Digital Electronic Circuits
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Lecture – 22
Negative Number and 2's Complement Arithmetic

Hello everybody. In this particular class, we shall look into representation of Negative Number and 2's Complement Arithmetic. So, we had got introduced to number system in the previous class, but in that we did not discuss how to represent negative numbers. So, we shall cover these concepts; how to represent is through sign magnitude representation; 1's complement, 2's complement; how to do binary addition and subtraction and how to perform 2's complement arithmetic.

So, Sign magnitude representation is very simple ok. So, we have a sign bit specially a you know placed which is towards the left hand left most side considered that is MSB Most Significant Bit and rest is the magnitude ok.

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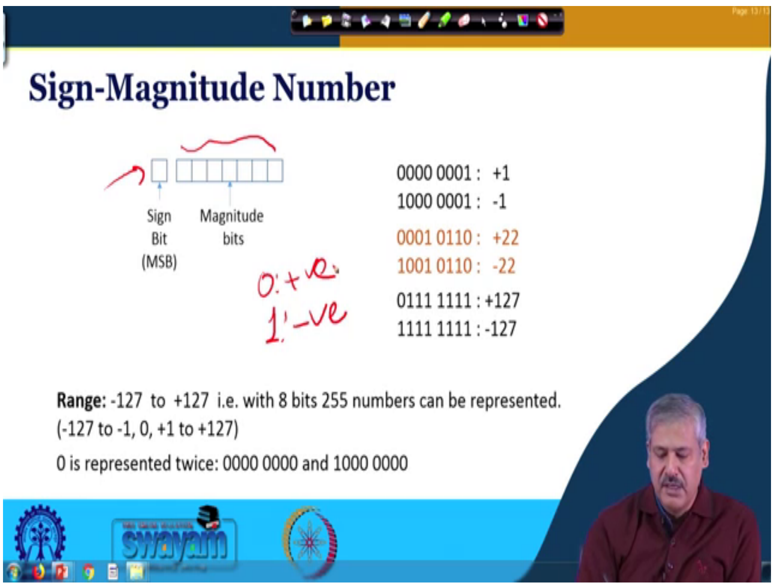

So, any number the magnitude part will be represented here right and the sign part will be represented here. So, sign if it is 0, we consider it is to positive; 0 is taken as positive and 1 is taken as negative ok. So, this is the convention. So, in these if we have got so, 8 bits to represent a number and we are considering say integers only and production can be taken up later so, this 0 0 0 0 0 0 0 1 ok. This 0, this particular bit is the sign bit. So, this 0 means, so this is positive and rest 0 0 0 0 or and all you know all these are 0 and this is 1.

So, in the from the previous class discussion we know that this gives a magnitude in binary which is 1 only because it is unit plus 2 to the power 0 multiplied by the coefficient 1 and rest of the cases the coefficient is 0. So, it is only 1. So, this is plus 1 ok. So, how will you represent a minus 1 in this convention, in this representation? So, the magnitude part will remain same; only the sign bit changes from 0 to 1 ok. So, if we have 1 0 0 0 and 0 0 1 like this 1 six 0's and 1, then we know that it is minus 1. So, this 1 over here is not associated with the place value that 2 to the power something that we have seen before ok.

So, it is not 2 to the power 7 right. So, this is only a sign bit right. So, similarly if we considered say this particular number, how do you what is this number. So, 0 is here. So, 0 is positive this particular sign bit. So, this is 1 so, 16 16. This is fourth place, 420 and this is 2's place so, 2; 16 16 plus 4 plus 2. So, 22. So, this is plus 22. So, other bits remaining the same; 0 becomes 1. This is minus 22. In the last class, we are seen if all these are 1 right. This is 127. You can also calculate you know like (Refer Time: 03:41) etcetera etcetera right. So, this is 0 means plus 20 plus 127. This is 1 means minus 127 ok. So, in this representation with 8 bits; what we can represent? We can represent from minus 27 to plus 127 right minus 1 0 etcetera etcetera.

So, that makes actually 2 into 127, 254 and 0. So, 254 number getting the presented by 8 bits and in this 0 is represented twice like one here, another there ok. So, one representation is mixed here. We mixed with 8 bits you can have 2 to the power 8, 256 representation. Here only 255 numbers get represented.

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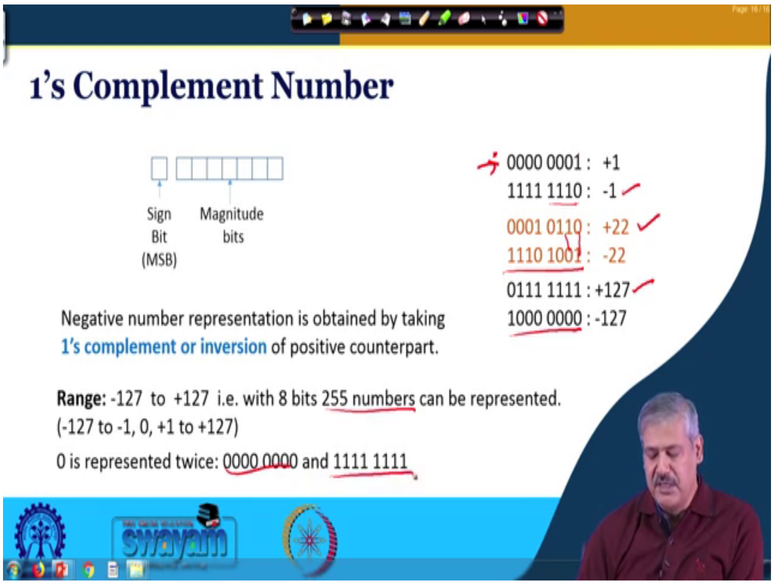

So, this is one way of representing it. Another representation is for negative number is called 1's complement number ok. So, in 1's complement, what do you do? We just take a inversion of the positive number. I mean you take a complement of the positive number. So, this is the positive number. So, first bit is the sign bit right. So, 0 0 0 0 0 0 0 1 plus 1 that is the way we have seen it before. So, then you just a inversion of it. So, 0 become 1; 0 become 1 right and this 1 becomes 0 over here. So, 1 1 1 1 and 1 1 1 0 this is the minus 1. Plus 22, we have seen before.

So, if you just take inversion of its. So, wherever 0 is there, it is 1 1 it is 0. So, this is the corresponding representation this is minus 22 ok. So, plus 127 we are seen. So, minus 127 is this representation. So, this is called 1's complement representation. So, in these also in this also we have got minus 127 to plus 127, 0 included. So, 255 numbers represented and 0 is represented twice; one by this, another by this.

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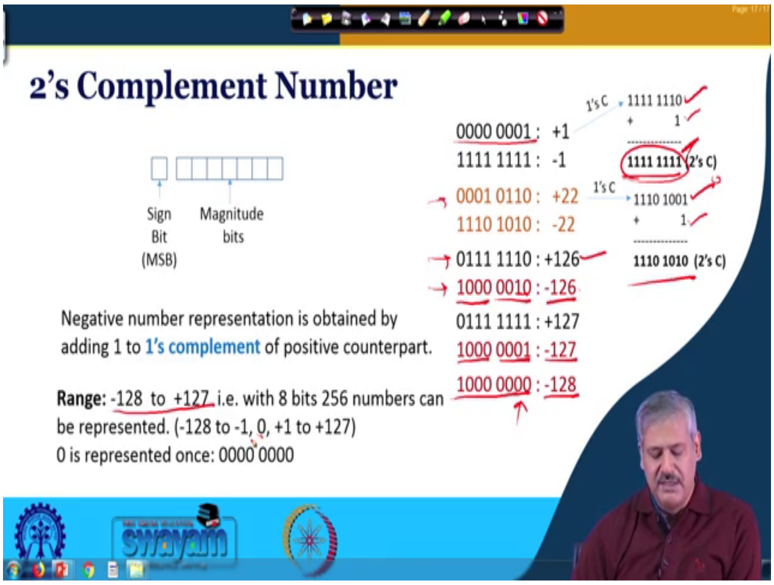

Next comes 2's complement representation. So, in 2's complement representation, the negative number is obtained by adding once 1 to the 1's complement. So, 1's compliment, first we are having we know how to get and after that you add 1 will get 1's 2's complement representation. So, for example, plus 1; this is plus 1 we have already noted. So, 1's complement we have seen before, add to it one. So, this is 2's complement. All 1's, all eight 1's this is 2's complement of representation of 1 that is minus 1. So, earlier all this 1 in 1's complement, it was another version of 0. So, this is not 0. So, this is different ok. Here all eight 0's are only 0; all eight 1's are not considered as 0 right.

Similarly, the same example of plus 22, if we see for minus 22 in 2's complement representation; 1's complement is this one, add to that add to that one. So, we get 2's complement representation of minus 22 right. So, that way we can continue and we have if you look at say plus 1 126, this is the representation minus 126 means you invert all the bits and then add 1, this is what you get for minus 126. 1 four 0 0 0 1 0 is minus 126.

1 0 0 0 0 0 instead of 0 0 1 0 0 0 0 1 is minus 127 and another representation that comes which comes within the range which was earlier outside the range 1 0 0 1 0 0 right, rest of the bits remaining same that is your minus 128 ok because now, 0 is not represented twice. 0 is represented not by this one, only by all 4 eight 0's. So, we could push one number over here right. So, that is the 2's complement representation of 128 that is

minus 128. So, the range here becomes minus 128 to plus 127, within that there is a 0. So, 256 numbers can be represented by this.

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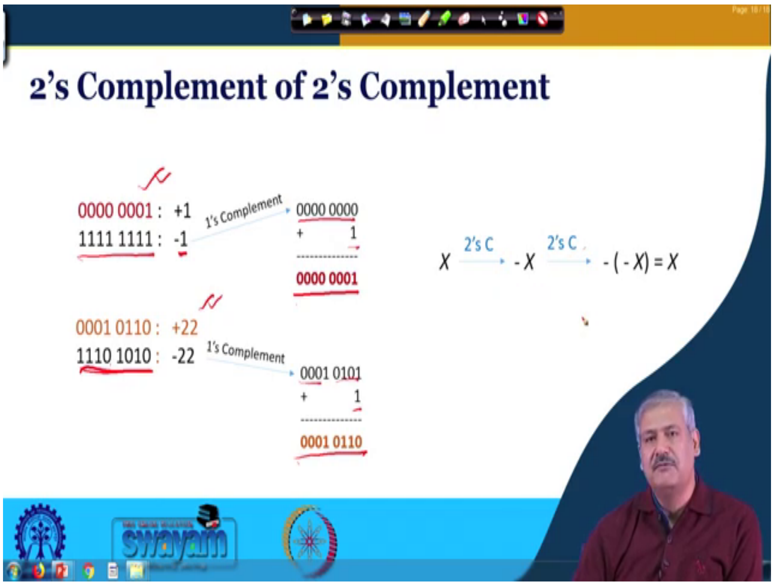

Now, if I 1's complement number, if you take another 1's Compliment, we get back the original number, is not it? So, that we have already we can understand. What happens if we have a 2's complement number that is a negative number. If you take again 2's complement of it; so, let us look at the example. So, this is minus 1; all eight 1's. If you take 1's complement of this we get all 0's and then, plus 1, it makes the 2's complement. So, this is 0 0 0 0 0 0 0 1 right. So, that is what? That is your original positive number plus 1.

So, 2's complement of minus 1, I mean the decimal version of the minus 1 right, we get decimal version of the plus 1; so, this was minus 22 all right in 2's complement and if we a look at its 1 1's complement right. So, this is 0 0 0 1 all right and 0 1 0 1 add to that 1 0 0 0 1 0 1 1 0, what is that? That is your plus 22 right so, 2's complement. Again, you do 2's compliment, you get back the original number right. So, minus again another minus so, you get the original number back. So, this we take note of.

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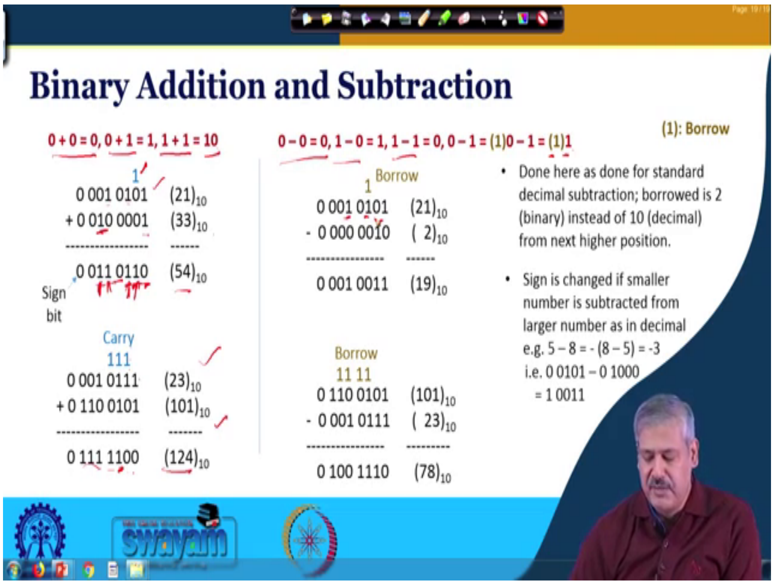

Now, let us consider how we can do Binary Addition and Subtraction ok, in the binary system. So, similar to decimal number addition and subtraction do you usually do, we can follow the same thing over here. So, first we look at the addition part. So, 0 plus 0 is 0; 0 plus 1 is 1 right and 1 plus 0 is also 1 and when we add 1 and 1 right, it will become 2; 2 in binary is 1 0 right.

So, this is called carry also you know, this is the 1 units place and this is 2's place ok, carry is there right. So, when we add say 21 with 33; just you know 21 in decimal 33 in decimal right. So, 21 we represent in this manner; 16 4 and 1 ok. This is the place these 1's are there, 33 right. So, this is 32 2 to the power 5 and this is 1; so, 33 right. So, when you add them up 1 plus 1 is 1 0. So, this 0 comes here, 1 is the carry. So, 1 plus 0 plus 0 this is 1. This is 1 over here, then 0 comes here. This is 1 and this is 1.

So, 1 0 0 1 1 0 1 1 0 so, this is the number that we get right. So, this is your fifth 2 to the power 32 plus this is 16; that is 48; this is 4; that is 52 and this is 2; that is 2; 32 plus 254. So, you get this is the decimal equivalent is 54 and you can see if you do add additional you get 54 right. Similarly, you can do it for 23 and 101. As long as the result is within the range, we do not I mean we can go on doing this, when it goes out of range we have to we cannot get the right result, valid result ok. So, this example you can also see that 1 plus 1 is 0. So, carry is 1. So, carry 1 1 again 0 and another carry is there so, three 1's.

So, three 1’s means 1 1 is the result. So, 1 is here and carry is 1 and this 1 0 0 this is 1, then rest are 1 1 1.

And again, if you convert to this to binary decimal you get 124 ok and for subtraction. So, 0 to 0, it is 0. 1 to 0, it is 1. 1 to 1, it is 0 and when you subtract 0 to 1 right. Then, you have to Borrow a 1 from the previous stage previous stage. So, that will make it 2. So, 2 minus 1 that we will make it the answer will be 1, but there is a Borrow 1 which need to be subtracted from the 2’s place, I mean the next higher rated place that need to be subtracted that we need to take note of that is concept of Borrow ok. So, you look at one example. So, 21 minus 2 right so, 16 4 and 1. So, 21, we have seen it before and this is 1 0 2 ok. So, how do you do it? So, 1 minus 0, 1.

So, it is 1, then 0 2 1 ok. You take a Borrow ok. So, when you Borrow, this is 1 0 become becomes 2. So, 1 0 that is 2 minus 1 is 1. So, there is a Borrow; this Borrow is to be subtracted from the next higher bit over here. So, this 1 is subtracted from 1 you will get 0 right. So, 0 0 and this is 1. So, this is 16. This is 2 and this is 1. 16 plus 2 plus 1, 19 so, you can see 19. Similarly you can look at other example right and this similar to decimal addition and subtraction.

And when the sign I mean when a smaller larger number is subtracted from the smaller number ok, as is the case of say an example 5 minus 8, what we do in the normal decimal subtraction? We subtract from the we put a minus sign and subtract from the larger number ok. So, basically minus of 8 minus 5 right; so, this number comes as 3 and this is minus 3 right. So, similarly in this case also if you required, you can do it in this manner right. So, you just see that this is the larger number, then you subtract it see this is larger number. So, sign is coming here; minus sign is coming here because this is the sign bit.

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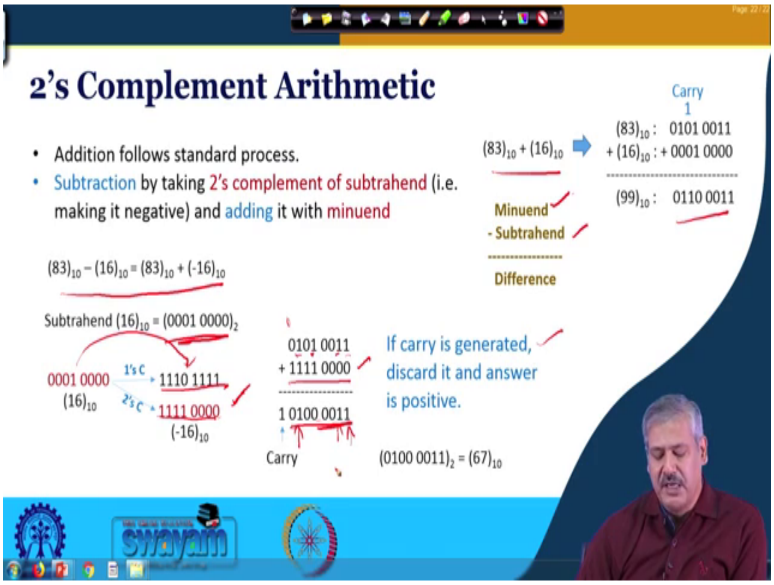

Now, in this digital electronics or in the binary system, when we perform arithmetic and we have a circuits building blocks using gates and all logic gates, then we prefer 2's complement Arithmetic for some in urgent advantages ok. So, we shall discuss 2's complement arithmetic now. 2's complement number representation, we have already seen. So, in this addition is normal standard process and in subtraction when you have to do this subtraction part what we do? We get 2's complement of the original number to make it negative. So, subtraction is a number that is getting subtracted and then, we add these 2 numbers. So, basically subtraction here also becomes a addition process right.

So, a addle block a adder circuit can perform both the job of addition as well as subtraction right. For that when we are talking about subtraction, we have to represent the number in 2's complement form ok. To get make a negative number out of it. So, this is the basic concept. This is the basic concept and how it gets done? So, this is the minuend and this is the subtrahend. So, subtrahend we are taking the 2's complement ok. And then, we are getting the difference.

And addition is simple. Addition is normal process, the way we do it say example is 83 plus 16, we can just you have done it before; you can do it the same manner. We shall see that 99 is obtained. Now, let us look at how 83 is 16 is subtracted from 83 using 2's complement ok. So, 83, 16 here when we talk about this number to associate the value of this number, but decimal systems; what actually you will be performing the operation using binary. So, first of all these 16 is Subtrahend. So, Subtrahend we have to to convert it to negative number by 2’s complement right.

So, this is the number in 16 when it is positive. So, 16 plus 16; plus 16 its 1’s complement is this one right. Only this 1 place become 0 right; rest are 1 and then you add 1 to it; it becomes 2’s complement. So, this is your minus 16. So, this is the first step. So, then this number 83 right so, 83 is this 1. 16 plus 64 plus 16 that is 82 and 1.

So, this is 83 ok. You look at the weights and then you add. So, then added eight to it this minus 16; four 1’s and 0’s right and if you add it, the normal addition operation. So, 1 1 this is 0 0; then 1 1 0. This is 1 1 0. This is 1 1 1. This is 1; there is a 1 carry over here. So, this carry 1 1 0 and this is a final carry 1 is what you get and if carry generated, discard it and the answer that you get is positive. So, the result that you get from this operation is to be finally, understood by this manner; if there is a carry generator, discard it right.

And rest of the number is to be taken as the final answer which is positive. So, if you look at it this is 64 right and this is 2 and this is one 63 ok. So, that is what you get right. This is 64 2 66 and 67 sorry this part ok. Is this clear? Right 67 in decimal now you have to look at other combinations. So, for example, subtracting larger number from smaller number and other things we shall look into that aspect now.

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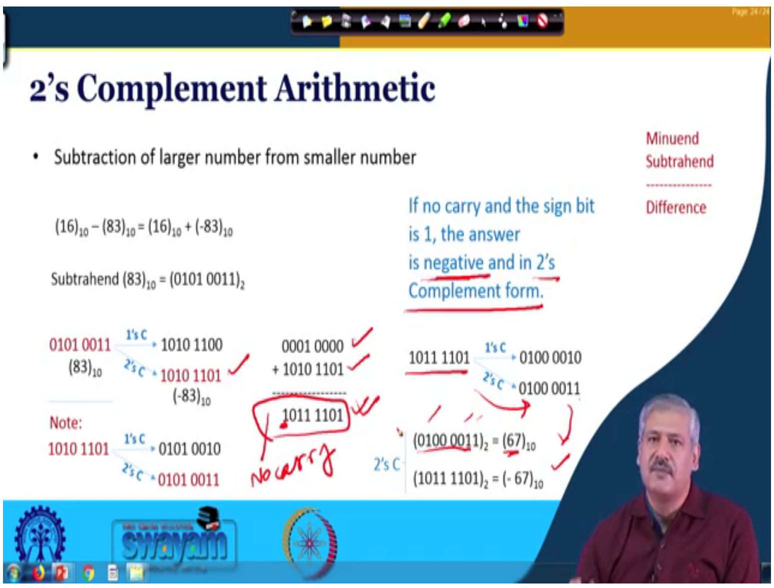

So, 16; early it was 83 minus 16; now it is 16 minus 83. So, for that what we will do? 16 will go as 16 like this; 83 you have to get minus 83 that is the Subtrahend now. So, we have to get minus 83. So, minus 83 you can get by 2's compliment. Again we perform the 2's complement you get this 1 right and then, this 83 we this minus 83, we add and this is what we get and interestingly in this case you see that carry is not generated ok. So, no carry ok.

So, no carry is generated right and the sign bit that you see here is say 1. So, we know that the answer that we have got is negative and in 2's complement form. So, this answer is of course, because of the presence of 1, it is negative and entire thing is 2's complement form. So, if you want to know its magnitude 2's complement. If you not very conversant, then we have to do 2's complement of this number only to get the corresponding positive part of it the magnitude part of it.

So, this is what you get here 1 0 1 1 1 1 0 1 if you perform 2's complement of it right, then you get 0 1 0 0 0 0 1 1. So, this is 0 0 1 0 0 0 1 1. So, this is 64 2 and 1 so, 67. So, that gives you the magnitude part, but you already know the answer is negative ok. So, when you are doing the binary to decimal conversion right, then you convert it back to the normal number representation right; do not use the weights on the 2's complement, the negative number.

And you know the numbers are negative number is negative ok. So, carry is not there. Sign bit is 1. The answer is negative and magnitude can be obtained by again for performing 2's complement. So, 16 minus 16 minus 83 we get minus 67 right. Result we shall get in this form. We know the answer is negative, no carry. Sign bit is 1 and to get the magnitude, we performance other 2's complement.

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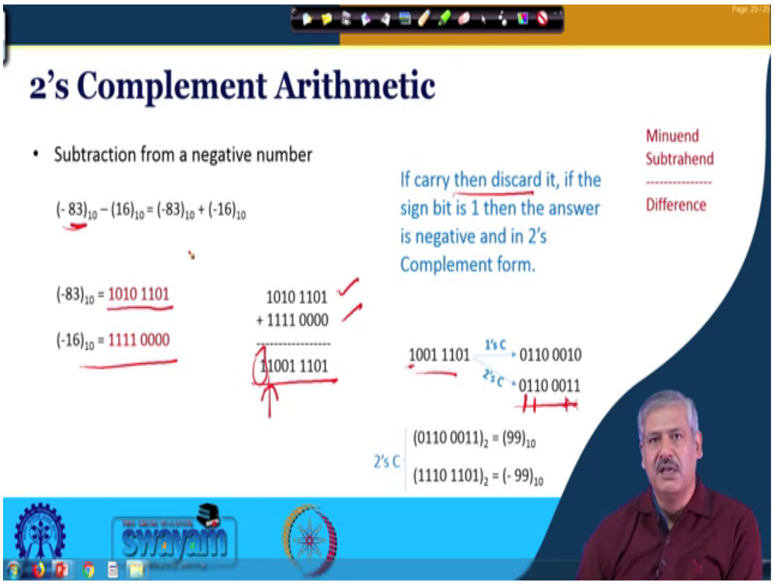

Now, if we want to perform subtraction from a negative number that is also possible that is also possible right. So, how we can do that? So, the negative number in the first place, it is 83 ok. Same example we are continue with different you know combinations. So, minus 83 we have already found to be 1 0 1 0 1 1 0 1 in the previous example.

So, this is minus 83 and 16 is subtracted ok. So, basically it is minus 83 plus of minus 16. So, minus 16, we have already calculated minus 16 before. So, this is your minus 16; you add them up. So, this is what you get here right. So, we have got a carry right, we discard this carry and if we see the sign bit here is 1 right. We know the answer is negative and when the answer is negative, it is in 2's complement form.

And do you want if you want to know the magnitude of the value, I mean you need to convert from binary to decimal; then we have take to another 2's complement of it to find the magnitude. So, 1 0 0 1 1 1 0 1 if you take 2's complement of it you get 0 1 0 0 0 0 1 1 right. So, this is your 64. This is your 16..

This is your 32 rather. So, this 2 to the power 5 32 64 plus 32 is 96, this is 298 and this is 199 ok. So, 99 in decimal and the number is negative because sign bit is negative right. So, this is the 1; whenever you are adding you know positive with positive and here effective bit is negative with negative, you have to be you have to care be careful that it doesn't go out of the range ok. So, if it is positive and negative. And negative and positive, it is always within the range right. So, this is something that we take note of.

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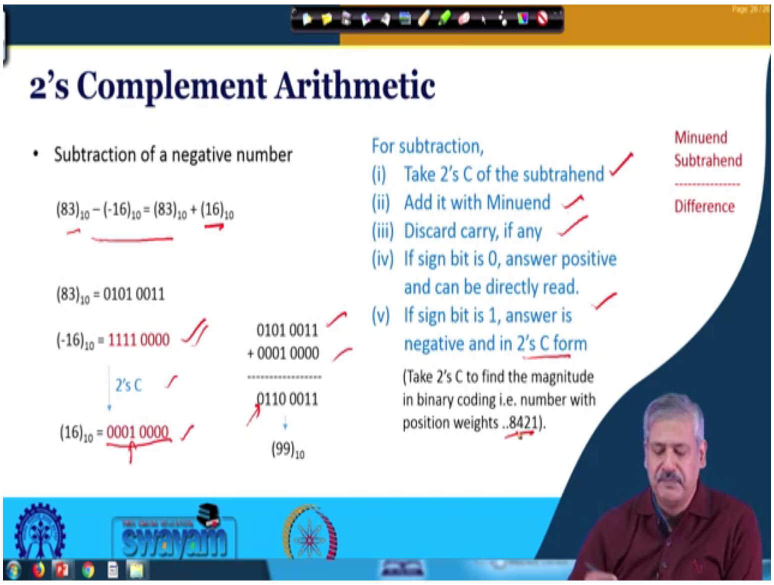

And if you want to subtract a negative number; so, negative number subtraction is what? I mean it is basically addition of 2 positive numbers. So, this is minus 16 to first place this is. So, this is your minus 16 right. So, since it is a subtrahend, it is there in a subtrahend we take twos complement of it then what will happen we will get back the original number original positive number. So, that is your 16 ok. So, this is 16th place 2 to the power 4 right. So, this 16 and this is 83; this is 83.

And this is 16. So, if we add them up you get corresponding decimal equivalent of 99 right and you see that this is sign bit is positive right. So, if we then, combine the different things that you have seen this is effectively a addition only for subtraction takes 2's complement of the subtrahend; add it with the minuend; discard carry if there is any.

If the sign bit is 0, the answer is positive and can be directly read and interpreted. If sign bit is 1, answer is negative and in 2's complement form ok. If you want to know the magnitude; that means, you want to convert from binary to decimal equivalent the magnitude part, then you have to take the 2's complement of this number. And the number will come in the form of the way the weights of placed 1 to 4 8 and then, we can convert it and you can get the corresponding number.

So, this is how we can go ahead with 2's complement arithmetic which we shall be using. In the previous class, you have to see the normal integer representation fixed point representation. So, all these are very easily done permanently done using 2's point arithmetic.

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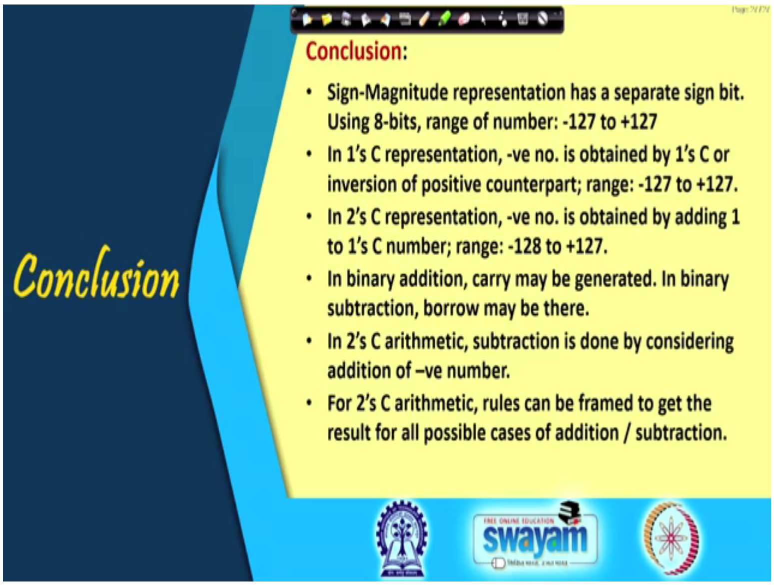

To conclude sign-magnitude representation as a separate sign bit. Using 8-bits, the a range of numbers that can be represented using sign-magnitude representation is minus 127 to plus 127. So, 256 255 different numbers can be represented using 8-bits. In 1's complement representation, negative number is obtained by 1's complement or inversion of positive counterpart ok. The range is again from minus 127 to plus 127; 255 different numbers can be represented by 8-bits.

In 2's complement representation negative number is obtained by adding 1 to 1's complement number and the range we have seen here is minus 128 to plus 127. And when we perform binary addition, carry may be generated and in a binary subtraction borrow may be there ok. So, these are the things that is similar to decimal addition subtraction, but here you are doing it with base 2.

In 2's complement, arithmetic subtraction is done by considering addition of negative numbers. So, basically first you convert it to a negative number, the number that is there in the subtrahend by taking 2's complement of it then we add it with the binary. For 2's complement arithmetic, rules can be framed to get the result of all possible cases of addition subtraction; that means; a smaller number is subtracted from a larger number; a larger number is subtracted from smaller number.

So, the presence of carry is ignored and if carry is not there in the sign bit is positive sign bit is 1, then the answer is negative and its magnitude is obtained by 2’s complement of the number. So, these are the different rules that can be framed and by which 2’s complement arithmetic can be performed

Thank you.