Power System Dynamics, Control and Monitoring
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Lecture – 13
Power System stability (Contd.)

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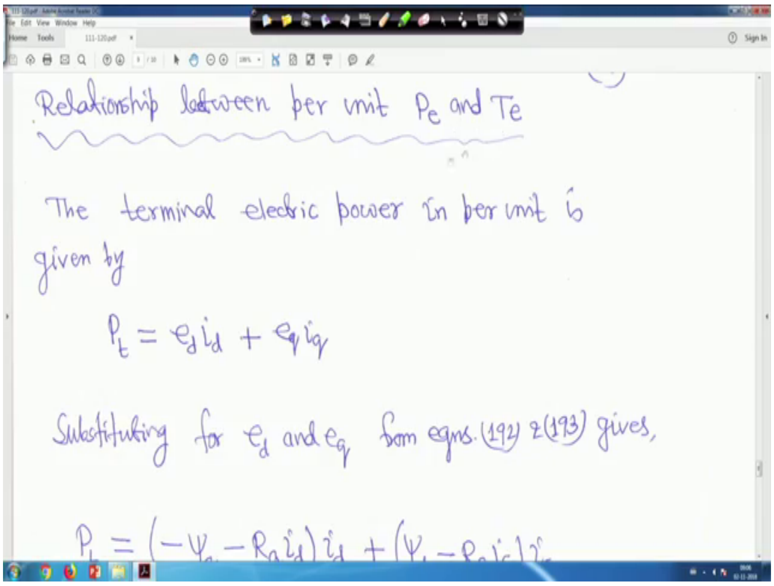

So we ended up in the previous lecture the relationship between per unit your P e and T e. So, the terminal electric power in per unit is given by that we have seen earlier that P t is equal to e d i d plus e q i q.

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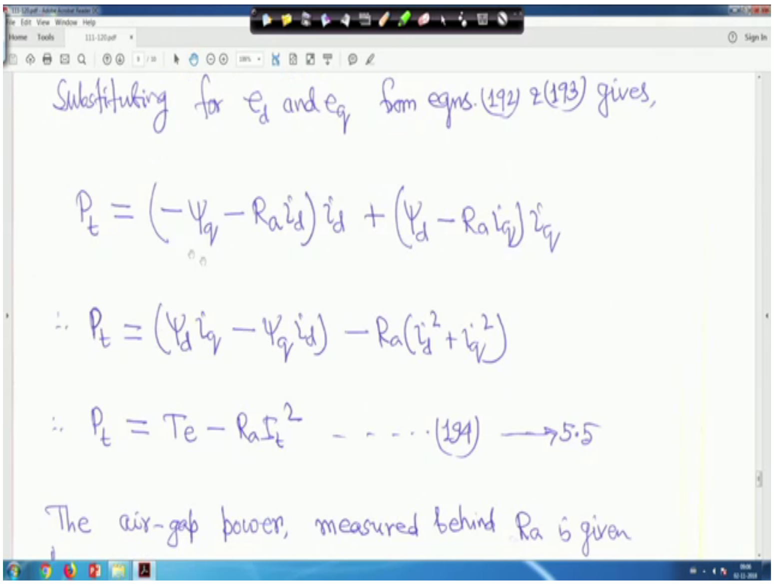

Now, substituting per e d and e q from equation 192 and 193 if you do so, it will become P t is equal to minus psi q minus R a i d bracket close into i d plus psi d minus R a i q bracket close into i q. Or P t is equal to you can write after collecting those terms psi d i q minus psi q i d minus R a into i d square plus i q square.

So, we know therefore this we know this is the torque T e, so P t is equal to T e minus R a I t square, because I t square is equal to I d square plus i q square, this is equation 194. And this is actually, this is actually nothing this is for my own reference.

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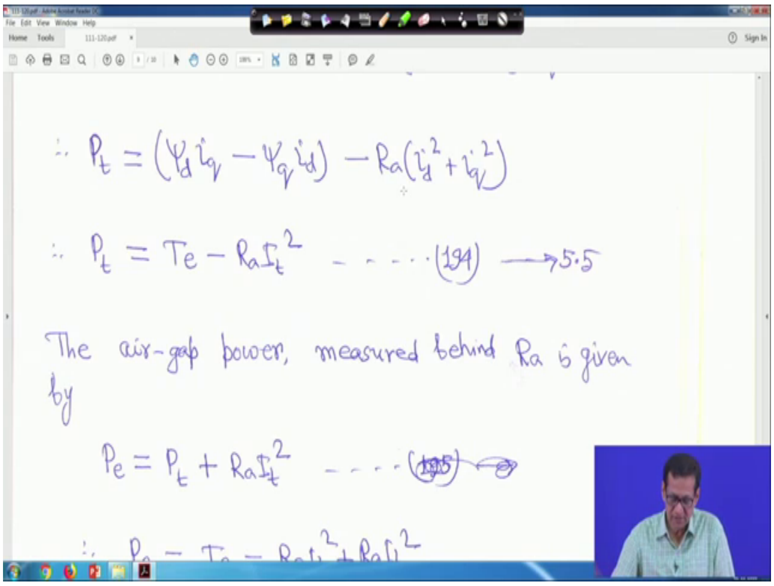

So, air-gap power measured behind R a, this also you have seen that P e is equal to P t plus R a I t square right. Therefore, whatever P t we have got, we will substitute here you this P t we will substitute here, so R a I t square R a I t square will be cancelled, finally we will find that P e is equal to T e right.

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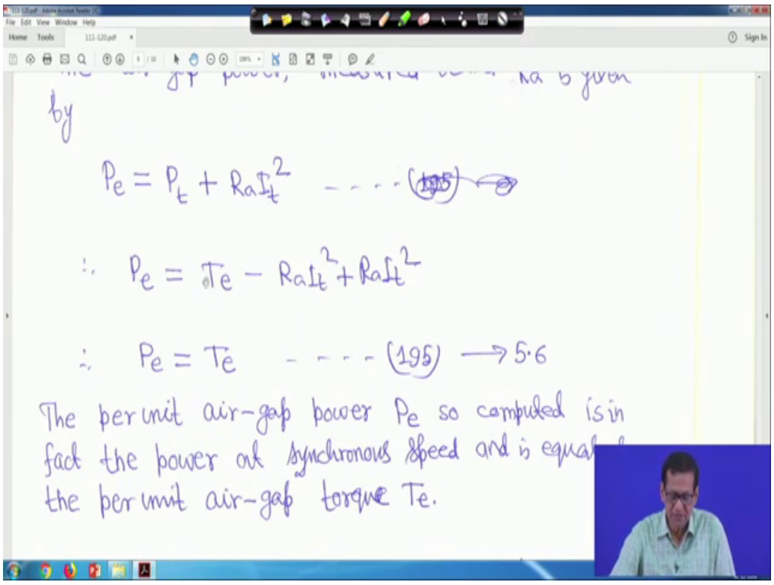

So, if you substitute, you will get P e is equal to T e, because R a I t square and minus R a I t square plus R a I t square will be cancelled right. Therefore, the per unit air-gap power P e so computed is in fact that the power at synchronous speed and is equal to per unit air-gap torque T e. So, in per unit value power and torque both are same right.

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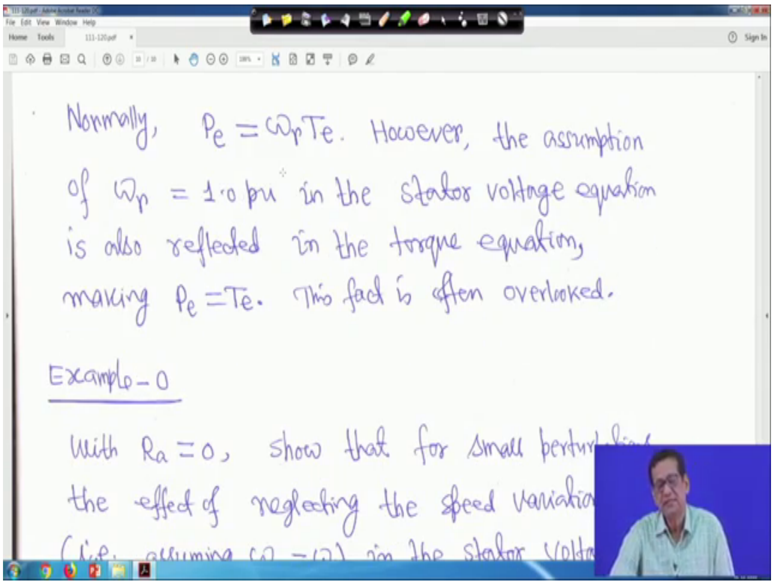

And we know the relationship normally that power is equal to you know angular speed into torque, the therefore P is equal to omega r into T e. However, the assumption of omega r is equal to 1.0 per unit in the stator voltage equation is also reflected in the torque equation, making P e is equal to T e. This fact is often overlooked right. Now, next one is this way this one this all this things, we have discussed in the previous lecture.

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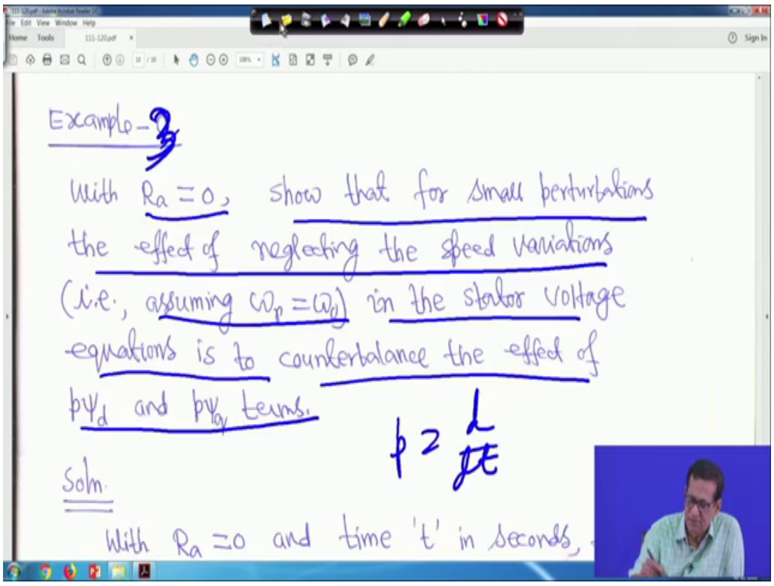

Now, next is I have written example-0, it will be example-3 I think, it will be example-3 right. So, this is a little bit understanding is required for this example, because this is no data, but some proof right. Now, with R a is equal to 0, suppose we are assuming R a is 0, because R a is very small right. Show that for small perturbations, the effect of neglecting, the speed variation that is assuming omega r is equal to omega 0 right. In the stator voltage equation is to counter balance the effect of P psi d and P psi q terms the meaning is that R a is equal to 0 say.

So, here we have to prove that for small perturbation, the effect of neglecting the speed variation that is assuming omega r is equal to omega 0 in the stator voltage equation is to counter balance the effect of the p psi d and p psi q terms. Psi d means, the this is a differential operator that is p is equal to d d t, so it basically psi d dot and psi q dot rough terms right.

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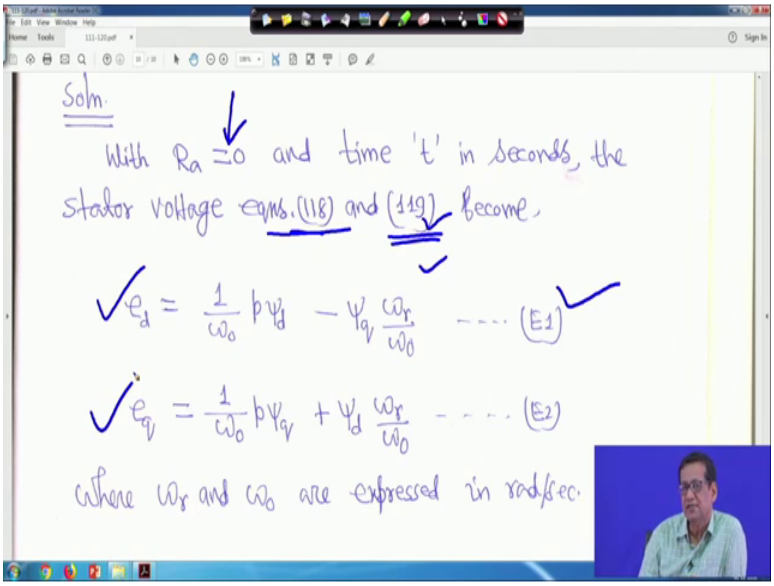

So, in this case like solution with R a is equal to 0 and time t if we write in second, right. Therefore the equation your stator voltage equation 118 and equation 119, it becomes e d is equal to 1 upon omega 0, p psi d minus psi q omega r by omega 0, this is I have make an equation E 1 right. And this is from equation 118.

And from 119 that e q is equal to 1 upon omega 0 p psi q plus psi d omega r by omega 0. In the 118 and 119 you substitute R a is equal to 0, then you will get equation your what you call equation 118 will be like this, and equation 119 will be like this, you put R a is equal to 0 right. Now, suppose and we have made some assumption that your that your omega r just hold on that assuming that omega r is equal to omega 0 right.

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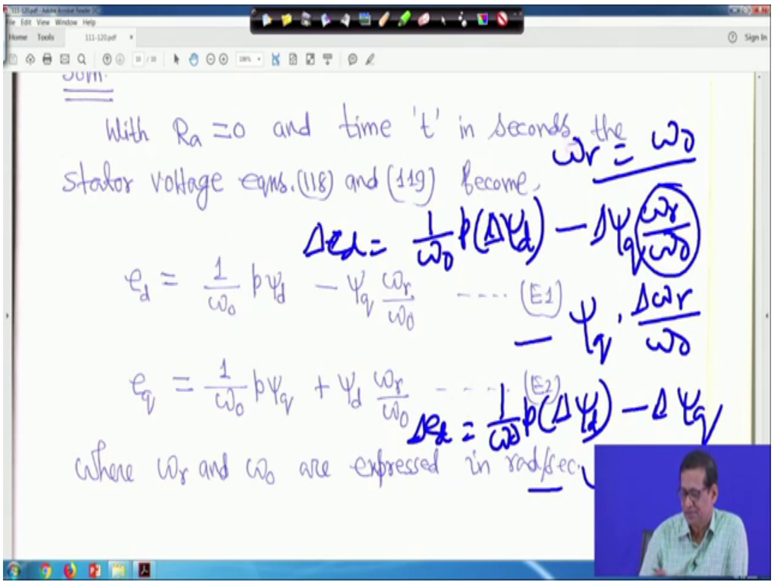

So, we if you now take the small perturbation of this one small perturbation of this equation in the next page; we will see that if you take the small perturbation of this one, so it will be delta e d it will be delta e d is equal to it is 1 upon omega 0 right p then delta psi d this one minus your it will be delta psi q into omega r by omega 0. But omega r is equal to omega 0 that we will write later, so this is minus delta psi q omega r by omega 0. Again minus your psi q right, we are taking small perturbation delta omega r by omega 0 right.

So, if you take the small perturbation per, but omega r is equal to omega 0 right that means, this omega r this term is unity omega r by omega 0 is equal to your 1, so that means delta I am writing somewhere here that means, my delta e d which actually is equal to 1 upon omega 0 p delta psi d right minus delta psi q, then minus psi q into delta omega r by omega 0. This is for small perturbation for delta e d. This thing had been retained in the next page right, but just making it.

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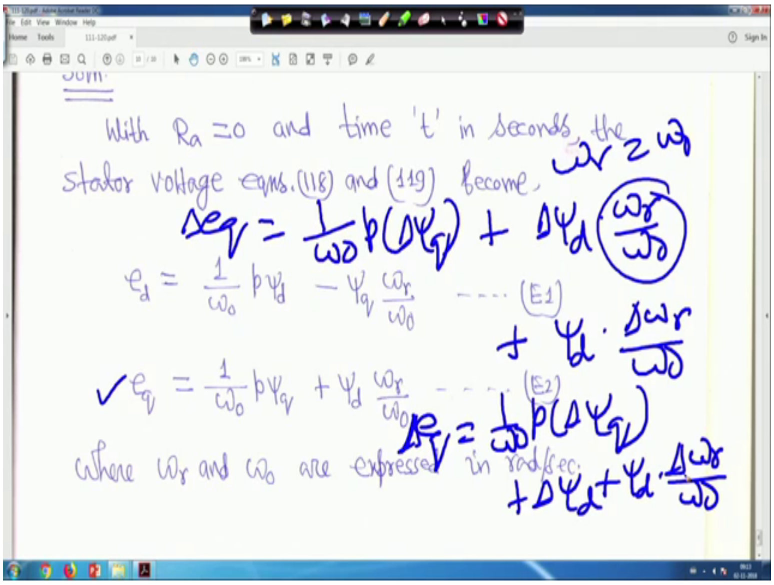

So, similarly if you take your e q delta e q, so writing here if you take small perturbation for equation e 2 that is this one for delta e q, then here also it will be like this delta e q is equal to 1 upon omega 0. Then p then delta psi q right plus your delta psi d then omega r by omega 0, then plus psi d into delta omega r by omega 0 right, so that means this one omega r is equal to omega 0 this we have assumed right.

So, if you put omega r is equal to omega 0, this term is one that means, here I am writing somewhere that my delta e q will be omega 0 then p delta psi q right plus delta psi d plus psi d into delta omega r by omega 0, this thing retain in the next page right. So, if we take this here what you call this small perturbation, so that will be like this, so omega r and omega 0 both are expressed in radian per second right just hold on, we will go to the next page so hold sorry just hold on.

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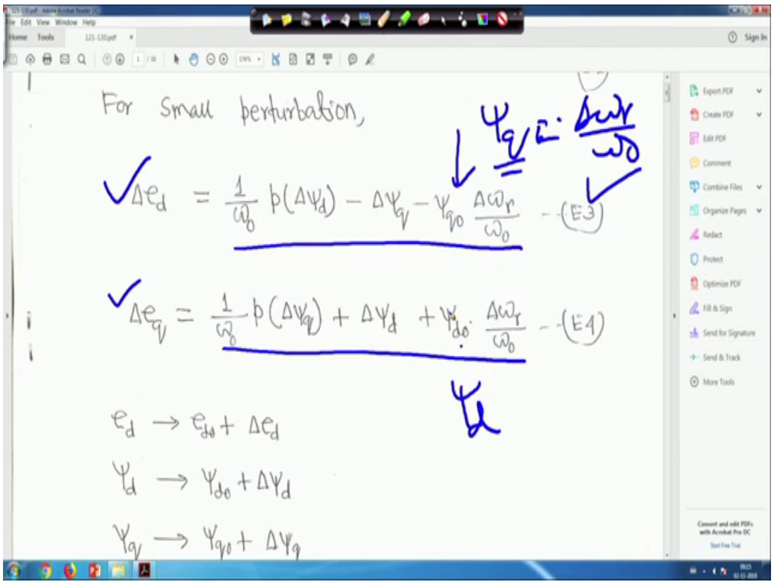

So, for small perturbation now that whatever I wrote previously right, so this your this equation that whatever I wrote in the previous page the delta e d will be this one, this is equation say E 3 right. Similarly, I wrote for delta e q, so it will be same equation, it will be this one right. So, in this case what is your in the previous case, what we made it that as it is a small perturbation, so there it is here what you call that your psi q into previous term that psi q into delta omega r by omega 0 right. But, when you make a when we are just making small perturbation, so psi q can be changed to psi q 0.

Similarly, psi d will be changed to psi d 0 right that is why, previous case I did not write in the previous page I am simply made your delta your psi q only and this one also only psi d. But, when you are perturbing around an operating point, so psi q 0, psi d 0 e d 0, e q 0 like this, so it will be actually psi q 0, and it will be actually psi d 0 rest are rest are ok. Now, how this psi q 0, psi d 0 coming just one step one complete calculation, I will show you.

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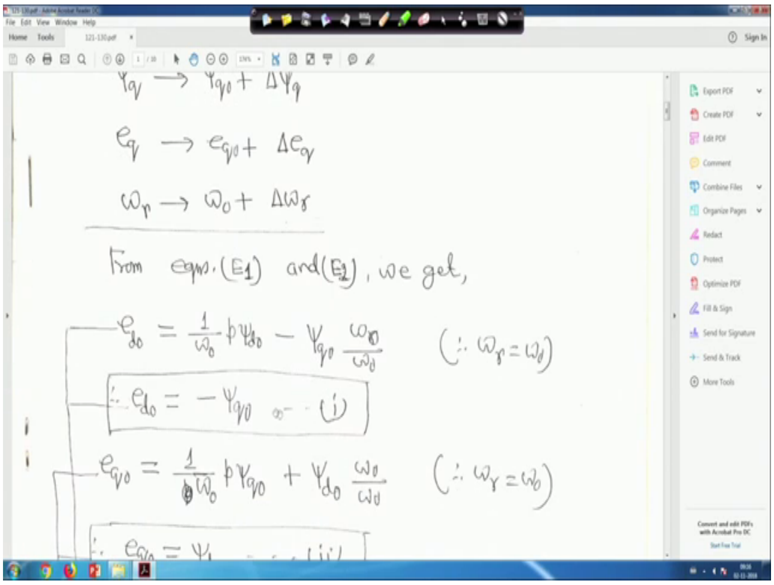

So, in this case what happens that you assume now that e d actually, suppose initially it was e d 0, psi d 0, psi q 0, e q 0 and omega 0, now it has been perturbed. Now, e d will become e d 0 plus delta e d, psi d will become psi d 0 plus delta psi d. Similarly, psi q will become psi q 0 plus delta psi q, and e q e q 0 plus delta e q, and omega r omega 0 plus delta omega r.

Now, in equation E 1 and E 2 if you substitute and simplify right, then how it will look like? Therefore, in equation eqo your equation E 1 and E 2, it can be written like this. So, e d 0 1 upon omega 0 p psi d 0 minus psi q 0, this is omega 0 by omega 0, so this term will be one, because omega r is equal to omega 0 I mean initially. So, if you put all the initial values only that means, your e d e d 0 will become your minus psi q 0, so e d 0 will become minus psi q 0, this is because we have perturbed from e d initially it was e d 0, now it is e d 0 plus delta e d, so if it is e d 0 is equal to this much.

And omega 0 is omega r is equal to omega 0 though this term is one right. And this is this is yours initially it has p psi is 0, therefore it is derivative will become 0 right, it was a study operating condition. So, it is derivative is 0 that is why, this term is also 0. Therefore, e d 0 is equal to minus psi q 0.

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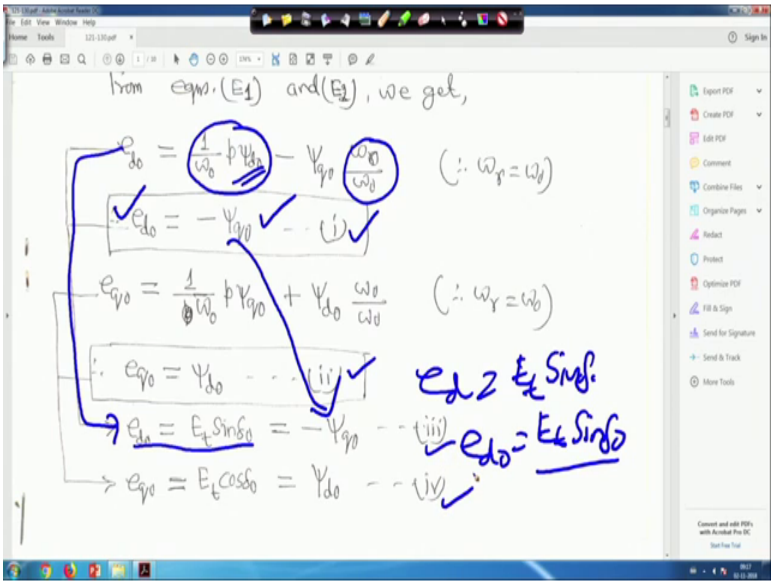

Similarly, and also before going to that also from here I have made it here that your e d 0 also we have seen earlier e d is equal to we have seen earlier that e e d is equal to E t sin your delta right.

And for initial condition that e d 0 will be is equal to E t initial operating condition will be e d sin delta 0 that is why, e d 0 is equal to written as E t sin delta 0, and that is nothing but this one minus psi q 0 right, so that is your this is what this is some equation is marked, this is equation-1, similarly this is equation-2 will come to that and this is 3, then this is equation-4 written like this.

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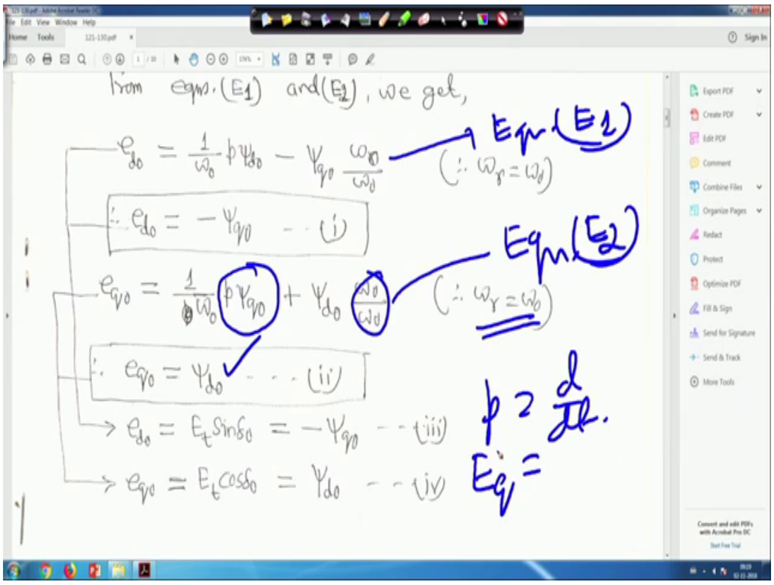

Now, similarly if you make it e q 0 now right, so from the same equation that is E2 from equation-E2, this is actually equation-E1 the previous page we have seen just make it all the 0 0 initial operating condition. Similarly, for this one this is actually equation-E2 right.

So, here also if you make like this, so e q 0 we will be 1 upon omega 0 p psi q 0 plus psi d 0 omega 0 upon omega 0, because omega r is equal to omega 0. Now, that means your this one that this is the initial condition right initial operating condition, so p psi q 0 it is derivative will be 0, because p is actually differential operator I told d by d t right, so this term will be 0. And this term is 1, so basically it will become psi d 0 right.

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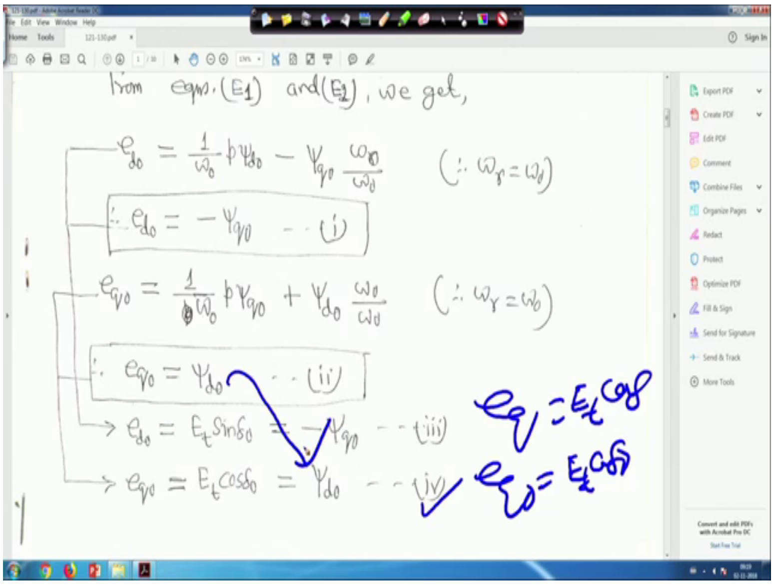

Similarly, we also knew that previously we have seen that e q is here what you call just one minute previously we have seen that e q is equal to E t your cos delta, therefore e q 0 is equal to E t cos delta 0 right. So, therefore this e q 0 is equal to E t cos delta 0, and that is nothing but psi d 0 this is equation-4, this is all this what you call that e d 0 e q 0 this can be simplified that how one can obtain this right. So, this is let me clear it.

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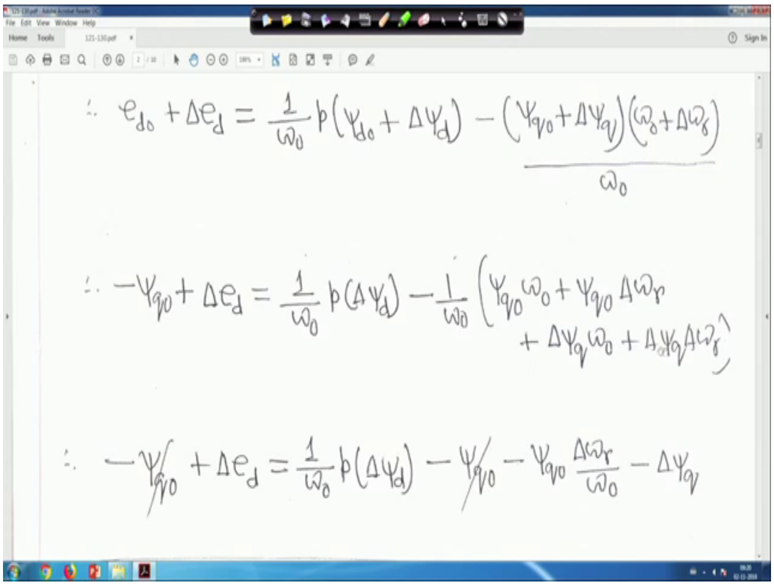

So that means, now this one also look at that this one also from in equation e 1 and e 2 you substitute all this relationship first input in e equation e 1 if you do so if you do so, so e d will become e d 0 plus delta e d and is equal to 1 upon omega 0 p psi d will become psi d 0 plus delta psi d minus this term was there that is your psi q will become psi q 0 plus delta psi q, and it will be your omega r will become omega 0 plus delta omega r divided by omega 0 right.

Now, if you simplify if you simplify, then it then straight forward we can write minus phi q 0 plus delta e d in between one and two step you do yourself, then some time will be safe for me right is equal to 1 upon omega 0 p delta psi d minus 1 upon omega 0, you multiply all right. And a simplify it will be psi q 0 omega 0 plus psi q 0 delta omega r plus delta psi q omega 0 plus delta psi q delta omega r right.

So, in this case what and e d 0 this there should not be any sort of confusion, and e d 0 we have seen e d 0 is equal to minus psi q 0 just in the previous page we you have seen e d 0 is equal to minus phi q 0, and that has been substituted here right, and this one multiplied this one multiplied, and is equal to your whatever it is coming right.

Now, further if you do so, it will be minus psi q 0 plus delta e d this side is equal to 1 upon omega 0 p delta psi d. And if you simplify this one, it will be minus psi q 0, then minus psi q 0 delta omega r, this is this one will be (Refer Time: 15:15) what you call this is my this term is there.

Then second term psi q 0 minus psi q 0 delta omega r by delta omega 0, and this term minus delta psi q, and this term product delta psi q and delta omega r is very small. So, we have neglected that term, because two small perturbation term delta, delta term. So, it is very small, so it has been dropped neglected right.

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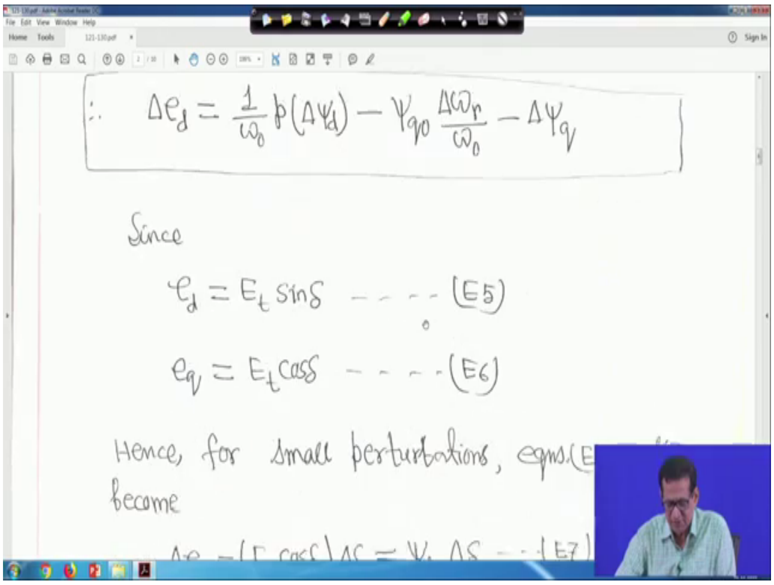

So, therefore we can write delta e d is equal to 1 upon omega 0 p delta psi d minus psi q 0 delta omega r upon omega 0 minus delta psi q, no equation number is given here right. Now, since e d is equal to e T sin delta, similarly way we can obtain delta e q also. Same way in similar way, you will substitute in equation-E2 in equation we will substitute E 2 that is your e q 0 plus delta e q right you substitute.

Similar way you can get the expression of your delta e q that will come later right. But, I am not derived that directly I have written, because one I have shown your complete derivation right. So, now since we know that e d is equal to E t sin delta that is equation E 5. And e q is equal to E t cos delta that is equation say E 6 right.

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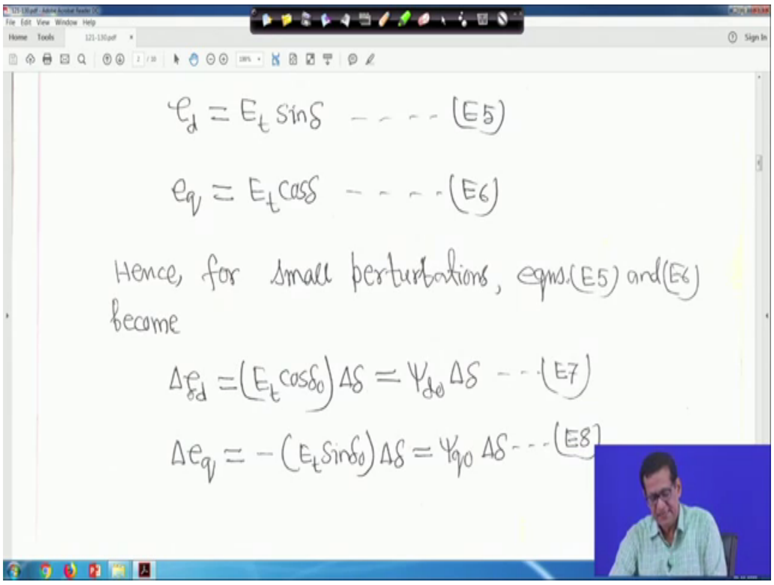

Hence, for small perturbation of equation E 5 and E 6, it will be something like this. So, it will be delta e d will be E t cos delta 0 into delta delta right. So, and we have seen that your E t cos delta 0 nothing but psi d 0, just in the previous your just a few minutes before, we have explained. So, e delta e d will be E t cos delta 0 delta delta, and E t cos delta 0 is nothing but psi d psi d 0 that we have seen right, so that is equation E 7.

Similarly, your delta e q will be minus E t sin delta 0 delta delta that is your and this one we have seen it will be psi q 0 right into delta delta, this is equation E 8 right. So, all these things what you call, because we have we have that psi q 0 is equal to your previous thing that your it will be minus E t sin delta your 0 right, so that is psi q 0 delta delta this is equation E 8.

Just see that whether if this e q 0, E t E t cos delta 0; and e d 0 is equal to E t sin delta 0 minus psi q 0 right, so that means all this things if you take this perturbation, so it will be psi q 0 your delta delta this is equation E 8 right. So, once again let us go to your what you call E t sin delta 0 here it is right that is your E t look here, E t sin delta 0 that is your minus psi q 0. So, that is why here we are substituting that E t sin delta 0 is minus psi q 0 that minus and minus plus, so psi q 0 delta delta this is equation E 8; do, understandable right.

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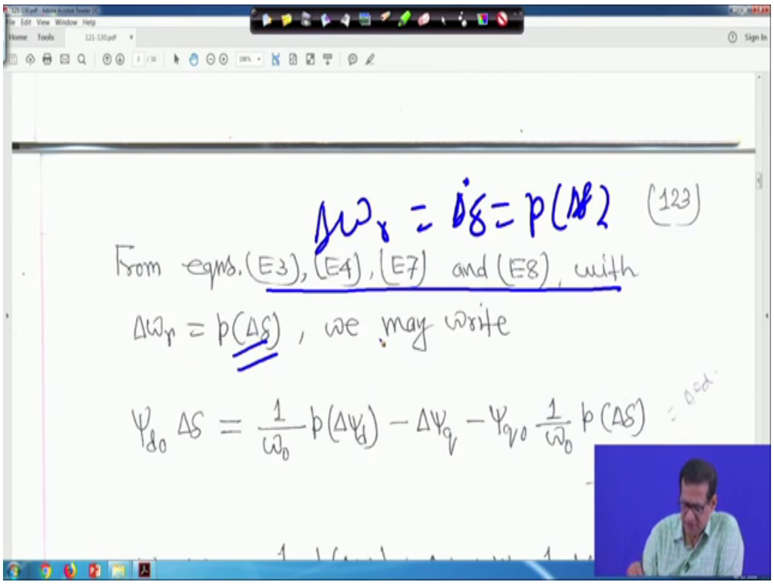

Now, from equation E 3, E 4, E 7, and E 8 with delta omega r is equal to p delta delta, because we know that delta omega r p is a d d t p is equal to d d t, therefore delta omega r is equal to delta delta dot right delta delta dot, so that is nothing but your p delta delta right. So, this is nothing but p delta delta that is delta omega r. So, from equation E 3, E 4, E 7, and E 8 if delta omega is this one, you substitute you substitute.

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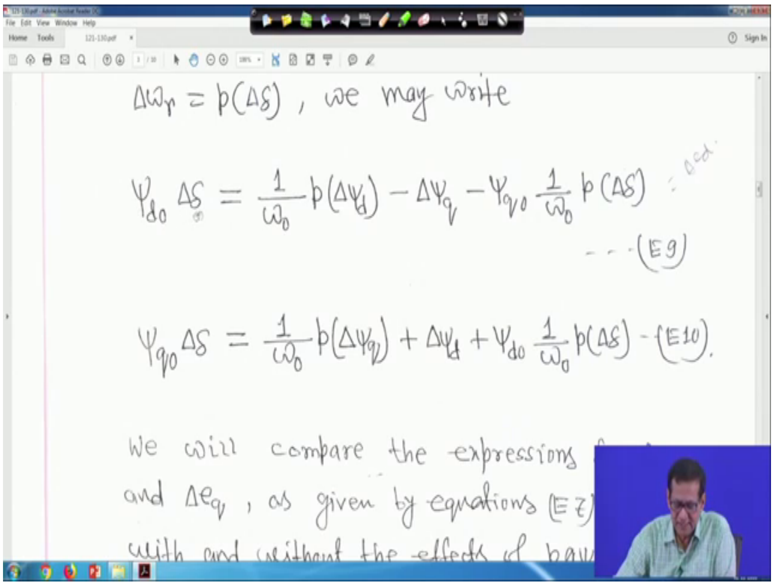

So, if you do so if you do so, then psi d 0 delta delta will become 1 upon omega 0 p delta psi d minus delta psi q minus psi q 0 1 upon omega 0 p delta delta, this is equation-E9 right. Similarly, similarly I told you that this step for e d, we have derived, you please derive e q for that also right.

So, if you do so, and finally same way if you proceed same way if you proceed, then what you will get that psi q 0 delta delta will be 1 upon omega 0 p psi q p delta psi q plus delta psi d plus psi d 0 1 upon omega 0 p delta delta, this is equation-E10 right.

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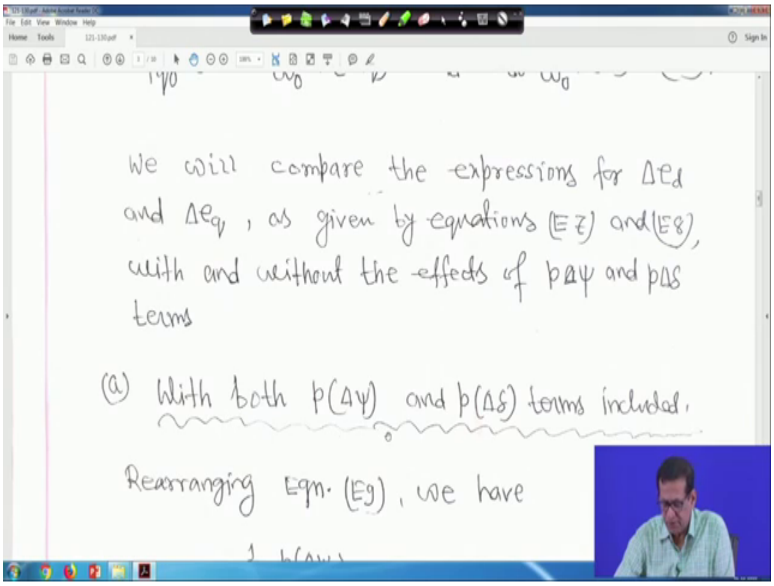

So, now we will compare the expression for delta e d and delta e q, as given by equations E 7 and E 8 with and without the effects of p delta psi and p delta delta terms right with p delta psi means, p delta psi d and p delta psi q right. And now part a with both p delta psi p delta psi means, p delta psi d and p delta psi q, so just we are putting like this and p delta terms included right.

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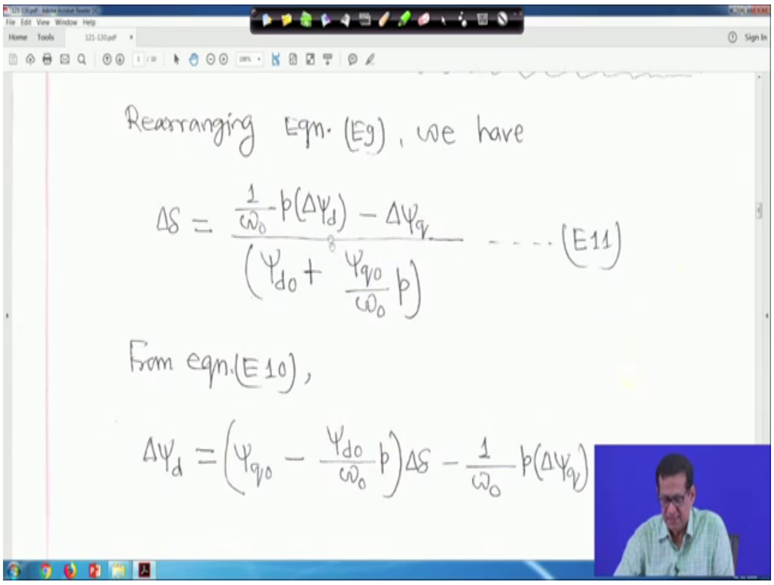

Now, rearranging equation E 9 if you if you just rearrange this equation E 9 that means, psi d 0 delta delta this equation if you rearrange right, it will become delta delta will become 1 upon omega 0 p delta psi d minus delta psi q upon psi d 0 plus psi q 0 upon omega 0 p, this is equation-E11 right.

Now, from equation-10 I mean from this equation from this equation equation-10 you rearrange, you will get your delta psi d will be psi q 0 minus psi d 0 upon omega 0 p delta delta minus 1 upon omega 0 p delta psi q, this is equation-E12 right.

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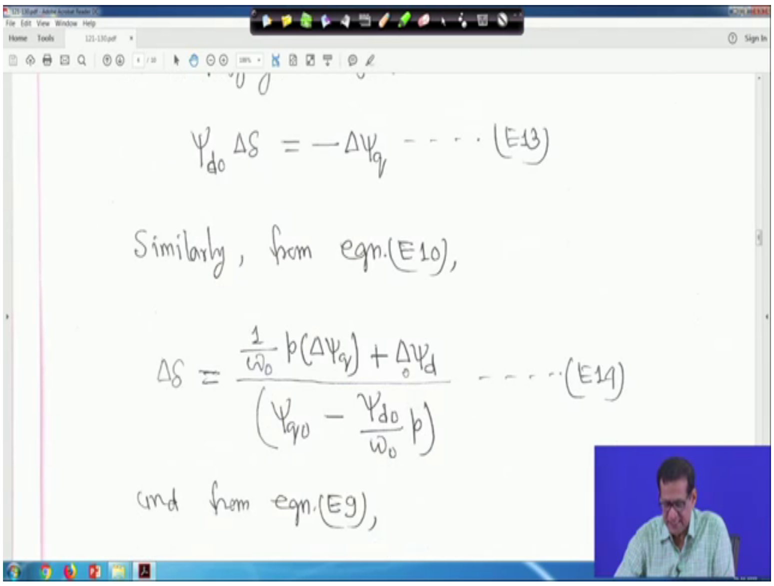

Now, substituting equation-12 in 11; so, this delta psi d term, you substitute in this equation equation-11 right. So, you just substitute if you do so if you do so, then what will happen? And you simplify, you will get psi d 0 delta delta is equal to minus delta psi q such a small expression, you will get you put it (Refer Time: 21:23) remember that all the derivative p p your psi q 0, p psi d 0 all these thing will become 0, because these are a study state operating condition right.

So, that means, you will get if you simplify psi d 0 delta is equal to minus delta psi q, this is equation E-13 right. Similarly, from equation-E10 if you make it equation-10 rearrange it, you will get delta delta is equal to 1 upon omega 0 p delta psi q plus delta psi d upon psi q 0 minus psi d 0 upon omega 0 p, this is equation-E14 right.

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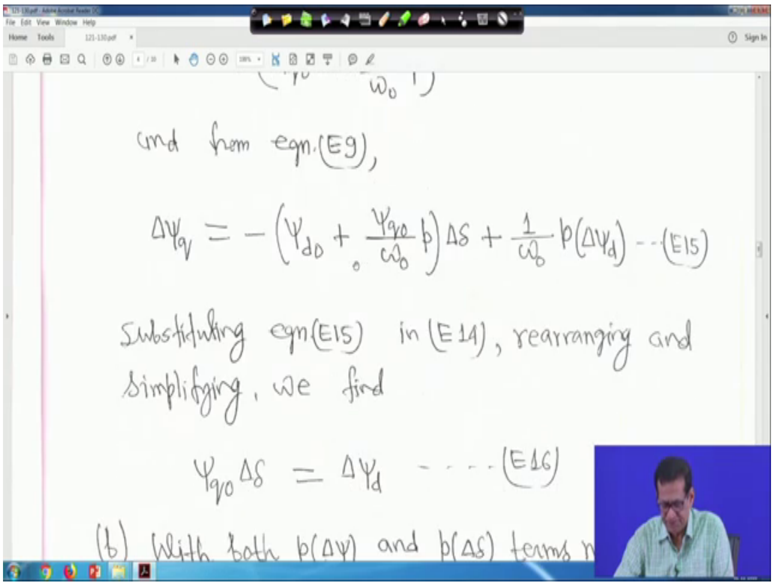

And from equation E9 that means from equation-E9 this equation right this equation from equation-E9 right, you will get delta psi q is equal to minus in bracket psi d 0 plus psi q 0 plus upon omega 0 p delta delta plus 1 upon omega 0 p delta psi d, this is equation-E15. Now, substitute this equation E15 that is delta psi q in this equation E14 you substitute and simplify.

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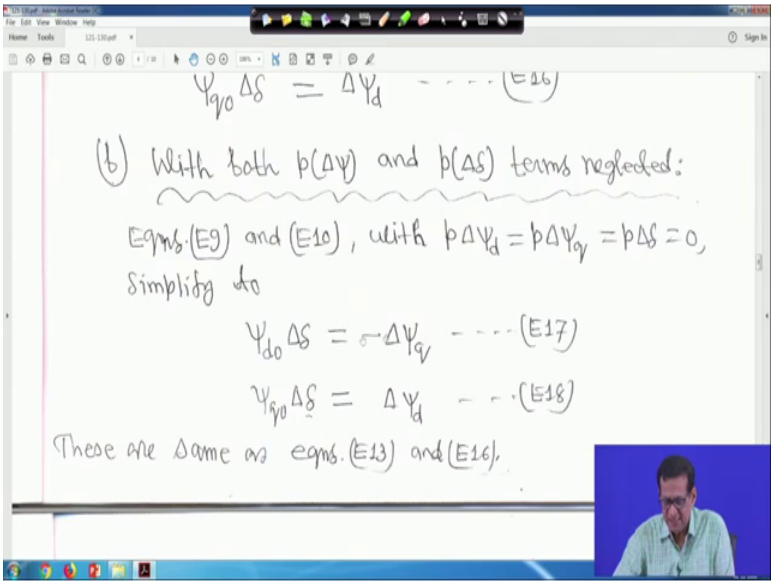

If you do so if you do so, you will get psi q 0 delta delta is equal to delta psi d that is with E16, this equation is E16 right. Now, this is I mean whatever we have made it right that means, considering both the here what you call that your considering your both p psi are (Refer Time: 22:51) included. When both this terms are included, we got this expression.

Now, so we got this E13 right, and we got this E16 these two equations we got. Now, with both p psi and p delta terms are neglected p psi p delta terms means, p delta psi d and p delta psi q terms are neglected along with p delta term, these are also neglected.

Now, that means in equation-E9 and E10 with p delta psi d is equal to p delta psi q is equal to p delta delta is equal to 0 that means, we are neglecting this. If you do so, you will get psi d 0 delta delta is equal to minus delta psi q; and psi q 0 delta delta is equal to delta psi d, this is E17 this is 18. And these two equations are same that is E13 and E16. If you go to that E13 and check with E17, This is your equation-E13 psi delta psi d 0 delta delta minus is equal to minus delta psi q right. And this is your psi d 0 delta, delta minus delta psi q, they are same.

And similarly if you see that psi q 0 delta delta and delta psi d, and this psi q 0 delta delta, delta psi d and psi q 0 delta delta, delta psi d, these two are same right. So, once that means including this terms and you are neglecting this term that they are appearing to be the same thing right. So, this is actually what you wanted to mean in the problem right.

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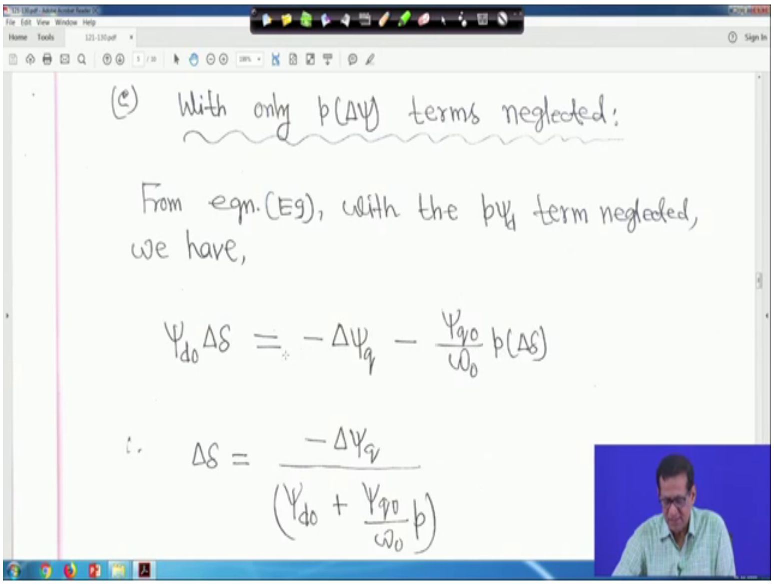

Now, now in the in this case now next case is suppose if we only p psi terms p delta psi terms are neglected in p delta psi d and p delta psi q terms here only it is neglected if we do so, then from equation-9 with the p delta psi d terms neglected, p delta psi means that is in general I am writing that is nothing but p delta psi d terms, and p delta psi q terms right.

So, with the neglected, then you will get psi d 0 delta delta is equal to minus delta psi q minus psi q 0 upon omega 0 p delta delta p delta delta right or if you rearrange delta delta will be minus minus delta psi q divided by psi d 0 plus psi q 0 upon omega 0 p, this is delta delta.

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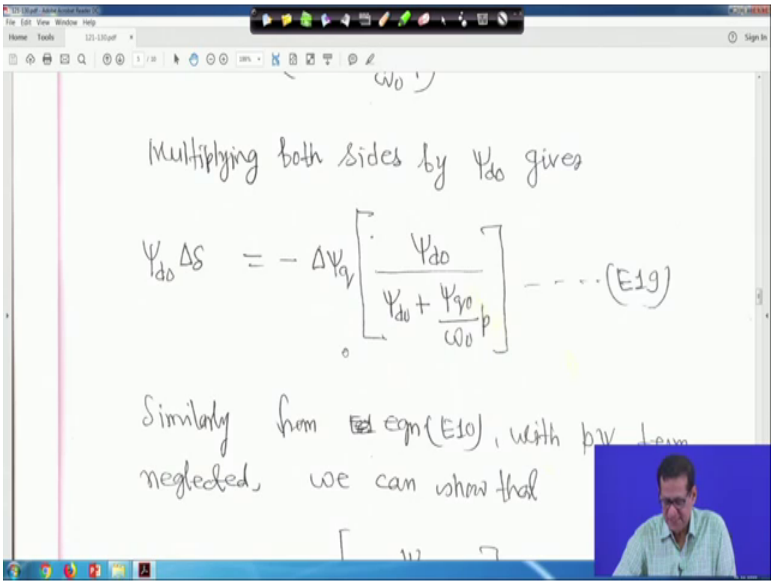

Now, multiplying both side, both side you multiply by psi d 0, this equation then you will get psi d 0 delta delta is equal to minus delta psi q, and this one can be written as your psi d 0 minus delta psi q, and this term into psi d 0 by psi d 0 plus psi q 0 upon omega 0 p right. p is nothing making it here it is nothing but that your differential operator this is equation-E19 right.

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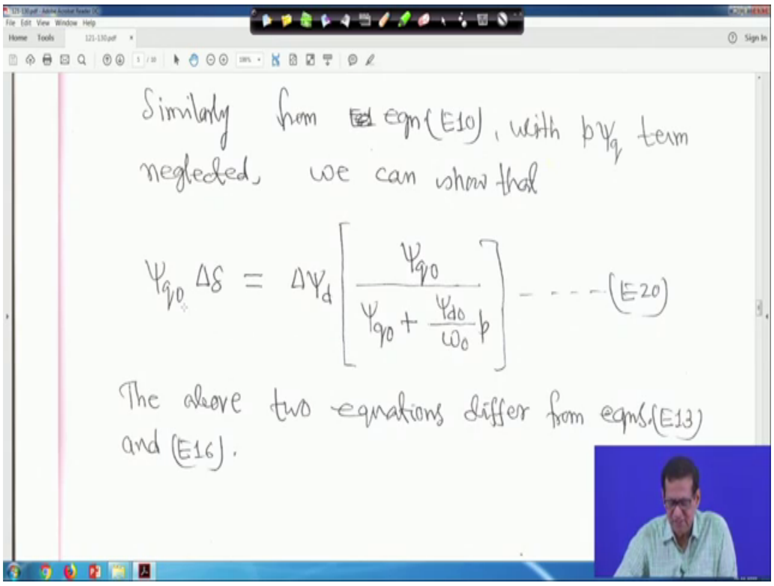

Similarly, from equation-E10 in equation-E10, you neglect p psi q term that is p psi q drop right you make p psi q 0 in equation-E10. And then you can we can show also that

that psi q 0 delta delta is equal to delta psi d, then psi q 0 upon psi q 0 plus psi d 0 upon omega 0 p right. So, if you if you look into if you look into this the above two equations differ from equation-E13 and E16, these two equations are not same like your E13 and equation- E13 and equation-E16 right.

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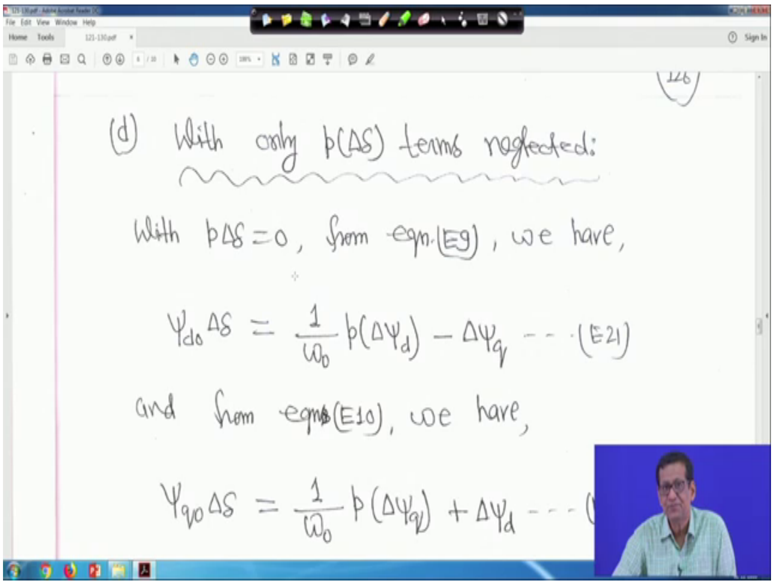

So, next is with only p delta terms neglected right with only p delta terms neglected. With p delta delta 0 from equation- E9, you put p delta delta 0. Then you will have psi d 0 delta delta is equal to 1 upon omega 0 p delta psi d minus delta psi q, this is equation-E21. And from equation E10, we have psi q 0 delta delta by putting your p delta delta term 0, 1 upon omega 0 p delta psi q plus delta psi d this is equation-E22 right.

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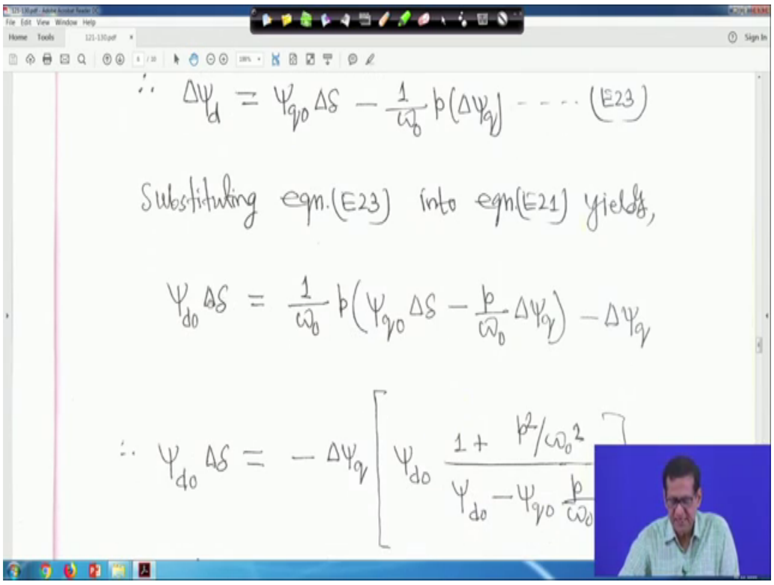

And therefore, delta psi d will be psi q 0 delta delta minus 1 upon omega 0 p delta psi q, this is equation-E23. Now, if you substitute equation-23 into equation-E21, this delta psi d term you substitute here and simplify, then you will get that psi d 0 delta delta is equal to 1 upon omega 0 p psi q 0 delta delta minus p omega 0 delta psi q minus delta psi q, just you know simplify, you will get this expression or psi d 0 delta delta, you will get minus delta psi q in bracket psi d 0, it is 1 plus p square upon omega 0 square divided by psi d 0 minus psi q 0 p upon omega 0, this is equation-E24.

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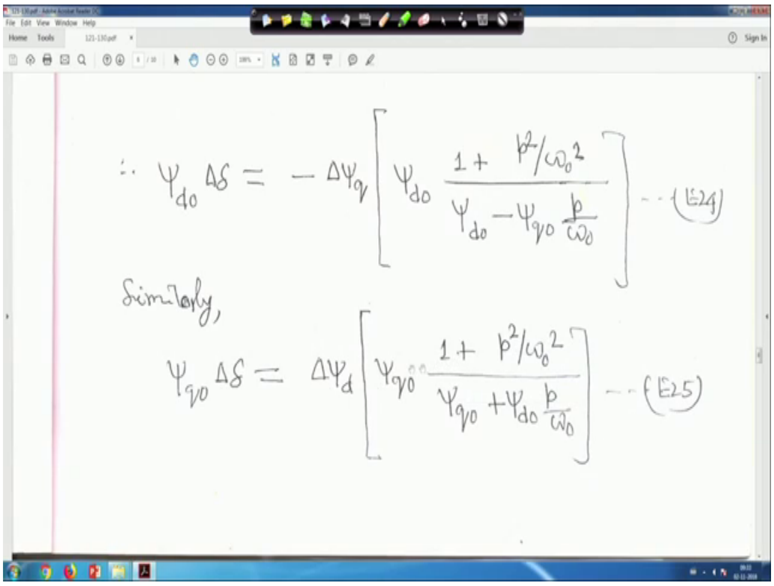

Similarly, you can get this one similar way you move right, and you will get this psi q 0 delta delta is equal to delta psi d psi q 0 1 plus p square upon omega 0 square divided by psi q 0 plus psi d 0 into p upon omega 0, this is equation-E25 right. So, little bit I I mean most of the things I have made it for you, but little bit you have to do up your own right.

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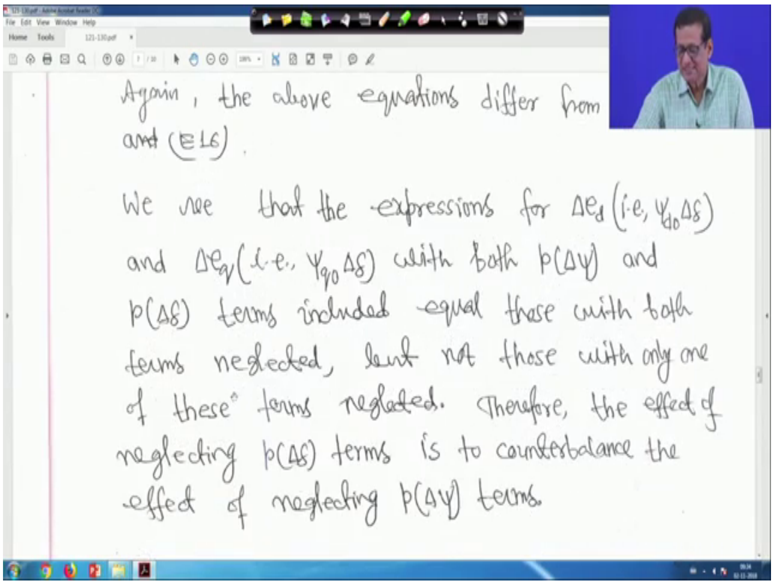

So, again the above equations differ from E13 and E16 right. So, you see that the expression for delta e d that is psi d 0 delta delta, and delta e q that is psi q 0 delta delta with both p delta psi and p delta terms included equal to this with both terms are neglected right. So, if you include or if you drop, it is same right, but not those with any one of these terms right neglected. Therefore, the effect of neglecting p delta delta terms is to counterbalance, the effect of neglecting p delta psi terms p delta psi means, p delta psi d and p delta psi q right.

Thank you very much, we will be back again.