Digital Electronic Circuits.
Prof. Goutam Saha
Department of E & EC Engineering
Indian Institute of Technology, Kharagpur

Lecture – 07
Fundamentals of Boolean Algebra

Hello everybody, in the last class we had a look at basic logic gates and we had seen representations of those logic gates or circuitry using Boolean algebra. So, we shall look more into the Fundamentals of the Boolean Algebra in this particular class.

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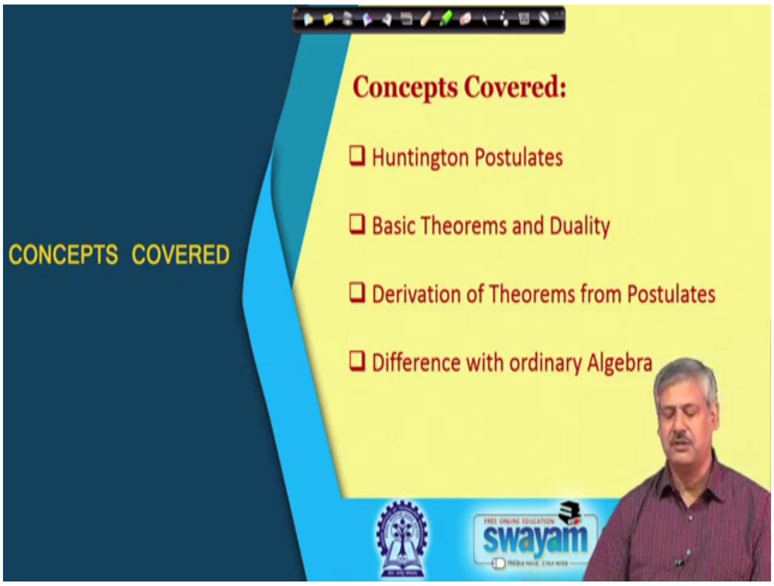

The concepts that we shall cover are Huntington postulates that that are the basics of this particular Boolean algebra. Basic theorems that are derived from it and the concept of duality we shall show some examples of these derivations of theorems and from the postulates and also we shall discuss it is difference with ordinary algebra.

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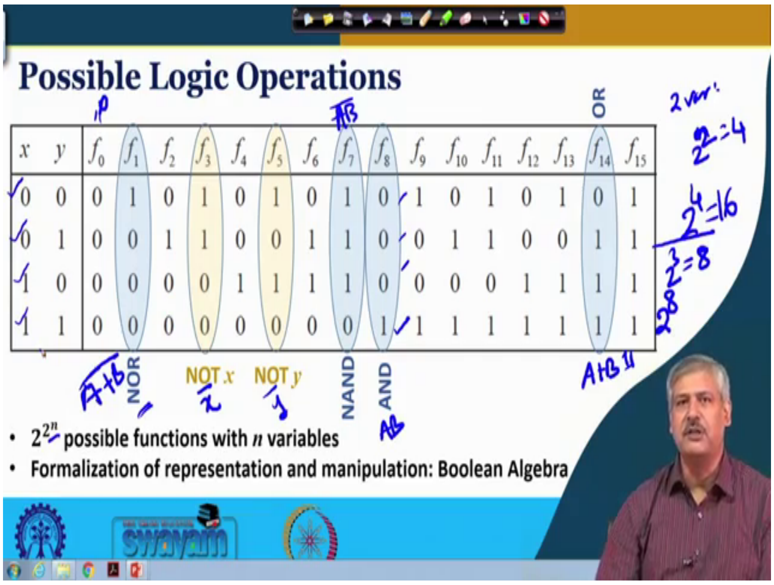

So, the in the previous classes we are looking at two variable examples two variable logic operations right. And if we look at the two variable representation so, with two variables we can have two variable we can have 2 to the power 2 2 that is 4 possible arrangements in the this truth table. So, this is 0 0 0 1 1 0 and 1 1 this 4 possible rows are there with 2 variables. And for each of this cases we can have many different way the functions Boolean functions can be represented right.

So, in one example say all of them the output could be 0, in 1 say output can be 1only for the case when 0 0 is 1 and rest of the cases it is 0, output can be 1 only for the case when input is 0 1 and rest of the cases it is 0. So, these are different possibilities and how many are possibilities are there since it is there are 4 so, 2 to the power 4 that is 16 such possibilities are there ok.

And with this among this 16 possibilities we are familiar with which operations? So, this operation we are familiar with NOR operation ok; that means, when 0 0 is the input and 1 is the output and rest of these things are 0. So, we know these operation as A plus B prime right and corresponding it is inverse is over here 0 1 1 right. So, we are familiar with these operation so, this is A plus B.

We are also familiar with this operation and when both of them are 1 output is 1 all right and rest of the cases output is 0 this is 0. So, we know that this to be AB this is next is NAND is just opposite of it we know this to be AB bar and some of this yellow marked one. So, you see that when x is 0 output is 1 irrespective of the value of y and x is 1 output is 0 irrespective of the value of y again this two cases. So, basically what you see here is x prime x complement similarly this is y compliment. So, that way all the different you know this combinations are possibilities each of the function can be represented this is 0 of course, and this is 1 ok, but rest of the cases we can you know have a corresponding Boolean expressions right.

So, this formalization of representation in manipulation is done by Boolean algebra and in this case we saw for you know two variables we had 16 such possibilities if there are 3 variables. So, how many rows will be there in the truth table 2 to the power 3 there will be 8 rows right and with 8 rows how many possible functions are there in this sight. So, for one such combination say a function is 0 that is all the outputs are 0 in another case only one case output is one rest of these cases are 0. So, that way 2 to the power 8 such possibilities will be there for 3 variable ok.

So, for any n variable cases will be having number of rows 2 to the power 2 n right here and with that you can have 2 to the power 2 to the power n possibilities of arrangement in the form of Boolean functions ok. So, this is we keep in mind and we understand the importance of their representation and manipulation and for which we shall have a look at Boolean algebra and more on that is we shall discuss late.

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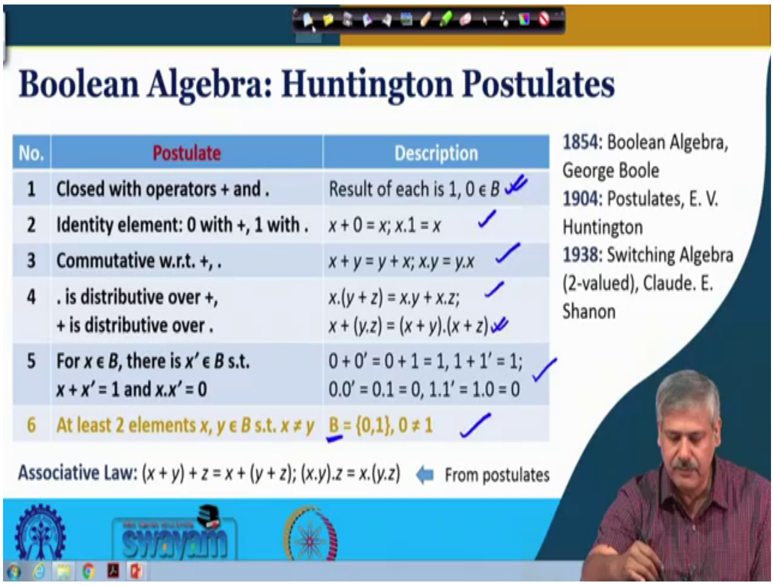

So, as I said Huntington postulates form the you know basic premise of this Boolean algebra. So, it has got 6 postulates and here we are talking about two valued Boolean algebra. So, it has got 2 elements and these 2 elements are 2 unique elements they are 0 1 this is the forming the set of B right and 0 of course, is not equal to 1. And postulate one says that it is closed with operators class that is OR and dot that is AND operator.

That means if you are having a operation with this two operators the result is unique and is a member of this set that is result will be 1 or 0 identity element. So, for or operation the identy identity element is 0; that means, x plus 0 or a plus 0 it is x or a ok. So, the it will be the variable itself and for AND operation the identity element is 1; that means, anded with 1 the it will be the variable itself. So, this is the second postulate commutative with respect to AND and OR operation all right. So, that is x plus y is equal to y plus x x y is equal to y x we have already seen this in you know the basic logic gate operation through truth table before.

Ah This is distributive over plus that is and is distributed over plus that is x ended with y plus z is x y plus x z x and y or x dot z ok. This is also we can all of this can be verified from the truth table and OR is also distributive over AND so; that means, x or y z is x or y and x or z ok. This is similar to what you see in ordinary algebra this does not look like you know that of an ordinary algebra expression this is one difference that we take note of more we shall take note later or ok. Again you can verify it by forming a three variable truth table and putting the values and then calculating the output corresponding output and you can see that that is what will arrive you will arrive at.

And finally, after this identity commutative distributive we come to complementarity postulate where it says that if x is an element of set B then there exist x complement which is such that x plus x prime is 1 and x x dot x prime x and or x prime is 0 ok. So, that is again one unique thing about a Boolean algebra unlike the ordinary algebra. And we already see the it is corresponding you know meaning significance so, 0 plus 0 prime ok. So, 0 prime is 1 it is complement is 1 0 plus 1 is 1 similarity 0 and 0 prime is 0 and it is 1 it is 0 ok. So, similarly will happen for 1 plus 1 prime case and 1 dot 1 prime case that will get 1 over here and 0 over there.

So, these are the basic postulates and we note that associative law that we are talked about that x plus y and anded oared with z separately and x order with y plus y order with z separately they are the same this is not part of basic postulate, but it can be arrived at from the postulates that we have discussed so far.

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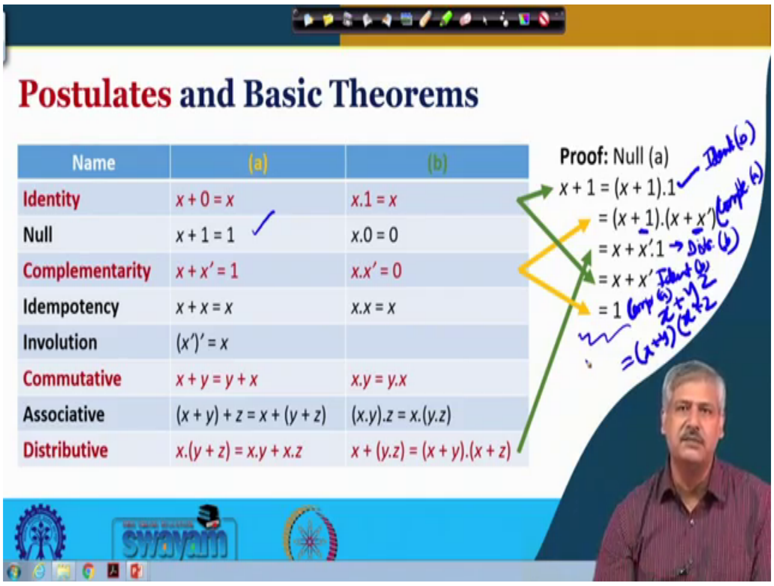

Now, with this postulates behind us we take note of in the form of a table some important identities which is very useful in case of this Boolean algebraic algebra based manipulation ok.

So, some of them are directly derived from the postulates some are represented as theorem which can be proved by the postulates. Postulates are the axioms self evident thing and theorems and other identities are actually the one that can be proved from the postulates. So, the that given different names which is useful in some sense later on you can refer to it when you are using a particular kind of postulate or theorem to prove one particular relationship or simplifying a certain relation certain algebraic expression.

So, identity you have already seen x plus 0 is x x and 1 is x null this is theorem null theorem so; that means, x if oared with 1 anything oared with one gets nullified I mean that has got no value. So, it will be the output will be 1.

So, x has no representation in the output and for the AND case it is x ended with 0 the output will be 0 ok. So, this is can be derived we can see later and complementarity x plus x prime is 1 x dot x prime is 0 coming directly from the postulates. So, the one that is in the brown are coming directly from the postulates and the rests are the theorems which can be derived from the postulate. So, idempotency Siemens it is remaining the same that is x plus x is equal to x x ended with x is also x involution. So, double prime double complement is this variable itself commutative we have already seen x plus y is same as y plus x and x y is y x ok.

Associative as I said it can be derived from this we have already noted distributive coming from the postulate which we have directly noted before. And as I said it can be the theorems can be developed from postulates more theorem more such theorems we shall see later in the next slide. So, for example, let us take this null a x plus 1 is equal to 1 ok. So, this is x plus 1 then it can be anded with 1 where from it is coming this is coming from the identity b this is coming from this one the arrow is there all right this what?

Then this one can be written as x plus x prime how can you write it from complementarity a ok. Then you can use distribute distributive property where x plus y z you are using x is there is equal to x plus y and x plus z. So, this is x this is your y and this is your z this is y and this is your z ok. So, y z getting multiplied over here and this is x according to this.

So, this x is there y z is multiplied x multiplied means ended so, x prime anded with 1 ok. So, then this is your by the this is your distributive b ok. And then again you use identity this identity b identity b you get x prime and x plus x prime is 1 from again your complimentarity complementarity a is it clear.

So, x plus 1 is equal to 1. So, that way all the this theorems can be proved and if it is asked you can write corresponding postulates or other basic theorem that you have used in the right hand side to explain the steps that you have taken to arrive at that particular proof ok.

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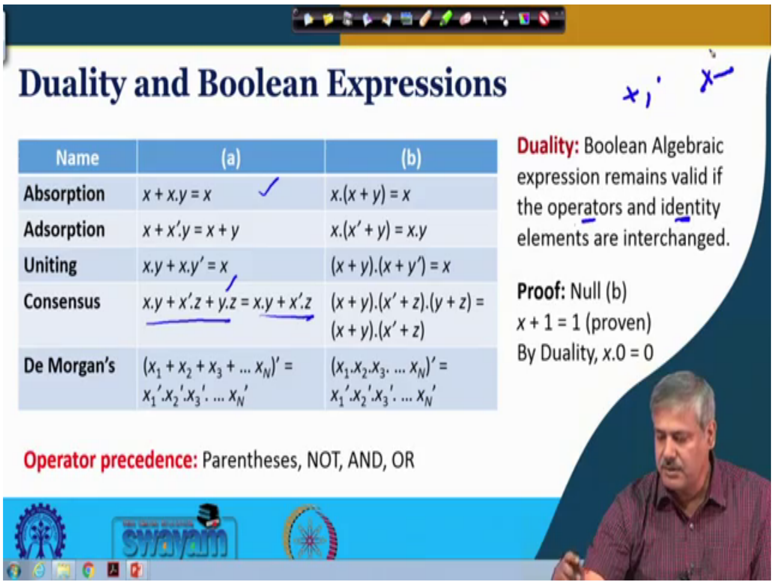

More theorems which is useful when it is called the absorption so, that is your x plus x y is x and corresponding form over there x anded with x plus y is also x adsorption x plus x prime y is x plus y x anded with all version of x prime plus y is x y uniting x y plus x y prime is x and x plus y anded with x plus y prime is x.

Consensus theorem it is interesting where you can see that there are 3 terms over there and there are only 2 terms over here. So, this term y z is not included in the right hand side. So, this is a consensus term; that means, what is explained by y z in a truth table is already explained by x y and x prime z for which it does not require another you know reference. So, that is why it can be simplified or it can be represented in this manner and De Morgan's theorem we have already seen and consensus theorem it is other version in the product of some version it is called product of some because you can see all the products are there and this is some version.

So, x plus y x anded with x prime or z anded with y prime y or z and it is y or z is not required in the final expression because this is a additional term which is already explain by other two terms. Now, in the previous case as well as in this case you see that there are 2 rows a and b one is mostly the one is representing in a SOP or sum of product form another is representing it in a product of sum forms.

So, basically the OR operation is done at the end and other case AND operation is done at the end ok. So, these two expressions if you just compare and if you look at another interesting theorem which is there for Boolean algebra all algebraic expressions remain valid if the operators and identity elements are interchanged ok.

Is a beautiful things about Boolean algebra; that means, operators and identity elements. So, what are the operators? Operators are AND AND OR and AND look at these are the two operators.

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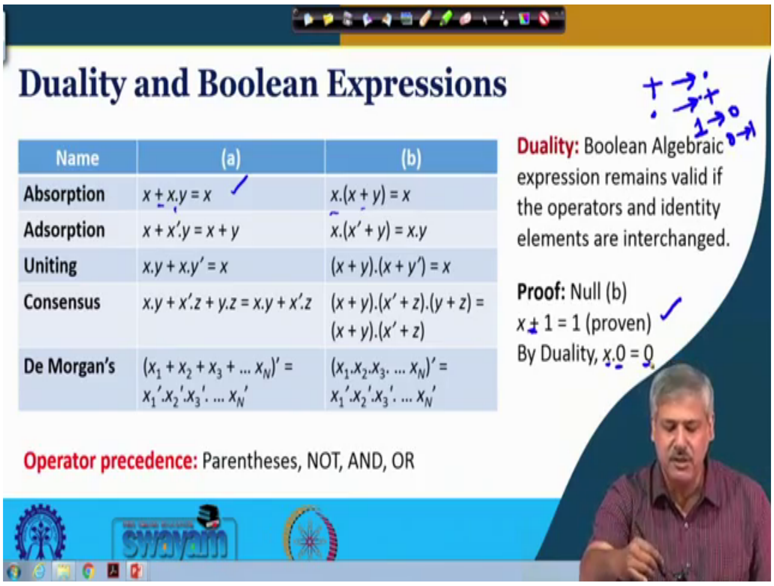

So, basically what you are telling by this that you convert OR to AND AND to OR identity elements 1 to 0 and 0 to 1 in a any algebraic expression right to get a equivalent I mean the identity will remain valid the resulting identity will remain valid ok. So, that is what this duality theorem tells and if you compare the left hand side and right hand side of the 2 tables that we have seen so, far it is just is talking about that ok.

I take this example say the 1st one over here all right. So, these OR is replaced with AND here AND is replaced with or and it is fine the whatever resulting expression identity you get it is also valid ok. So, earlier we have seen for null 1 x plus 1 is equal to 1 right we have already seen from basic postulates a valid in the scene.

So, now, what you can do if you apply duality. So, this x and changes to OR changes to AND over here 1 identity element 1 changes to 0 and 1 changes to 0. Will get another relationship that is x AND 0 with 0 AND with 0 results in 0 ok. So, this is also your that was there in the column b for the null. So, this also a very useful theorem which is which can used for simplification and other cases ok.

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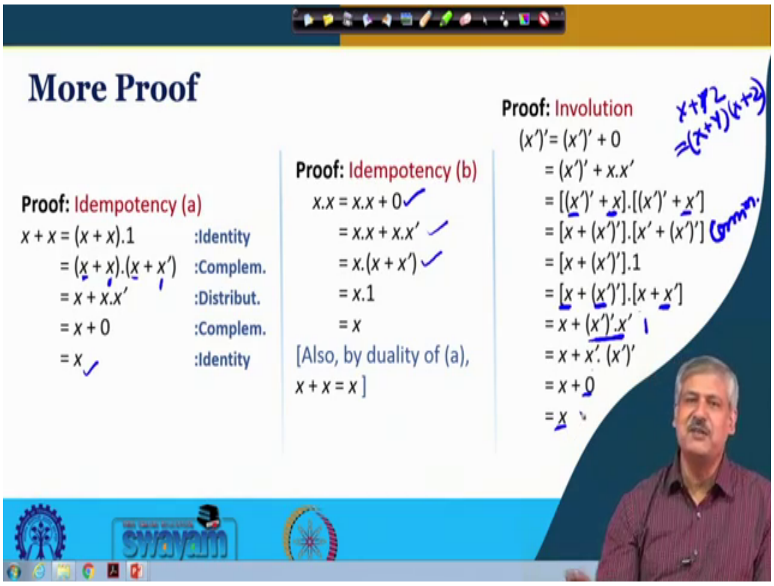

And to get you accustomed with the way you use these axioms or postulates and basic theorems we have few more you know prove that we can quickly discuss ok. So, idempotency that we started with that we can have see we can see how it can be used in this particular case right. So, this x plus x all right we would like to prove that is same as x all right. So, start with you and it with 1. So, that is the identity you postulate then one can be written as x plus x prime complementarity postulate all right and this is your x this is your x this is your you can consider this is y and this you can considered as z right then you can apply the distributive postulate distributive law and you can get x plus y is it y is your x over here and z over here.

So, x x prime you get and then x x prime is 0 from complementarity postulate all right and x plus 0 is x from identity postulate ok. So, the basic theorem related to idempotency can be arrived at by using the basic postulate ok. Similarly, idempotency b also you can arrive at starting with using the identity element then the complementarity ok. Then distributive the other version of the distributive one and the complementarity and then finally, the identity element we can get the all the final proof of this. And you can also get it from duality also that and is replaced with or there is no identity element. So, it remain same x and x is x clear.

So, similarly you can look at the involution the proof of involution that is double prime is the function itself ok. So, in this case again you start with the plus 0 that is identity element identity postulate. So, compliment that is 0 can be written as x x prime ok. So, now, this is the distributive law in the other direction. So, from x plus y z you are going to x plus y and x plus z ok. So, this is your x this is your y and this is your z. So, x this is your y corresponding y and this is your corresponding z ok. So, up to remember I mean you can sometimes you need to expand for which also you can use the distributive law from right hand side you go to left hand side ok.

Ah So, after that what you are doing now we just changing. So, basically it is commutative for which you can change. So, x come this side and x prime goes to the other side and similarly over here all right. So, when you do that you can use the complementarity a plus a prime what x plus x prime is equal to 1 ok. So, after doing that this one can be written just to you know help the help progressing with the proof one can be written as x plus x prime and then you can apply again distributive law over here. So, this is your x this is your y and this is your z.

So, x is there y is it comes over here ok. So, with that you get this complimentarity postulate you get 0 over here and x plus 0 with identity postulate you get back one x ok. So, using the basic postulates you can get all the theorems that we have discussed.

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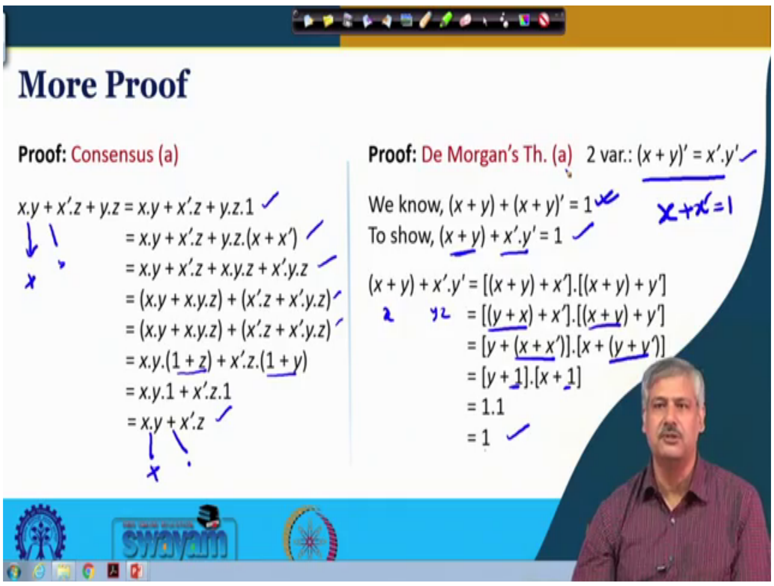

So, we shall look at just 2 more proofs the last two works so, the consensus theorem all right. So, in the consensus theorem we have left hand side we have this term so, y z we use the identity postulate. So, we include 1 then complementarity x plus x prime we put in this place then we just expand it and then we use the associativity ok. So, associativity is not part of basic postulate, but as I said the basic theorem can be also used for proving a more complex expression.

So, this is the way you are doing it all right and then if you just use take x y out then 1 plus z comes over here 1 plus y comes over here. So, again 1 plus z is 1 that is again a basic theorem we have derived from postulates ok. So, so, this is sorry this is your null postulate from that you will getting 1 this is also getting 1. And finally, you will arrived at a this particular version which is a simplified version of this. So, this is your consensus theorem on version another version you get similarly or through duality through duality just you have to change this AND to OR and this to AND and similarly here it will change to OR and this will change to AND.

So, basically if you just do it that way for all the cases we will get the corresponding version for the consensus theorem. And proof of De Morgan's theorem it is done again this is what you have seen in the previous case that you can use basic theorems also along with postulates to derive the identity and or the proof. And in the De Morgan's theorem we do it in an indirect way we can we can see that how you can be done in an indirect manner. So, the what we have to proof is x plus y prime is x prime anded with y prime ok. So, we already know that from the complementarity the basic complementarity x plus x prime is equal to 1.

So, x plus y as the whole you can consider as x. So, this is already known right and if we can prove that x plus y and x prime dot and with y prime this one that we seek to prove which to prove is equal to 1. Then we have or we can rest our case this is in your indirect way of doing it, but this is also can be used depending on the need ok. So, this x plus y and x prime dot y prime. So, basically we start with that 1 and again you use the distributive law. So, this is your x and this is your y z kind of thing. So, x and this is x plus y and x plus z this is the distributive law that your apply and then you are just it is commutative so, basically x plus y we can write y plus x and this remains the same.

Then you can apply associativity so, this is your associativity. So, basically instead of this grouping you are doing this grouping instead of this grouping you are doing this grouping all right and x plus x prime complimentarity is 1 and y plus y prime complementarity is 1 ok. So, you can then you can look at the identity sorry null relationship ok. So, this is 1 and this also one together you get one ok. So, now, we can see that this is true so, the other 1 is also true.

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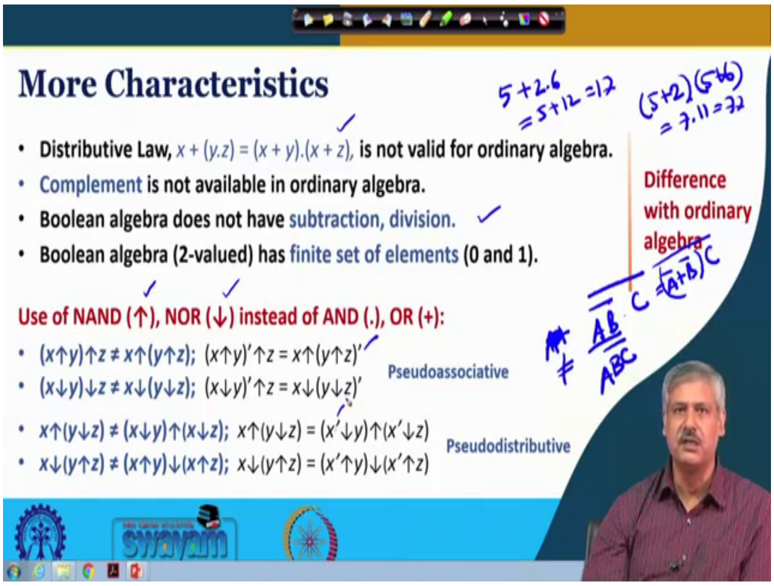

So, two and we shall just this part of the discussion we shall just look at some of the characteristics. So, what we have noted before that the Boolean algebra as certain differences with ordinary algebra this distributive law the way we have seen it and utilized in many different cases this is not valid for ordinary algebra. So, if you have got so, I like five plus 2 into 6 kind of thing.

So, 5 plus 12 is equal to 17 and here if it is five plus 2 multiplied by 5 plus 6 ok. So, you will get what 7 into 11 ok. So, that is not that is 77 so, it is quite different ok. So, this not valid for ordinary algebra complement is not available in ordinary algebra that we already know it does not have inverse is for addition and multiplication like subtraction and division this kind of inverses are not there in Boolean algebra which is there in ordinary algebra.

And Boolean algebra is this 2 valued Boolean algebra the one that we have seen has got only finite set of elements 0 and 1 for ordinary algebra that is infinite set of elements certain 0 1 2 3 4 the number can extend to any high value. And if you if somebody things that instead of AND and OR operation if we can develop a Boolean algebra using NAND and NOR it is not possible because the distributive law and some of these basic things postulates are not valid using NAND and NOR.

And earlier we had seen that we discuss that they are commutative, but they are not associative so, we can just the quick you know we now know how to use it. So, if you are talking about say a NAND operation and then see this one and if we try to you know check it with ABC and then NAND we shall see that they are not this. So, we can expand it using De Morgan's theorem and you can see that it will give you A bar plus B bar dot C. And then this bar and similarly over here if you just look at if you just expand it you will see the the both side do not match.

In fact, what will match is the relationship like this here we have use this operator for NAND and this operator for now for better understanding ok. And similarly for distributive I do not follow distributive law over each other NAND is not distributive over NOR or vice versa NOR is not distributive over NAND.

But, if you check the relationship then you will see that again using De Morgan's theorem if you just expanded and put it together instead of this x NAND y NOR z the way put it in parenthesis it is x prime NOR with y which y prime x prime NOR with z which is when there is a NAND operation between these two then we shall get whatever it is there in the left NAND side.

So, this prime operation the compliment operation comes here as well as here as well as here. So, that is why it is called pseudo associative or pseudo distributive ok. So, and we cannot use it in place of AND and OR operator in Boolean algebra.

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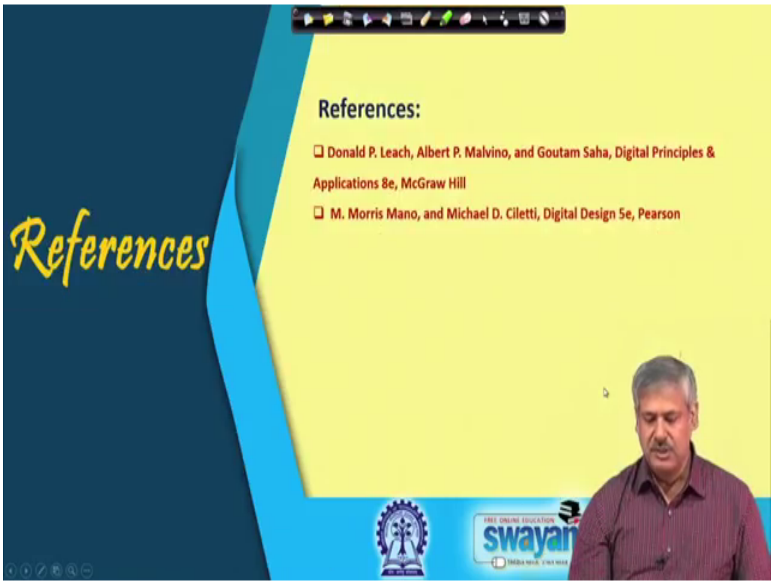

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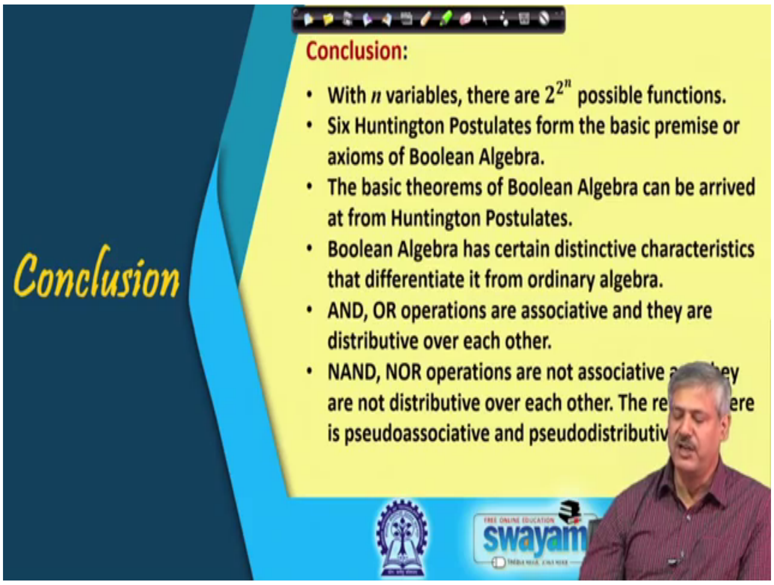

So, with this we conclude key points in variables we can have 2 to the power 2 to the power 2 n possible functions 6 Huntington postulates form the basic premise or axioms of Boolean algebra. And the basic theorems of Boolean algebra can be arrived at from Boolean standing. And postulates and this basic theorems and postulates can be used to simplify or proof different identities which is more complex in nature and we can refer to those basic theorems and postulates.

And it has got to Boolean algebra has got certain distinctive characteristics that differentiate it from ordinary algebra AND and OR operations are associative as well as distributive over each other ah, but NAND and NOR operations are not.

Thank you.