**Power System Dynamics, Control and Monitoring**

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**Lecture – 11**
**Power System stability (Contd.)**

So we are back again. So, although it is a dynamics course, but we have to see little bit of study state analysis right; so competition of steady state values.

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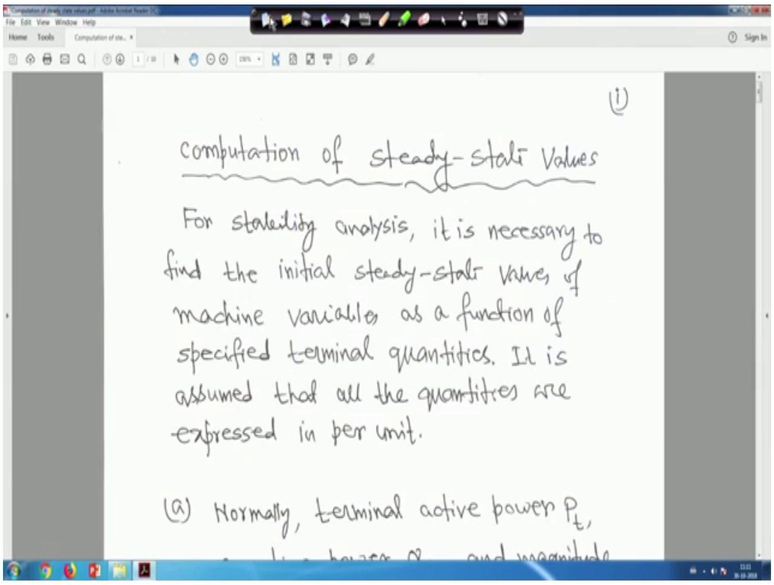


So, for stability analysis it is necessary to find the initial steady state values of machine variables as a function of specified terminal quantities right. So, it is assumed that all the quantities are expressed in per unit.

So, whatever analysis we will do we will make it in per unit right.

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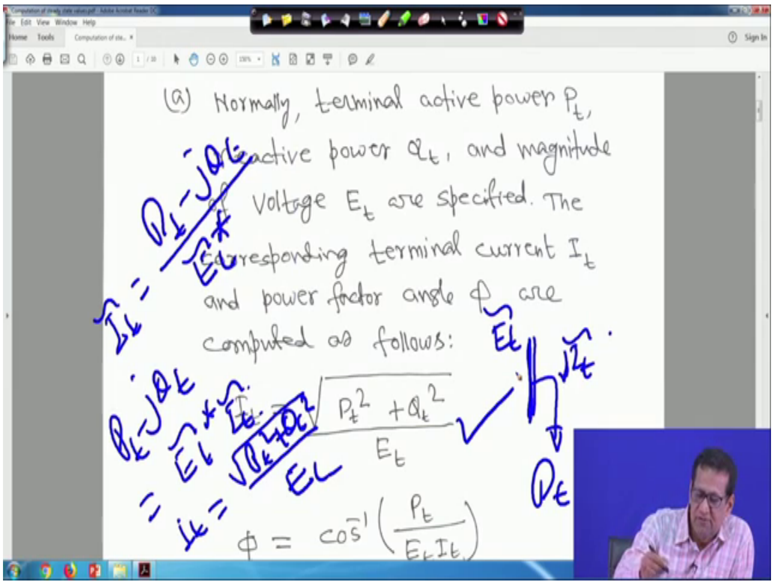


It is a normally terminal active power P t reactive power Q t and magnitude of voltage E t are specified. Therefore, the corresponding terminal current I t and power factor angle phi are computed as follows: So, I t will be root over P t square plus Q t square upon Et and I mean if suppose if you have a if you have a this suppose you have a bass bar right. And suppose here you have the power say P t plus J Q t say right and voltage here say E t right say E t tilde, the pressure quantity, and here what you call that current, current is a it is your I t right.

So, when you write your lower flow studies you have studied, so when you write the power equation that is your P t power injection equation P t minus JQ t we can right that E t tide conjugate I t right. So, these are all E t and I t, their pressure quantity. So, if you take the here what you call that I t what is that value of I t, then I can write that I t is equal to P t minus J Q t divided by E t tilde conjugate right.

So, I mean if you take the magnitude of this I t, then we can write say if we I t is the magnitude of the current right then we can write I t is equal to root over P t square plus Q 2 square divided by E t; that is what it has been written here right. So that means that is your I t.

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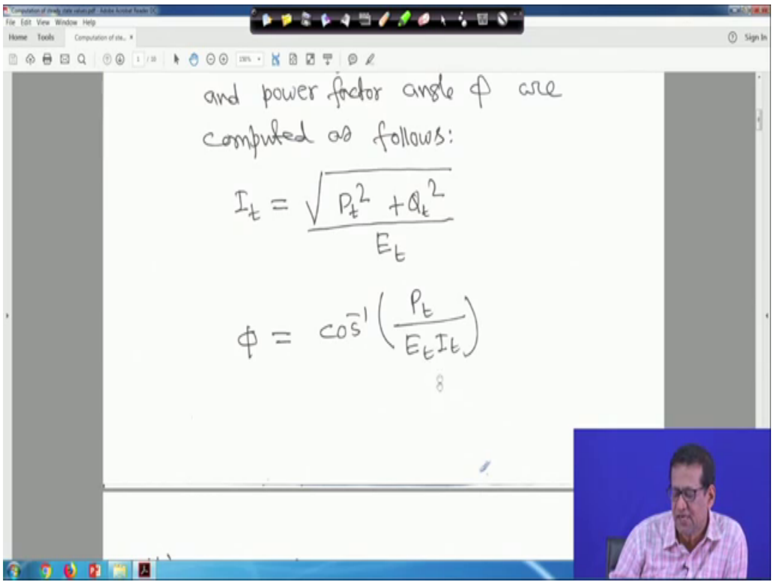


And similarly your power factor that cos phi will be P t upon E t, I t. E t is the magnitude, I t is the current. So, if P t is the real power and E t, I t is the volt ampere therefore, cos phi will be P t upon E t I t. Therefore, phi is equal to cos inverse P t upon E t I t right.

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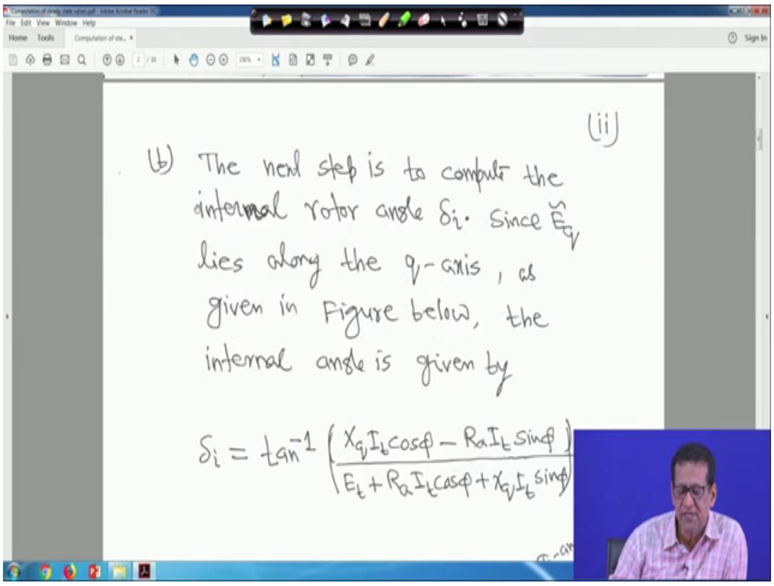


Now, this b part, the next step is to compute the internal rotor angle delta i. Since E q tilde lies along the key axis this we have seen earlier as given in figure below, the internal angle is given by, I mean this is that phasor diagram that E q this is by q axis say i have drawn like this is my q axis right.

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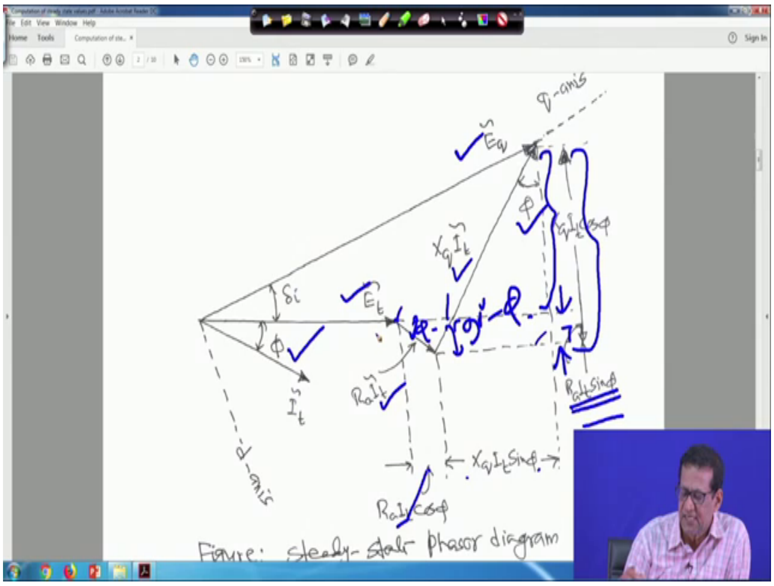


So, this is E q tilde is equal to your E t tilde plus Ra I t tilde plus J X q I t tilde. This we have seen right.

So, this current I t tilde (Refer Time: 03:39) from E t tilde by triangle phi and this is my d axis an angle between q and d axis is 90 degree right. Therefore, if you take the projection of this one, this is what you call this X q I t and this is by angle phi and this angle will be also phi from the simple geometry. Therefore, this portion this portion will be your what you call X q X q I t sin phi because this is phi, this is 90 degree, this is your 90 degree minus phi. So, these are this portion will be X q I t sin phi and this portion will be X q I t cos phi; I mean from here to here right.

Now, and this is my Ra I t this is phi, so also this angle is also phi. Therefore, from here to here this is your Ra I t cos phi right. And similarly your this portion I mean from it is it is marked here, I mean Ra this portion I mean from here to here, this is my Ra I t sin phi, this is written here Ra I t sin phi right. That means, from here to here it is X q I t cos phi minus Ra I t sin phi I mean from here to here it is X q I t cos phi minus Ra I t sin phi.

Similarly, from here to here it is X q I t sin phi plus Ra I t cos phi. Therefore, tan delta i will be easy tan delta i will be simply is equal to a your you take this vertical one X q I t cos phi minus Ra I t sin phi divided by E t plus Ra I t cos phi plus X q I t sin phi right. That is what has been just hold on, that is what has been written here right, that is del tan delta i will be X q I t cos phi minus Ra I t sin phi divided by E t plus Ra I t cos phi plus X q I t sin phi. That means, delta is equal to tan inverse X q I t cos phi minus Ra I t sin phi divided by E t plus Ra I t cos phi plus X q I t sin phi right.

So, this is that simply steady state phasor diagram that is how to compute actually delta i.

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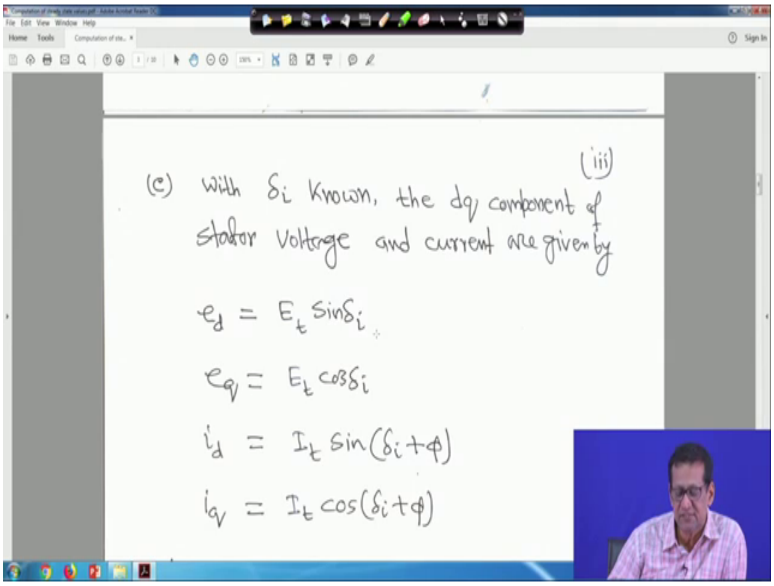


Now, with delta i known the d q component of stator voltage and current are given by this. We have seen earlier that e d is equal to E t sin delta i if delta i is known and e q is equal to E t cos delta i right. Similarly earlier we have seen that i d is equal to I t sin delta i plus phi and i q is equal to I t cos delta i plus phi. No, equation numbers given here because all these things we have seen in previous lecture right.

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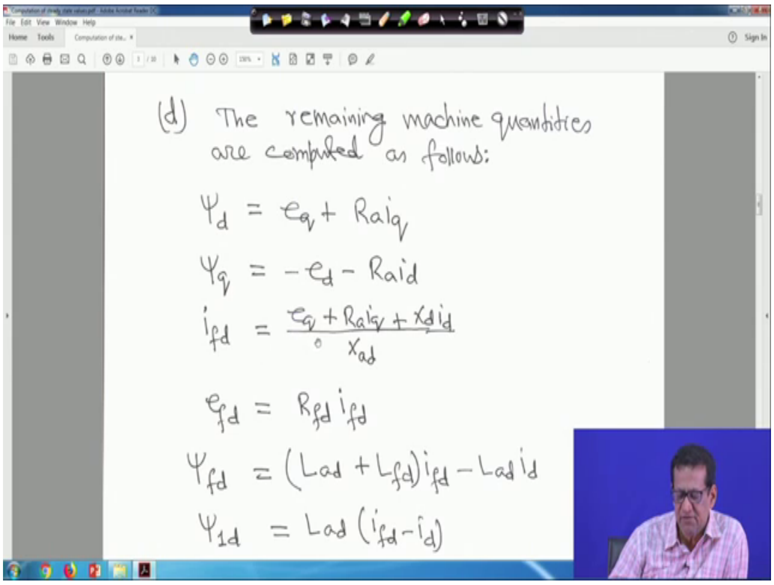


Therefore, the remaining machine quantities are computed as follows; therefore, already we have done this. Therefore, psi d will be e q plus Ra i q psi q will be minus e d minus Ra i d and this i f d expression also given before. So, i f d will be e q plus Ra i q plus X d i d upon X a d right. Therefore, and e f d there is a field voltage is equal to R f d into i f d. Equation numbers not given here because all these things are given before.

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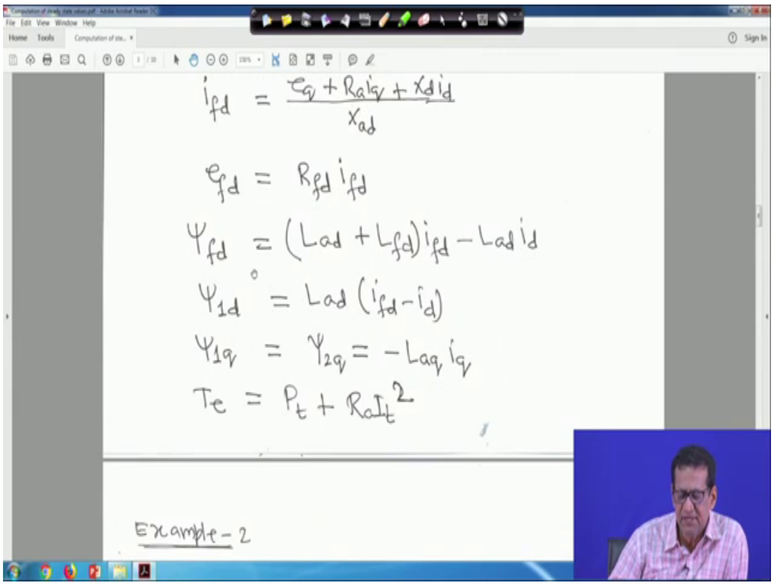


Now, psi f d will be now is equal to L a d plus L f d i f d minus L a d i d. Similarly psi 1 d we are assuming that your 1 I am also d axis it will be L a d into i f d minus i d similarly psi 1 q will be is equal to psi psi 2 q is equal to minus L a q i q right and torque T e is equal to P t plus Ra I t square. This all this equations whatever is written here that has been given that developed and given in the previous lectures right.

So, with this we will take one small examples, now this is example 2.

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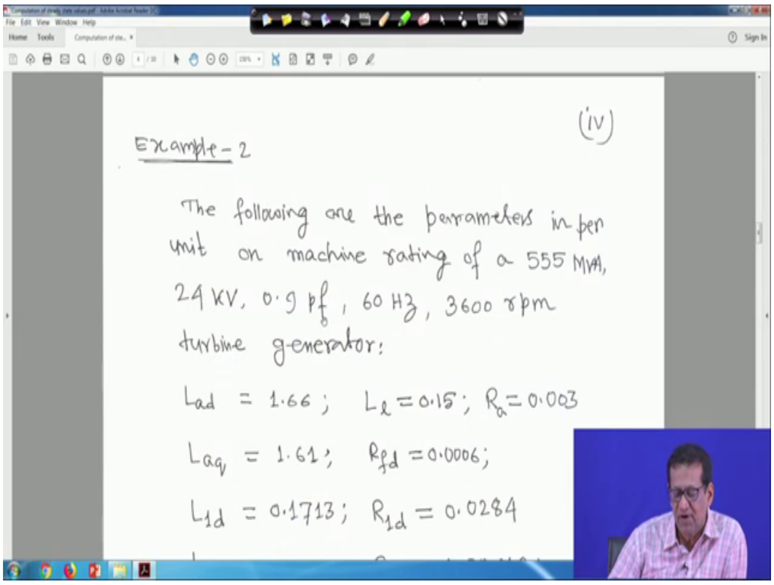


The following are the parameters in per unit on machine rating of a 555 MVA 24 KV 0.9 power factor, 60 hertz 3600 rpm turbine generator. It is all these parameters are given in per unit, so every place I did not write p u p u like this right; it is it is understandable that it is in per unit.

L a d is given 1.6.6, L l is given 0.15, Ra 0.003, all are in per unit. L a q is 1.61, R f d 0.666 L1 d 0.1713, R 1 d 0.0284 L 1 q 0.7252 R 1 q 0.00619.

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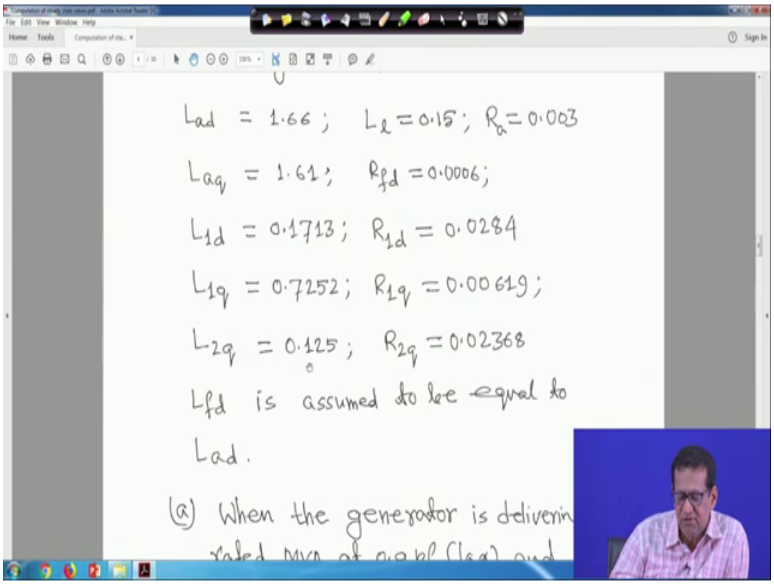


L 2 is given 0.125, R 2 q 0.02368 right. L f d is assumed to be equal to L a d right. So, I mean L L f d is equal to your L a d. So, here L a d is given. So, your this is assumed that L f d is equal to L a d right.

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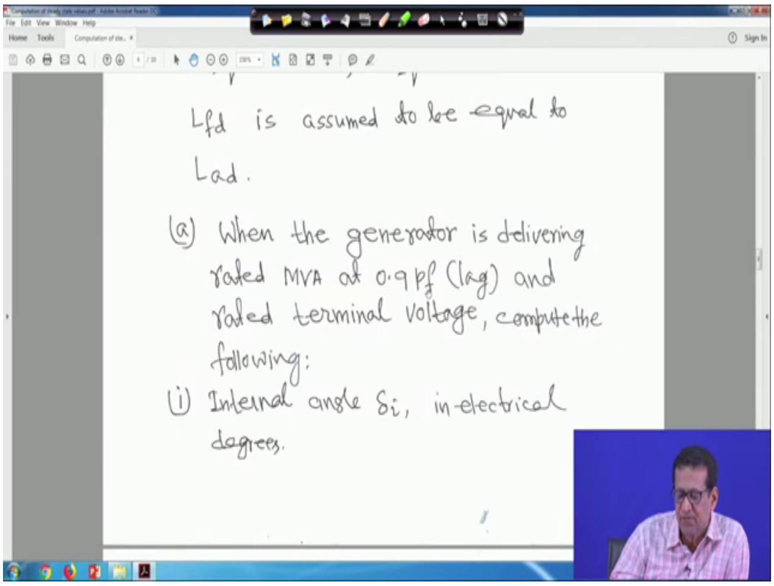


Now, part a when the generator is delivering rated MVA at 0.9 power factor lagging and rated terminal voltage compute the following. One: internal angle delta in electrical degrees that you have to determine.

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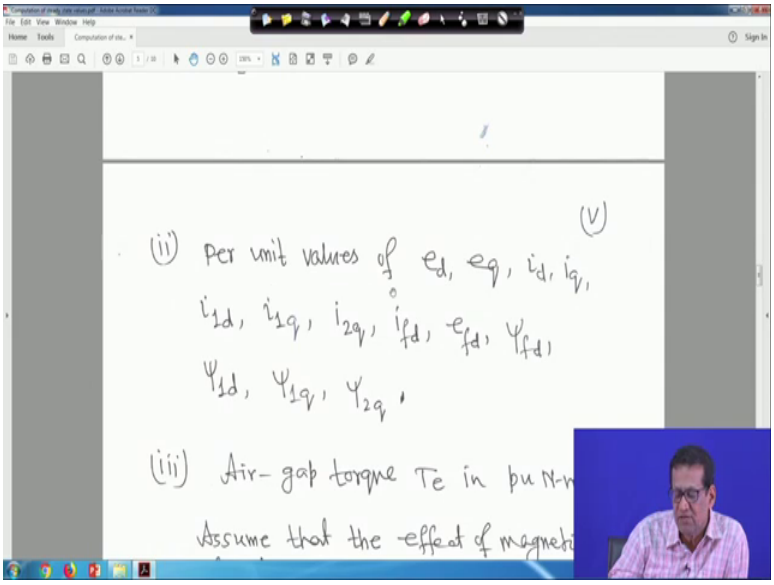


Then, 2 per unit values of e d e q i d i q i 1 d i 1 q i 2 q i f d e f d psi f d psi 1 d psi 1 q and psi 2 q.

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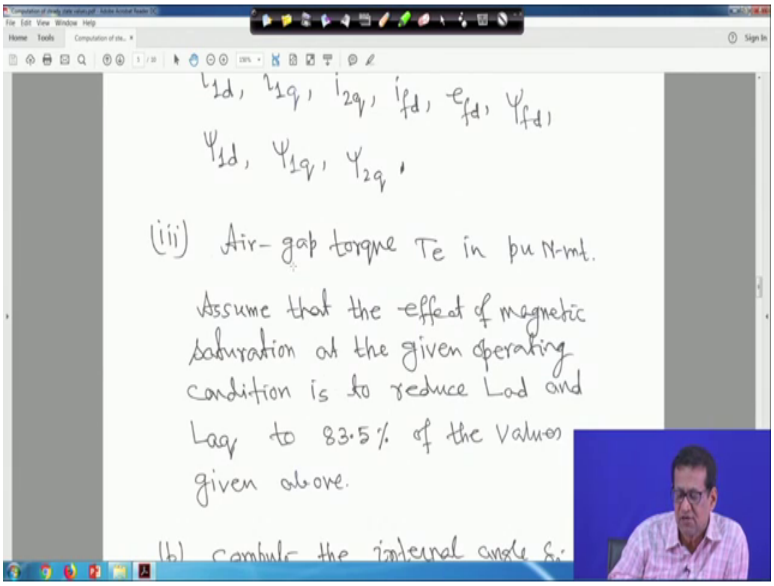


Part 3, that air gap torque Te in per unit in per unit newton meter right. Assume that the effect of magnetic saturation at the given operating condition is to reduce low your L a d. And L a q to 83.5 percent of the values given above that we will see when we solve this problem.

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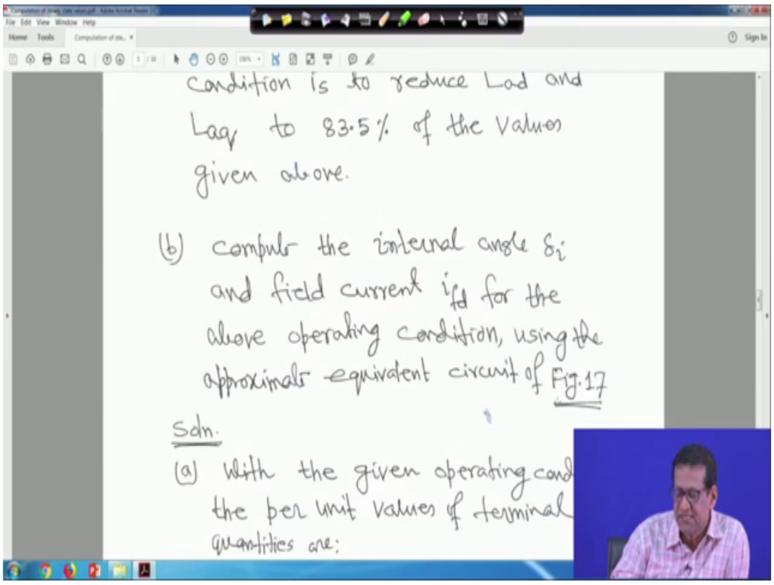


And part b compute the internal angle delta i and field current i f d for the above operating condition using the approximate equivalent circuit of figure 17 right.

So, this figure 17 already given before.

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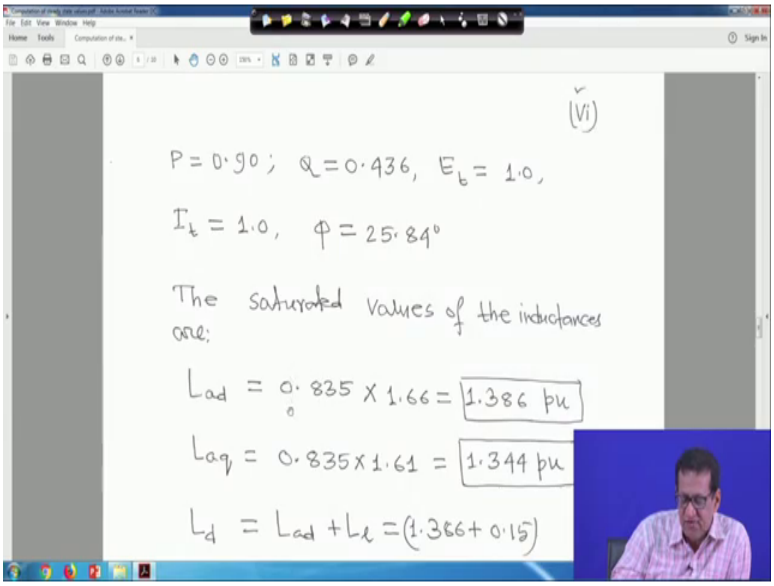


Now, solution with the given operating condition the per unit values of terminal quantities are it is P is equal to actually 0.90 per unit, q is equal to 0.436 per unit E t is equal to 1.0 per unit. Although not retained many places, but understandable there in per unit values. I t is equal to 1.0 per unit and phi will be 25.85 degrees because power factor is given 0.9 lagging right.

So, the saturated values of the inductances are L a d it is actually given here in the problem that is your 83.5 percent of the values given above. So, it is L a d will be 0.835 into 1.66 that is your 1.386 per unit. L a q is equal to 0.835 into 1.61 it is 1.344 per unit right. L d is equal to L a d plus L l that is 1.386 plus 0.15 because L l is given 0.15. So, it comes about 1.536 per unit right.

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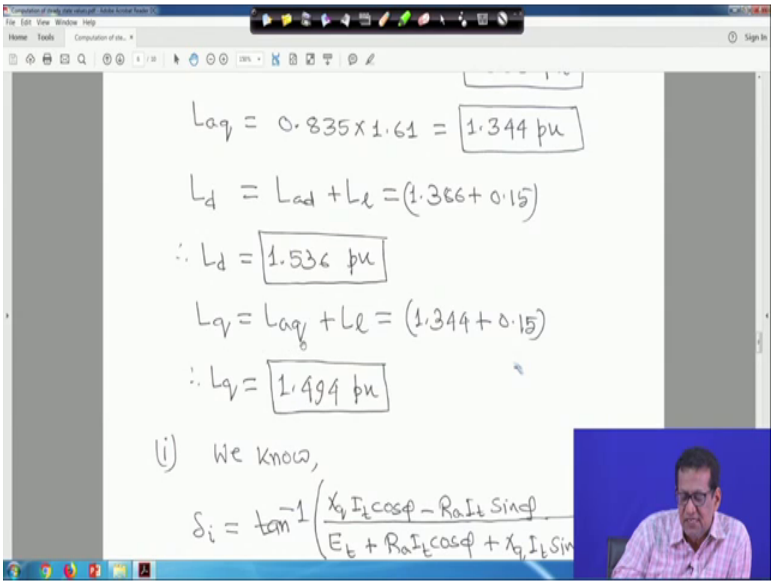


Similarly, L q is equal to L a q plus L l. So, it is 1.344, because here it is 1.344 we have computed plus 150.15. So, it is L q 1.4944unit.

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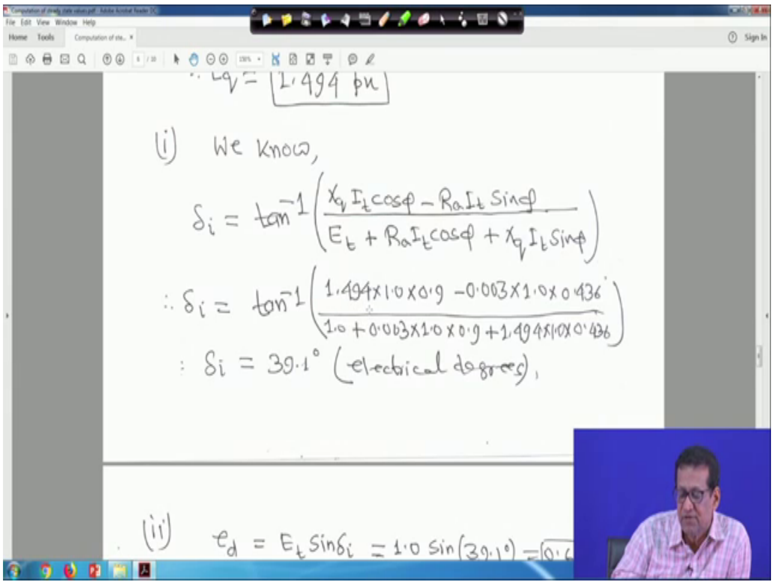


Now, first one is internal angle delta i for this. So, we know delta is equal to tan inverse X q I t cos phi minus Ra I t sin phi divided by E t plus Ra I t cos phi plus X q I t sin phi right Now therefore, delta is equal to tan inverse x q is given 1.494 I t is one per unit and cos phi is 0.9 it is given.

Now, minus R a, Ra is given 0.003, I t is 1 and sin cos phi is 0.9. So, sin phi will be 0.436 right divided by E t. E t is one plus Ra I t, Ra is 0.003 into 1, I t is 1 into cos phi that is 0.9 plus 1 X q is 1.494 into 1.0 into sin 5.436. If you solve it, delta will be 39.1 electrical degrees right that is the internal angle of the machine. So, only thing is that just you have to compute in correct fashion.

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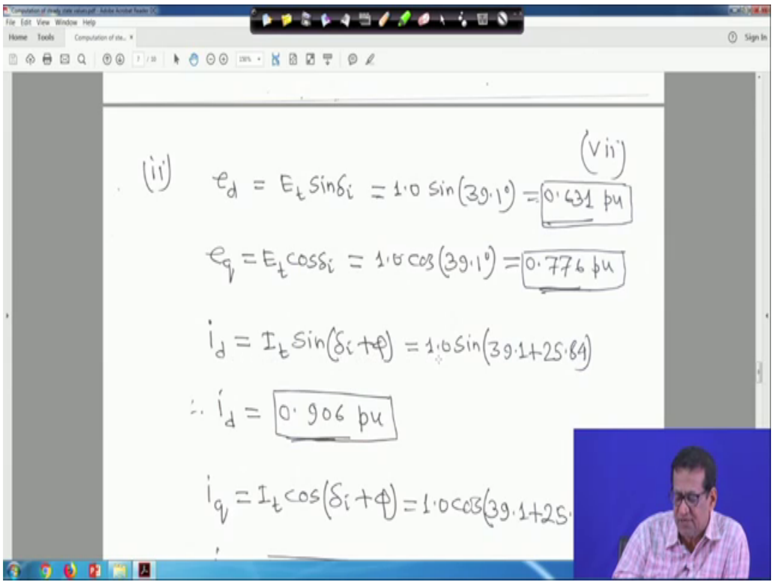


Now, part 2 e d we know e d is equal to E t sin delta i. So, E t is 1 and sin 39.1 degree that is 0.631 p u that is your e d. e q you know at E t cos delta i, so 1.0 cos 39.1 degree, So, 0.776 per unit that is my e q. And i d is equal to we know I t sin delta i plus phi. So, I t is 1 per unit delta i is 39.1 degree and phi is equal to 25.84 degree.

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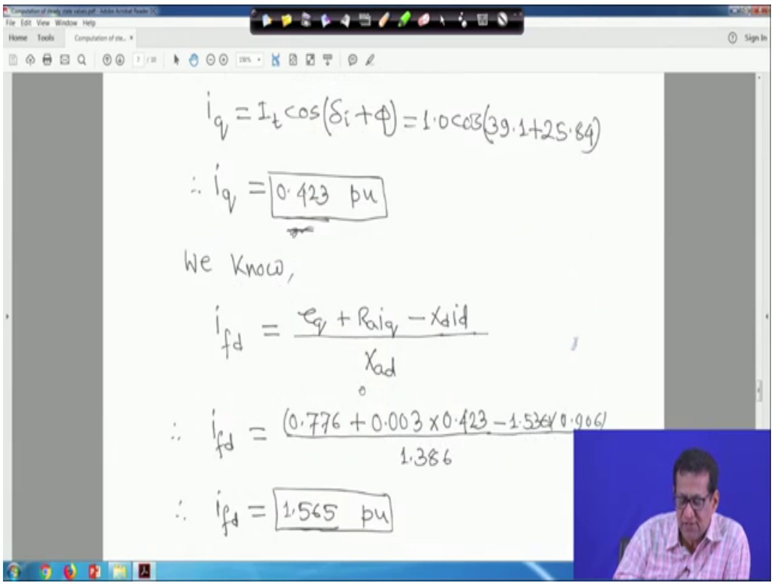


If you compute i d is 0.906 per unit right. Similarly i q is equal to I t cos delta i plus phi that is one, because I t is one cos39.1 plus 25.84 they are degrees So, i q will be 0.423 per unit right.

We also know the field current this formula or equation we have derived before. So, i f d will be e q plus Ra i q minus X d i d upon X a d, e q is we have got 0.776 right. So, it is 0.776 and e d is 0.631.

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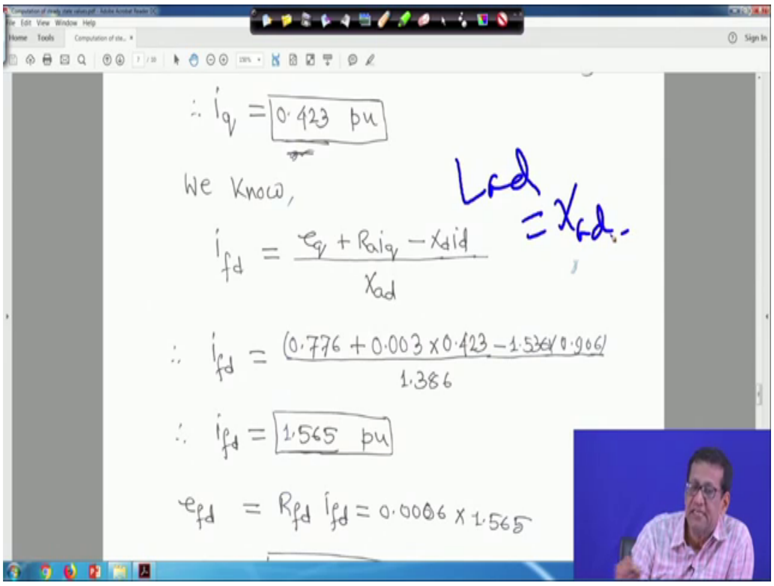


So, e q is 0.776 Ra is 0.003, i q we have computed 0.423 minus x d is given 1 phi 36 into i d, that is 0.906 divided by X a d we have computed 1.38. Actually in per unit L a d will be is equal to X a d. I told you in per unit values your inductance and your reactants all are same. So, in per unit value L a d is equal to X a d right.

So, sorry so that is why it is 1 point divided by 1.386. If you compute you will get i f d is equal to 1.565 per unit right.

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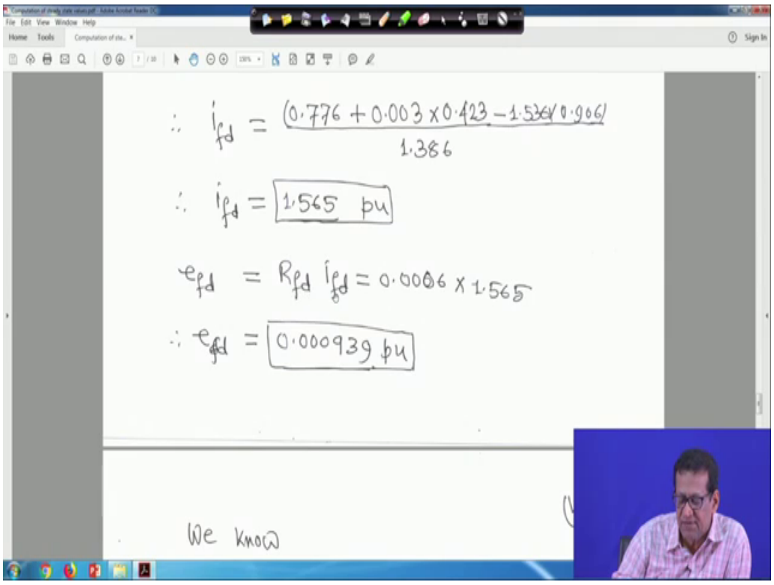


And we know e f d is equal to R f d into i f d. So, R f d is 0.666 into 1.565. So, e f d will be0.000939 per unit right.

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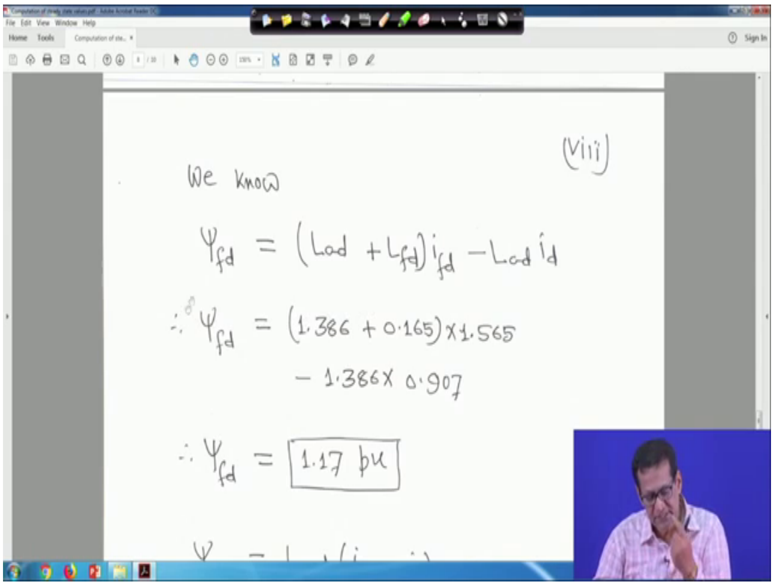


So, we also know that psi f d is equal to L a d plus L f d into i f d minus L a d into i a d i d right. So psi f d is equal to L a d 1.386 plus L f d 1.165 into i f d we have computed, 1.56 as minus. I told you L a d and X a d same right So, L a d is given 1.386 into i d 0.907. So, this comes psi f d 1.17 per unit right. We also know psi 1 d is equal to L a d i f d minus i d. So, psi 1 d will be 1.386 into in bracket i f d is 1.565 minus i d 0.909.

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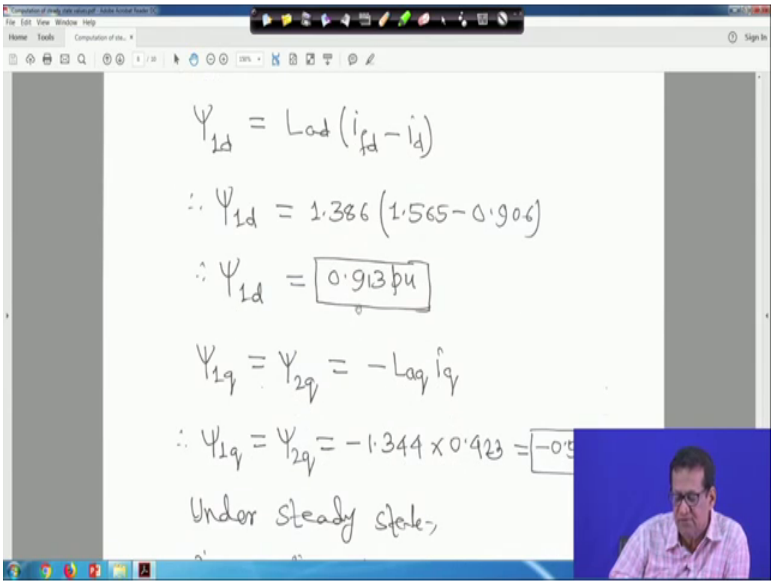

So, psi 1 d will become 0.913 per unit right and psi 1 q is equal to psi 2 q is equal to minus L a q i q. So, that means, psi 1 q is equal to psi 2 q L a q is equal to minus L a minus L a q is equal to 1.344 i q is equal to 0.423. So, it is actually minus 0.569 per unit right.

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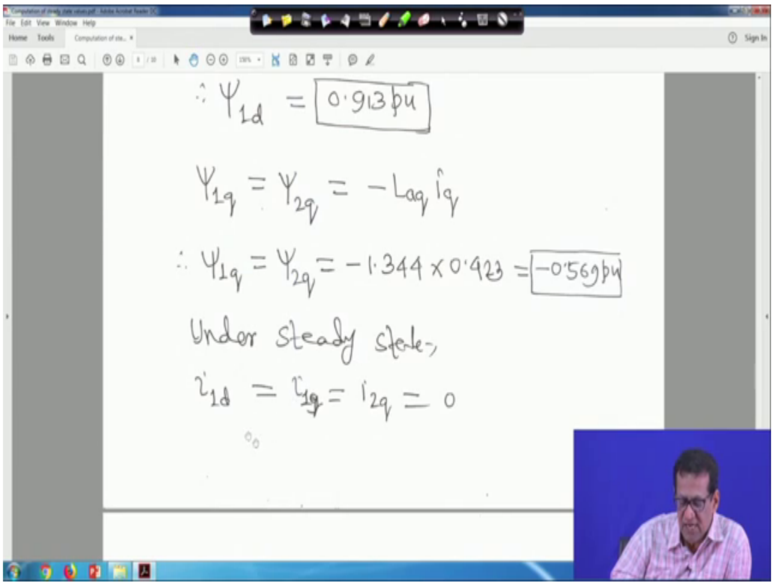

Now, under steady state i 1 d is equal to i 1 q is equal to i 2 q will be 0 right. So, because there only one amounts of winding is there on the d axis and 2 are on the q axis and on when it is as reach steady state, because their close winding when it is steady state all this currents are equal to 0 right.

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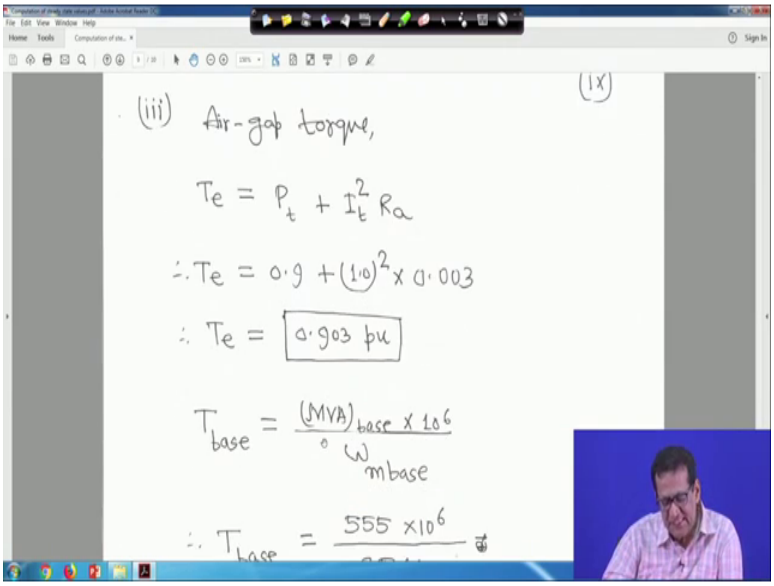


Now, number 3, the air gap torque, we know Te is equal to P t plus I t square r a. So, P t is 0.9 and I t is 1. So, 1 square and Ra is 0.003. So, basically it is coming t is equal to 0.903 per unit. If you neglect this loss then t approximately equal to P t because these are they are very close right. Now next is the torque base, the t base. Torque base is MVA base into 10 to the power 6 divided by omega mechanical base right.

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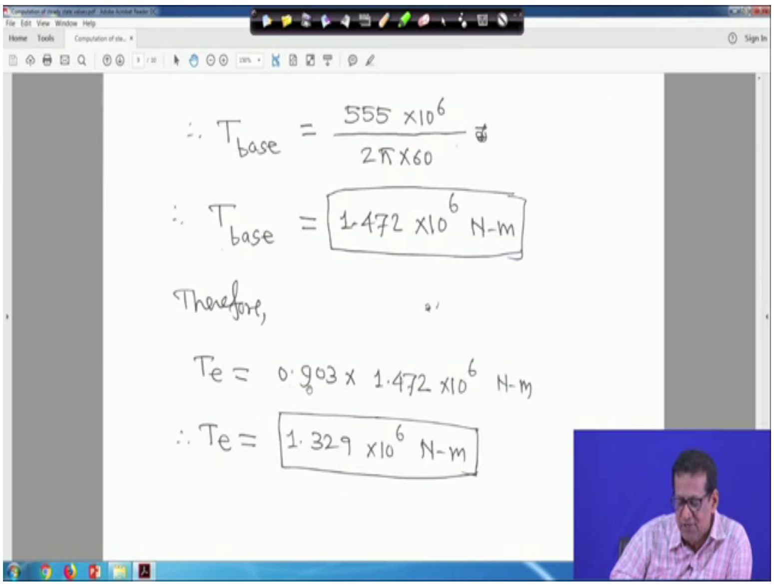

So, it is MVA base is 555 MVA into 10 to the power 6. We are actually converting to volt ampere right divided by omega mechanical base 2 pi f, f is 6 t hertz right So, if you compute torque base that is 1.472 into 10 to the power 6 Newton meter this is the base value.

Therefore, t e is equal to its 0.903 we have got per unit value 0.903 and T e is equal to you multiple by this base value t base, you will get 1.329 in to 10 to the power 6 Newton meter right.

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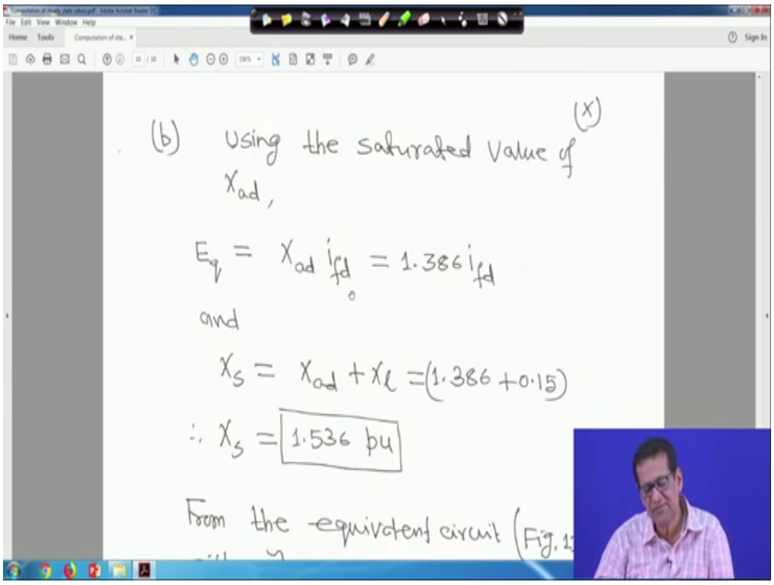


And part b using the saturated value of X a d we know that earlier we have seen e q is equal to J X a d i f d, but we are just thinking about the magnitude. Therefore, e q is equal to X a d into i f d that is 1.386 i f d right and X s will be X a d plus X l. So, X a d is 1.386 plus 0.15. That is X s is equal to 0.5536 per unit we call it as synchronous reactance right.

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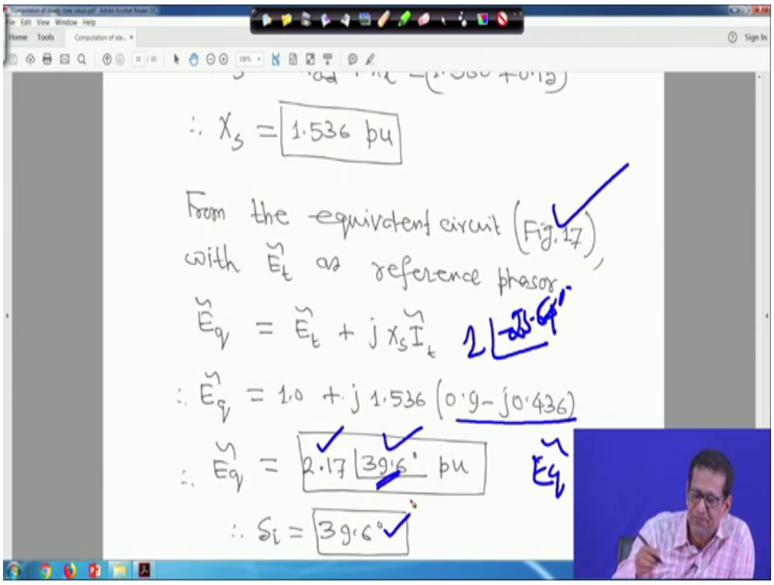


From the equivalent circuit just go back to figure 17 once, this figure 17 with E t tilde as reference phasor the simple single line circuit. if you see you see e q tilde is equal to E t tilde plus j X s into I 2 I t tilde right.

So, I am not going to figure 17, just have a look right. So, there everything is given. So, e q tilde E t tilde is 1 angle 0. So, it is 1.0 plus J x s, we got 1.536 and I t tilde it is one per unit. So, basically it is your and what you call that power factor is 0.5 lagging.

So, basically it is one angle your minus 25 84 degree it is lagging right. So, basically it will become point 9 minus j 0.436. So, if you compute e q tilde will become 2.17 angle, 39.6 degree. And if you go back to this figure 17, it is given e q tilde right if that is your angle delta i, so this delta I this is the magnitude of e q e q values and delta i is 39.6 degree right that is the answer.

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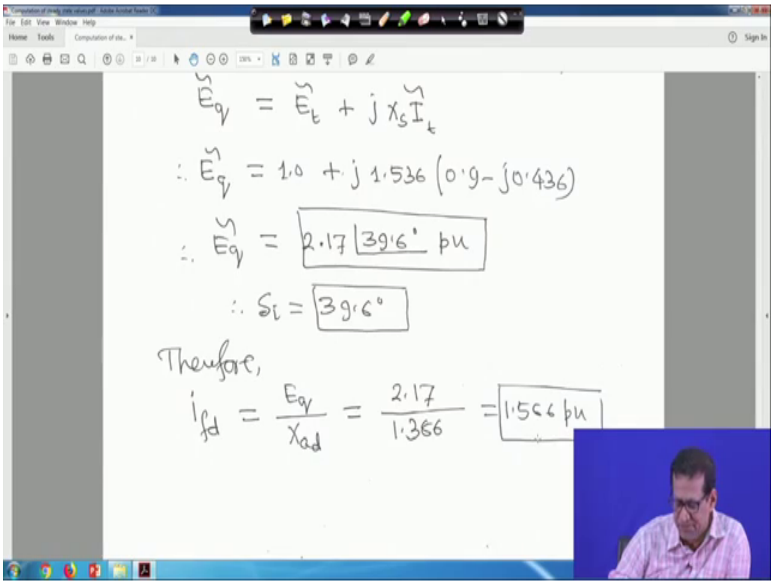


Therefore i f d magnitude that i f d e q upon X a d, so e q we got 2.17 and X a d we have computed 1.386. So, it gives actually 1.56 six per unit right that is the current. So, now with this example with this example whatever little bit steady state analysis we have done that will what you call that; we will clear our here what you call doubts right.

So, these are the simple example and simple phasor diagram just this kind of phasor diagrams you have already done in power system as well as in machine analysis So, it is very simple. Next we will move to your just hold on.

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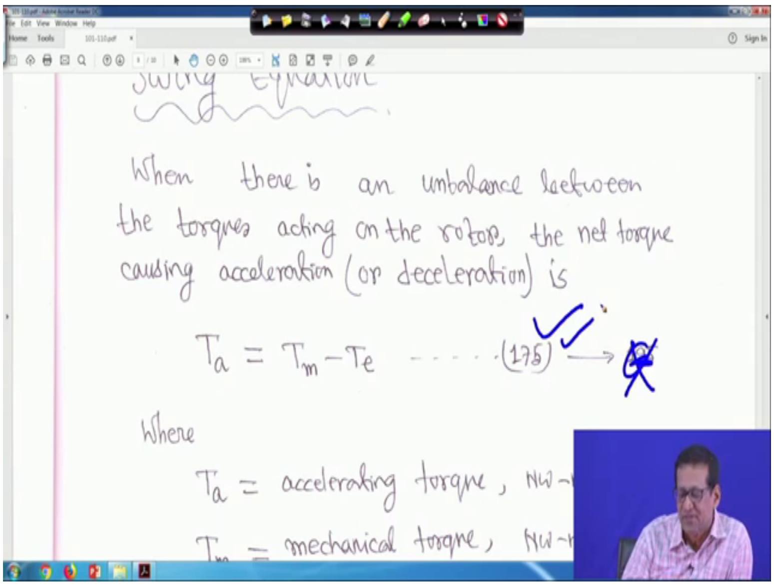

Next we will move to your swing equation right. So, because now slowly and slowly we will enter into the dynamics of the synchronous machine you are aware of this. This here what you call this swing equation, so here also we will see the same thing, but step by step, because we have to develop a your synchronous machine model in a I mean as per as it is possible as a class room exercise that kind of model slowly and slowly we have to step by step we will develop this. So, we will start from swing equation swing equation because that here what you call that most important for synchronous machine.

So, swing equation, so already you have studied in power system this single machine infinite bus the transient stability analysis there you have studied, in machine dynamics also little bit you have studied. So, let us see how actually things are. So, when there is an unbalanced between the torques acting on the rotor the net torque causing acceleration or deceleration that you know accelerating torque is equal to T a minus T e right.

So, this equation you should not see this number no need only you will follow this one right. So, this is not need this is for my own clarification. So, this equation is 175 that is T a is equal to T m minus T e, where T a is equal to accelerating torque that is in Newton meter and T m is mechanical torque that is also in Newton meter right.

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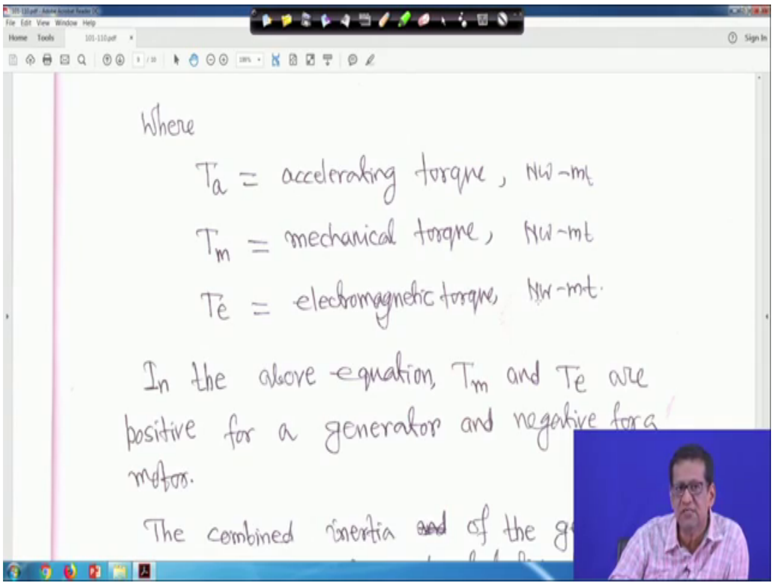


And T e is equal to electromagnetic torque that is also in Newton meter right.

In the above equation T m and T e are positive for a generator and negative for a motor i mean in the above the T m and T e they are positive for a generator and negative for a motor. That means, this just hold on this equation this equation the T a is equal to I mean they will be positive for generator and if its become negative then it will become T e minus t m for motor right.

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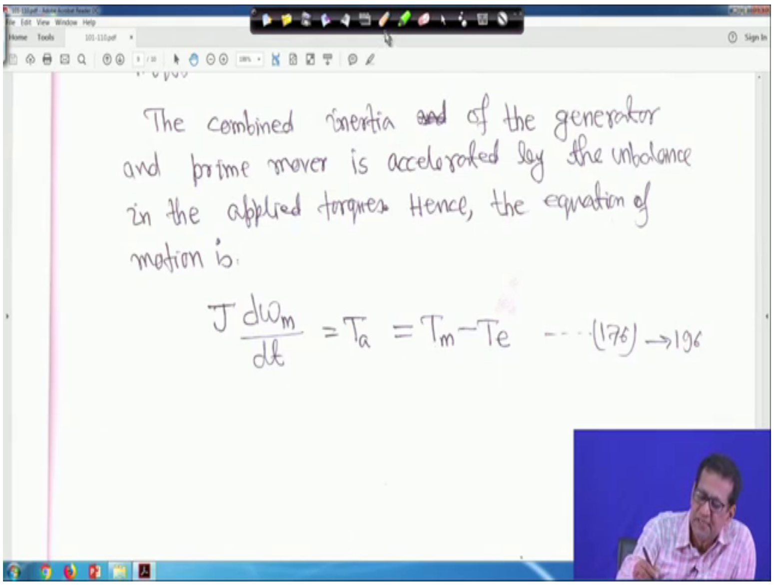


So, now the combine inertia of the generator and prime mover, prime mover means in power plant it is a turbine right. So, combine inertia of the generator and prime mover is accelerated by the unbalance in the applied torques. Hence, the equation of motion is given by J d omega m d t is equal to accelerating torque T a is equal to T m minus T e. This is equation 76 right. So, this one you should not read, this is equation 176 right.

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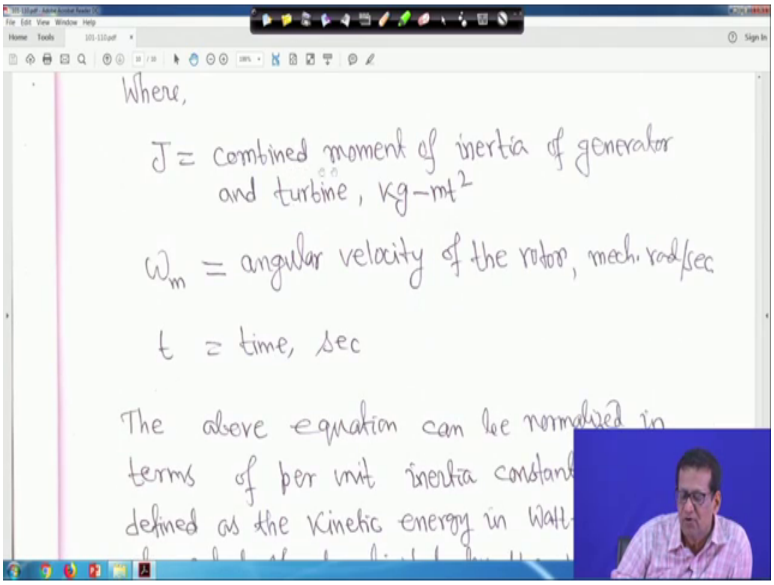


Now, where J is equal to combined moment of inertia of generator and turbine; that is in kg meter square right and omega m that is angular velocity of the rotor in mechanical radian per second and t is equal to time in second right. So, this equation you are familiar with this equation from your power system studies as well as machine stability studies.

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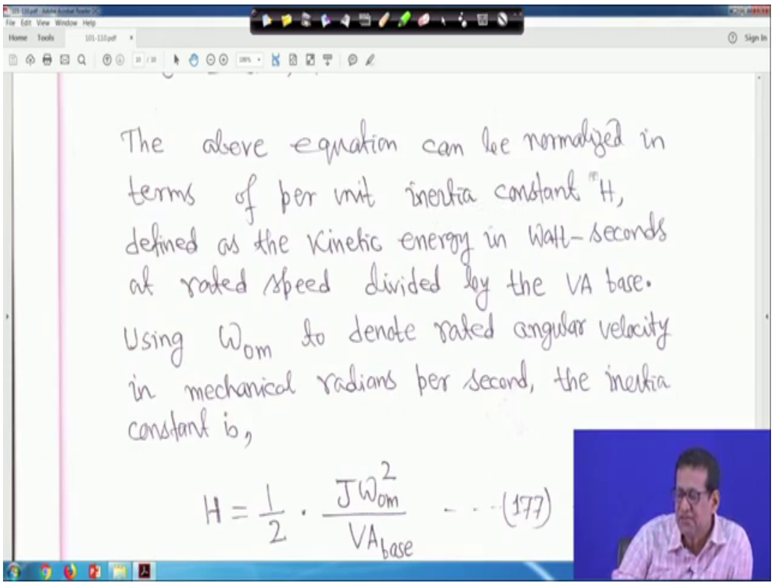


So, the above equation can be normalized in terms of per unit inertia constant H, that H also you have studied the inertia constant and its unit is second your rapids. So, define sorry define as the kinetic energy in watt second right watts, because that joule per second is equal to watt energy unit of energy is joule. So, joule is equal to watt second because joule per second is equal to watt.

So, energy in watt second at rated speed divided by the volt ampere base. Using omega 0 m to denote rated angular velocity in mechanical radians per second; the inertia constant is given by H is equal to your half into J omega 0 m square divided by volt ampere base right.

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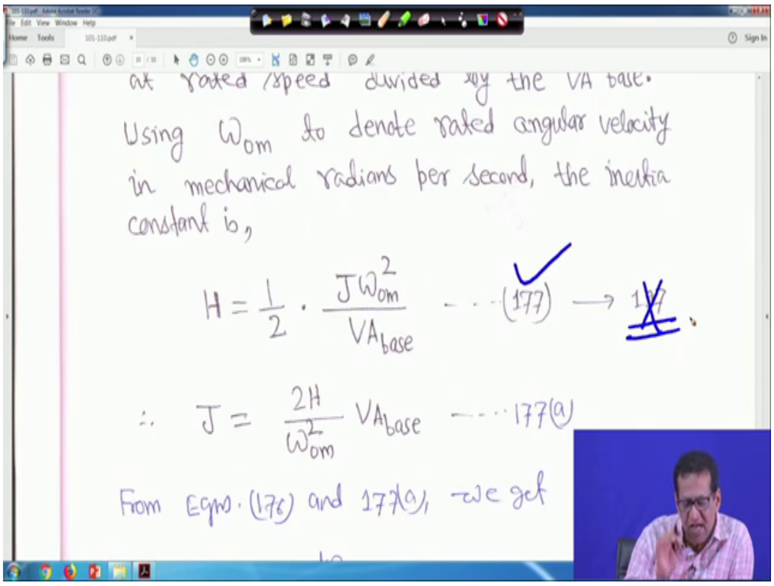


This equation this is equation 177, this you should not read.

So, H is equal to half J omega 0 m square upon volt ampere base, because half J omega 0 m square is that your what you call that you are aware of these the kinetic energy right So, that is why and divided by the volt ampere base. So, H is given like this right or just hold on or J is equal to 2 H from this equation only 2 h upon omega 0 m square into volt ampere base. So, this is equation 177 a right. So, H is defined as the kinetic energy in watt second at rated speed divided by the volt ampere base.

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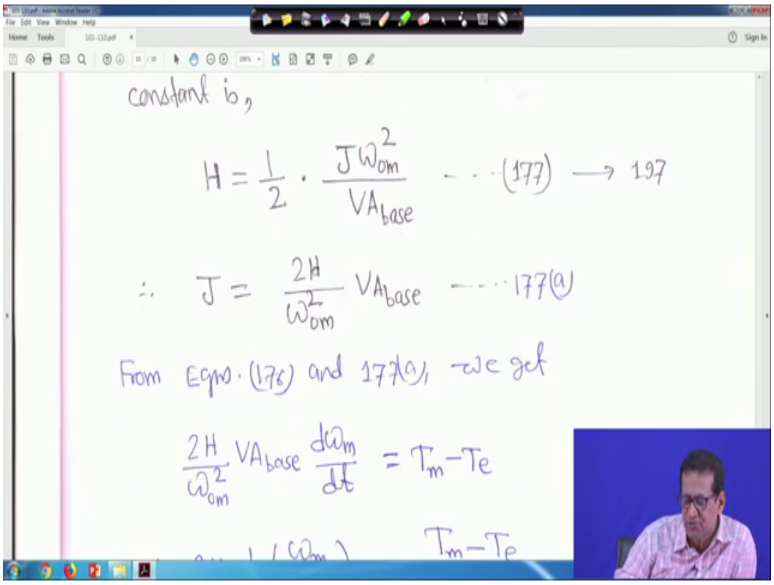


So, half J omega 0 m square is that kinetic energy and divided by volt ampere base.

That means, J is equal to 2 H upon omega 0 m square into VA base from this equation only this is equation 177 a right. So, from equation 176 and 177 a we get, I mean this J value you substitute here, j value you substitute here in 176, you substitute here.

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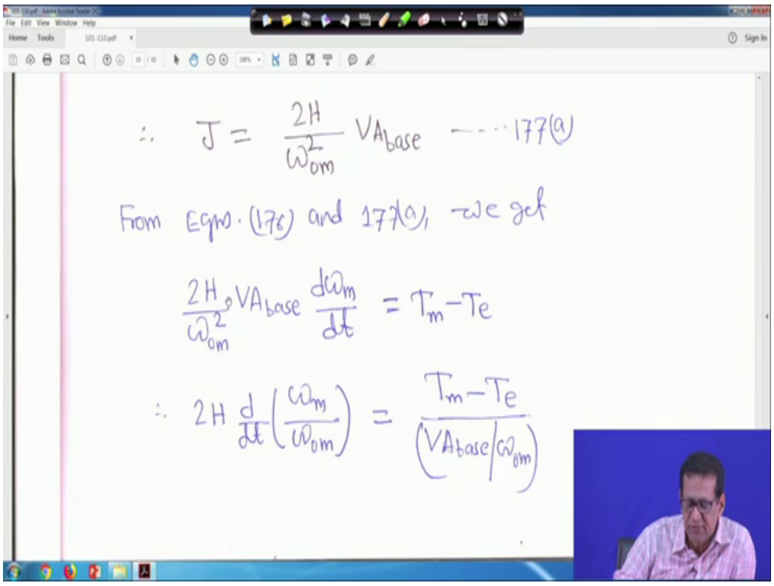


So, if you do so, you will get 2 H upon omega 0 m square volt ampere base into d omega m by d t is equal to T a minus T e right or you can write 2 H your 2 H ddt omega m upon omega 0 m slight at that this thing is equal to T a minus t upon VA base divided by omega 0 m.

What we are doing actually this 2 H is there then d d t omega 0 m square is there. So, what we are doing, we are diving 0 by omega 0 m another omega 0 m because, omega 0 m into omega 0 m this part we are writing T m minus T T e right. And this is VA base this VA base, we are doing a divided by omega 0 m right.

So, this is just a just to represent, this your what you call this a mathematical equation, just to make it in per unit values everything will be in per unit values.

Thank you very much. We will be back again.