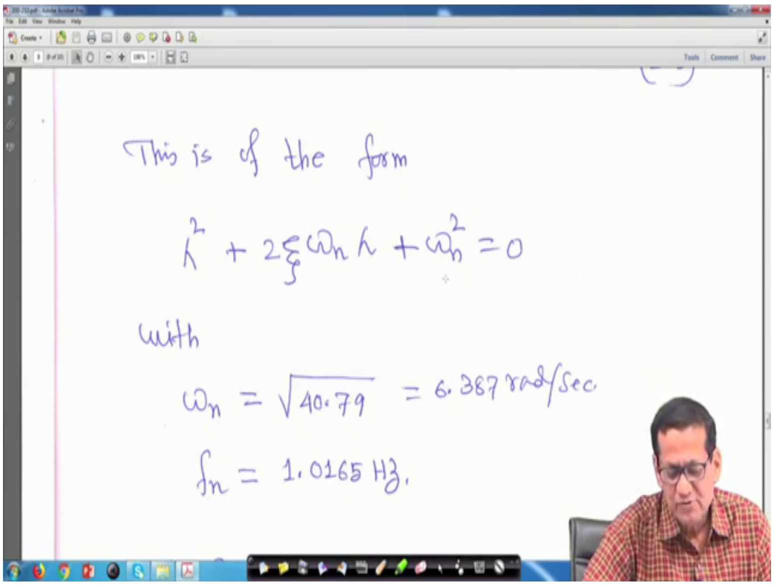

Now, this equation is of the form of we can write lambda square plus 2 xi omega n lambda plus omega n square is equal to 0. I mean this equation we can put it like this.

So, if you compare, then with omega n here it is we have made omega n square is equal to 40.79. So, omega n is equal to root over 40.79 that is 6.387 radian per second, right and if you divide by 2 phi then, f n is equal to natural frequency will be 1.0165 hertz, right. And 2 xi omega n is equal to nothing, but 0.143 K D, right and first we have to see K D is equal to 10. Three values are given K D is equal to 0, K D is equal to minus 10 and K D is equal to plus 10, we will take first K D is equal to plus 10 right.

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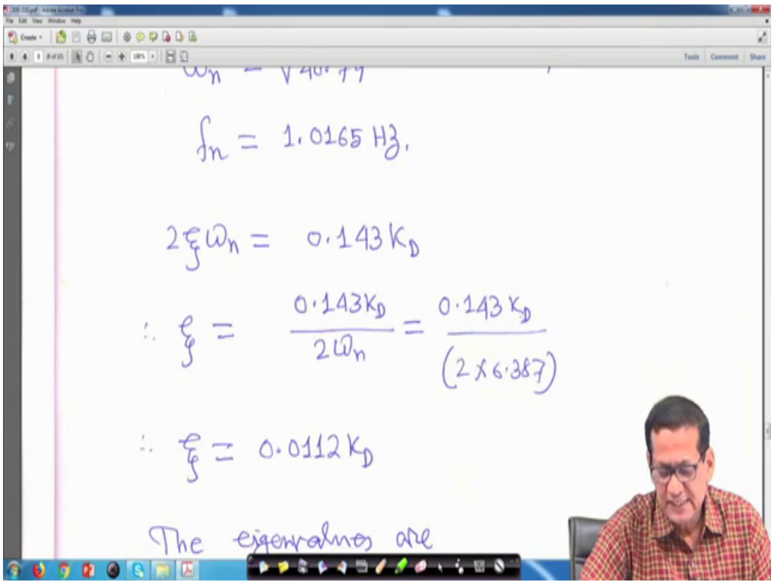

So, and 2 xi omega n is equal to 0.143 K D or xi is equal to 0.143 K D upon 2 omega n. So, omega n we have got it 6.387 therefore, xi is equal to 0.43 K D upon K D upon 2 into 6.387 and this comes actually point 0.0112 K D that is equal to xi, right.

Now, in terms of K D; if you put K D is equal to 10 it will be 0.112 if you K D is equal to 0 then xi will be 0. If you put K D is equal to minus 10 it will be minus 0.112, right.

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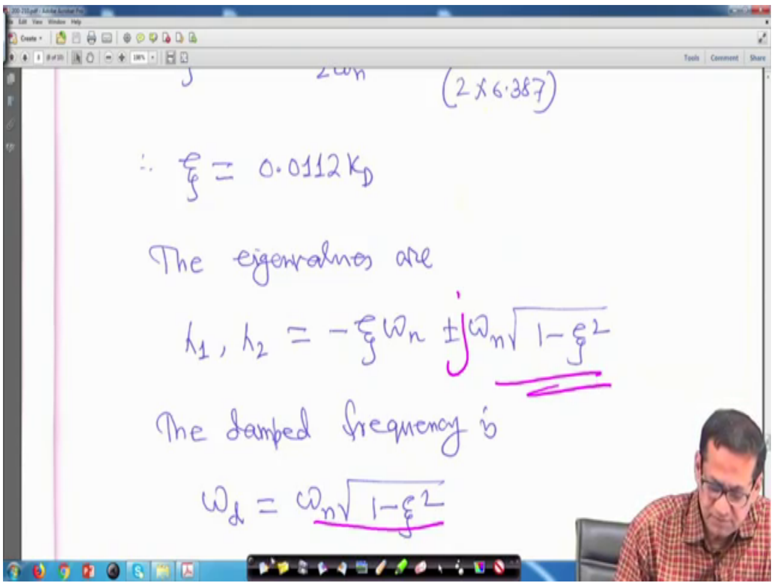

So, Eigenvalues are lambda 1, lambda 2 it can be written as minus xi omega n plus minus one thing is there that is your j is missed, right. So, minus xi omega n plus minus j omega n root over 1 minus xi square, right.

So, the damped frequency is that omega d is equal to omega n root over 1 minus your xi square that is this one this one, right and so, I have corrected that one.

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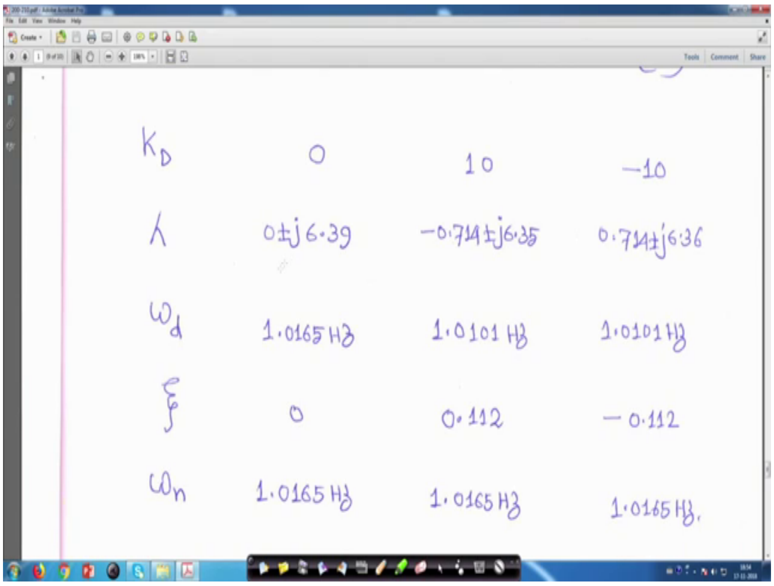

And, now for K D is equal to 0, 10 and minus 10 that lambda will be like this. So, when K D is equal to 0 lambda will be it is it is a 2 into 2 system only two Eigenvalues. So, lambda will be 0, real part is 0 plus minus j 6.39. Now, when K D is equal to 10, it will be minus 0.714 plus minus the complex conjugate pair j 6.35 if K D is equal to minus 10 it is 0.714 plus minus j 6.36; that means, with K D is equal to minus 10 actually system becomes unstable.

Now, with that if you calculate omega d, so, it is 1.0165 hertz when it is K D your is equal to 0, but when it is 10 or minus 10 it will remain same 1.0101 hertz, it is also 1.0101 hertz, right and for K D is equal to 0 xi will be 0 here it will be 0.112 and here it will be minus 0.112 and omega n will be all this case 1.0165 hertz, 1.0165 hertz, 1.0165 hertz. So, these results are given in tabular form, right.

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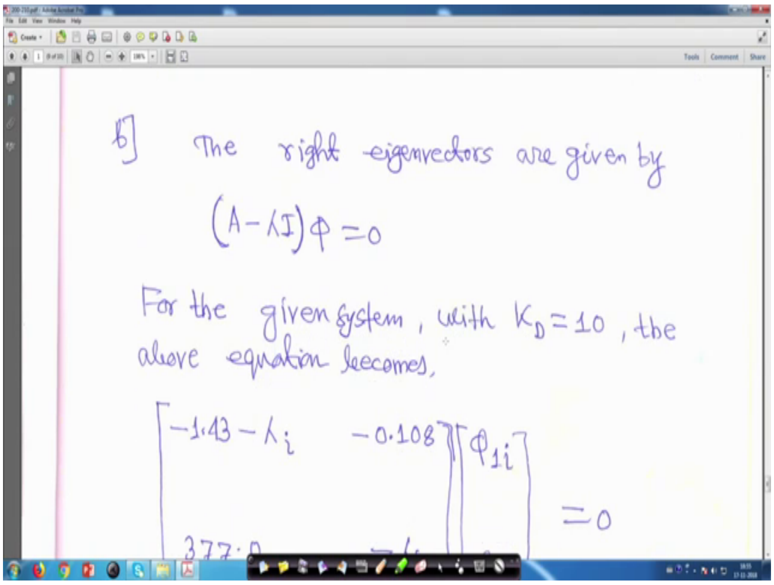

Now, part b: the right eigenvectors are given by that is A minus lambda I into phi is equal to 0. This we have also seen to get this you know where the beginning the right Eigenvector left Eigenvector, right. So, if you get the right Eigenvector you will get the left Eigenvector also.

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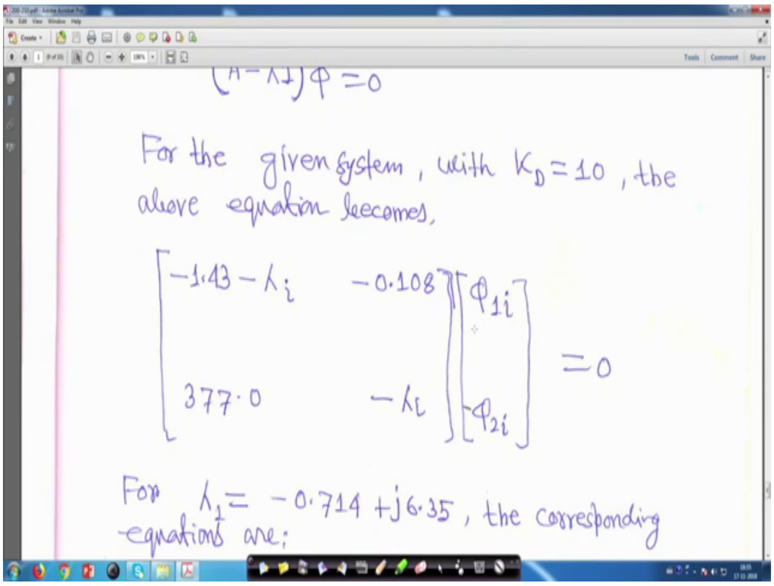

So, from the given system with K D is equal to 10, the above equation become. So, when K D is equal to 10. So, this equation I mean this a minus lambda i into phi becomes that minus 1.43 minus lambda i minus 0.108 377 and minus lambda and here it will be phi 1i, phi 2i, right this is for your what you call for i-th Eigenvalue say right for is equal to 0, but K D is equal to 10 means we will consider this Eigenvalues, right.

So, now for lambda 1 we are considering your positive imaginary part that is minus 0.714 plus j 6.35. The corresponding equations are if you assume lambda 1 is equal to this and write down the equation, right.

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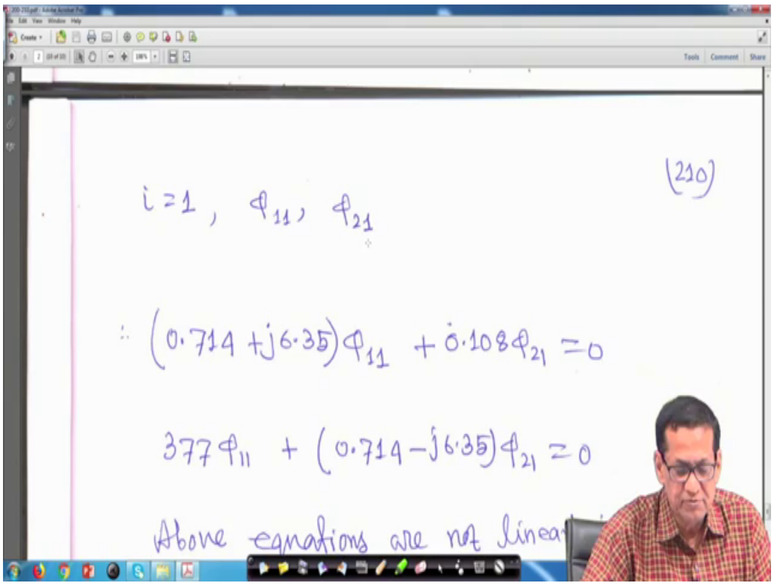

Then, you will get for i is equal to 1, i is equal to 1 means and there should not be any confusion this is lambda one the first Eigenvalue; that means, i is equal to 1, right. So, that is why that is why this is phi 11 and this is phi 21, we have written phi 1i phi 2i is equal to 0, right. So, that is phi 11 phi 21.

So, if you write this equation this is a that means, for i is equal to 1, this is phi 11 this is phi 21 ; that means, minus 1.43 minus lambda i into phi 11 minus 0.108 into phi 21 is equal to 0. This is one equation, right.

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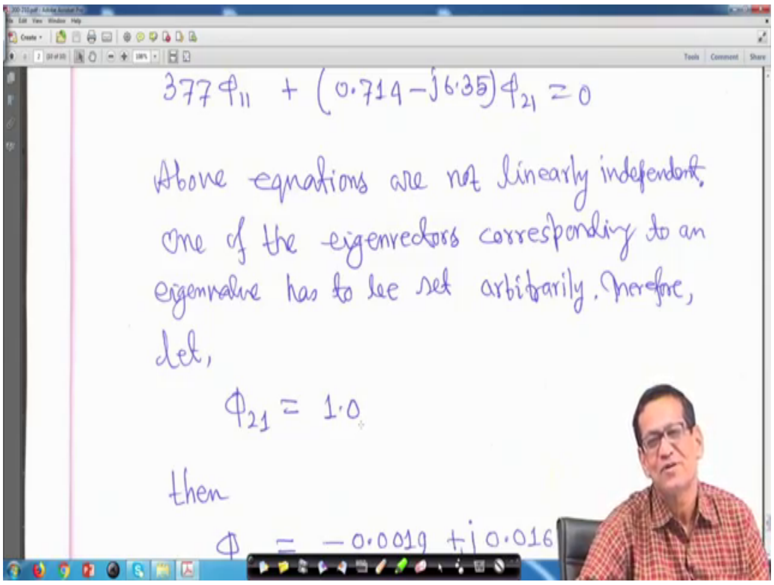

So, your the and the your what you call I mean if you simplify this will be plus because here it is minus is here minus both the cases it is minus is here minus is here. So, ultimately it will become 0.714 plus j 6.35 phi 11 plus 0.108 phi 21 is equal to 0. This is one equation.

Second equation also 377 phi 11 plus I mean this one 377 phi 11, right I mean this equation that is phi 11, right and your lambda 1 is equal to minus 0.714 plus j 6.35. So, this one become actually plus 0.714 minus j 6.35 phi 21 is equal to 0. These two equation first you write down, right. First you write down these two equations, equation numbers are not given, but we have written this one, but above equations are not linearly independent, right. One of the Eigenvectors corresponding to an Eigen value has to be set arbitrarily I mean phi 11 and phi 21 you have to choose one of them arbitrarily, right.

Therefore, we assume that phi 21 is equal to 1. This is normal practice such that calculation will be easier for this small problem.

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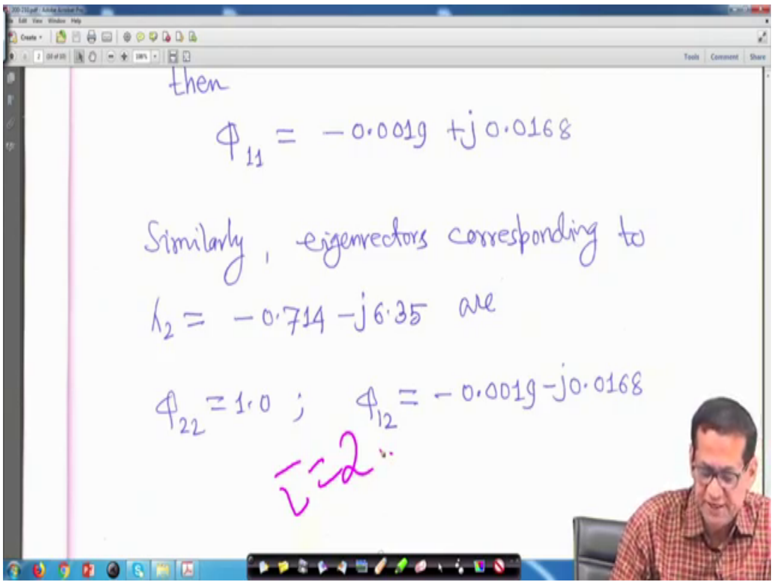

So, if you choose phi 21 is equal to 1, then you will get your what you call that phi 11 will become minus 0.0019 plus j 0.0168 this is phi 11, right. Similarly, Eigenvector corresponding to your psi 2 lambda 2 that is second Eigenvalue minus 0.714 minus j 6.35; that means, that means, here I is equal to 2, right, I is equal to 2; that means, if you go back to that equation if you go back to that equation for I is equal to 2, here I is equal to 2 means phi 12 and this is phi 22 and this will be lambda 2, right. Because I is equal to 2 mean this will be lambda 2 and lambda 2 is equal to you have taken this one minus 0.714 minus j 6.35. Previous one was plus j 6.35, this is minus j 6 and you substitute.

And, if you substitute and if you just your what you call that assume phi 22 one. Similarly, like similarly your what you call like this like this you please write other two equation in terms of 21 and 22 you write down the other two equations, right for that lambda 2 values and then you assume that phi 22 is equal to I think we have assumed phi 22 is equal to 1, right that is phi 22 is equal to 1 then phi 12 can be calculated minus 0.0019 minus j 0.0168, right.

So, we got all the four values. Therefore, we will go to the next page. So, just hold on right.

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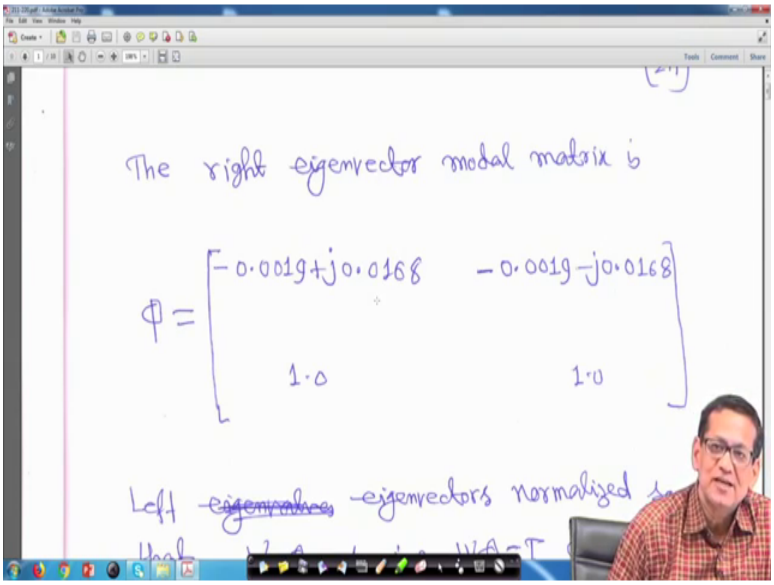

So, right Eigenvector modal matrix is phi is equal to then given as phi 11, then phi 12 then phi 21 and phi 22 this is my phi, right this is that right Eigen right Eigenvector modal matrix. Now, we know phi is that you know that your phi psi phi that is your is equal to 1 in general we have seen that psi i phi i is equal to 1 or in general psi phi is equal to I, the identity matrix. Therefore, psi is equal to phi inverse; that means, if you take the inverse of this one, you will get the left Eigenvector.

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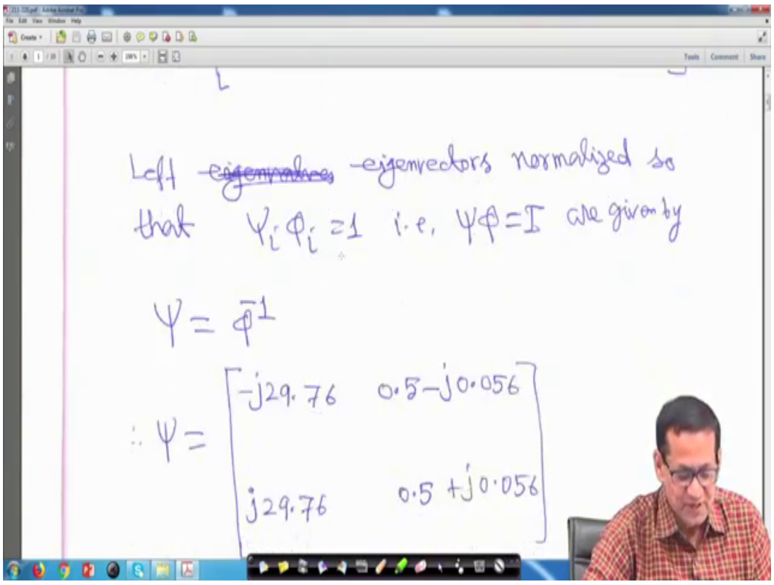

So, psi is equal to if you take the inverse of this matrix it will come like this that minus j 29.76 and 0.5 minus j 0.056, right and this one also j 29.76 and 0.5 plus j your 0.056. So, this is left Eigenvector, right. Now, we got right Eigenvector, we got the left Eigenvector, right.

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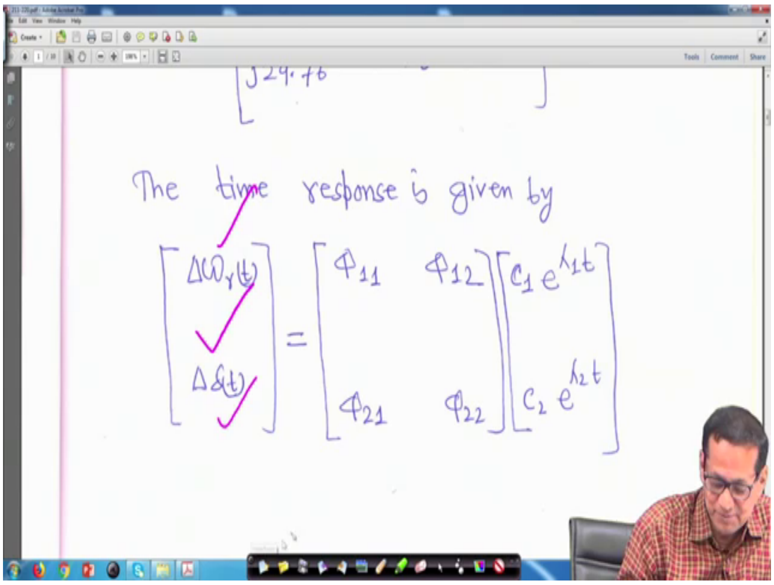

So, time response is given by earlier we have see in general so, delta omega r t delta delta t will be phi 11, phi 12, phi 21, phi 22 then C 1 e to the power lambda 1 t, C 2 e to the power lambda 2 t. If you go back to that equation that your time response of delta is we wrote in terms of phi and C 1 for a particular variable i. But, here you have two variables that is one is delta omega r another is delta delta t, right.

So, that is why we putting this one first 1 phi 11, C 1 e to the power lambda 1 t and phi 12 C 2 e to the power lambda 2 t. Similarly, for i is equal to 2, the second variable that is your delta delta t phi 21 C 1 e 2 to the power lambda 1 t plus phi 22 C 2 e to the power lambda 2 t, right. So, this is that general expression for your I think slowly and slowly your understanding will be clear, right.

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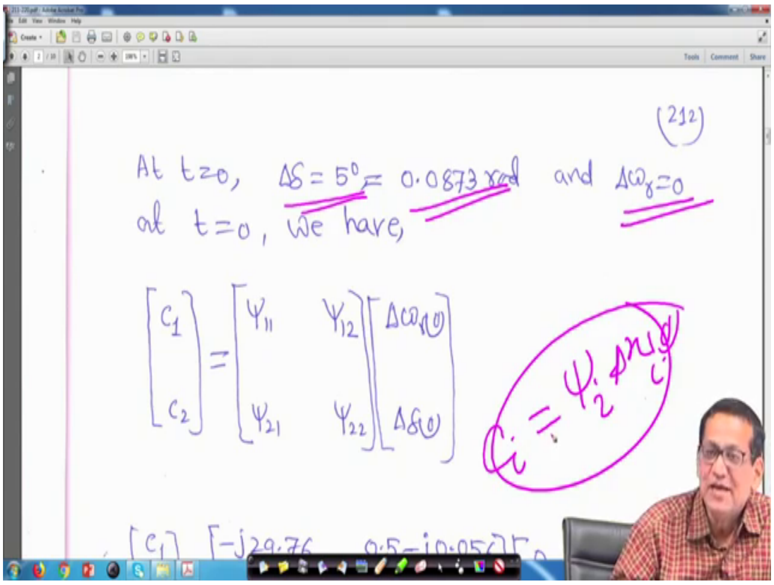

So, so, this one. If it is so then at t is equal to 0, this initial conditions were given that at t is equal to 0 that delta delta is equal to given 5 degree is equal to 0.0873 radian and delta omega r also given initial condition 0. Therefore, we have seen in general that C is equal to your psi delta x 0 that was that C i I mean in general we have saw we have seen that initial condition this C i is equal to your what you call that psi i then delta x i 0. This way this way you saw the initial conditions, right, but that was mathematics that is representation of this one.

But, in general actually C 1 and C 2 you can easily put it in your mind it is left Eigenvector into delta omega 0 delta. Easy to easy to remember is that C 1, C 2 or C 3 whatever may be the number of state variables it will be your left Eigenvector.

So, psi 11 psi 12 psi 21 psi 22 into that delta omega r 0 delta delta delta omega r 0 delta delta 0. This way you can you can put it in your memory the way it has been given in the mathematical form, right.

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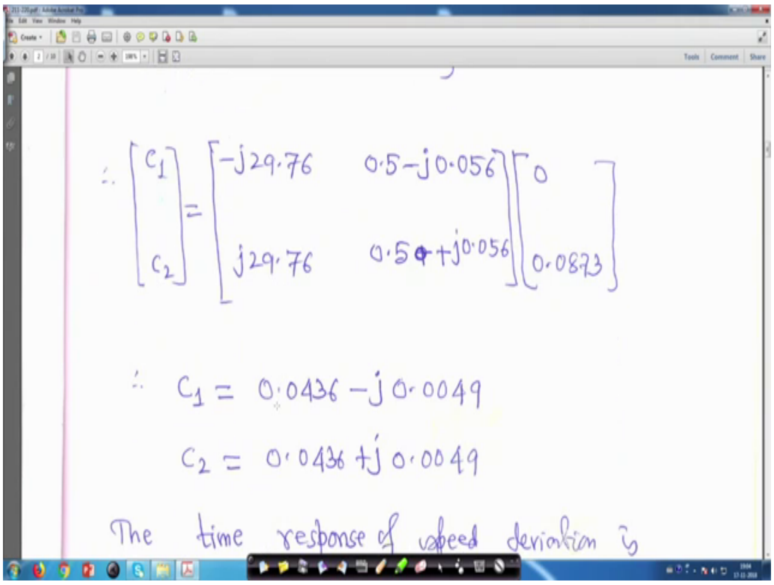

If you do so, if you do so, then it will become that your psi we got this matrix that psi matrix the left Eigenvector. So, it is minus j 29.76 0.5 minus j 0.056 and this is j 29.76 and it is 0.5 plus j 0.056 and initial value at t is equal to 0 delta omega r 0 and delta delta is 0.08 your 73 radian.

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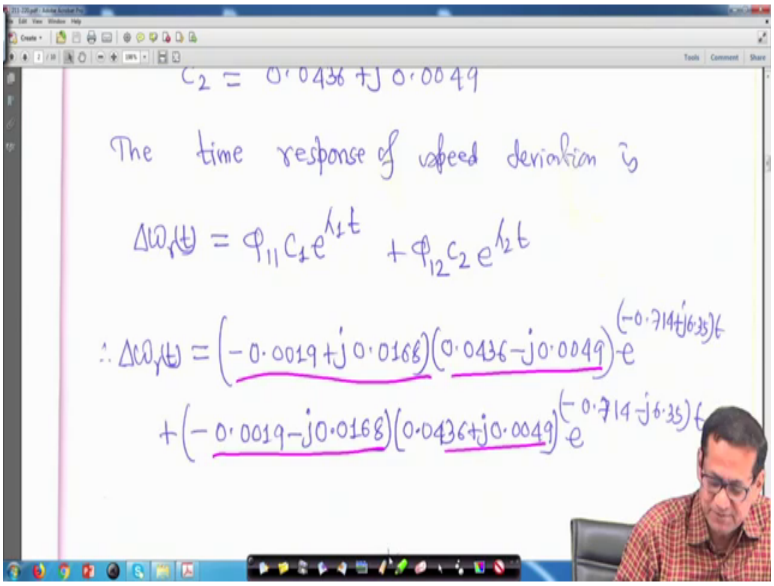

So, we put these values we put these values and just simplify then you will get C 1 is equal to 0.0436 minus j 0.0049 and C 2 we will get 0.0436 plus j 0.0049, right. Therefore, the time response of speed deviation is given by from this equation from these equation from this equation right from these equation that your delta omega r t is phi 11 C 1 e to the power lambda 1 t plus phi 12 C 2 e to the power lambda 2 t.

You substitute the value of phi 11 C 1 and lambda 1, right. So, it will be in this form that very complicated form that phi 11 is this much minus 0.0019 plus j 0.0168 and C 1 is this much 0.0436 minus j 0.0049 then e to the power lambda 1 is minus 0.714 plus j 6.35 into t plus that your phi 12 minus 0.0019 minus j 0.0168, right then your C 2 point 0 0.0436 plus j 0.0049 e to the power minus 0.714 minus j 6.35 t, right.

So, this is response of delta omega r t time response. If you simplify this one if you just hold on if you simplify this one you will find that how response will come.

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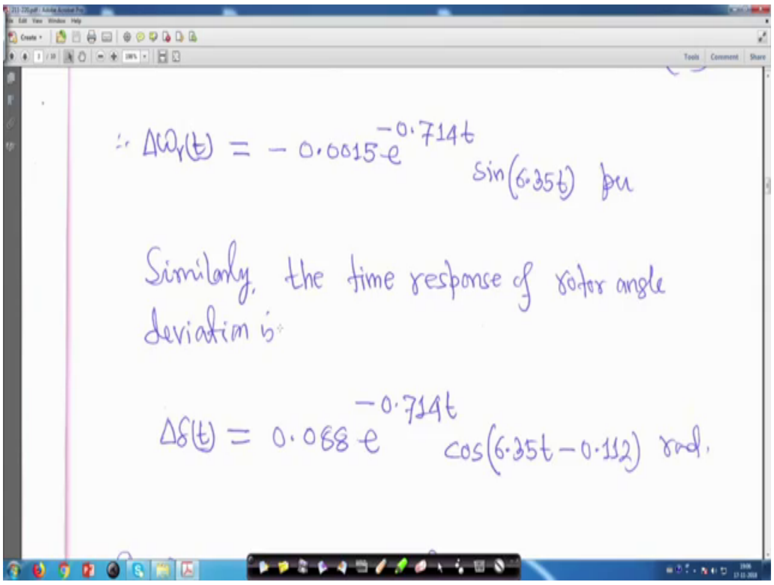

And, if you simplify it will come in this form delta omega r t is minus 0.0015 e to the power minus 0.714 t into sin 6.35 t; I mean if you simplify all these things we will find there will be no complex number involved, right, nothing will be there and whatever I am giving this answer is correct one, right. But, all derivation cannot be put here it will consume lot of time, but this is the form for delta omega r t that minus 0.0015 e to the power minus 0.714 t into sin 6.35 t as a check at t is equal to 0 this sin term will become 0 therefore, delta omega r at t is equal to 0 will become 0, right.

Similarly, the time response of the rotor angle deviation, right. Similarly, delta delta I mean here from this equation that delta delta t is equal to phi 21 phi 21 C 1 e to the power lambda 1 t, I mean I have given the final form only.

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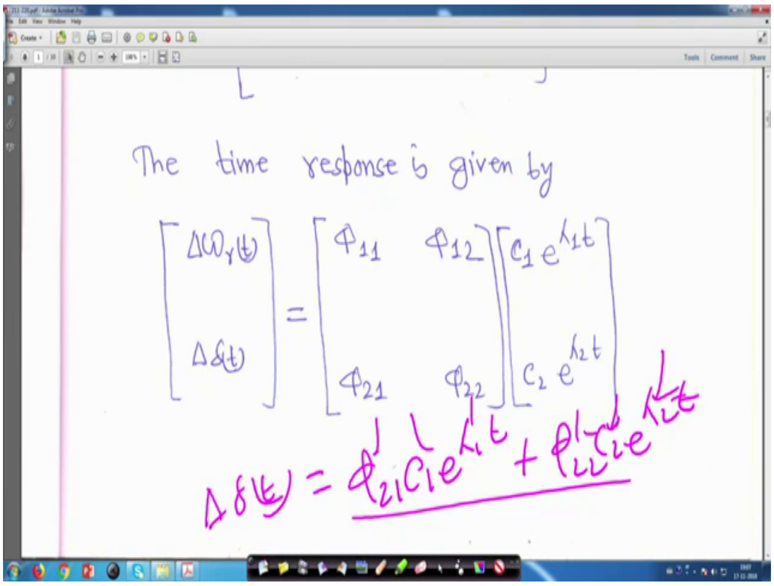

But, if I write on this delta phi t will be phi 21 C 1 e to the power lambda 1 t plus phi 22 C 2 e to the power lambda 2 t, right.

So, if you put phi 21 value C 1 value lambda your what you call lambda 1 value phi 22 value C 2 value and lambda 2 value then you will get response for the delta delta, right.

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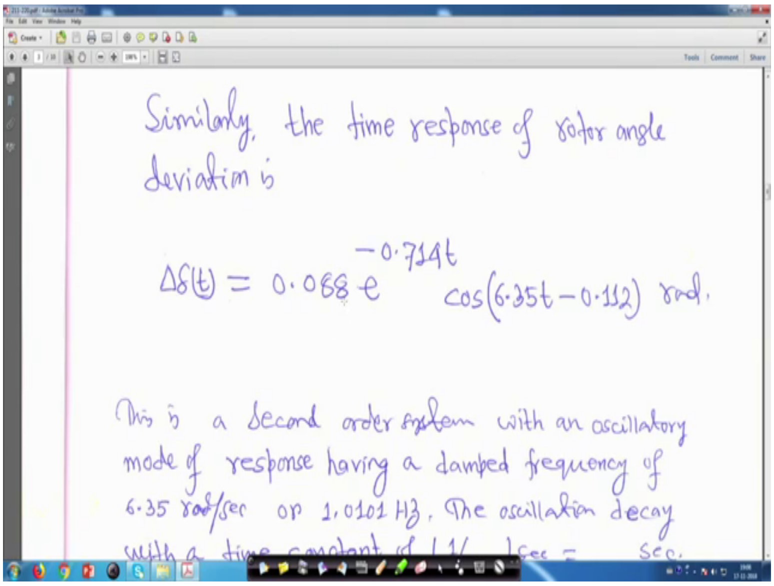

Now, therefore, similarly delta delta you will get 0.088 e to the power minus 0.714 t cosine of 6.35 t minus 0.11 t. Now, if you put at t is equal to 0, you will find delta delta at t is equal to 0 will become 5 degree or what you call that you have convert it to the radian 0.0873 radian, right, from that you can make out that your response is correct. So, this is delta delta t and this is your delta omega r t the time response, right. So, from that you can make out that this response actually is stable, right.

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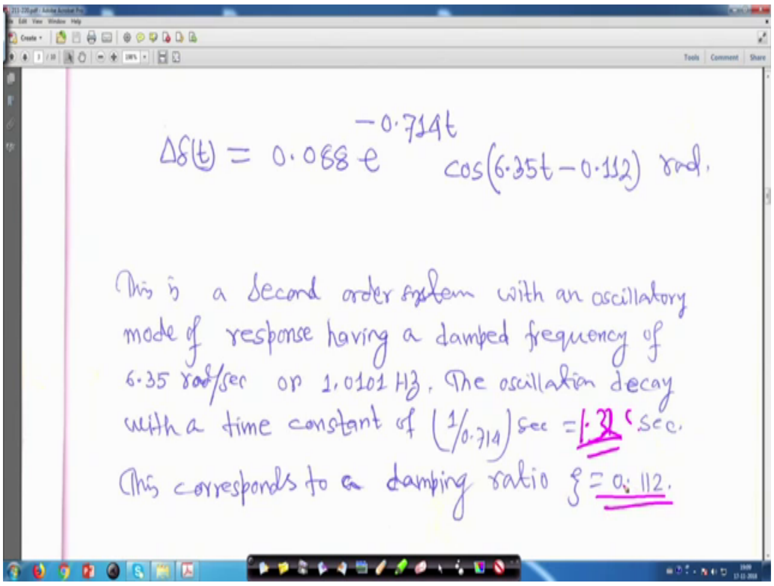

So, this is a second order system with an oscillatory mode of response having a damped frequency of 6.35 radian per second every every term delta delta or delta (Refer Time: 17:40) it is attached basically which is sinusoidal term multiplied right or 1.0101 hertz the oscillation decay with a time constant 1 upon I have not calculated it that 1 upon yours 0.714, right. So, it will be roughly approximately 1.31 or 32 second I did not put it, I missed it actually, right.

So, because your that e to the power minus 0.714 is coming. So, with a time constant will be 1 upon 0.714. So, roughly 1.3 or 32 second, right. This correspond to a or this correspond to a damping ratio there xi is equal to 0.112, right. So, just hold on. So, if you I mean from this thing your what you call from this thing you can find out the time constant right you know that it is a basically it is e to the power minus t upon tau, right. So, so, it is 1 upon 0.714 whatever it comes, right.

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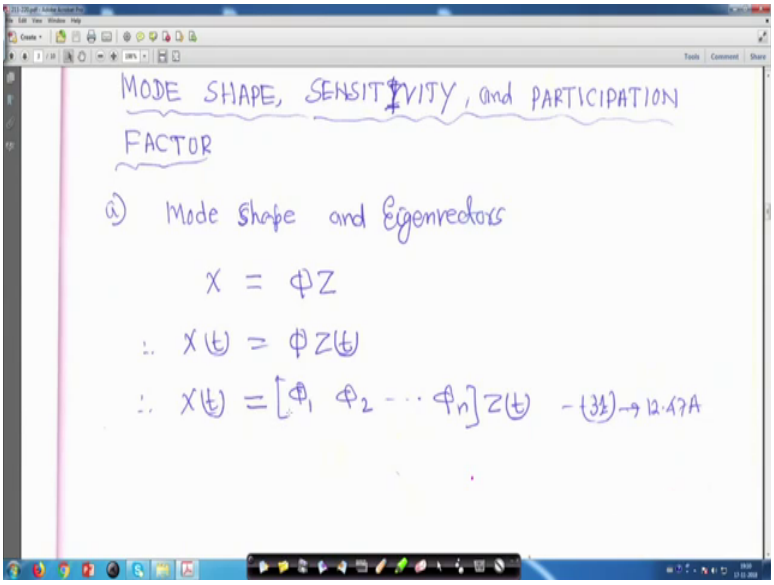

Next is that mode shape sensitivity. These are all example. I will tell I take you some other example mode shape sensitivity and participation factor. Now, mode shape and Eigenvector, suppose x is equal to phi Z, right. Earlier also we have taken like this. Now, therefore, we can write X t is equal to phi into Z t. So, therefore, X t is equal to phi 1, phi 2, phi n, Z t this is say equation 31, right.

So, this is nothing this is just hold on this is actually nothing, this is for my own reference, right.

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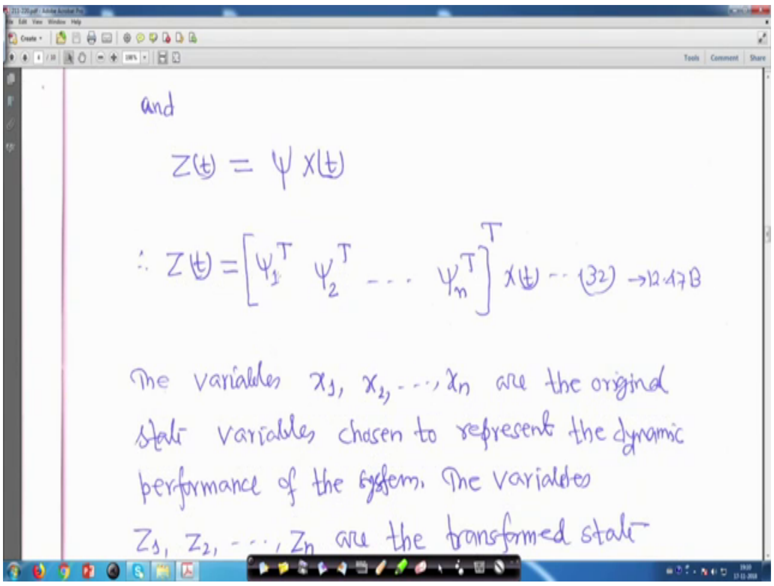

So, so, we can and Z t is equal to psi X t. This is also we have earlier we have seen. If Z t is equal to psi X t then mathematically it can be written as Z t is equal to psi 1 transpose, psi 2 transpose up to psi n psi n transpose to the transpose X t and this is actually not a vector it is a matrix because each one is a your what you call that column vector, right. This is equation 32.

Now, the variables x 1, x 2, x n are the original state variable chosen to represent the dynamic performance of the system, this we know and Z is the transform variable, right. The variable Z and Z 1, Z 2 and Z n are the transformed state variables.

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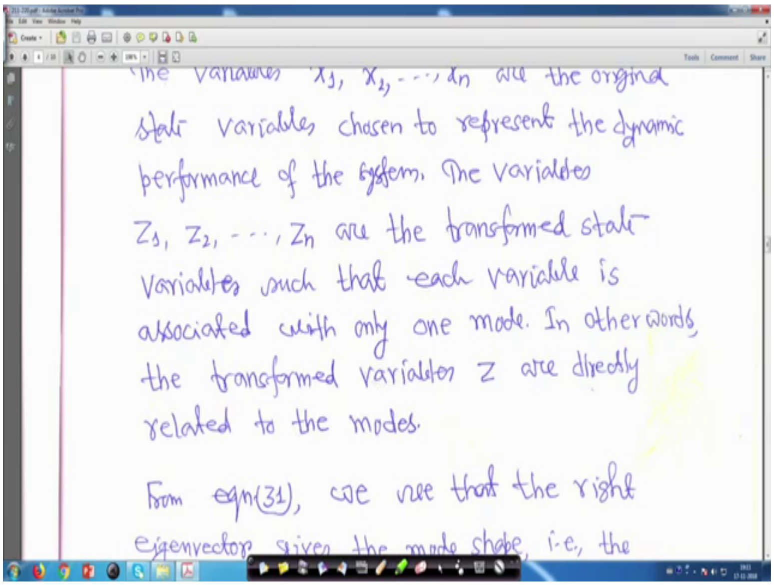

Such that each variable is associated with anyone mode. In other words, the transformed variable Z are directly related to the modes, right.

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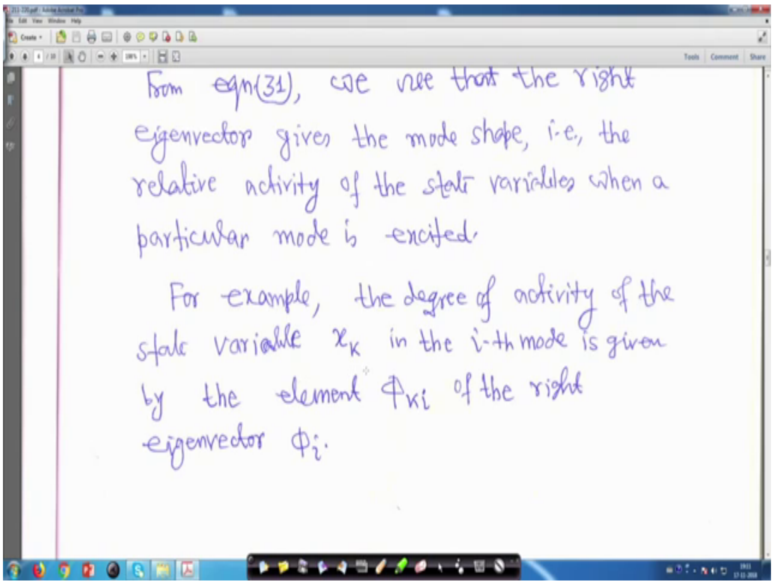

From equation 31, we see that the right Eigenvector give the mode shape that is the relative activity of the state variables, right that is from equation 31; that means, from this equation from this equation and phi is actually is the right Eigenvector, right. So, so, from equation 31, you see that right Eigenvector gives the mode shape that is the relative activity of the state variable when a particular mode is excited. For example, the degree of activity of the state variable x k in the i-th mode is given by the element phi k i of the right Eigenvector phi i, right.

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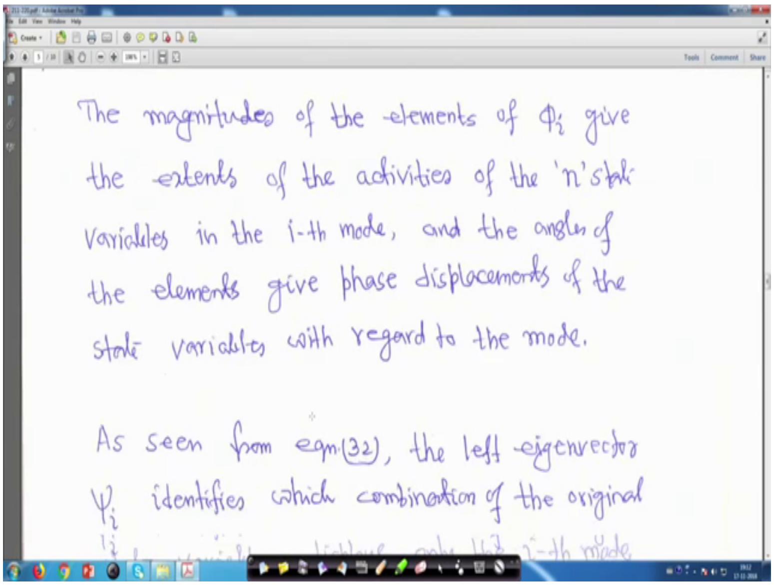

That means the magnitudes of the element of phi i give the extents of the activities of the n state variable in the i-th mode. Later we will see this exactly what it is meaning and the angles of the elements give phase displacement of the state variable with regard to the mode, right.

So, as seen from equation 32; that means, this equation; that means, this equation this is function of actually your left Eigenvector, right.

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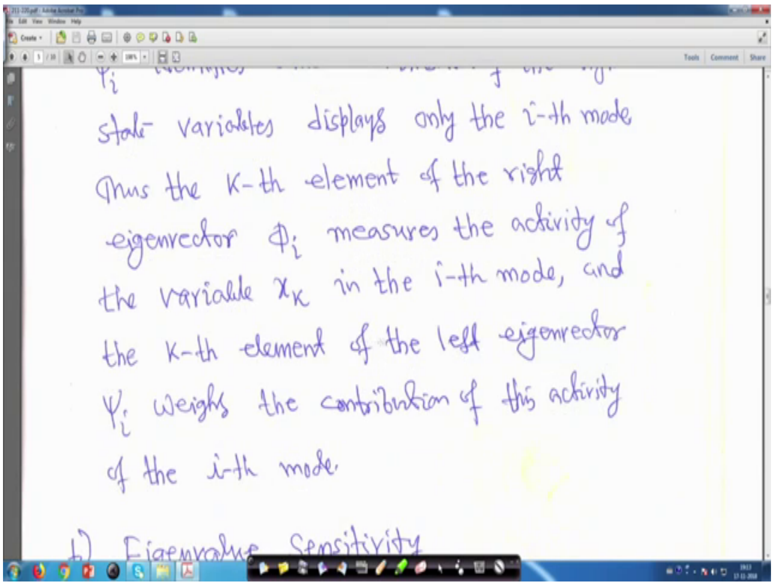

So, from equation 32 right the left Eigenvectors psi i identifies which combination of the original state variables displays only the i-th mode thus the k-th element of the right Eigenvector phi i measures the activity of the variable x K in the i-th mode and the k-th element of the left Eigenvectors psi i weighs the contribution of this activity of the i-th mode, right. This is the meaning.

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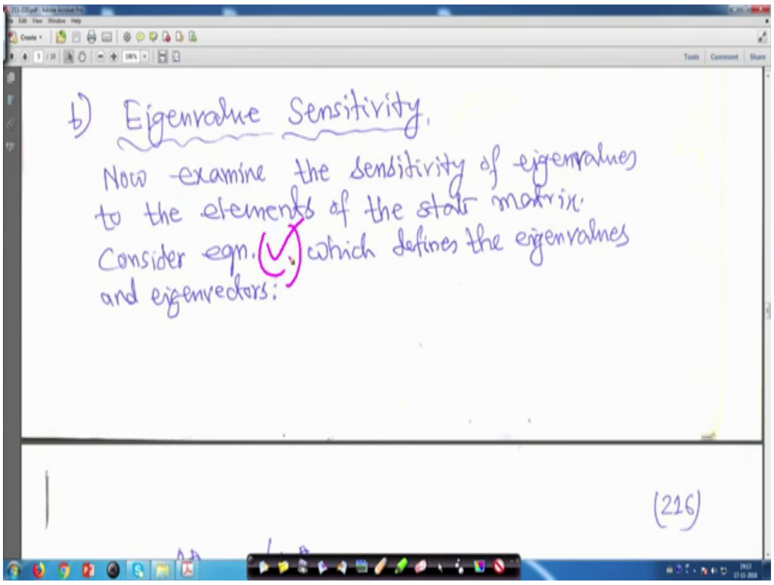

Now, that Eigen value sensitivity from these thing slowly and slowly we will go to the participation factor. Now, examine the sensitivity of the Eigen values to the elements of the state matrix consider equation which defines the Eigen value and Eigenvectors. Actually this is actually equation number I forgot to write previously we have taken that that in the dynamic system may be just it is equation this equation is rewritten here of course, right.

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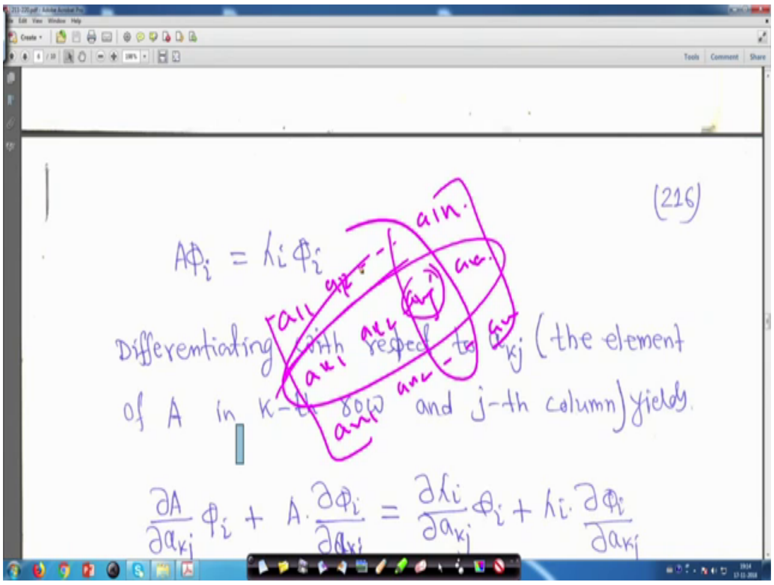

That your this equation that A phi i is equal to lambda i phi i, right. This is the this equation.

So, differentiating with respect to a kj the element of a in the k-th row and j-th column. Suppose you have the matrix now things will be clear to you just hold on, right; just hold on. Suppose, suppose you have the matrix right you have the matrix. So, it is actually spelling that a kj that is in the k-th row; suppose a 11, a 12, a 12 up to a 1n then suppose it is a k1, a k2 up to a kn, right it is a k-th row and suppose it is coming a n1, a n2 up to a nn, right it is k-th row and say some j-th column somewhere some somewhere a 11, a n2, a k1, a k2 some elements a k a k j. So, this is the thing and this is the your k-th row and your j-th column. So, this is your say a kj, right.

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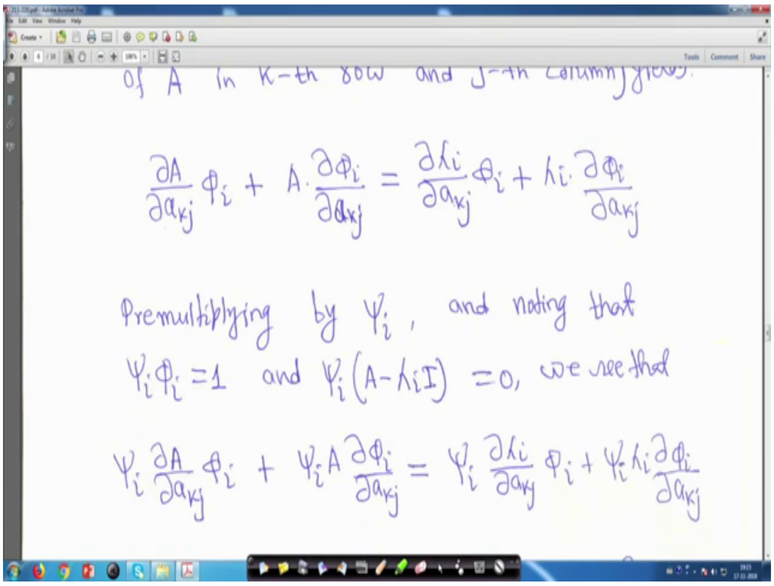

So, and if you take the derivative of this one then this equation if you take the derivative a phi i is equal to lambda i phi i then your it will be delta a delta a lj phi i plus A into delta phi delta a kj, this is the left hand side, right is equal to the right hand side if you look into this delta lambda i upon delta a kj phi i plus lambda i into delta phi i upon delta a kj, right.

Now, pre multiplying by psi i and noting that psi phi is equal to 1 we know I identity matrix. So, psi i phi i is 1 and psi i A minus lambda i is equal to 0. So, this we have seen it before, right. So, multiply both side by psi i this equation. So, if you do so, it will be psi i delta A upon delta a kj phi i plus psi i A delta phi i upon delta a kj is equal to psi i delta lambda i by delta a kj phi i plus psi i lambda i delta phi i upon delta a kj, right.

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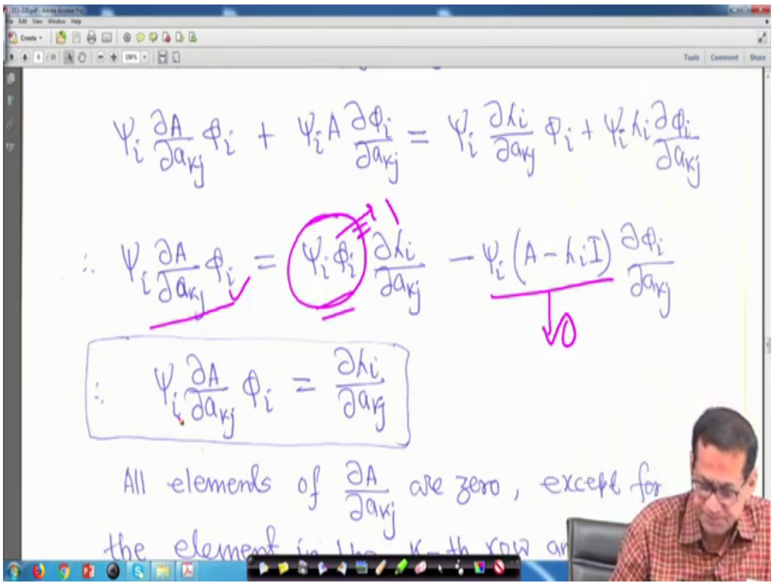

Now, upon simplification you will get this that psi i delta a upon delta a kj phi i is equal to psi i phi i delta lambda i upon delta a kj minus psi i A minus lambda i into I that is identity matrix bracket close delta phi i upon delta a kj, right or psi i delta a upon delta a kj phi i is equal to delta lambda i upon delta k j because from this definition we know psi i phi i is equal to 1 and from this definition this 1 we know 0, and this 1 we know it is equal to 1 and this is 0.

Therefore, psi i delta a upon delta a kj phi i is equal to delta lambda i upon delta a kj, right. Now, let me clear it equation number I have not given here, but I think it is understandable for you.

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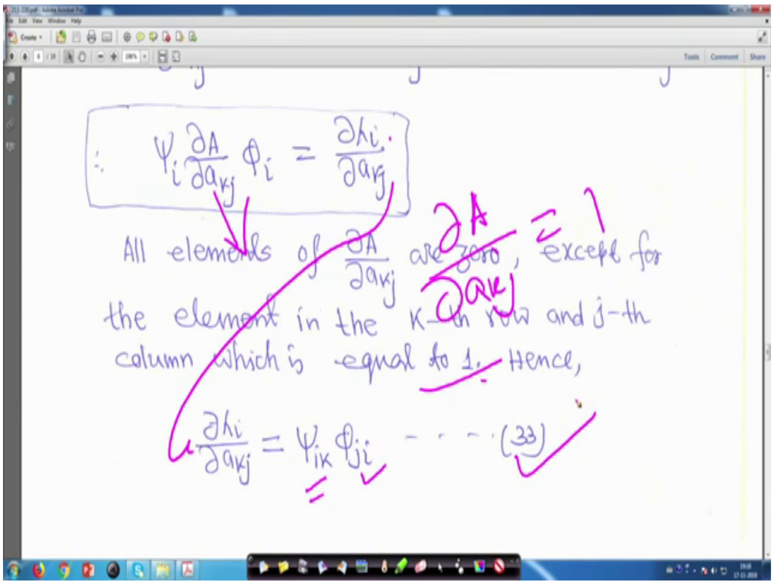

Now, all elements of delta A upon delta a kj are 0 except for the element in the k-th row right. So, because we have taken that derivative with respect to a kj right and j-th column which is equal to 1 right because you have taken the derivative that is your k-th row and j-th column.

So, delta A upon delta your a kj, right it is basically we are taking the derivative the with respect to a kj that particular element so, it will be 1. So, that means, your delta A upon delta a kj your what you call this term this term actually it will be 1 delta A and with respect to this your k-th row and that element that is your k-th row and j-th column and this should be is equal to 1. So, if it is so that is equal to 1 that means, my delta lambda i this one delta lambda i upon delta a kj is equal to then psi ik then psi ji because k-th row and j-th columns.

So, instead of writing all psi i your psi i phi i we will write only that your psi ik and phi ji, right that is k-th row and j-th column. So, that is why psi ik phi ij this is equation 33, right.

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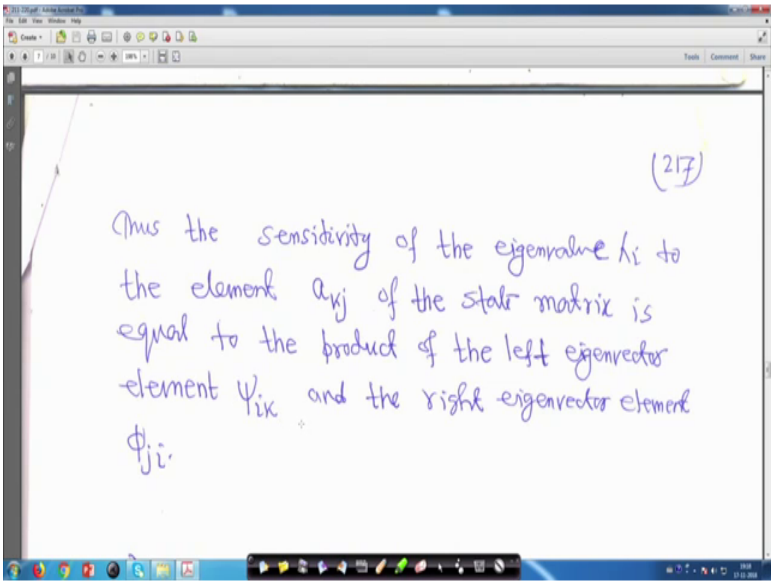

Thus the sensitivity of the Eigenvalue lambda i to the element a kj of the state matrix is equal to the product of the left Eigenvector element psi ik and the right Eigenvector element your phi ji, right. So, that sensitivity lambda i with respect to that element a kj is basically product of the left Eigenvector and the right Eigenvector, right.

So, that is your psi ik and the right Eigenvector element phi ji, right.

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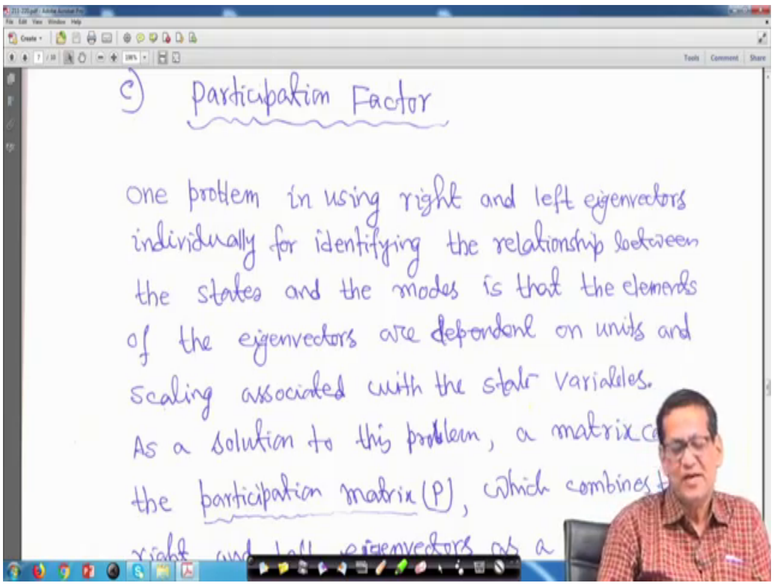

Next, the most important thing, this regarding participation factor. This participation factor at the end also we will see some your what you call some couple of more problems and elaborately I will take a separate thing and I will tell after making this derivation another thing and we will take one more example and again we will come back to participation factor such that your in another form such that there should not be any your confusion in your mind.

So, problem in using right and left Eigenvectors individually for identifying the relationship between the states and the modes is that the elements of the Eigenvectors are dependent on the units and scaling associated with the state variables.

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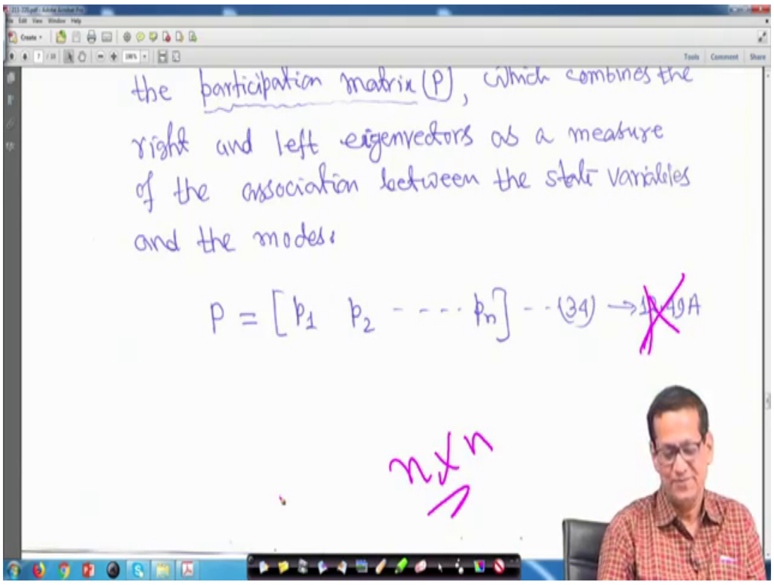

As a solution to this problem a matrix called participation matrix P which combines the right and left Eigenvalues right and left Eigenvectors as a measure of the association between the state variable and the modes, right.

Therefore, P can be given as p 1, p 2, p n this is equation 34, right. This is for my reference, right. Actually p 1, p 2, p n it is a column vector and P basically P actually it is a n into n matrix, right and these are all column vector, right.

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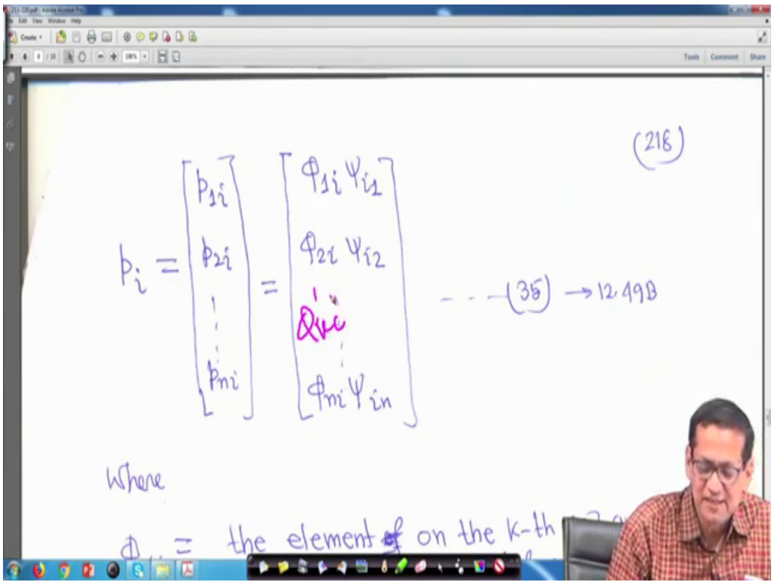

So, that means, in general p i is given as your p 1i, p 2i up to p ni is equal to from the relationship only that psi 1i phi 1i, psi i1, phi 2i, psi i2, phi ni, psi in this is equation 35, right where phi ki, right; that means, in general 1 to n I mean in general it is 1 to n, so, some term should be there say phi ki, right.

So, that means, that phi ki is that element on the k-th row and i-th column of the modal matrix phi is equal to k-th entry of the right Eigenvector phi i, right. Similarly your psi ik the meaning is same, but your definition is just different. The element on the i-th row and k-th column of the modal matrix phi psi your what you call is equal to k-th entry of the left Eigenvector your psi i, right.

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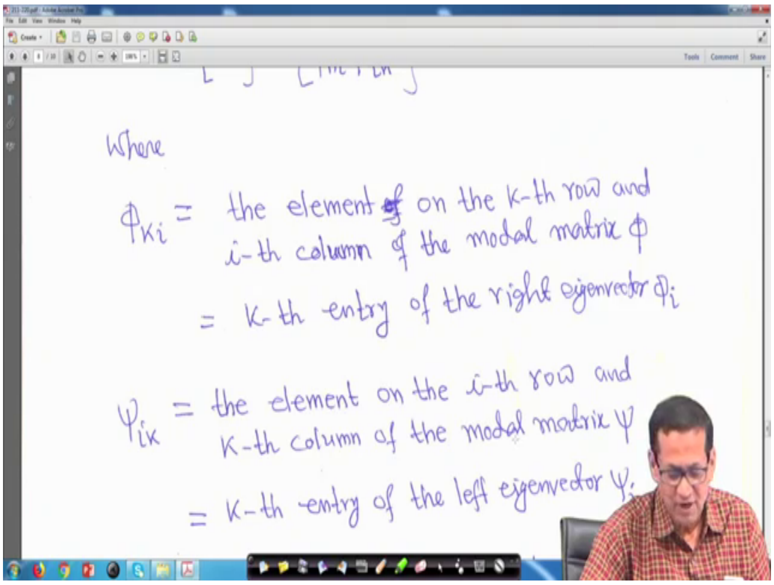

So, this is the meaning.

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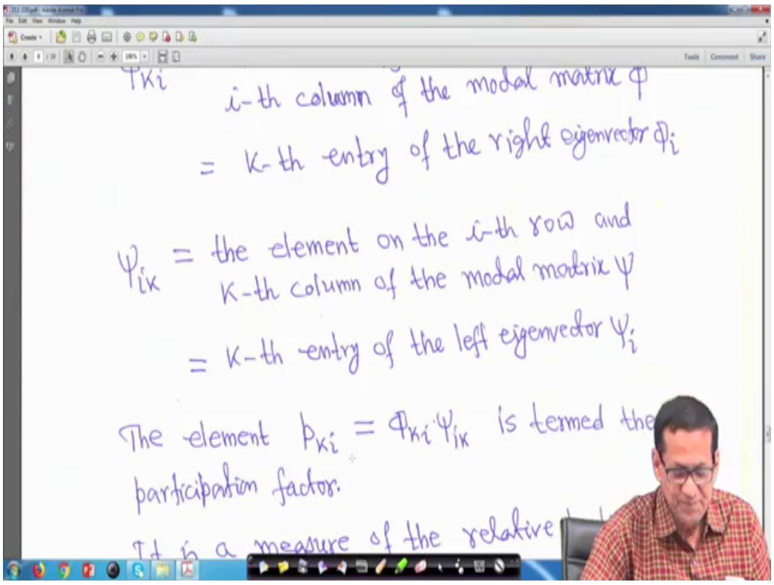

So, and the element p ki is equal to phi ki psi ik is termed as the participation factor, right.

So, thank you very much. With much more detail we will come back in the next lecture.