

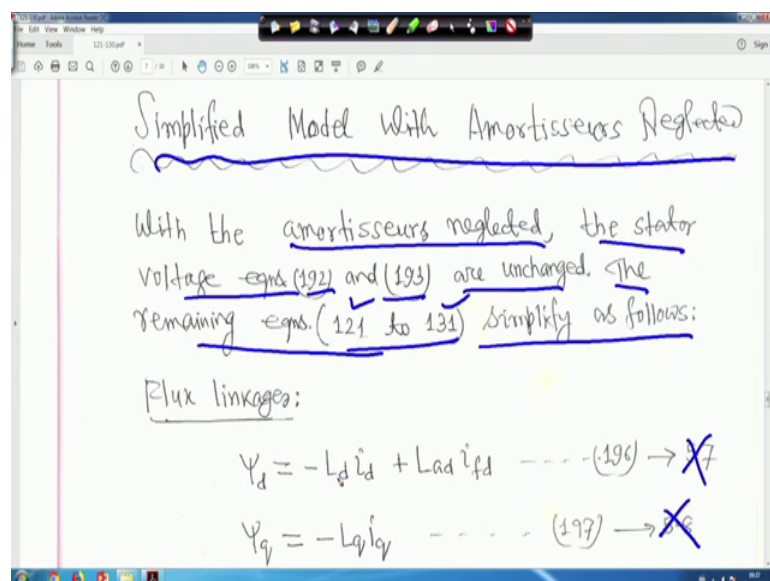
Power System Dynamics, Control and Monitoring
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Lecture – 14
Power System stability (Contd.)

Ok, so next so previous example a little bit of you know nothing to be confuse actually just see very carefully it is a if this example is a very unique example actually. And just you try yourself to derive that thing and most of the things I have derived and I have told you how to do it.

But if you give elaborate your derivation then it will not be it will be very your lengthy procedure and it will consume more time in a video course and video lecture class right.

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So, next is that your simplified model with amortisseurs neglected right. So, first we will we will take a simplified model. So, with the amortisseurs neglected the stator voltage equation that is equation 192 and 193 are unchanged. But the remaining equation 121 to 131 simplify as follows right.

So, your what you call that your from equation 121 to 131. So, total 11 equations right so because we are neglecting the amortisseur terms. If you do so this is not required for you

this is for my reference right. So, ψ_d is equal to ok, let me move let me move little bit up.

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Remaining eqns. (121 to 131) simplify as follows.

Flux linkages:

$$\psi_d = -L_d i_d + L_{ad} i_{fd} \quad \text{--- (196) } \rightarrow \text{X}$$

$$\psi_q = -L_q i_q \quad \text{--- (197) } \rightarrow \text{X}$$

$$\psi_{fd} = -L_{ad} i_d + L_{ffd} i_{fd} \quad \text{--- (198) } \rightarrow \text{X}$$

So flux linkage first, so ψ_d is equal to minus $L_d i_d$ plus $L_{ad} i_{fd}$ with amortisseurs neglected. This is not for you this is for my reference right. So, ψ_q is equal to minus $L_q i_q$ this is equation 197 and ψ_{fd} is equal to minus $L_{ad} i_d$ plus $L_{ffd} i_{fd}$ this is equation 198. All these nomenclatures all this nomenclatures have been given before so right.

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Rotor Voltage:

$$e_{fd} = b\psi_{fd} + R_{fd} i_{fd}$$

or

$$b\psi_{fd} = e_{fd} - R_{fd} i_{fd} \quad \text{--- (199) } \rightarrow \text{X}$$

Alternative form of machine equations

Eqs. (196) to (198) are often written in terms of field variables:

So, next is the rotor voltage equation with amortisseurs neglected. So, in this in here e_{fd} will be $p \psi_{fd}$ plus $R_{fd} i_{fd}$ right or this $p \psi_{fd}$ or $p \psi_{fd}$ is equal to just rewrite this equation is equal to e_{fd} minus $R_{fd} i_{fd}$ this is equation 199. This is not required this is for my reference right. So, next we will make that alternative form of machine equations. Because we have to make it in till that block diagram whatever you have started it will slowly and slowly it will block diagram will grow later you will see.

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Alternative form of machine equations

Eqns. (196) to (198) are often written in terms of the following variables:

- ✓ $E_I = L_{ad} i_{fd} = \text{Voltage proportional to } i_{fd}$
- ✓ $E_q' = \frac{L_{ad}}{L_{ffd}} \psi_{fd} = \text{Voltage proportional to } \psi_{fd}$
- $E_{fd} = \frac{L_{ad}}{R_{fd}} e_{fd} = \text{Voltage proportional to } e_{fd}$

Now, alternative form of machine equations now equations 196 to 198 this we will write little bit different way are often written in terms of the following variables right. That is E_I we define a variable E_I ; E suffix capital I is equal to $L_{ad} i_{fd}$; that is voltage proportional to i_{fd} right. Another term we write capital E_q dash is equal to L_{ad} upon L_{ffd} ψ_{fd} that is voltage proportional to ψ_{fd} . Another term we write capital E_{fd} L_{ad} upon R_{fd} into your e_{fd} that is voltage proportional to e_{fd} . Because we have to simplify we have to simplify the whole mathematical derivations right. So, we represent like this.

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$E_I = L_{ad} i_{fd} = \text{voltage proportional to } i_{fd}$
 $E_q' = \frac{L_{ad}}{L_{ffd}} \psi_{fd} = \text{voltage proportional to } \psi_{fd}$
 $E_{fd} = \frac{L_{ad}}{R_{fd}} e_{fd} = \text{voltage proportional to } e_{fd}$

In terms of the new variables, eqn.(196) becomes

$\psi_d = -L_d i_d + E_I \dots (200) \rightarrow 5.11$

multiplying eqn.(198) by $\frac{L_{ad}}{L_{ffd}}$ throughout and

So in terms of the new variables equation 196 will become; that means, whatever we have seen here equation 196, 197 and 198 right. All these things in terms of new variables, it will become that your ψ_d will be minus $L_d i_d$ plus E_I this is equation your this is equation 200 right. So, this is not required this is for my reference.

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$\psi_d = -L_d i_d + E_I \dots (200) \rightarrow 5.11$

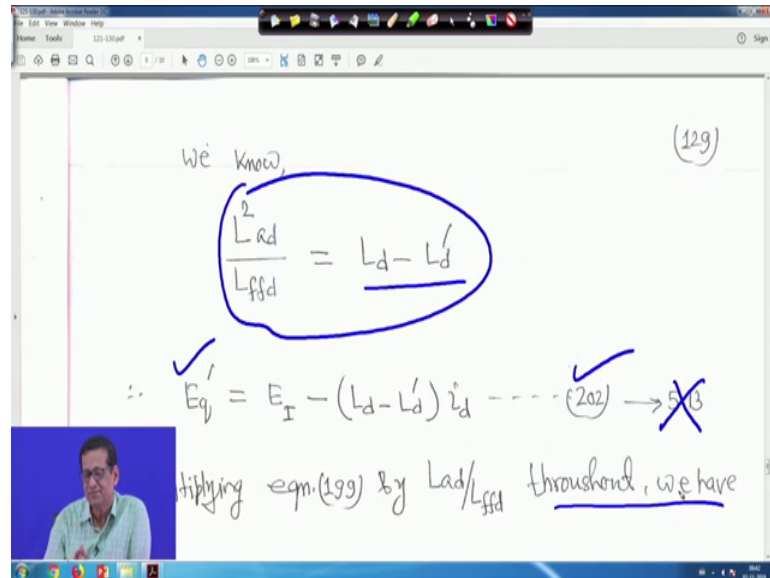
Multiplying eqn.(198) by $\frac{L_{ad}}{L_{ffd}}$ throughout and expressing in terms of the new variables, we get,

$E_q' = -\frac{L_{ad}^2}{L_{ffd}} i_d + E_I \dots (201) \rightarrow 5.12$

So and multiplying equation 198, you multiply equation 198 by L_{ad} upon L_{ffd} right. And throughout and expressing in terms of the new variables, then you will get E_q' will be minus L_{ad}^2 upon L_{ffd} i_d plus E_I . You just equation 198 multiplied

both side by L_{ad} upon L_{ffd} and then you simplify and whatever assumptions we have made right and then you can get E_q' will be $\frac{L_{ad}^2}{L_{ffd}}$ minus L_{ad} square upon L_{ffd} id plus E_I . This is equation 201 right.

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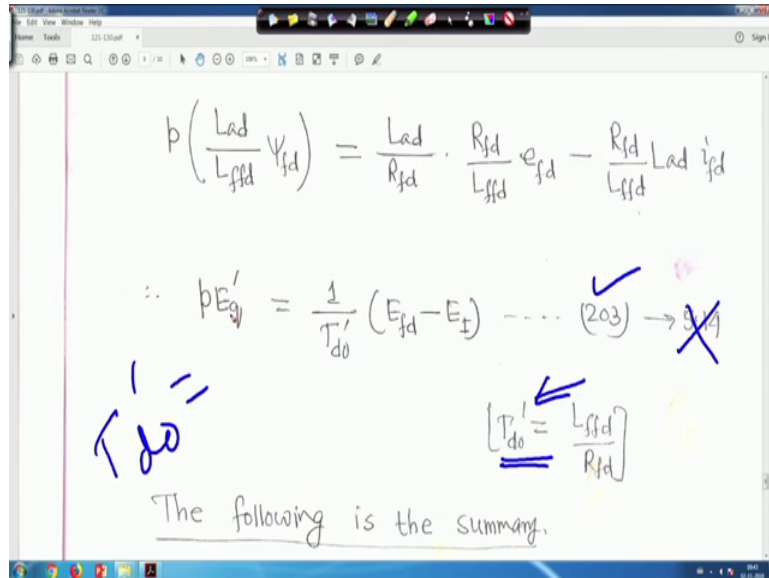
So, now this one we cannot we here what you call we cannot we are not interested to prove this one right. So, we assume $\frac{L_{ad}^2}{L_{ffd}}$ is $L_d - L'_d$. This is actually it can be derivation is there, but I will keep that one we will assume that $\frac{L_{ad}^2}{L_{ffd}}$ is $L_d - L'_d$ right. And when you will listen to the lecture if you need the your what you call that expression of this one, then we will put it in to the forum right.

But otherwise what will happen the continuity of this your topic will be lost right. That is why $\frac{L_{ad}^2}{L_{ffd}}$ it is $L_d - L'_d$ like your X_d double dash X_d dash half transient, transient steady state reactance's right. So, it can be proved $L_d - L'_d$ dash. So, $\frac{L_{ad}^2}{L_{ffd}}$ so, but if you have anything you put in the forum we will explain, but here we will we will not then continuity will be lost right. So, that is why I have written we know $\frac{L_{ad}^2}{L_{ffd}}$ is equal to $L_d - L'_d$.

This also you should keep it in your memory right. So, if it is so then E_q' it can be written as E_I that is capital E suffix capital I minus in bracket $L_d - L'_d$ id this is equation 202 right. This is other things for my reference right. Now, what you do now

multiply equation 199 by L_{afd} upon L_{ffd} throughout right. I mean left and left I mean left and right side both right.

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$$p \left(\frac{L_{ad}}{L_{ffd}} V_{fd} \right) = \frac{L_{ad}}{R_{fd}} \cdot \frac{R_{fd}}{L_{ffd}} e_{fd} - \frac{R_{fd}}{L_{ffd}} L_{ad} i_{fd}$$

$$\therefore pE_{q'} = \frac{1}{T'_{do}} (E_{fd} - E_I) \quad \dots (203) \rightarrow \text{X}$$

$T'_{do} = \frac{L_{ffd}}{R_{fd}}$

The following is the summary.

If you do so then we have that is p into L_{ad} upon L_{ffd} ψ_{fd} is equal to L_{ad} upon R_{fd} into R_{fd} upon L_{ffd} e_{fd} minus R_{fd} upon L_{ffd} L_{ad} into i_{fd} right. Or this term this term we have as you know if this thing is it will be $p E_{q'}$ dash will be is equal to 1 upon actually T'_{do} dash into E_{fd} minus E_I . And this is T'_{do} dash is equal to L_{ffd} upon R_{fd} . I am not telling you know this term so this is question to you what is what is your T'_{do} dash?

It is time constant of course but what is the full form right it is time constant. So, this is a small question to you that T'_{do} dash is equal to L_{ffd} upon R_{fd} right. And if you multiply and simplify this we will if this equation will simplify become that 1 upon T'_{do} dash E_{fd} minus E_I this is equation 203 right. So, this is your what you call that your basically it is nothing, but $E_{q'}$ dash dot right. So, now the following is the summary.

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The following is the summary.

$$\Psi_d = -L_d i_d + E_I$$

$$\Psi_q = -L_q i_q$$

$$E_q' = E_I - (L_d - L_d') i_d$$

$$p E_q' = \frac{1}{T_{d0}'} (E_{fd} - E_I)$$

So, summary is that ψ_d is equal to minus $L_d i_d$ plus E_I ; ψ_q is equal to minus $L_q i_q$. E_q' will be E_I minus in bracket L_d minus L_d' i_d right. And $p E_q'$ will be 1 upon T_{d0}' in bracket E_{fd} minus E_I this is the summary right. Now, phasor diagram for transient condition right.

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Phasor Diagram for Transient Conditions:

In order to do this, it is first necessary to express E_q' , E_I and E_q in terms of d- and q-axis components of terminal voltage and current.

Since in per unit, $X_d = L_d$, from eqns. (193) and (196)

So, in order to do this; this is first necessary to express E_q' , E_I and E_q in terms of d and q axis is component of terminal voltage and current. Because we have to this representation that your phasor diagram for transient condition we want. So, if you do so

that that your express E_q and E_q in terms of d and q axis component of terminal voltage and the current right.

So, in per unit X_d is equal to L_d I told you in per unit reactants and inductance are same. So, from equation 193 and equation 196 right we will write like this. So, we will replace L_d by X_d , X_d right. So, it will be something like this.

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CURRENT

Since in per unit, $X_d = L_d$, from eqns (193) and (196),

$$e_q = \psi_d - R_a i_q$$

$$\therefore e_q = -X_d i_d + X_{ad} i_{fd} - R_a i_q$$

$$\therefore e_q = -X_d i_d + E_f - R_a i_q$$

Therefore,

That e_q that in equation 193 to 196 e_q will be ψ_d minus $R_a i_q$ this is same as before right, e_q will be instead of L_d we will write X_d minus $X_d i_d$ plus instead of L_{ad} we will write $X_{ad} i_{fd}$ minus $R_a i_q$ right because in per unit inductance and reactants are same right.

So, e_q will be minus $X_d i_d$ plus capital E suffix capital I minus $R_a i_q$ this first you will rewrite right. Therefore, we can write therefore, E_f is equal to e_q plus $X_d i_d$ plus $R_a i_q$ right.

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$$e_q = V_d - R_a i_q$$

$$\therefore e_q = -X_d i_d + X_{ad} i_{fd} - R_a i_q$$

$$\therefore e_q = -X_d i_d + E_I - R_a i_q$$
 Therefore,

$$E_I = e_q + X_d i_d + R_a i_q$$
 Multiplying by j , we have,

$$jE_I = j e_q + j X_d i_d + j R_a i_q$$

Therefore E_I will be e_q plus your; I mean from this equation E_I will be e_q plus I mean from this equation it is coming from this equation only here it is writing E_I is equal to e_q plus $X_d i_d$ plus $R_a i_q$ right. Now, multiply both side by j right that is the complex operator.

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$$E_I = e_q + X_d i_d + R_a i_q$$
 Multiplying by j , we have, $\tilde{E}_I = j E_I$

$$j E_I = j e_q + j X_d i_d + j R_a i_q$$

$$\tilde{e}_q = j e_q$$
 In terms of phasor notation,

$$\tilde{E}_I = \tilde{e}_q + X_d \tilde{i}_d + R_a \tilde{i}_q$$

$$\tilde{i}_d = j i_d$$

$$\tilde{i}_q = j i_q$$

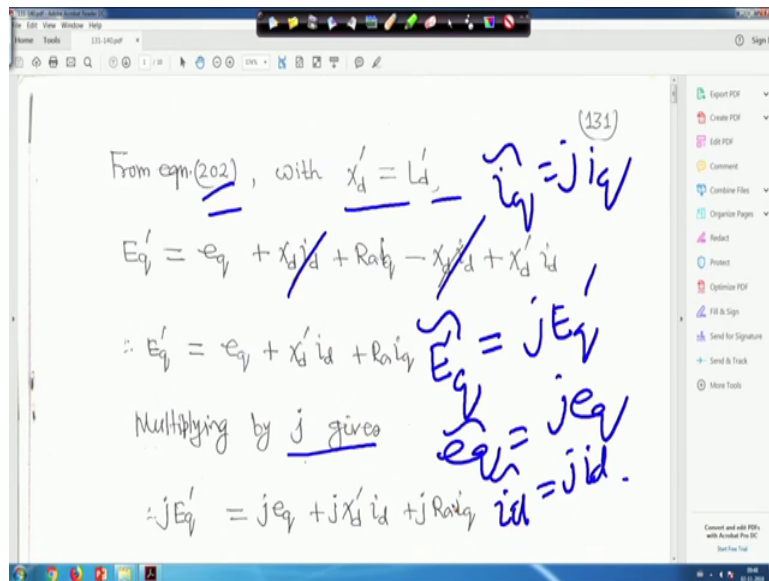
If you do so it will be written as $j E_I$ is equal to $j e_q$ plus $j X_d i_d$ plus $j R_a i_q$ right. So, in terms of phasor notation then, this one can be written as E_I tilde is equal to $j e_q$ can

be written e_q and $j X_d i_d$ can be written $X_d i_d$ it is not there this term this term will be actually $X_d i_d$.

So, it will be $X_d i_d$ this j is not there, it is when I writing by mistake I made it. So, it will $X_d i_d$ and $j i_q$ can be made $R a i_q$. That means, your for your understanding that $E i$ this is actually $j E i$ right because we have to represent by phasor quantity. Similarly, your small e_q is equal to $j e_q$ right.

Similarly your i_d is equal to $j i_d$ right. And similarly your i_q is equal to $j i_q$ right. So, just to represent that the all our tilde is given these are phasor quantities. So, this way we can write and this is nothing this is for my reference. And this is actually equation 204 right. So, we will go to the just hold on we will go to the next page.

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So, similarly you're for equation 202 right; with X_d is equal to L_d because I told you in per unit reactance and inductance are same right. Therefore, you can write if it is so that E_q will be e_q plus $X_d i_d$ plus $R a i_q$ minus $X_d i_d$ plus $X_d i_d$. So, this $X_d i_d$ and $X_d i_d$ will be cancelled; that means, E_q will be e_q plus $X_d i_d$ plus $R a i_q$.

Now, if you multiply both side by j that complex operator. So, it will be $j E_q$ it will be $j e_q$ plus $j X_d i_d$ plus $j R a i_q$. Now before going to the next page so, we will define that your same as before the phasor quantity say E_q your E_q it is

\tilde{E}_q will be $j E_q$. Similarly your \tilde{e}_q will be e_q will be $j e_q$ right. Similarly your \tilde{i}_d will be i_d equal to $j i_d$ right.

And similarly here I am writing that \tilde{i}_q will be $j i_q$ right. So, because we are because we have to represent now your, we have to draw the phasor diagram and for transient condition. So, we have to put them in the phasor form. So that way you can write if you do so then next equation will be in phasor notation.

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$$jE'_q = jE_q + jX'_d i'_d + jR_a i_q$$

In phasor notation,

$$\tilde{E}'_q = \tilde{E}_q + X'_d \tilde{i}'_d + R_a \tilde{i}_q \quad \text{--- (205) ---} \quad \checkmark$$

We see that the phasors \tilde{E}_I and \tilde{E}'_q both lie along the q -axis. We have also seen that \tilde{E}_q also lies along the q -axis.

Rearranging eqn. (167) and substituting \tilde{E}_I for

It will be your \tilde{E}_q \tilde{E}_q is equal to e_q I told you this j should not be there. It will be $X'_d \tilde{i}'_d$, your \tilde{i}'_d this should not be there right. Then plus $R_a \tilde{i}_q$ this is equation 205, this is nothing this is for my reference right.

So, that way first we make this one after this we see that the phasors \tilde{E}_I and \tilde{E}'_q both lie along the q axis because multiplied by j only right. So, we have also seen that the \tilde{E}_q also lies along the q axis because all you multiplied by j and all are lying actually along the q axis right. So, what you can do is that if you rearrange equation 167.

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We see that the phasors \tilde{E}_I and \tilde{E}_q' both lie along the q -axis. We have also seen that \tilde{E}_q' also lies along the q -axis.

Rearranging eqn. (167) and substituting \tilde{E}_I for $X_d i_f d$, we get,

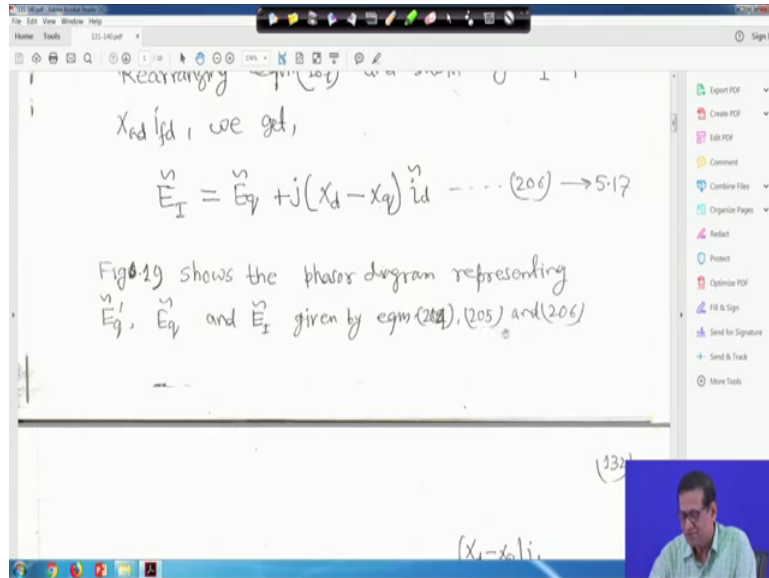
$$\tilde{E}_I = \tilde{E}_q' + j(X_d - X_q) i_d \quad \dots (206)$$

Fig. 19 shows the phasor diagram representing \tilde{E}_q' , \tilde{E}_q and \tilde{E}_I given by eqn. (204), (205) and (206)

And substitute E_I for $X_d i_f d$ that go to equation look this is this again and again it is difficult to go back to that equations. Then this is for my reference nothing required that it is difficult to go back again and again. But when we will when you are reading this, I mean listening this lecture at that time notes also will be available right. So, everything will be uploaded.

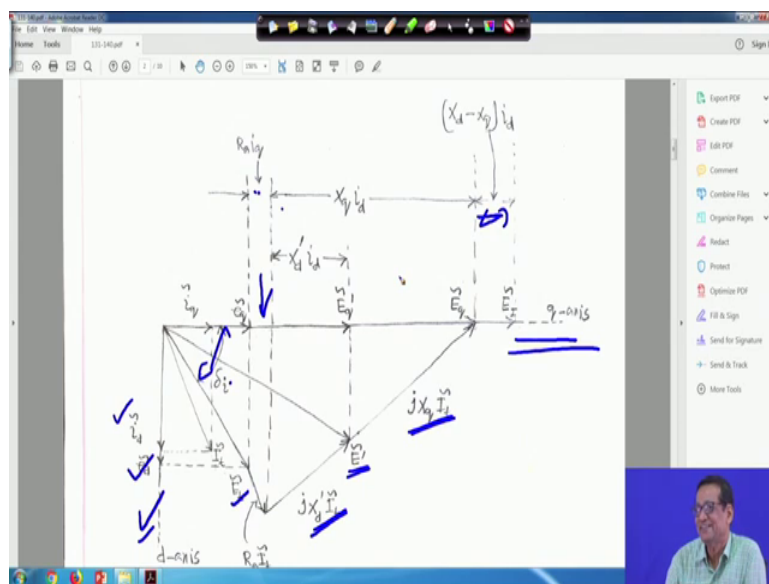
So, at that times see the equation 167 other and substituting E_I for $X_d i_f d$. Then you will get E_I tilde is equal to E_q tilde plus $j X_d$ minus X_q id tilde this is very simple thing actually I mean just you put it and you will get it. By chance if you stuck somewhere you just put a question to the forum a answers will be given there right and absolutely there will be no problem right. So, in that case you will get E_I tilde is equal to E_q tilde plus j into X_d minus X_q id this is equation 206 right.

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Now after doing all this figure 19 actually shows the phasor diagram representing \vec{E}_q , \vec{E}_q and \vec{E}_I given by equation 204, 205, and 206. So, this 204 this equation previous 204 has gone to the previous page 205 and 206 right.

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If you draw the phasor diagram the phasor diagram will look like this right. I mean just hold on if I can reduce the percentage little bit then all so here it is accommodated right. So, in this case if you see this that, this is my q axis for my own drawing this is has been taken q axis such that and this is your d axis this is your d axis right. Such that drawing will be phasor diagram drawing will be here you know like easier way.

So, this $E I \tilde{E}_q \tilde{E}_q \text{ dash } \tilde{E}_q$ and $E_q \tilde{E}_q$ and of course, $i_q \tilde{E}_q$ all are your, what you call lying on that your q axis right. And if you look into that that $E I \tilde{E}_q$ is equal to $E_q \text{ plus } X_d \text{ minus } X_q \text{ id}$ because just know we have seen that $E I \tilde{E}_q$ is equal to $E_q \text{ plus } X_d \text{ minus } X_q \text{ id}$. So, this is your this portion is $X_d \text{ minus } X_q \text{ id}$. And from here to here this is the, and this voltage this voltage is $E_t \tilde{E}_q$ right.

And this voltage is $E \text{ dash } \tilde{E}_q$ right and this angle that is between your what you call this $E \tilde{E}_q$ your what you call that $E_t \tilde{E}_q$ and $E_q \tilde{E}_q$ or $E I \tilde{E}_q$ or $E_q \text{ dash } \tilde{E}_q$ whatever it is. This is that your δ i this angle is δ i right this we have seen earlier also and this is the current $I_2 \tilde{E}_q$. So, current is lagging from $E_t \tilde{E}_q$ right because this is $E_t \tilde{E}_q$ and this is $I_t \tilde{E}_q$. So, this current is lagging and this is your $i_t \tilde{E}_q$ and e_d this is d axis right. So, that when this and this current is $i_q \tilde{E}_q$ all are along the q axis.

So, if you now and this term this your from here to here from here to here this term is $X_q \text{ id}$ this we have seen just look at those three equations you will get it this term is E_q . And this term is $R a_{iq}$ right. This term is $R a_{Iq}$ I mean from here to here because this is my I_t and this is my $E_t \tilde{E}_q$. So, this term is actually this term is $R a_{iq}$ just see those equations its I mean you will simply get it like E_q like $E_q \text{ dash } \tilde{E}_q$ is equal to $e_q \tilde{E}_q$ plus $R a_{iq}$ plus $X_d \text{ dash } \text{id}$ and all this things are there in the equations.

So, I could just see the equation and just make this make this phasor diagram. And this term your $j X_d \text{ id}$ $I_t \tilde{E}_q$ right and from here to here this is $j X_q$ your, $I_t \tilde{E}_q$ right. So, just see those E three equations and just make this phasor diagram this phasor diagram is under transient condition right. So, all these things are drawn just you have to open the notebook and you have to make it right from this three equations.

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$$E_t = e_d + j e_q = e_d + e_q$$

$$E_t = \tilde{E}_t + (R_a + jX_d) \tilde{I}_t$$

$$\tilde{E}_q = \text{q-axis component of } \tilde{E}_t$$

$$= \tilde{e}_q + R_a \tilde{i}_q + jX_d \tilde{i}_d$$

$$\tilde{E}_q = \text{voltage behind } R_a + jX_q$$

$$= \tilde{E}_t + (R_a + jX_q) \tilde{I}_t = \tilde{e}_q + R_a \tilde{i}_q + jX_q \tilde{i}_d$$

$$\tilde{E}_t = \tilde{E}_t + i(x_s - x_d) \tilde{i}_t$$

So, just for your reference I have written that E_t , E_t your what you call that your I am now little bit zooming it. So, E_t just hold on so E_t tilde is equal to e_d plus $j e_q$ is equal to e_d tilde plus e_q tilde this way we write this way we write right so whatever has been drawn here. And similarly E_t tilde E_t dash tilde is equal to E_t tilde plus R_a plus $j X_d$ dash I_t because a you are we have we have replaced L_d dash by X_d dash right.

I_t tilde and E_q dash tilde that is q axis is component of E_t tilde dash or E_d dash your E_t dash tilde right is equal to e_q tilde plus $R_a i_q$ tilde plus $j X_d$ dash i_d your what you call here one thing is there it should not be there it is X_d dash i_d tilde right this should not be there. Now, at E_q tilde is equal to voltage behind R_a plus $j X_q$ it is voltage behind R_a plus $j x_q$. So, here also one j should not be there it is E_t tilde plus R_a plus $j X_q$ oh no here I have multiplied by this thing sorry, sorry. Here also here also sorry. Just hold on just hold on here actually it is multiplied.

So, R_a here what you call your q axis component that e_q tilde $R_a i_q$ plus $j X_d$ id here tilde right. And similarly your this thing your E_q tilde voltage behind R_a plus it is E_t tilde plus R_a plus $j X_q I_t$. So, it is E_q tilde plus $R_a i_q$ no this should not there it is correct it is correct right it is correct. Because we are making already i_d tilde and this is i_q tilde and this is i_d tilde and this is i_q tilde e_q tilde all are in phasor quantity right so, this j should not be there.

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$$= \tilde{E}_q + Ra \tilde{i}_q + jX_d' \tilde{i}_d$$

$$E_q^s = \text{Voltage behind } Ra + jX_q$$

$$= \tilde{E}_t + (Ra + jX_q) \tilde{i}_t = \tilde{E}_q + Ra \tilde{i}_q + jX_q \tilde{i}_d$$

$$E_I^s = \tilde{E}_q + j(X_d - X_q) \tilde{i}_d$$

Fig. 19: Synchronous machine phasor diagram in terms of \tilde{E}_q , \tilde{E}_I and \tilde{E}_q'

So, similarly your E_I this thing your tilde is equal to E_q tilde plus $j X_d$ minus X_q i_d tilde right. So, this is your figure 19 that is this is the figure 19 right. And this is your synchronous machine phasor diagram in terms of E_q E_I and E_q dash right. So, that is your what you call that is your phasor diagram and this is your X_q and this is i_d let us me have a look $X_d - X_q$ i_d dash i_d tilde so this is the phasor diagram.

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Constant Flux Linkage Model

For studies in which the period of analysis is small in comparison to T_{d0} , the machine model is often simplified by assuming E_q' (or ψ_d) constant throughout the study period.

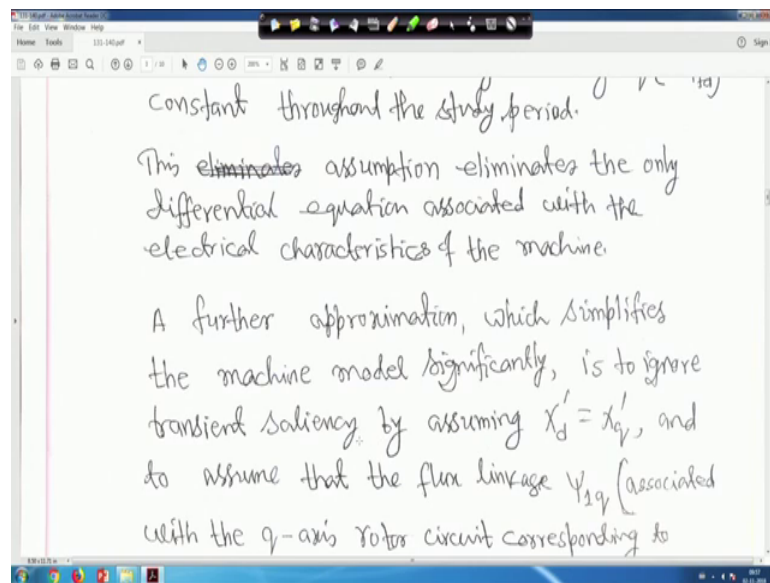
This ~~eliminates~~ assumption eliminates the only differential equation associated with the electrical characteristics of the machine.

Now, next is that constant flux linkage model right. For studies in which the, it is constant flux linkage model. So, before going to your constant flux linkage model I will suggest that just have a look all these things once again right. And throughout all the derivations are there it is basically you know that power system dynamics control course

is basically synchronous machine course and it may huge mathematics is involved later we will see much more mathematics right.

If you see by chance if have made any error even anything is there you please put the question in the forum such that I can rectify myself right. So, this is one thing so many things we have made j operator and other thing if you see anywhere I have made any error or anything you just please here what you call put that in the forum right. So, next is constant flux linkage model.

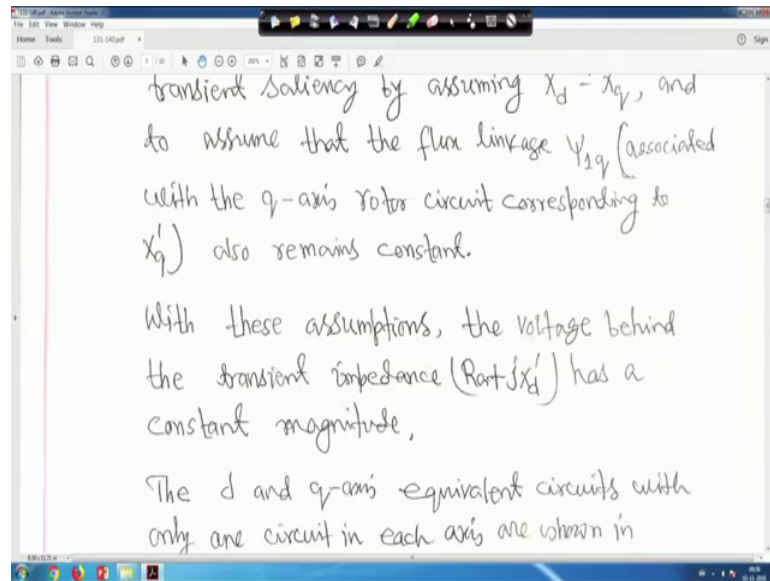
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So, for studies in which the period of analysis is small in competition to T do dash the machine model is often simplified by assuming E_q dash or ψ_{fd} constant throughout the study state period right. Now this assumption eliminates the only differential equation associate with associated with the electrical characteristics of the machine.

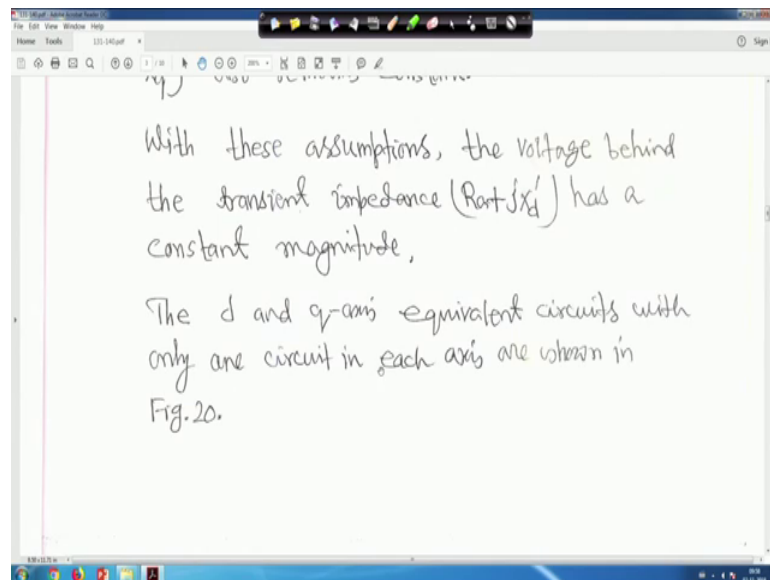
A further approximation which simplifies the machine model significantly is to ignore transient saliency by assuming X'_d dash is equal to X'_q dash and to assume that the flux linkage ψ_{lq} associated with the q axis rotor circuit corresponding to X'_q dash also remain constant right because we are going for your constant flux linkage model right.

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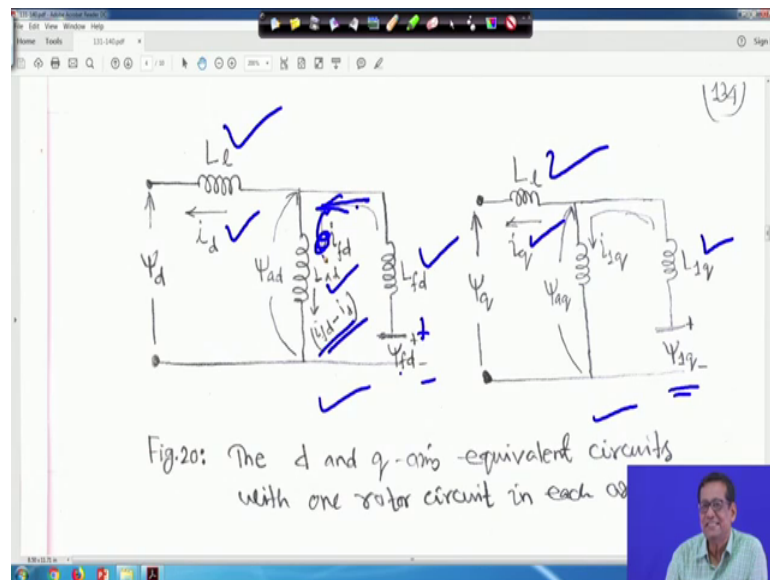
Now, with these assumptions the voltage behind that transient impedance that is $R + jX'_d$ has a constant magnitude right.

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That d and q axis equivalent circuit with only one circuit in each axis are shown in figure 20 right. So, if you if you that d axis d and q axis equivalent circuit with only one circuit in each axis are shown in figure 20.

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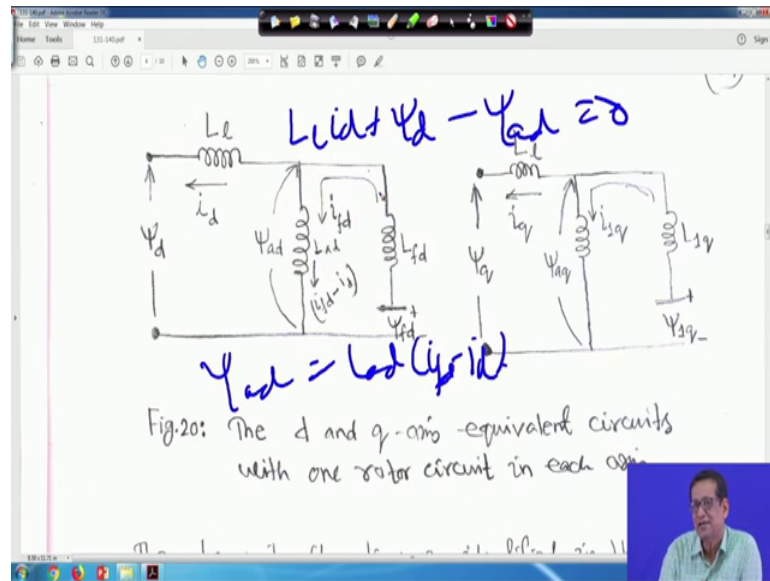


That means this actually it is a analogous to your what you call the dc circuit. Where this is ψ_d whenever you are showing a arrow means it is positive and no arrow means it is negative. And this is ψ_{ad} and this is actually this is this is actually ψ_{fd} and current. So, this is actually $i_{fd} - i_d$ right. And this is actually plus minus this is ψ_{fd} and this is L_{fd} .

So, it is it is something like your what you call that analogous to dc circuit. So, this is d axis and this is q axis this is ψ_{1q} , this is L_{1q} and this is ψ_{aq} , this is ψ_q and this is i_q and this is L_l . And this is also capital L suffix L and this current is i_d this current is i_q and this is actually L_{ad} right. So, the d and q axis equivalent circuit one with one rotor circuit is each axis. I mean whatever equations we have seen before this way you can represent I mean if we can put the circuit like this. So, it will be analogous to the dc circuit right.

So, from this also I mean from this also you can write all that equation. For example, sorry for example, ψ_{ad} will be $L_{ad} i_{fd} - \psi_d$ right and you can apply k v L also if you apply k v L for example, if you apply k v L say here if you apply k v L.

(Refer Slide Time: 24:45)



So, it will be $L_d i_d + \psi_d - \psi_{ad} = 0$. This equation you have seen. And similarly if you write ψ_{ad} from here it will be $L_{ad} i_{fd} - i_d$ and here also you can apply KVL in this mesh also you can apply KVL right. So, whatever equations you have written this way you can represent the circuit right.

So, now the per unit flux linkage identified in the d axis is given by your ψ_{ad} is equal to $-L_{ad} i_d + L_{ad} i_{fd}$. That means, this one we are writing directly you can write ψ_{fd} is equal to $i_{fd} - i_d$ then you multiply. And you simplify it will be a ψ_{ad} is $-L_{ad} i_d + L_{ad} i_{fd}$.

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$$\Psi_{ad} = -L_{ad} i_d + L_{ad} i_{fd} \quad \text{--- (207) \rightarrow X}$$

$$\Psi_d = \Psi_{ad} - L_e i_d \quad \text{--- (208) \rightarrow X}$$

$$\Psi_{fd} = \Psi_{ad} + \underline{L_{fd} i_{fd}} \quad \text{--- (209) \rightarrow X}$$

From eqn. (209),

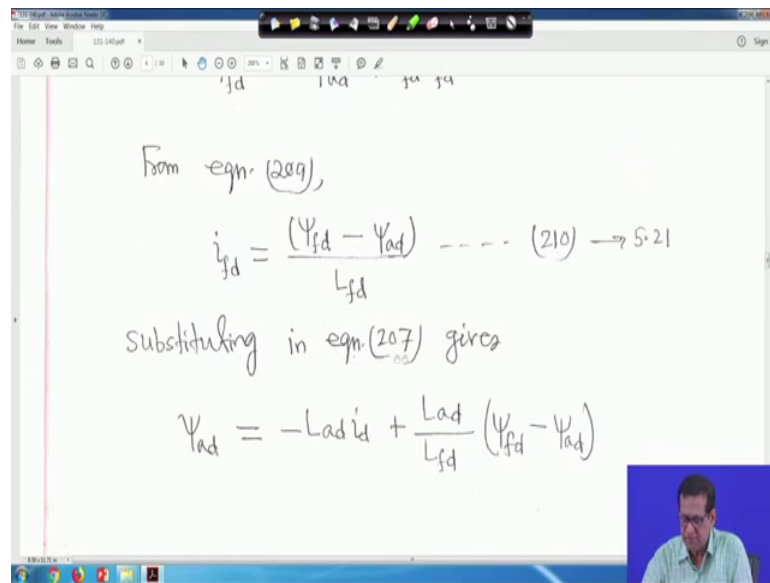
$$i_{fd} = \frac{(\Psi_{fd} - \Psi_{ad})}{L_{fd}} \quad \text{--- (210) \rightarrow}$$

Similarly ψ_d will be I told you that $k v L$ that $\psi_a d$ minus $L l i_d$ and similarly $\psi_f d$ will be $\psi_a d$ plus $i_{fd} l_{fd}$. I mean here I mean here there should not be any confusion here, this current is i_{fd} actually this in this branch current is i_{fd} minus i_d .

Now, if you apply your $k v L$ right if you apply $k v L$ say anticlockwise then your it is L_{fd} then your $i_{fd} l_{fd} i_{fd}$ plus L_{ad} then i_{fd} minus i_d this term. Then minus $\psi_f d$ is equal to 0 because this plus this is minus minus $\psi_f d$ is equal to 0 and you simplify right.

So, if you do so if you do so you will get $\psi_f d$ is equal to $\psi_a d$ plus $L_{fd} i_{fd}$. This is equation your what you call 209. So, this is this is nothing this is for my own reference this is nothing this is 207 208 209 right.

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From eqn. (209),

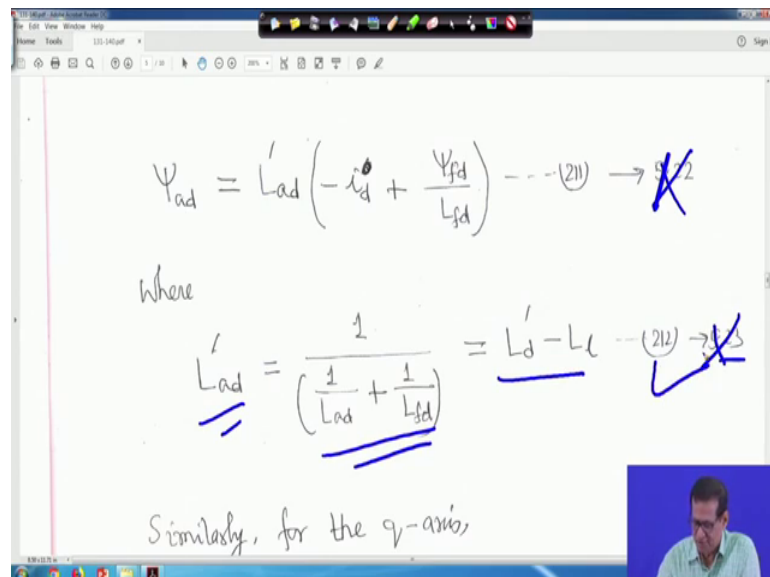
$$i_d = \frac{\psi_{fd} - \psi_{ad}}{L_{fd}} \quad \text{--- (210) } \rightarrow S.21$$

substituting in eqn. (207) gives

$$\psi_{ad} = -L_{ad} i_d + \frac{L_{ad}}{L_{fd}} (\psi_{fd} - \psi_{ad})$$

And from equation 209 from this equation you can write i_d is equal to ψ_{fd} minus ψ_{ad} upon L_{fd} this is equation 210 right. So, if you substitute in equation this one in equation 207 that mean this equation you substitute for i_d . If you do so you will get ψ_{ad} is equal to minus $L_{ad} i_d$ plus L_{ad} upon L_{fd} in bracket ψ_{fd} minus ψ_{ad} right.

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$$\psi_{ad} = L_{ad} \left(-i_d + \frac{\psi_{fd}}{L_{fd}} \right) \quad \text{--- (211) } \rightarrow X$$

where

$$\underline{L_{ad}} = \frac{1}{\left(\frac{1}{L_{ad}} + \frac{1}{L_{fd}} \right)} = \underline{L_d - L_l} \quad \text{--- (212) } \rightarrow X$$

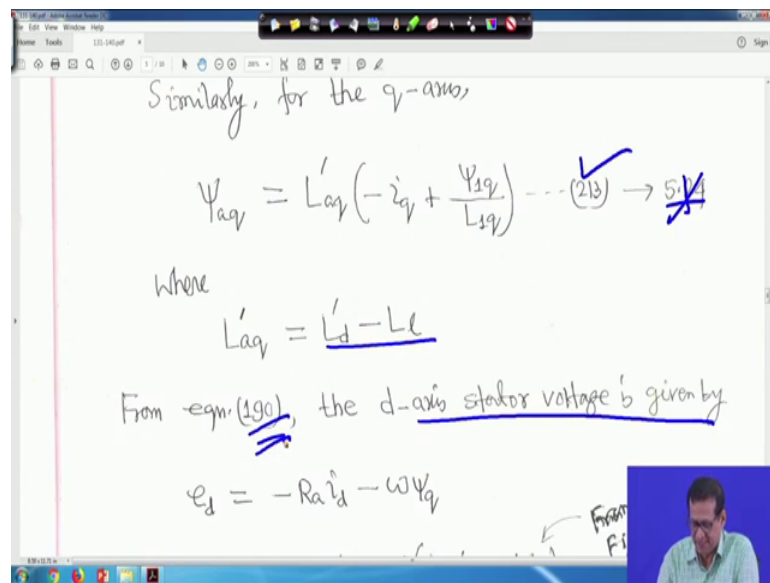
Similarly, for the q-axis,

So, therefore, we simplify this one we can write this equation like this that ψ_{ad} is equal to L_{ad} dash in bracket minus i_d plus ψ_{fd} upon L_{fd} , this is equation 211 right. I just hold on so this is for my own reference this is nothing for you. So, where L you put it

and simplify you will get L_d you can write 1 upon 1 upon L_d plus 1 upon L_l . And this is nothing but your L_d minus your L_l this is equation 12 right.

Similarly for the q axis I mean for this circuit similarly for the q axis again you apply k v L you please do yourself right I mean. So, all these things are derived before, but just we have put it that your at the analogous this is analogous circuit right for that flux linkage equation. So, this is from this your, what you call from this figure only you can get like this right this one.

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Similarly, for the q axis only that from the figure you write those equations and simplify you will get Ψ_{aq} is equal to L'_{aq} in bracket minus i_q plus Ψ_{lq} upon L_{lq} this is equation 200 your 13 213 right. This is, this is nothing this is for my reference where L'_{aq} is equal to L_d dash minus L_l right, same as before. So, from equation 119 that d axis stator voltage is given by e_d is equal to minus $R_a i_d$ minus $\omega \Psi_q$. We go back to equation 190 right.

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$L_{aq} = L_d$
 From eqn(190), the d-axis stator voltage is given by

$$E_d = -R_a i_d - \omega \psi_q$$

$$\therefore E_d = -R_a i_d + \omega (L_l i_q - \psi_{aq})$$
 (From Fig-20)
 where $\omega = \omega_r = \omega_0 = 1.0 \text{ pu.}$
 Substituting for ψ_{aq} from eqn(213) gives

$$\psi_{d1} = -L_{ad} i_d + L_{ad} \cdot \psi_{r1} - \frac{L_{ad}}{L_{ad}} \cdot \psi_{ad}$$

So, and therefore e_d is equal to minus $R_a i_d$ plus $\omega \psi_q$ is equal to $L_l i_q$ minus ψ_{aq} here what you call ψ_{aq} from that q axis that your analogous circuit diagram is there from there just you apply $k_v L$ that ψ_q is equal to is equal to $L_l i_q$ minus ψ_{aq} this is from figure 20. I have written here; that means, from here ; that means, from this figure from here right from this figure.

You apply it will look it will if you apply $k_v L$ I am telling from my mouth it will be $L_l i_q$ into i_q plus ψ_q minus ψ_{aq} is equal to 0 from there you substitute right. From there you substitute that your this thing this is that is why it is written from figure 20 and where ω is equal to ω_R is equal to ω_0 say 1 per unit all are same. So, if you and substituting for ψ_{aq} from equation 213 right you do this you will get ψ_{d1} .

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$$\therefore e_d = -R_a i_d + \omega (L_l i_q - \psi_{1q})$$

From Fig-20

Where $\omega = \omega_r = \omega_0 = 1.0 \text{ pu}$.

Substituting for ψ_{1q} from eqn(213) gives

$$\psi_{1d} = -L_{ad} i_d + L_{ad} \frac{\psi_{fd}}{L_{fd}} - \frac{L_{ad} \cdot \psi_{fd}}{L_{fd}}$$

$$= L_{ad} i_d + \frac{\psi_{fd}}{L_{fd}} - \frac{\psi_{fd}}{L_{fd}}$$

Here what you call that this is nothing this is no you need not you this is nothing for you this is no need right you better you skip this one go to the next page this is nothing for you right. So, go to next page.

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$$e_d = -R_a i_d + \omega L_l i_q - \omega L_{aq} \left(-i_q + \frac{\psi_{1q}}{L_{1q}} \right)$$

$$\therefore e_d = -R_a i_d + \omega (L_l + L_{aq}) i_q - \omega L_{aq} \left(\frac{\psi_{1q}}{L_{1q}} \right)$$

$$\therefore e_d = -R_a i_d + x'_{q} i_q + E'_d \dots (214) \rightarrow 5.25$$

Then you will see that e_d will be is equal to minus $R_a i_d$ plus $\omega L_l i_q$ minus $\omega L_{aq} \left(-i_q + \frac{\psi_{1q}}{L_{1q}} \right)$. Or you will see e_d is equal to minus $R_a i_d$ plus $\omega (L_l + L_{aq}) i_q$ minus $\omega L_{aq} \left(\frac{\psi_{1q}}{L_{1q}} \right)$.

dash in bracket ψ l q upon L l q or or it can be written as $\text{minus } R a i d \text{ plus } X q \text{ dash } i q$ plus $E d \text{ dash}$.

So, next is phasor diagram for transient conditions right. So, in order to do this it is first necessary to express that $E q \text{ dash}$, $E I \text{ dash}$, and $E q \text{ right}$. In terms of d and q axis components of terminal voltage and current right. So, we know that in per unit that reactance is equal to inductance. So, $X d$ is equal to $L d$ therefore, from equation 193 and 196 right. So, whenever you will go through it that all the previous lecture notes will be available to you. So, immediately you will open that equation 193 and 196 right.

So, $e q$ will be small $e q$ will be $\psi d \text{ minus } R a i q \text{ right}$ or $e q$ is equal to substitute expression for ψd right previously we have derived that. So, $\text{minus } X d \text{ id plus } x a d I f d \text{ minus } R a i q \text{ right}$. So, this terms small $e q$ is equal to $\text{minus } X d \text{ id plus capital } E \text{ suffix } I \text{ minus } R a i q \text{ right}$. Therefore, we can write that $E I \text{ this } E I$ is equal to $\text{capital } E I$ is equal to $e q \text{ plus } X d \text{ id plus } R a i q \text{ right}$.

Now, question is that you multiply both side by j . If you do so it will be $j E I$ is equal to $j e q \text{ plus } j X d \text{ id plus } j R a i q$. Now in terms of phasor notation in terms of phasor notation your then basically it is $E I$. So, whenever we are doing this I mean before explaining this one right just hold on before explanation of this one we will go to the next page right. So, I am not multiply both side by j so it its matter that j means that is your $q x$ is actually leading the d axis by 90 degree right.

So, whenever you make it so basically $E I E I \text{ tilde}$ is equal to your is equal to your $E I$ right. This is angle 90 degree that is nothing, but your $j E I \text{ right } j E I \text{ right}$. So, similarly your $e q \text{ tilde}$ also is equal to your $e q \text{ angle } 90 \text{ degree}$. So, $j e q$, but this is this j will be there $j X d \text{ id}$ because your id actually if aq actually leading the here what you call the d axis by 90 degree.

So, that is why if this $id \text{ tilde}$ then your this $i d \text{ tilde}$ will be is equal to $i d$ then angle is what? Because $i d$ is q your q is leading that here what you call id if you take that your $i d$ is a reference one then it will be angle 0 right. But this j should be there from the phasor notation and $i q$ also because this $i q$ also along the q axis. So, it is $j i q$ we are taking that is your $i q \text{ angle } 90 \text{ degree}$ so it will be actually $j X d \text{ id tilde}$ right. And if you think that id is the, your d axis is the reference one the q is actually leading the d axis. So, this j should be there right in the phasor in the in terms of the phasor notation.

So, next we will go to the next one phasor diagram right. So, now from equation 202 with $X d \text{ dash}$ is equal to $L d \text{ dash}$. Therefore, $E q \text{ dash}$ can be written as your $e q$ then you then it will be plus $X d i d$ plus $R a i q$ minus $X d i d$ plus $X d \text{ dash} i d \text{ dash}$. So, $X d i d$ and $X d i d$ will be cancelled Therefore, $E q \text{ dash}$ will be $e q$ plus $X d i d$ plus $R a i q$. Here also you multiply both side by j . So, in that case it will be $j E q \text{ dash}$ is equal to $j e q$ plus $j X d \text{ dash} i d$ plus $j R a i q$ right.

So, in phasor notation from the same philosophy right from the same philosophy this j should be there. So, it is $E q \text{ dash tilde}$ is equal to $e q \text{ small } e q \text{ tilde}$ plus $j X d \text{ dash} i d$ plus $R a i q \text{ tilde}$; that means, your here I making it; that means, your $E q \text{ dash}$ $E q \text{ dash tilde}$ is equal to basically your $E q \text{ dash}$ $E q \text{ dash}$ and angle 90 degree that is $j E q \text{ dash}$ this is $j E q \text{ dash}$. Similarly for $e q \text{ tilde}$ right similarly for $i q \text{ tilde}$ and I told you that $q X$ is leading this one. So, it we will be $j X \text{ dash} i d \text{ tilde}$. So, this is your phasor this is for my reference nothing this is equation 205 right.

Now, we see that just hold on let me clear it. So, we see that the phasor just hold on. We see that the phasor $E I \text{ tilde}$ and $E q \text{ dash tilde}$ both lie along the q axis we have also seen that $E q \text{ tilde}$ also lies along the q axis just on the previous page. So, rearranging equation 167 right. So if you that is and substituting $E I$ for $X a X a d I f d$ right. We get that $E I \text{ tilde}$ you go back to equation now 167 and you your $X a d i f d$ you replace by $E I$ then you will be $E I \text{ tilde}$ will be $E q \text{ tilde}$ plus $j X d$ minus $X q i d \text{ tilde}$ this is equation 206 right.

So, figure 19 shows the phasor representing $E q \text{ dash tilde}$ then capital $E q \text{ tilde}$ then $E I \text{ tilde}$ given by equation 204 205 and 206. Now if you draw the phasor node diagram this is your q axis this is your d axis. So, this is for easy understanding this thing q axis actually leading the d axis by 90 degree. So, this is your capital $E I \text{ tilde}$ look at that all these things right. So, $E I$ just now we have seen know equation 216 $E I \text{ tilde}$ is equal to $E q \text{ tilde}$ plus $X d$ minus $X q$ into $i d$ right.

Similarly, your this is your $E q \text{ tilde}$ this is $E q \text{ dash}$ your $tilde$ right. So, it is basically $E q \text{ small } e q \text{ tilde}$ plus $R a i q$ plus $X d \text{ dash} i d \text{ dash}$ right. And from here to here this distance is $X q i d$ and this is that your current $I t \text{ tilde}$ right and. And this is your voltage $E t \text{ tilde}$ and this is this portion is $R a I t \text{ tilde}$ this portion is $X d I t \text{ dash}$ your $tilde$ into j . And this is your $E \text{ dash} \text{ tilde}$ and this is your $j X q I t \text{ tilde}$ right.

So, this is the, and this angle is δ ; earlier we have explained about δ . And this is your q axis component I_q of I_t this is q axis is component and d axis is component for this one it is i_d right. Similarly for your E_t that d axis is component E_d direct axis component and for E_t it is E_q and this this one your x axis component is your R_a right.

So, this is the complete phasor notation from this your three equation 204, 5 and 6 previous page 204 is there. This is 205 and here also j should be there this j should be there right this j should be there this is phasor notation. So, so that is why your this is your \hat{a} ; that means, at the bottom the E_t is given $E_d + j E_q$ is equal to $E_d + j E_q$. So, I told you previously E_q is nothing, but $j E_q$. Similarly E_t is $E_t + R_a + j X_d I_t$.

So, most of the cases actually we will find R_a is very small right and it is neglected. So, I will suggest one thing that neglect R_a and redraw the phasor diagram just neglect R_a . And E_q is q axis component of your E_t . So, it is E_q look at the phasor diagram everything is given there $R_a + j X_d I_t$ right. Because, because this j should be there because if you take that i_d is a reference though i_d will be actually i_d angle 0 degree right.

So, and E_q will be voltage behind $R_a + j X_q$ that is also given. It is $E_t + R_a + j X_q I_t$ that is $E_q + R_a + j X_q I_t$ all these things are given in the phasor diagram and this equations also shown. And E_t is equal to $E_q + j X_d I_t - X_q I_d$. This is the summary of this phasor diagram all the equation and this is synchronous machine phasor diagram in terms of E_q , E_t and E_q . I think this portion is now understandable for this phasor diagram right.

Just see this equation 204, 205 and 206 and just see how it has been drawn. The way you draw the single phase (Refer Time: 39:42) circuit or three phase (Refer Time: 39:43) circuit phasor diagram right the way you draw single phase circuit phasor diagram only three equations you put in front of you and just see how it is drawn. So, automatically you can easily draw this phasor diagram ok. So, next will be this constant next it will be your constant flux linkage model.