Power System Dynamics, Control and Monitoring
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Lecture – 22
Power System stability, Eigen properties of the state matrix

Ok we are back again. So, before moving to that your Eigen value analysis participation factor right so all dynamic performances. So, before that just a little bit of one or two small example and that and if you want to convert the unit in English unit that is British units right. So, little bit of calculation of inertia is from WR square in English unit we call right.

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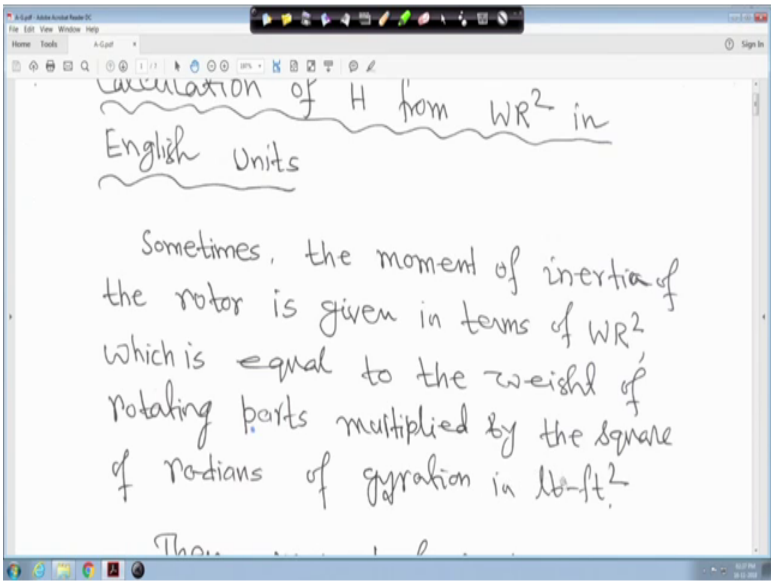

So, sometimes the moment of inertia of the rotor is given in terms of WR square which is equal to the weight of the rotating parts multiplied by the square of your, what you call radius of gyration in lb that is pound feet square right.

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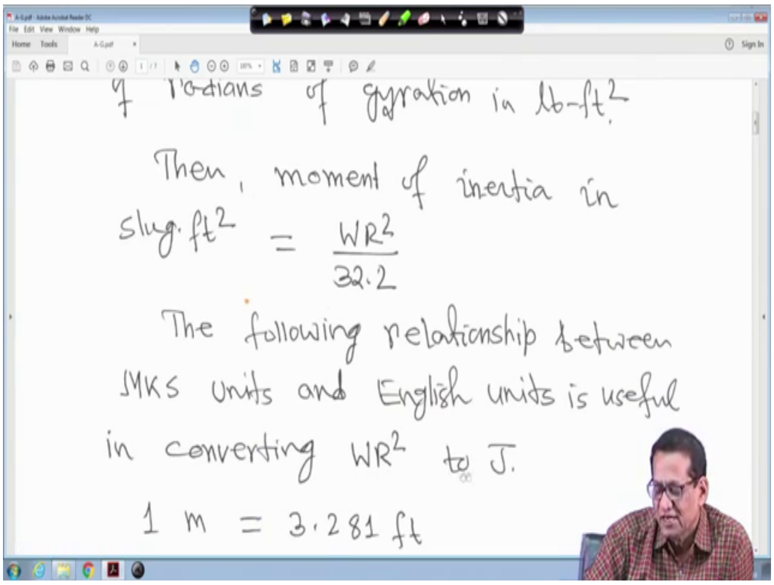

Now, in moment of inertia in slug into feet square is equal to WR square by 32.2 that is the value of g right the following relationship between MK's unit and English unit is useful in converting WR square to J that is moment of inertia.

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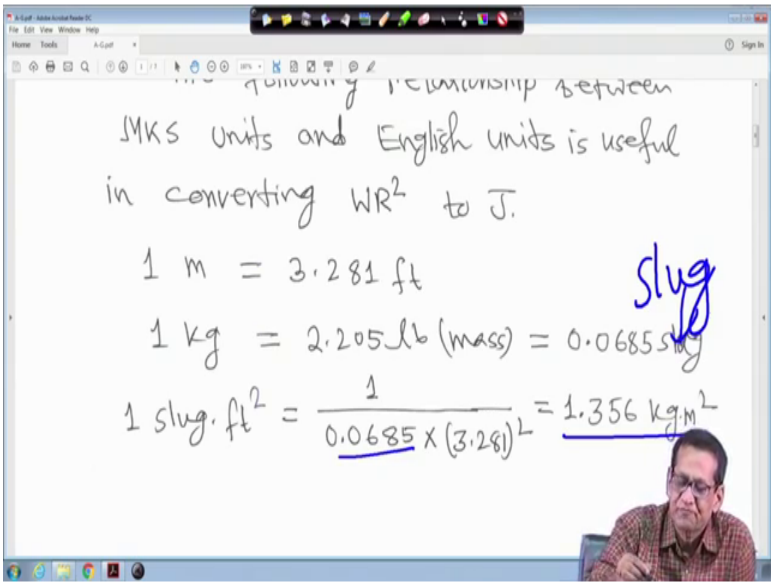

For example 1 metre is equal to 3.281 feet right. And 1 kg is equal to 2.2085 pound lb that is mass that is equal to 0.0685 that is kg right. So, you are sorry the your what you call that is 0 your what you call 0.068 slug sorry not kg this is actually slug right.

So, now 1 slug into feet square is equal to 1 upon 0.0685 into 3.281 square that is equal to 1.356 kg metre square right. So, English unit to your MK's unit right sometimes we convert this.

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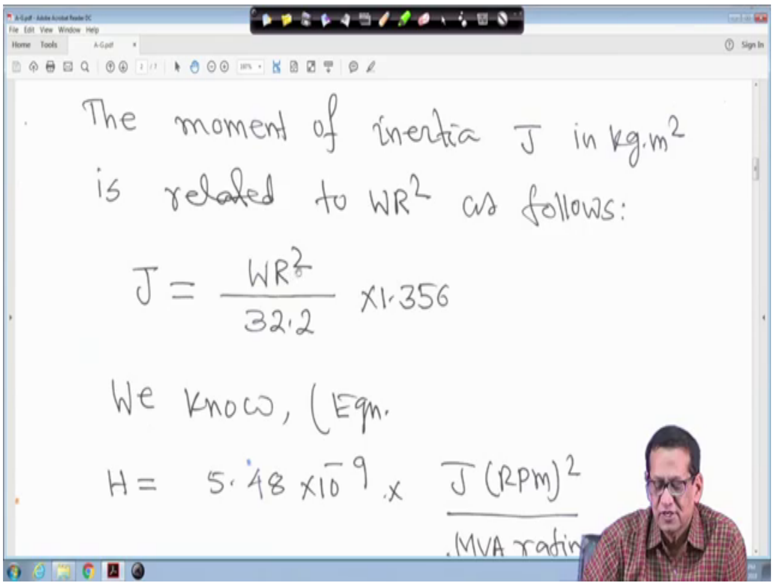

So, in this case now the moment of inertia J in kg per metre square is related to WR square as follows. Therefore, J is equal to WR square upon 32.2 into 1.356 right.

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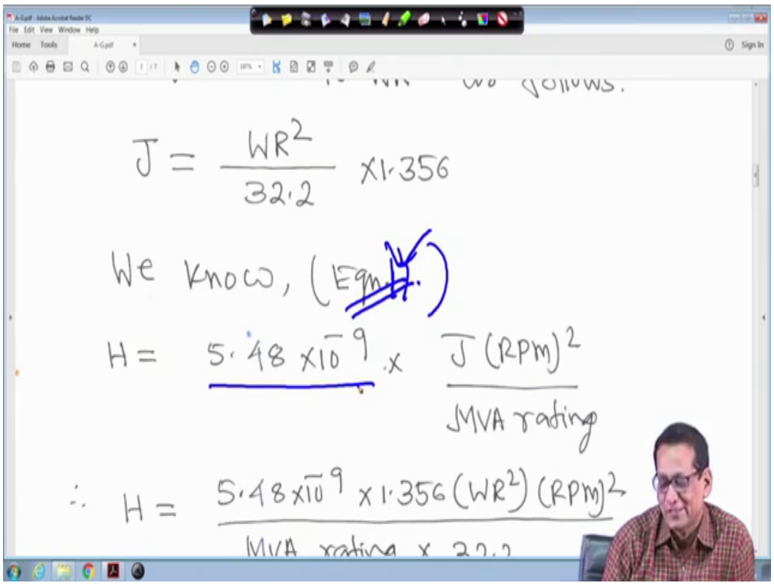

Therefore we know we know that is this equation number right I cannot recall this equation number this one we have derived know; that in the swing equation H is equal to

5.48 into 10 to the power minus 9 into J into RPM square by MVA rating I cannot I have forgot to write the equation number.

But you just find out that in swing equation this derivation is if I try to find out here it will take time, but this has been derived for H right. So, before we have done it and we have done one or two small example also.

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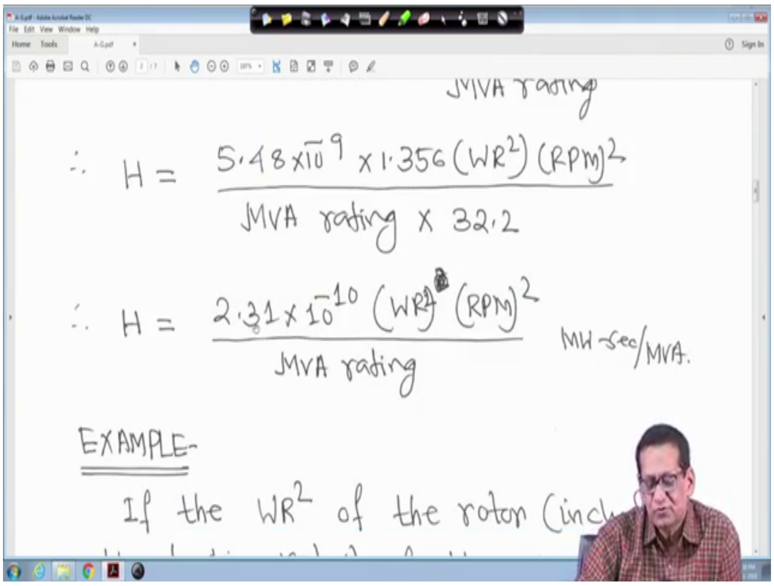

Now, in this case what will happen that your H is equal to that 5.48 into 10 to the power minus 9 into 1.356 and J is equal to W 1.356 WR square by 32.2 here we have made it know J is equal to WR square by 32.2 into 1.356. So, 1.356 WR square by 32; 32.2 and rest is square by MVA rating it is there. So, if you simplify H will be 2.31 into 10 to the power 10 WR square into RPM square divided by MVA rating; this is megawatt second per MVA right.

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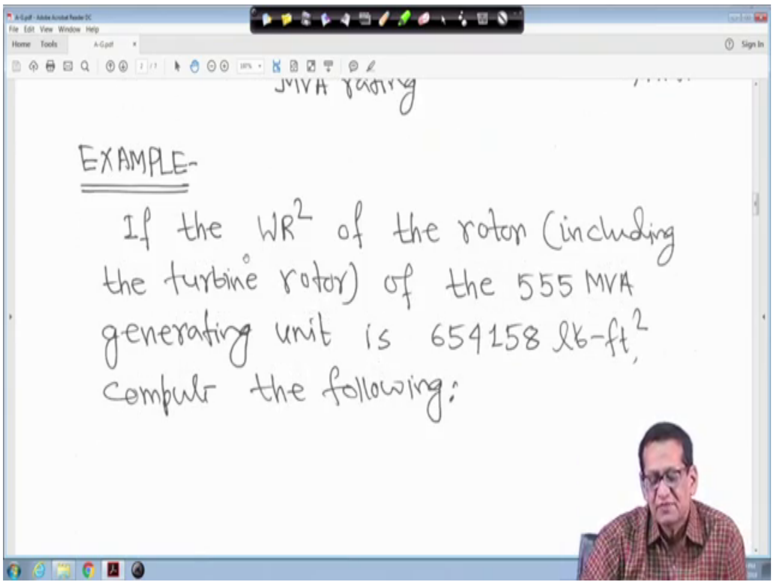

So, take one small example; if the WR here I have not written example number if the WR square of the rotor including the turbine rotor of the 555 MVA generating unit is your 654158 pound feet square compute the following right it is given.

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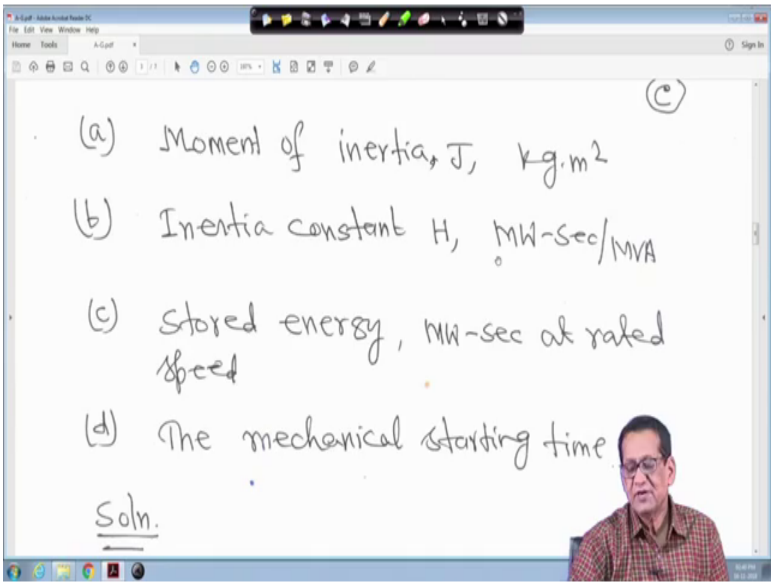

Therefore you have to calculate moment of inertia J in kg metre square. Inertia constant H is megawatt second upon MVA; that is basically second right because megawatt upon MVA is a dimensionless so H unit is second. Stored energy that is megawatt second at your rated speed and now last one the mechanical starting time right.

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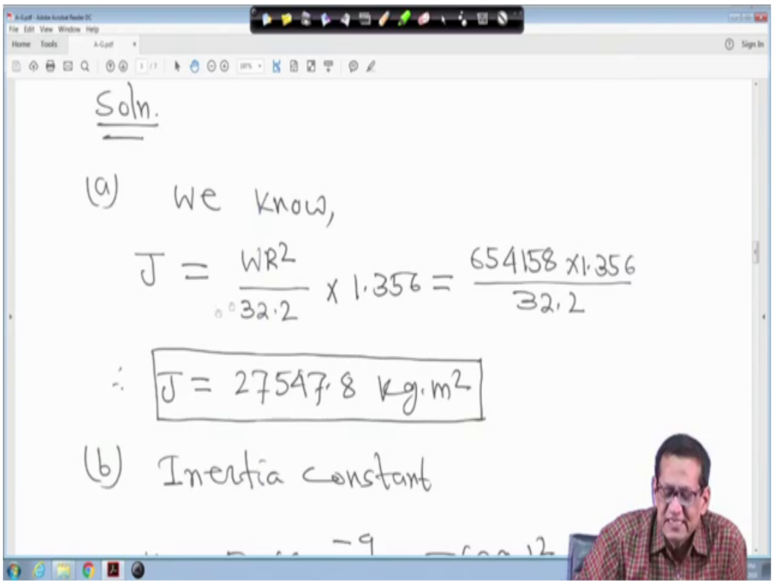

Now, we know that J is equal to WR square upon 32.2 into 1.356 this just we have derive. So, WR square is given 654158 it is given into 1.356 upon 32.2. So, this comes actually J is equal to 27547.8 kg meter square right.

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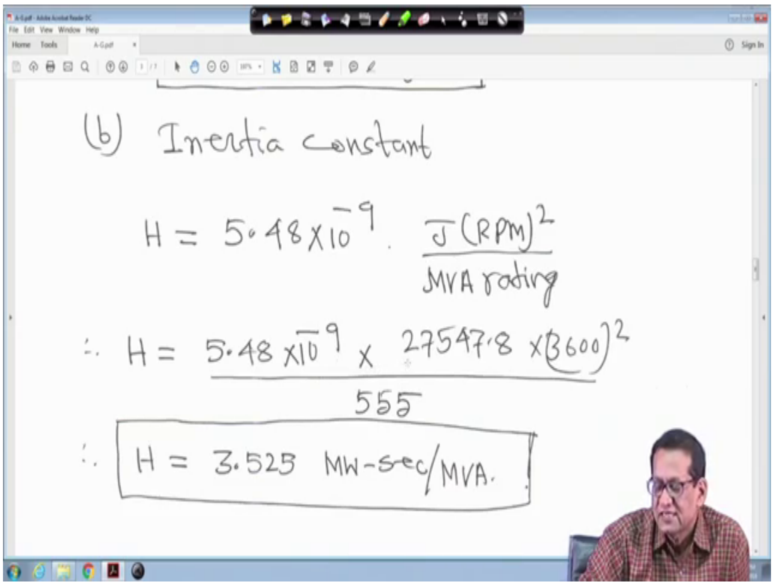

Now, part b inertia constant we know this formula H is equal to 5.48 into 10 to the power minus 9 J into RPM square upon MVA rating right. So, 5.48 into 10 to the power minus 9 then J we have got 27547.8. So, 27547.8 and RPM revolution per minute it is 3600 it is 60 hertz you have consider 60 hertz right. If it is a 50 hertz, then it will be 3000 it is 60 hertz divided by 555. This H is equal to then 3.525 second or megawatt second per MVA, but you can write second also right.

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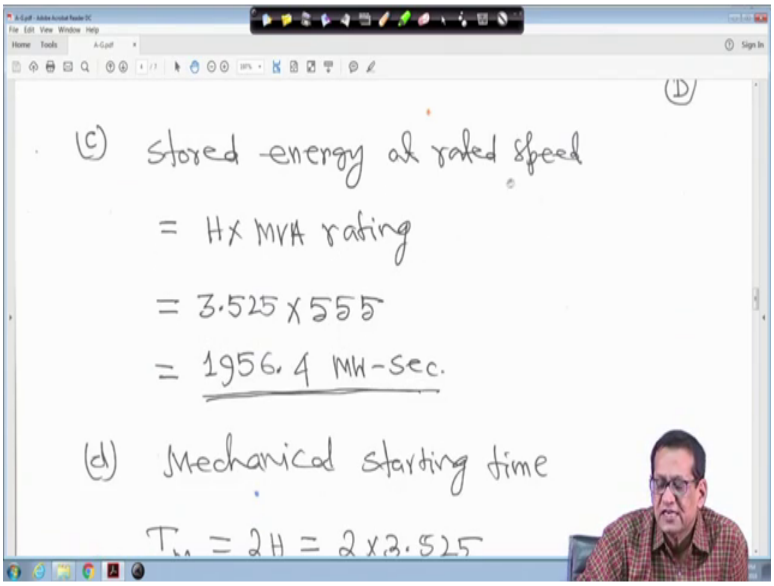

Now, numbers third party stored energy at rated speed you know H into MVA rating. So, H is equal to 3.525 we got and MVA rating was 555. So, it will be 1956.4 megawatt second right.

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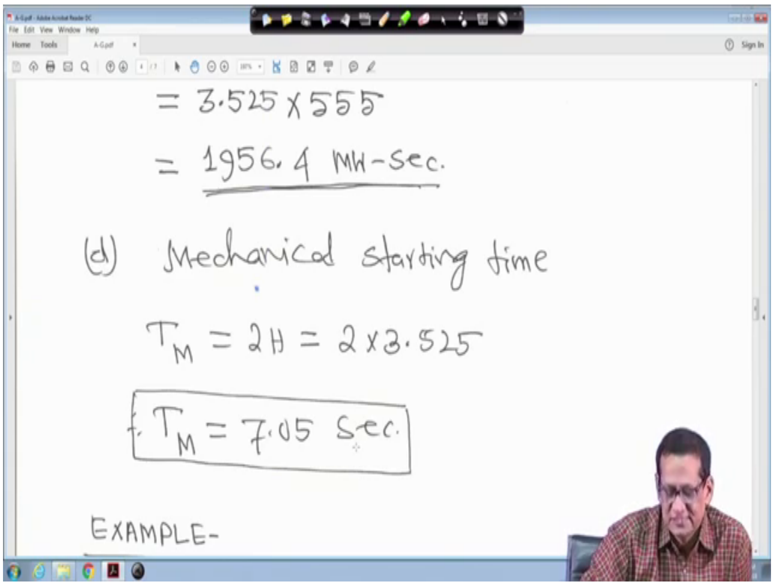

And d the mechanical starting time you know T M is equal to 2H. Earlier we have made this one know earlier we have seen it when you are doing swing equation there we have seen T M is equal to 2 H it is 2 into 3.525 that is T M is equal to 7.05 second right this is the answer.

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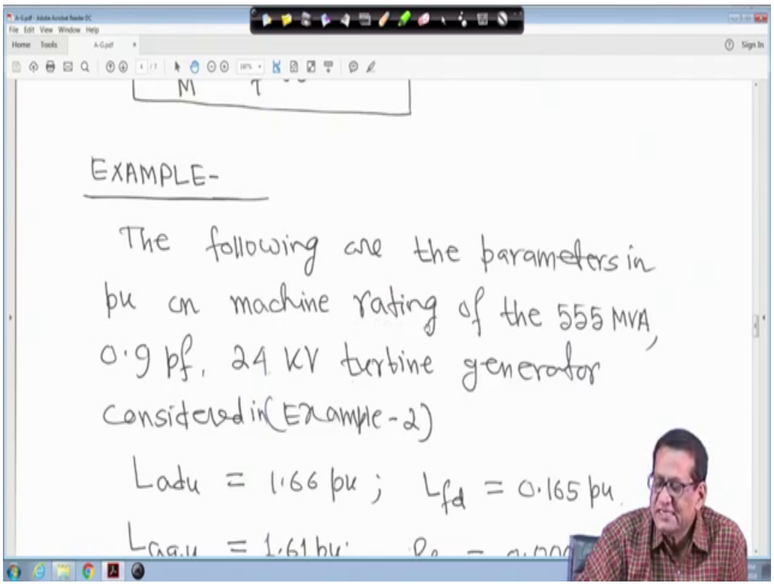

Now, another example we have taken the following are the parameters in per unit right and in per unit and your machine rating of the 555 MVA 0.9 power factor 24 kV turbine generator considered in example 2.9 power factor means lagging right.

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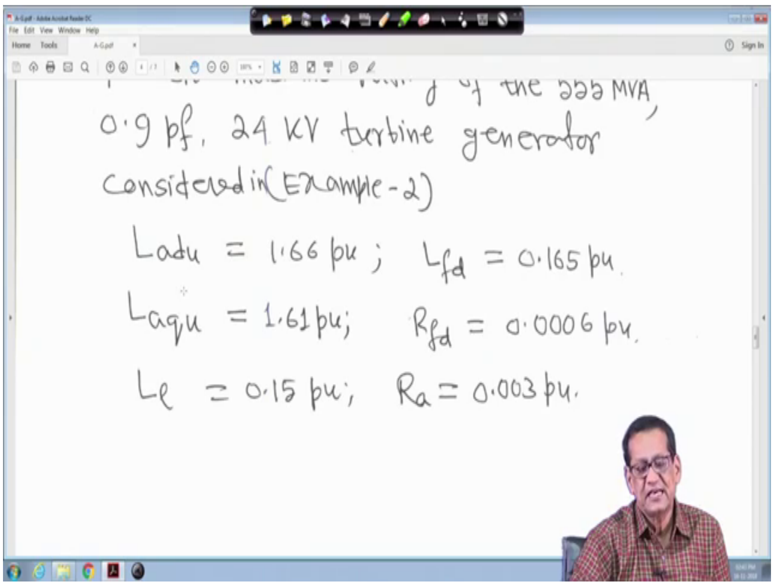

So, following parameters also are available in that example, but rewritten here Ladu is equal to 1.66 per unit. Lfd is equal to 0.165 per unit Laqu is equal to 1.61 per unit. And

Rfd is equal to 0.0006 per unit and Ll is equal to 0.15 per unit and Ra is equal to 0.003 per unit right.

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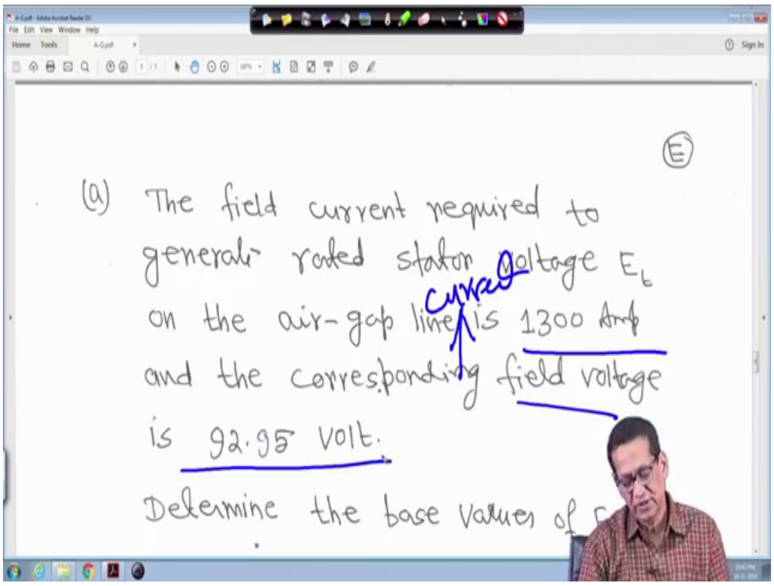

So, now first part is the field current that is part a field current required to generate rated stator voltage Et on the air gap your air gap line current it is air gap line current means ttr is it is air gap line current is 1300 ampere and the corresponding field voltage is 92.95 volt right.

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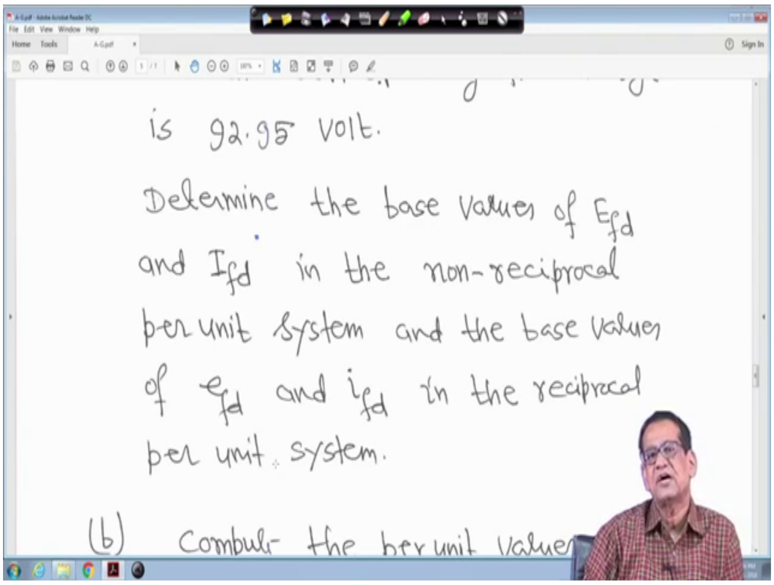

So, so what you have to what you have to determine that determine the base value of Efd and Ifd in the non reciprocal per unit system and the base value of Efd and Ifd in the reciprocal per unit system. The reciprocal and non reciprocal in the beginning of the course the per unit system everything is derived in detail right. So, I got this example I thought we should try to solve it a very simple one right.

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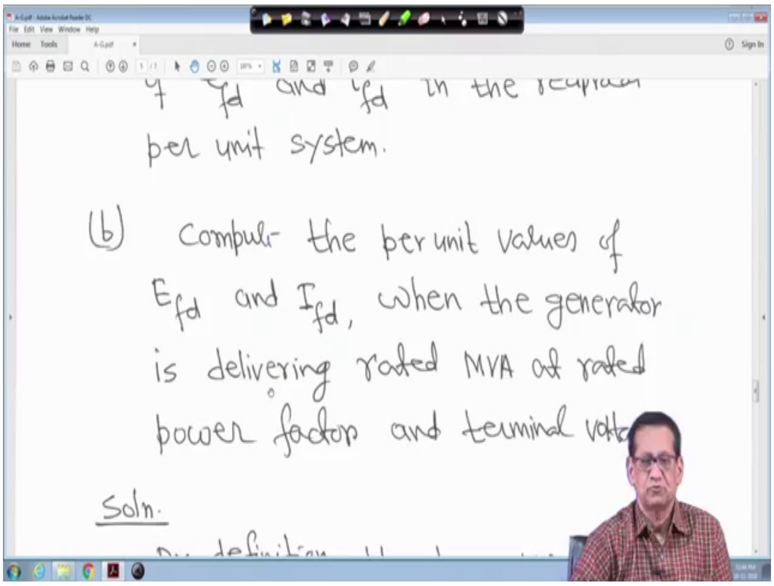

Now, part b is the compute the per unit values of Efd and Ifd when the generator is delivering rated MVA at rated power factor and terminal voltage right the store.

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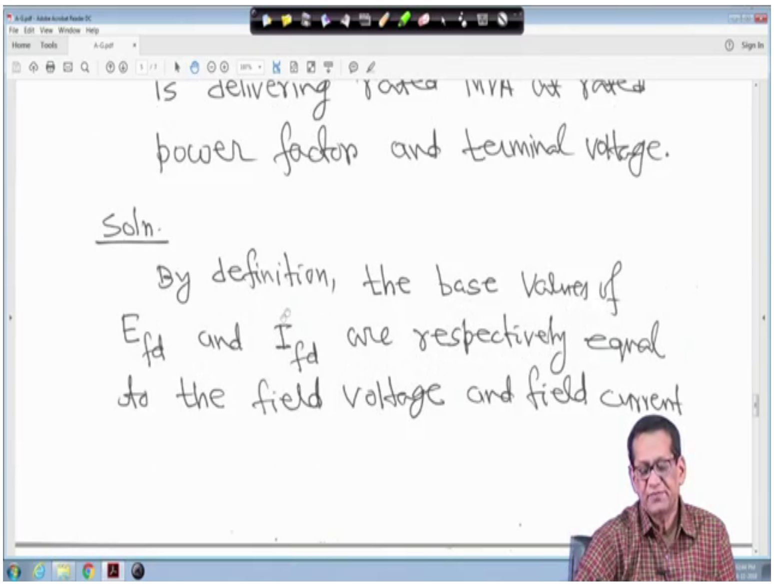

So, solution by definition the base values of Efd and Ifd are respectively equal to the field voltage and field current right.

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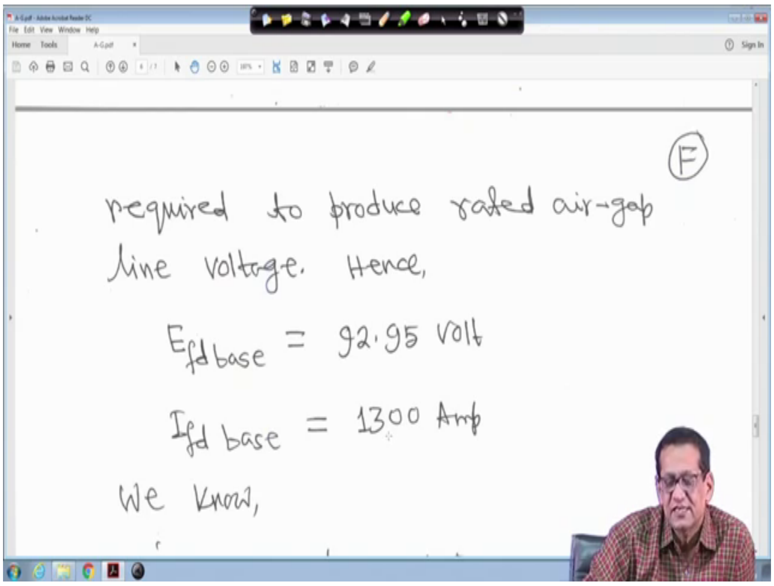

So, earlier when you are going for deriving that per unit system just open those things once right because it was done in the early beginning of this course to produce your rated air gap line voltage. Hence that Efd base actually you can take 92.95 volt this was given and Ifd base is 1300 ampere.

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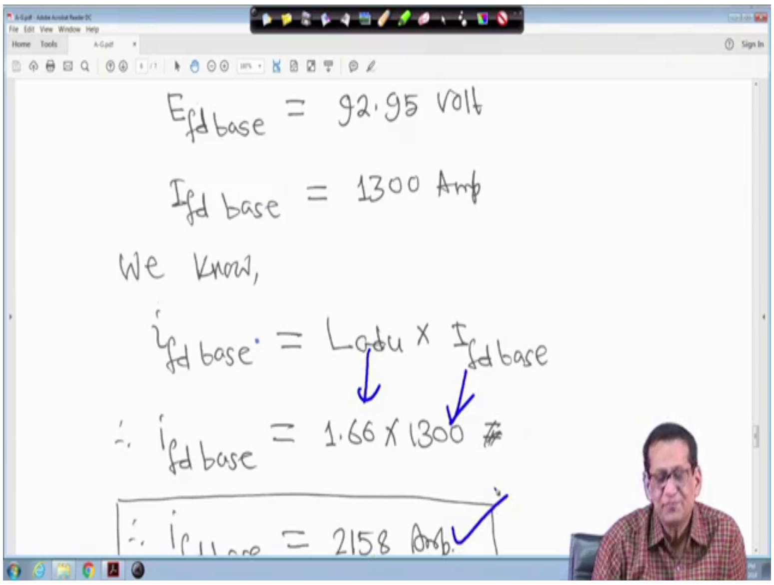

Now, we know that Ifd base is equal to Ladu into Ifd base right because that in your in per unit system that your inductance and your reactance they are same in per unit I told you that is why instead of writing in terms of x, I have written Ladu into Ifd base so understandable. Therefore, Ifd base will be 1.66 which data it is data this data were given right this data were given that 1.66 and it is 1300 right So, Ifd base will be 2158 ampere right.

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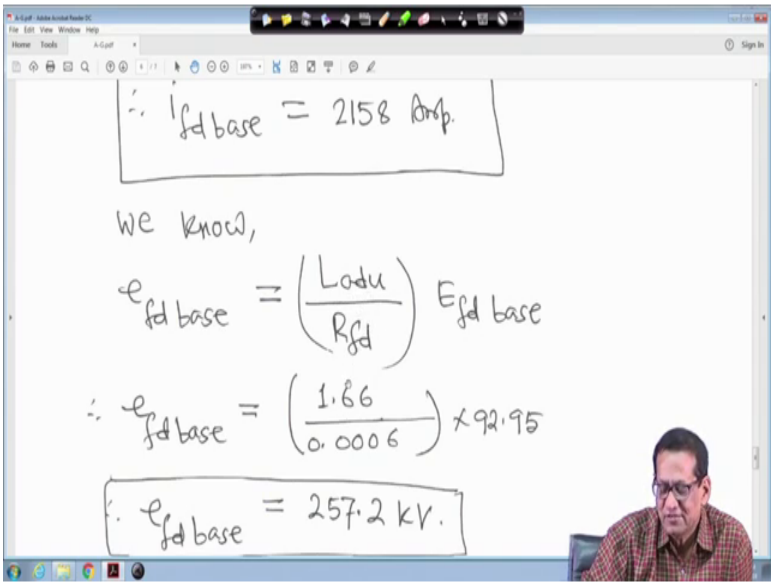

So, next is we also know that Efd base is Ladu upon Rfd into Efd base. Again I suggest you go back to this when you are trying to base quantities there you see this right that is why I have written we know that right. So, if the base is equal to Ladu upon Rfd into Efd base. So, Ladu is 1.66 and Rfd is 0.0006 into Efd base is 92.95 volt. Therefore, Efd base is equal to 257.2 kV right.

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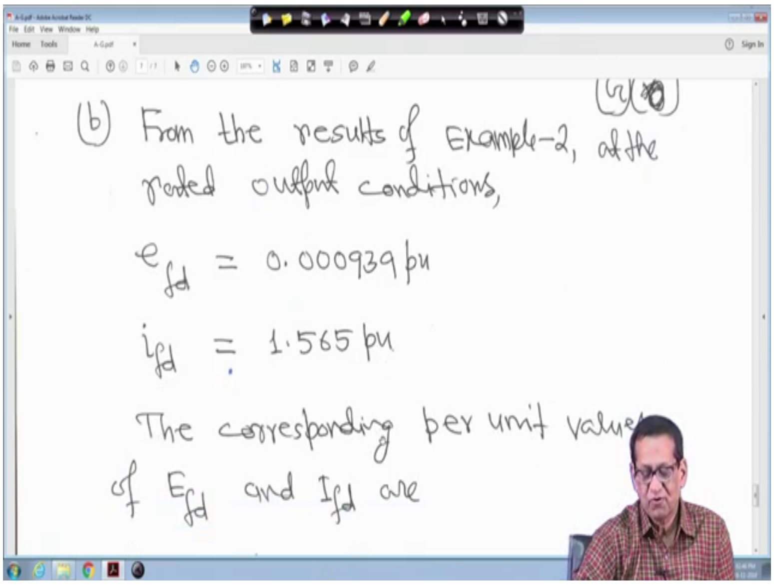

Now, part b from the results of example two at the rated output condition that example to you go back whatever you have done that Efd you got 0.000939 per unit and Ifd we got 1.565 per unit right.

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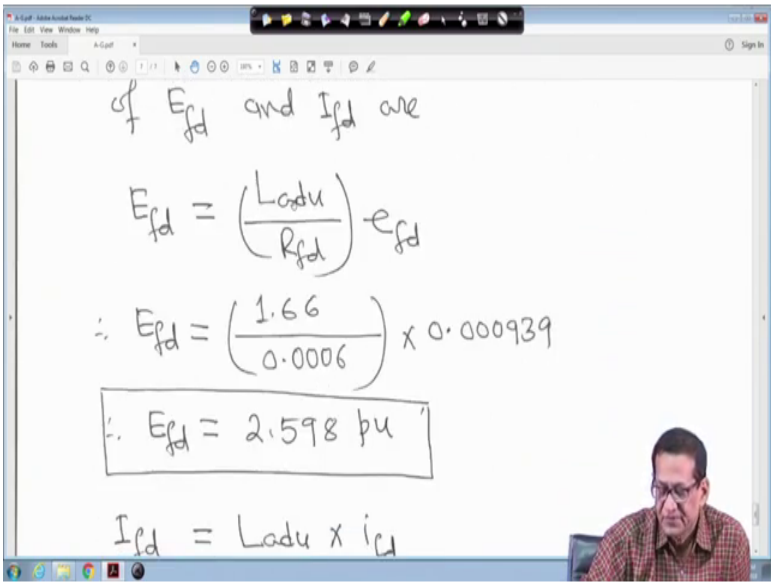

The corresponding per unit values of Efd and Ifd are that Efd is equal to Ladu upon Rfd into Efd. Therefore, Efd is equal to 1.66 upon 0.0006 into 0.00093 000939 this Efd value this you got from example two right. Therefore, it is coming 2.598 per unit.

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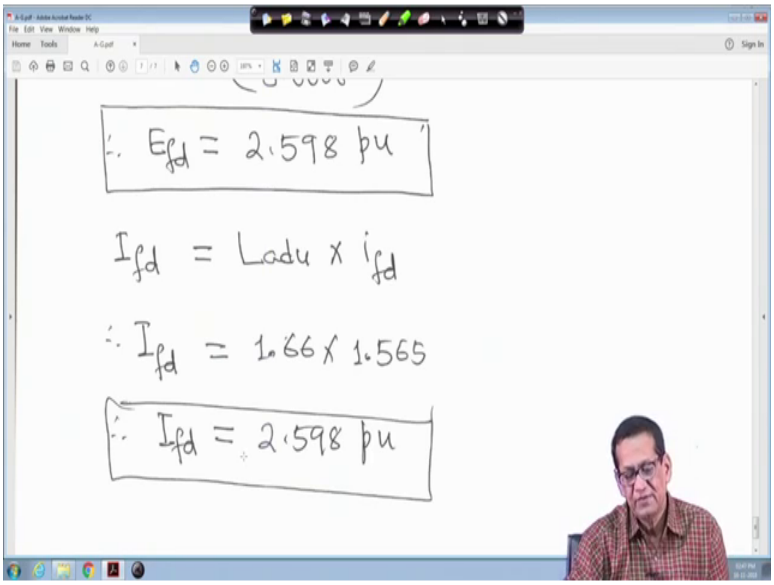

Similarly, Ifd is equal to Ladu into ifd. So, Ifd is equal to 1.66 into 1.565. So, this is actually Ifd is equal to 2.598 per unit. So, Efd and Ifd in terms of per unit for this case it is coming same right. I thought this one or two small example I should give you just for the purpose of understanding right. So, next will go to the next topic, just hold on.

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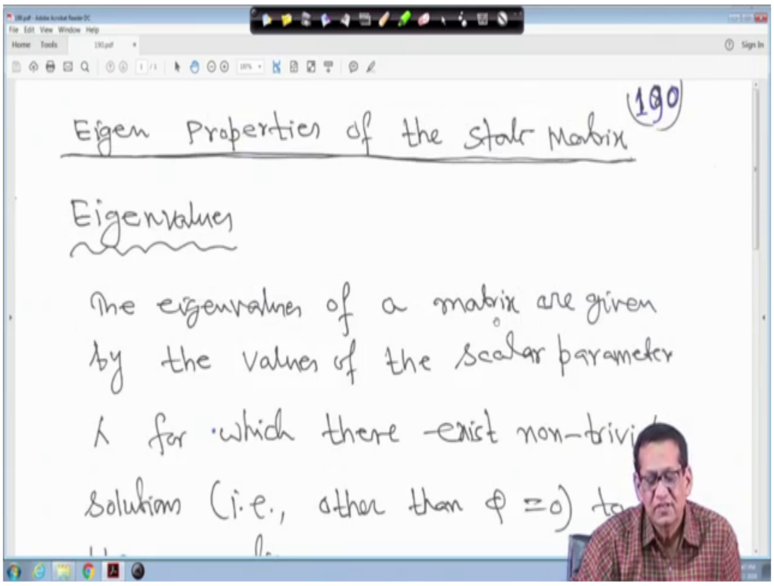

Next now we will come to the Eigen properties of the state matrix that is first will be Eigen values and then slowly and slowly you will see that your what you call that your participation factor right and dynamic responses right. So, the eigenvalues that you know eigenvalues other things you have studied right.

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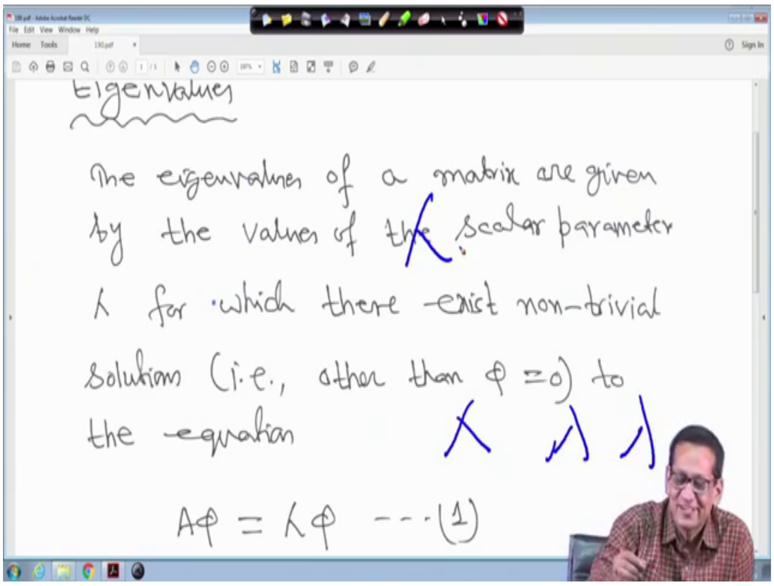

So, the eigenvalues of a matrix are given by the values of the scalar parameter lambda actually my habit had become to write lambda like this. Actually it is like this right actually lambda is like this right, but my habit has become lambda like this. So you should forgive me for that right. So, because now question is that the lambda for which there exist non trivial solution that is other than 5 is equal to 0 to the equation; that means, if your matrix A.

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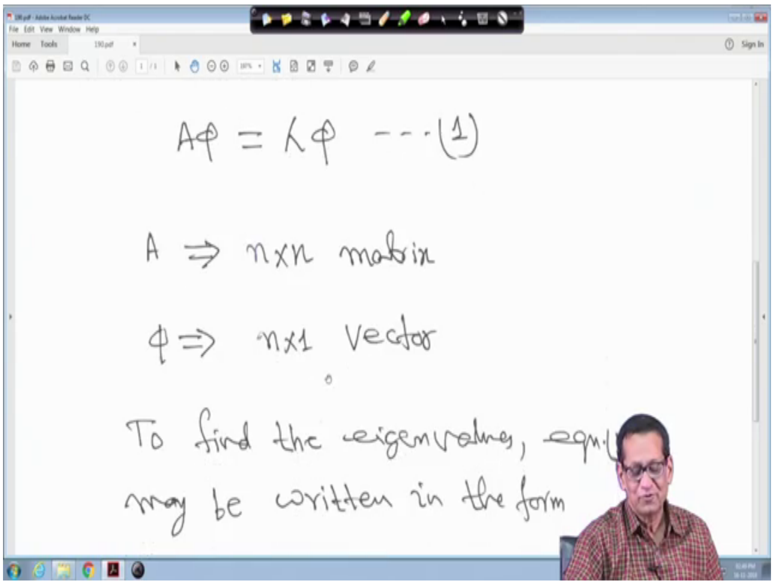

Then A phi is equal to say lambda phi question 1. A is equal to your this you know n into n matrix right and phi actually n into 1 vector right. So, now, to find eigenvalues of equation 1 that you know.

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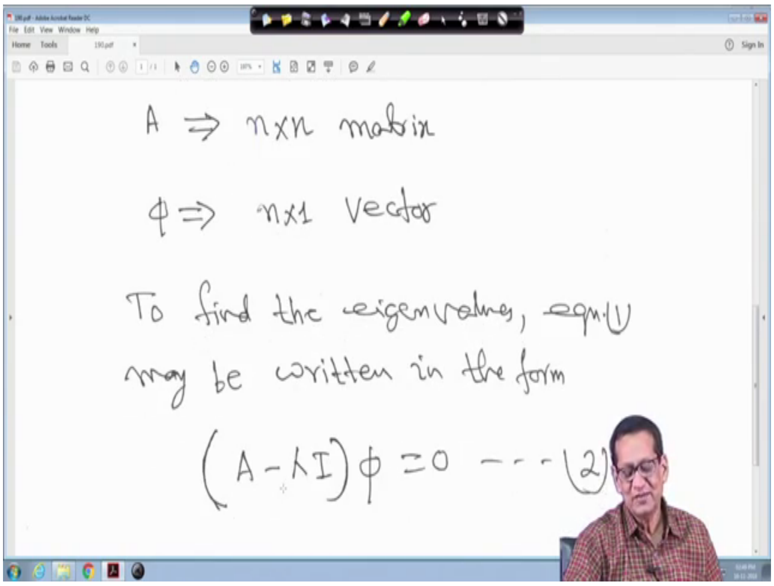

That may be written in the form that A minus lambda I bracket close phi into phi is equal to 0; this is equation 2 this you know right. So, next we will go to this next page just hold on right.

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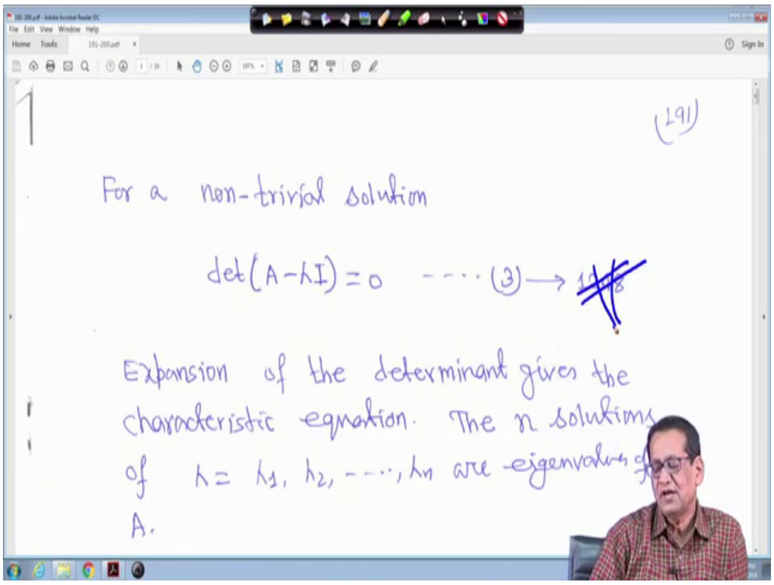

Now, for a nontrivial solution for a non trivial solution determinant of a minus lambda is equal to 0 this is actually equation 3 right. This is no need for you this is for my reference right. So, expansion of the determinant actually gives the characteristic equation this you also know. The n solutions of lambda is equal to lambda 1, lambda 2, up to lambda n are the eigenvalues of A right.

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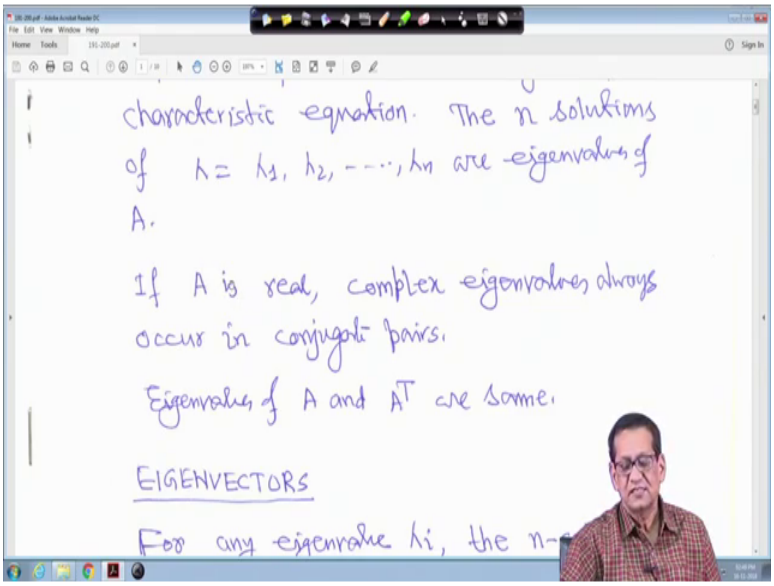

So, if A is real then complex eigenvalues always occur in conjugate pairs. One thing is there if A is real and symmetric, then all eigenvalues are real right. just repeat I am repeating if A is real and your symmetric matrix right then all eigenvalues are real. But if it is not symmetric then eigenvalues may be real maybe complex conjugate or both right.

So, complex eigenvalues always occur in conjugate pairs that also you know. But in this case matrix a will never become your symmetric matrix like your Aij naught is equal to Aji for all cases right. So, eigenvalues of another thing is that eigenvalues of A and A transpose are same right so this you know.

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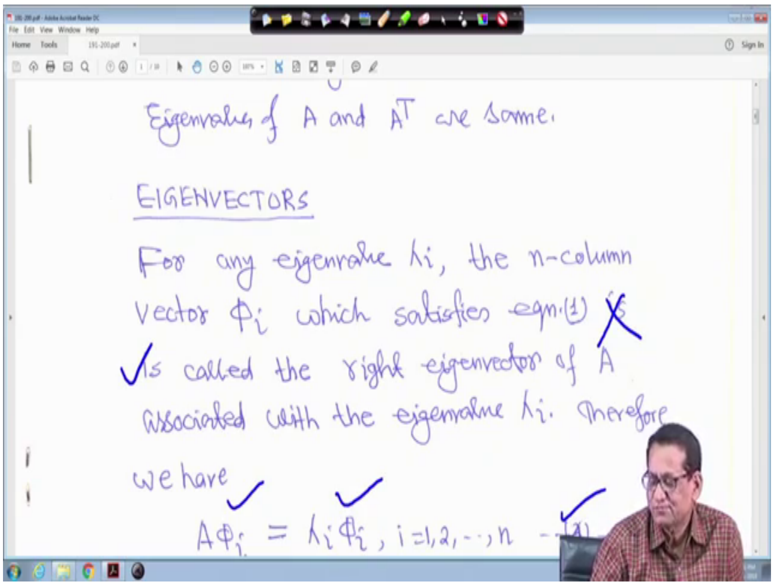

Next we need both right eigenvector as well as left eigenvector. So, first eigenvectors; for any eigenvalue lambda i the n column vector that is phi i which satisfies equation 1 is called the right eigenvector of a associated with the eigenvalue lambda I just repeat this you know it. But for any eigenvalue lambda I the n column vector phi I which satisfies equation 1 that is we have given is actually is has been written twice right. So, it should not be there one is here right is called the right eigenvector of a associate with the eigenvalue lambda i. Therefore, we have a phi i is equal to lambda i phi i for i is equal to 1 to n this is equation 4; this is for my reference right.

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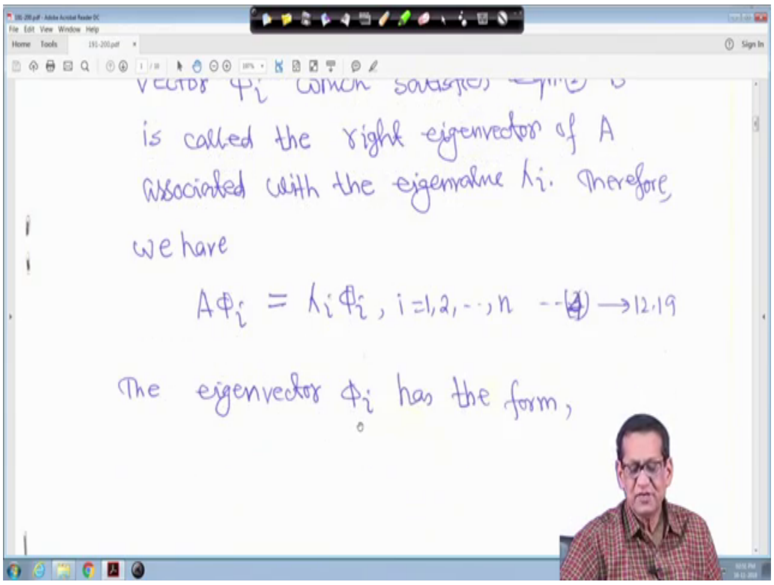

So, the Eigen vector phi i it has the form like this right.

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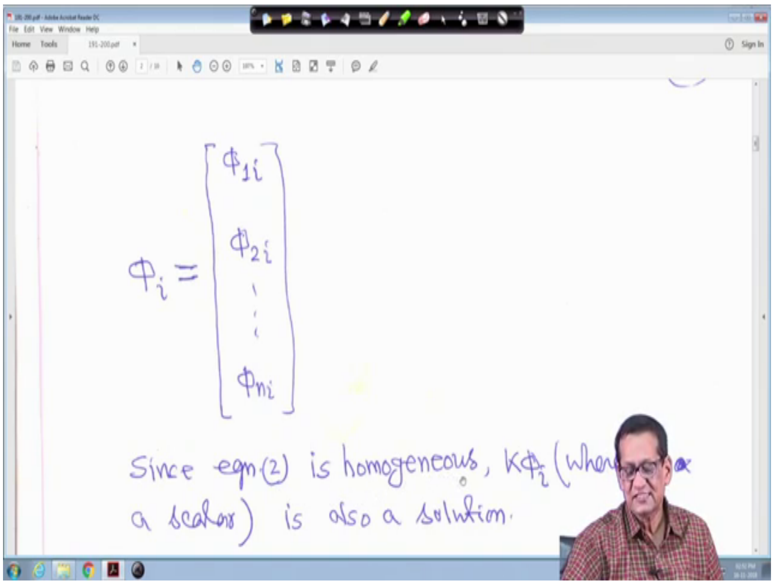

Write phi i is equal to phi 1i that is phi 2i up to phi ni. Since equation 2 is homogeneous right therefore, K phi i where K is A scalar is also a solution right.

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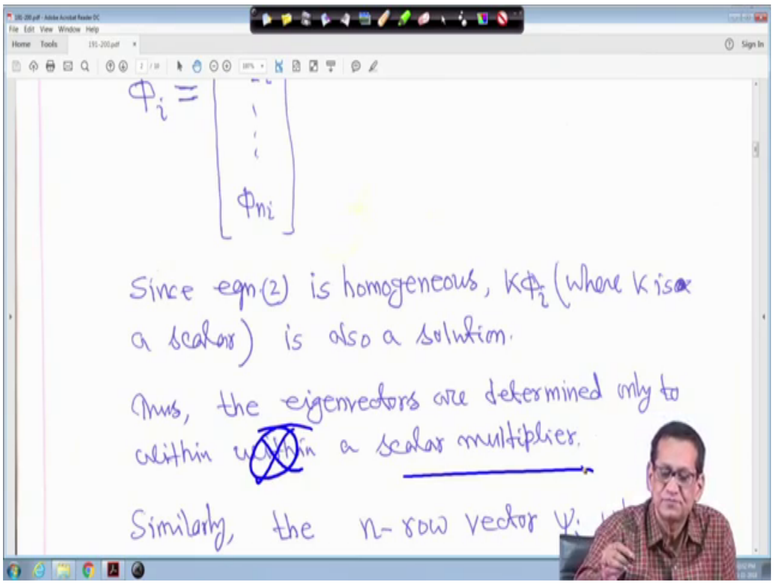

So, thus the eigenvectors are determined only to here also twice I have within forget about this. Thus the eigenvectors are determined only to within a scalar multiplier right.

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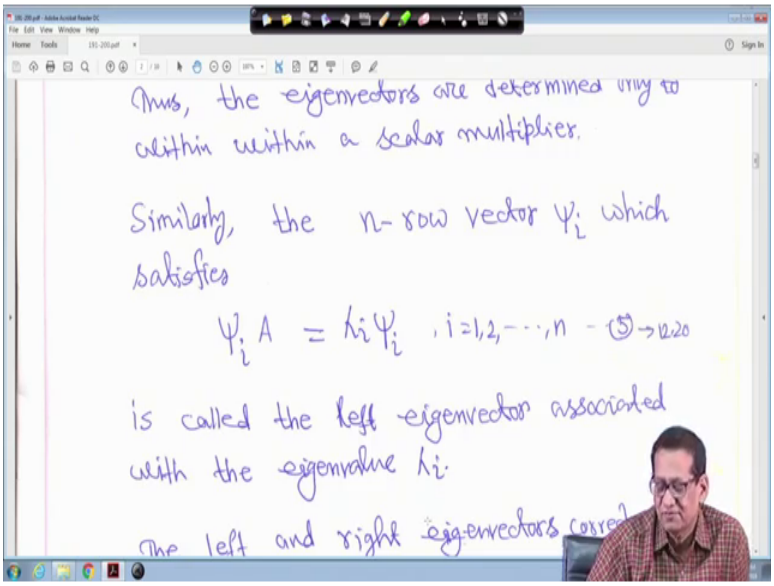

Similarly the just hold on similarly the n row vector your what you call psi i which satisfies psi i into A is equal to lambda i psi i right for i is equal to 1 to n this right this is do not see it is equation 5 this is for my own reference it is call the left associated with the eigenvalues lambda I right. Now the left and right eigenvectors corresponding to the different eigenvalues are the corresponding to different eigenvalues are orthogonal.

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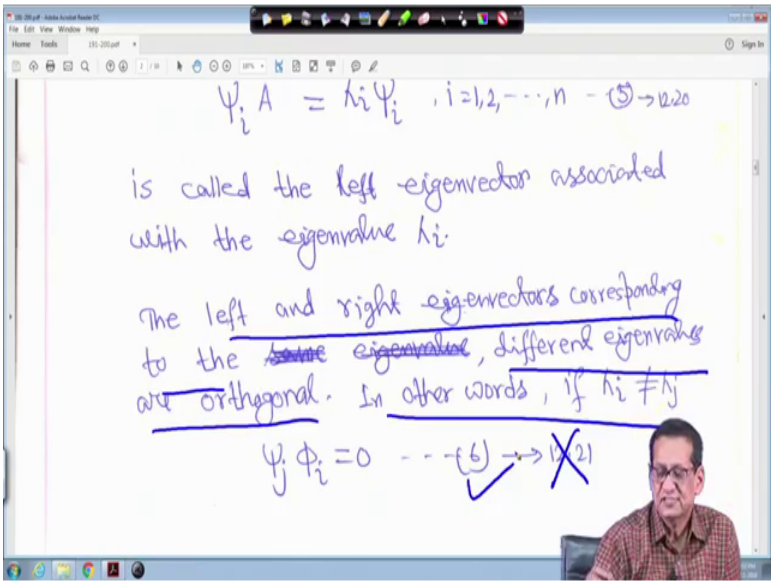

In other words if lambda is i naught is equal to say lambda j then psi j phi i is equal to 0 this is equation 6 right. So, this is you forget it right I believe you have little bit of knowledge on linear control system so only these are the preliminary things. Because the left and the left and right corresponding to the different eigenvalues are orthogonal that is; in other words if lambda i naught equal to lambda j psi J phi i is equal to say 0 this is equation 6 right.

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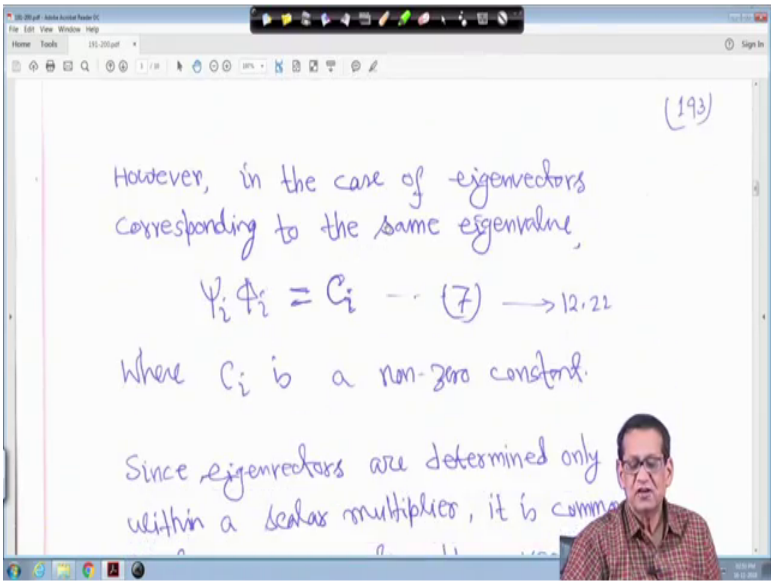

So, however, in the case of eigenvectors corresponding to the same eigen value we can write that psi i phi i is equal to Ci where Ci is a non zero constant right. This is equation 7 right.

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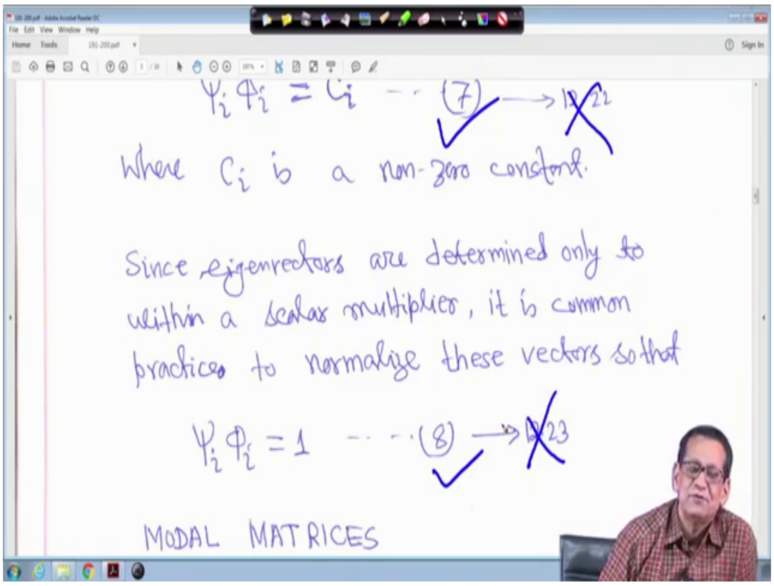

Since eigenvectors are determined only to within a scalar multiplier, it is common practice to normalize these vectors. So, that psi i phi i is equal to one or in other general psi phi is equal to i the identity matrix that we will see later. So, if you normalize these thing so psi i phi i is equal to 1; this is equation 8 right. And these things are for my reference this is not for you; this is 7 this is 8 right. So, psi i; phi i is equal to 1 right. So, these are the preliminary things before entering into that eigenvalue analysis.

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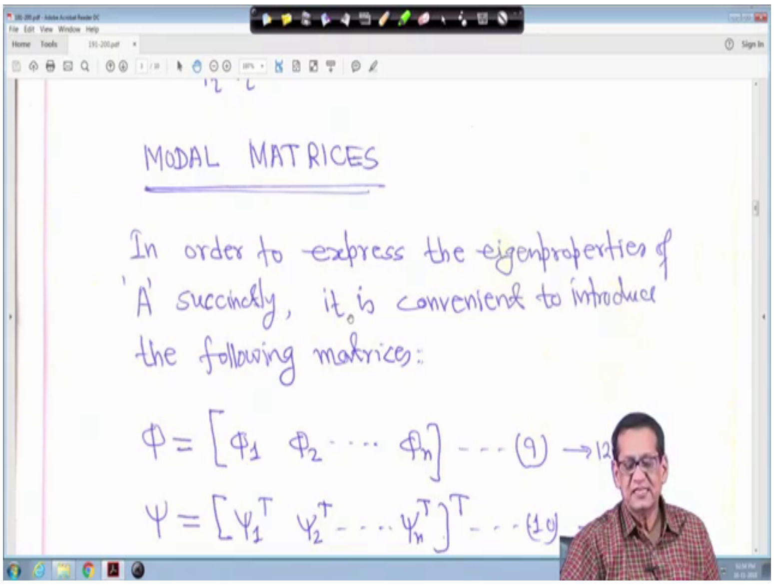

Now modal matrices in order to express the eigen properties of a succinctly it is convenient to introduce the following matrices.

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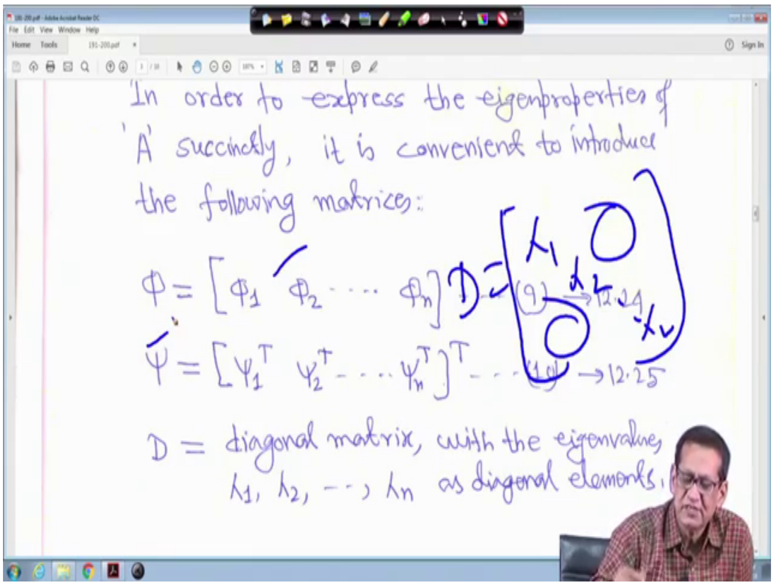

For example phi is equal to phi 1 phi 2 phi n this is equation 9, this is actually not a vector right it is a matrix later you will see this is not for you; this is equation 9. And psi is equal to psi 1 transpose we have taken psi 2 transpose psi n transpose and whole transpose right this is transpose and whole this is equation 10 right.

So, it is introduce the; this is actually matrix is an all column vector right it is a matrix. And D is equal to a diagonal matrix with the eigen value lambda 1 lambda 2 lambda n as diagonal elements right. That means, your D is equal to just hold on; that means, D is equal to actually it is a diagonal matrix lambda 1 lambda 2 up to lambda n other elements are 0 right. And this phi 1 phi 2 phi n or psi 1 transpose psi 2 transpose up to psi n transpose these are all column vector these are actually total it is a matrix phi psi later we will see it is a matrix, but we are representing like this right.

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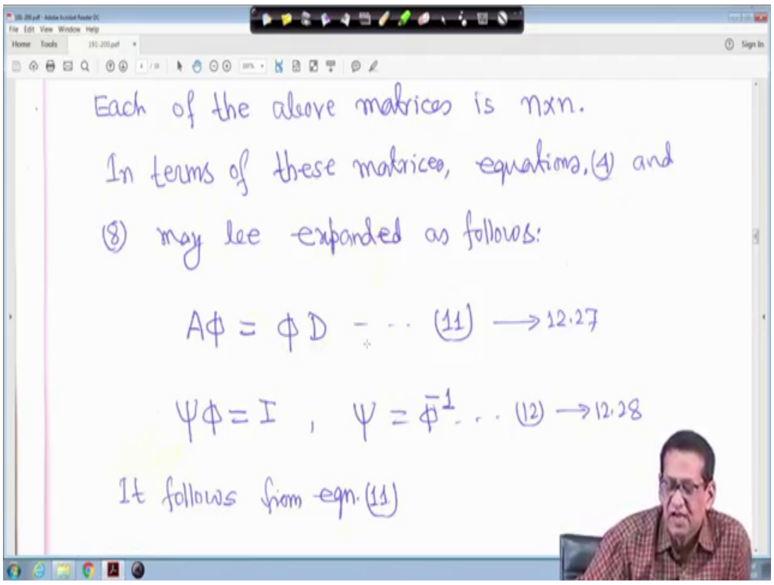

Each of the above matrices is n into nth order right your matrix order a is n to into n though this is also n into n diagonal D also n into n. Because you have all n diagonal your, what you call element that is nothing, but the eigenvalues. In terms of this matrices equation 4 and 8 may be expanded as follows. So, equation write A phi is equal to your phi D equation 4 and 8 right. If you go back to equation 8 this condition is satisfy psi phi is equal to 1 and if you come to your equation 4 right. Here it is equation 4 that A phi is equal to lambda lambda lambda phi in general A phi is equal to lambda phi right. And that is A I phi i is equal to your lambda i phi i.

So, it is written that a phi i is equal to lambda i phi that your what you call a phi i is equal to lambda i phi i is written. But just let me explain here that it is written here in equation 4 that your a phi i is equal to lambda i phi i in general a phi is equal to we are writing phi D because D is a diagonal matrix that is it is only all diagonal elements are eigenvalues lambda. So, you can write a phi is equal to phi D. That is why you are writing here in this equation that your a phi is equal to phi D this is equation 11 right.

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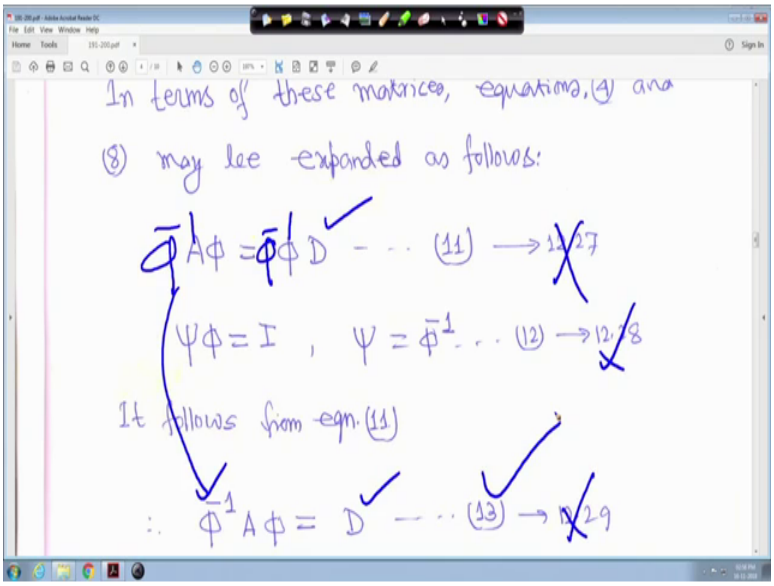

And we have since phi i is equal to 1 this is not for you right so for my own reference. So, psi phi is equal to identity matrix or psi is equal to phi inverse right. So, now this equation this equation both side this equation both side you multiply by phi inverse. If you multiply this by phi inverse and this side by phi inverse. So, it is phi inverse a phi it is here coming and phi inverse phi is nothing, but identity matrix that is nothing, but then it is D only right. This is actually equation 13. So, phi inverse a phi is equal to nothing, but the your D matrix that is your diagonal matrix sorry diagonal elements are your eigenvalues right of the matrix A right.

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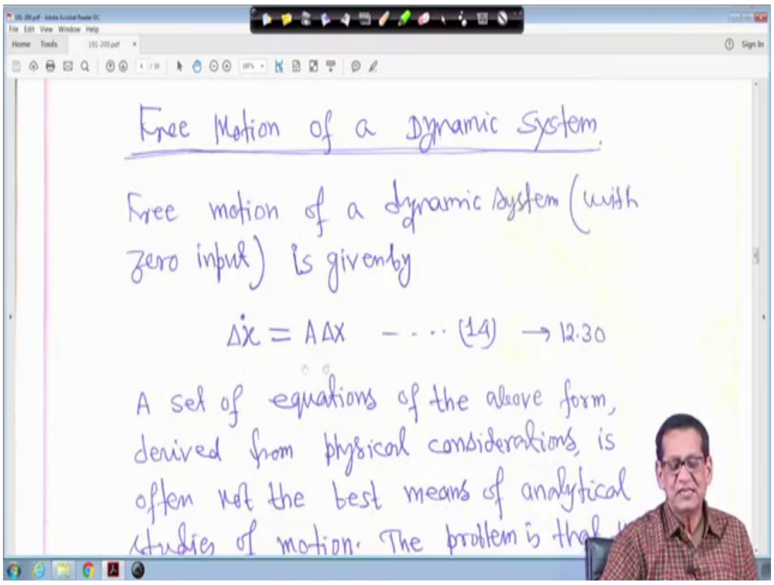

So, see now with this with this basic thing now we will move slowly and slowly to the other thing right; with this thing in our mind because these thing we will apply later. Now free motion of a dynamic system right now free motion of a dynamic system say with 0 input can be given as delta x dot is equal to A delta X this is free motion of a dynamic system from that we will start right.

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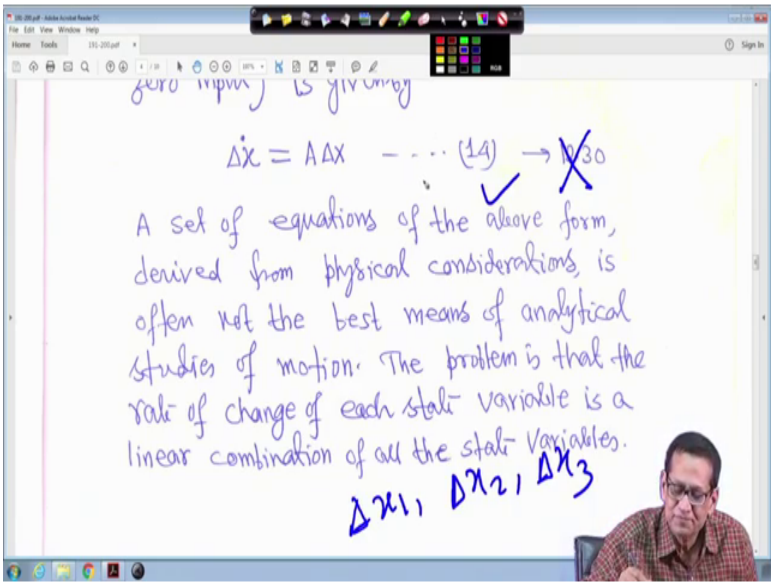

So, we can write the delta x dot is equal to A delta X. So, A set of equations of the above form derived from physical consideration is often not the best means of analytical studies of motion. The problem is that the rate of change of each state variable is a linear combination of all the state variable. The meaning is something like this suppose for example, I am just writing somewhere this delta X suppose you have three state variable right. For example, suppose delta x 1 delta x 2 and delta x 3. Suppose you have three variables and a is a 3 into 3 matrix right.

So; that means, let me here it is a blue color. So, just hold on let me change the your what you call that color right just hold on right. So, it will be easy for me to the put it as there is a blue color. Now suppose you have a three state variable then I have just over writing on it.

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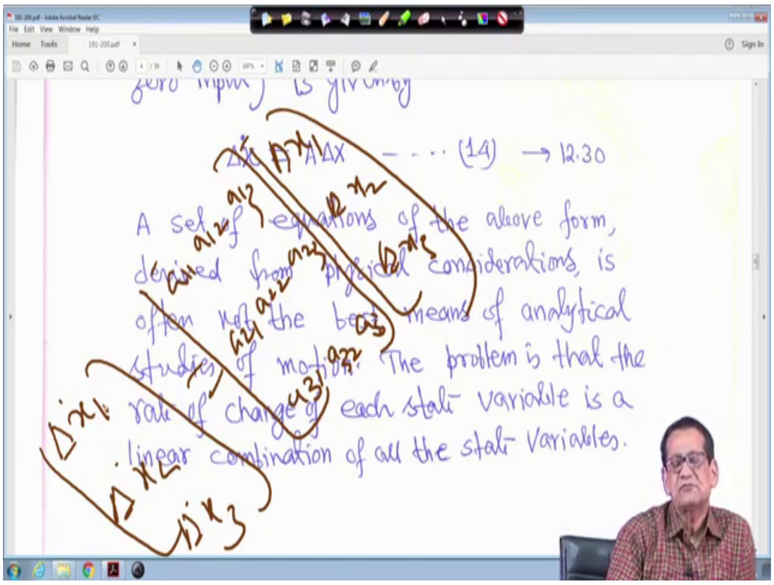

Say then this is my delta x 1 dot delta x 2 dot and delta x 3 dot right and this side is a matrix that is your a11 a12 a13 a21 a22 a23 say it is a full matrix some of the element may be 0. a 32 and a 33 and this side is your delta x 1 delta x 2 and delta x 3 right.

So, if you write delta x 1 dot, say in this case delta x 1 dot will be function of all delta x 1 delta x 2 delta x 3. Some of the element may be 0 it does not matter, but delta x 1 dot is equal to a 11 delta x 1 ,a12 delta x 2 then a 13 delta x 3 like this. Delta x 2 dot also your what you call a 21 delta x1, a 22 delta x 2 a 23 delta x 3 similarly delta x 3 dot also will be your a 31 delta x 1 a 32 your delta x 2 a 33 delta x 3; that means, this state variables there it is for example, this left hand side that delta x 1 dot is also related to apart from delta x 1 it is also related to delta x 2 and delta x 3 right.

So, that is why the it is sometime that is why it is your what you call it is not often the best means of analytical studies of motion. So, the problem is that the rate of change of each state variable is a linear combination of the state variable. That is delta x 1 dot delta x 2 dot like this right.

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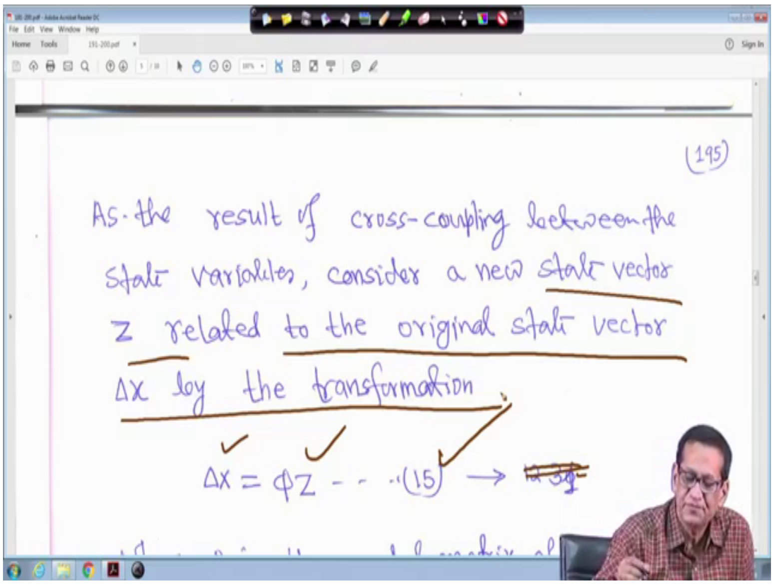

So, as a as the result of cross coupling between the state variables consider a new state vector z related to the original state vector delta x by the transformation. So, what we will do we consider a new state vector z right we do a new state vector z related to the original state vector delta x by the transformation. That is we are assuming say delta x is equal to so you are assuming phi into z this is equation 15 right. So, you are assuming some new state vector.

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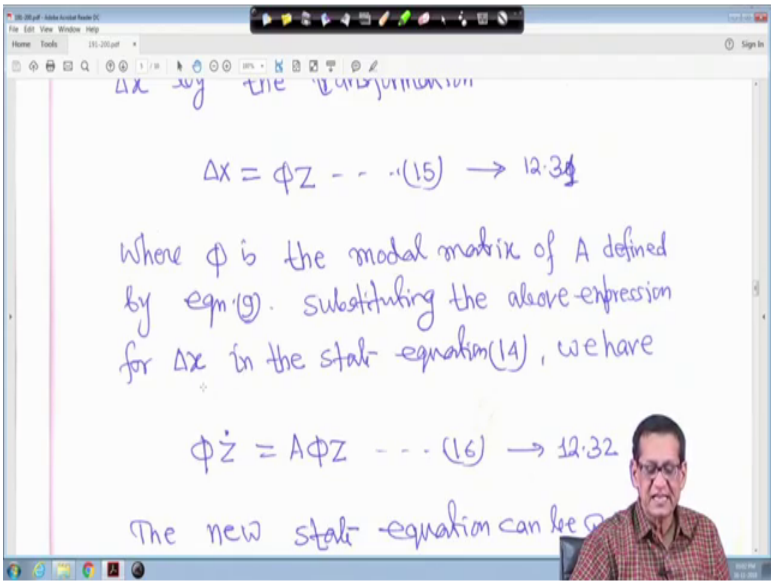

So; that means, where phi is the model matrix of A defined by equation 9 this you have seen earlier. Substituting the above expression for delta x in the state equation 14. So, this equation here your delta x dot is equal to A delta X here you substitute this right. So, if you substitute delta X is equal to phi z; that means, my delta X dot will be phi Z dot right. So, it is phi Z dot is equal to A phi Z because delta X your delta X dot is equal to X and then you substitute delta X is equal to phi Z. So, you will get phi into Z dot is equal to A phi Z this is equation 16 right.

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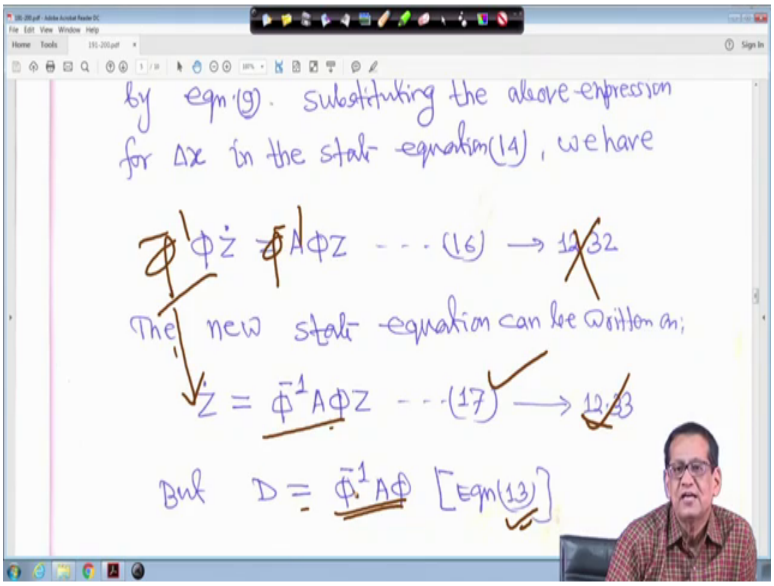

Or this is not for you or both. Now both side if you multiply by phi inverse if you multiply by phi inverse. So, phi inverse phi is equal to identity matrix. So, left hand side it is Z dot is equal to phi inverse A phi Z this is equation 17. But earlier we have seen the D is equal to phi inverse A phi this is from equation 13 this we have seen right. This is nothing but D is the diagonal matrix right that is all diagonal elements are the eigenvalues of the A matrix right. So, that means, we can write Z dot is equal to D Z.

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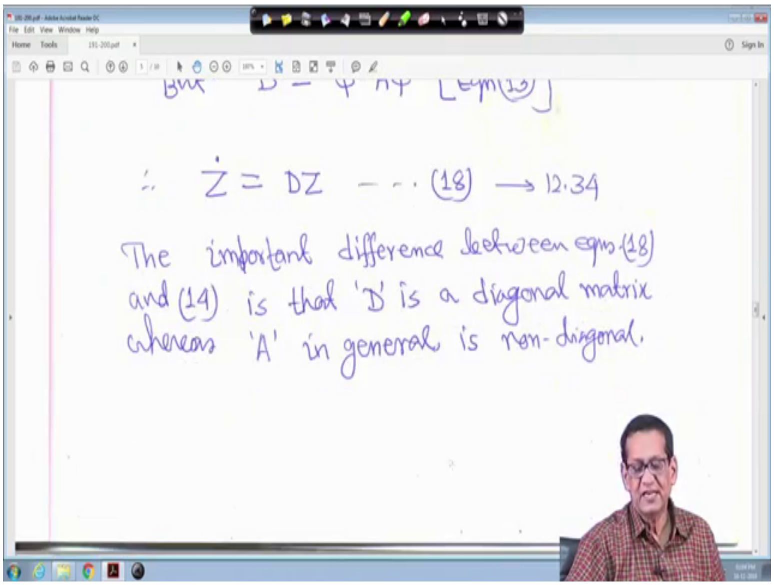

So, we can write Z dot is equal to D Z this is equation 18 right. So, the important difference between equation 18 and 14 is that D is a diagonal matrix whereas, A is general is non diagonal right. That means, what happen? D is a diagonal matrix; that means, equation is lambda 1 lambda 2 lambda 3 lambda n with Z 1 dot will become lambda 1 Z 1 Z 2 dot will become lambda 2 Z 2 like this. So, other a other things are not involve right. That means, this make our your what you call analysis simper right.

With this thank you very much we will be back again.