Digital Electronic Circuits
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Lecture - 11
Karnugh Map to Entered Variable Map

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Hello everybody. We are into week-3 of this particular course. So, in this particular lecture, we shall have a quick recap of what we discussed in week-2. And then we shall discuss five-variable Karnaugh map, and entered variable map for SOP simplification and POS simplification will also be discussed.

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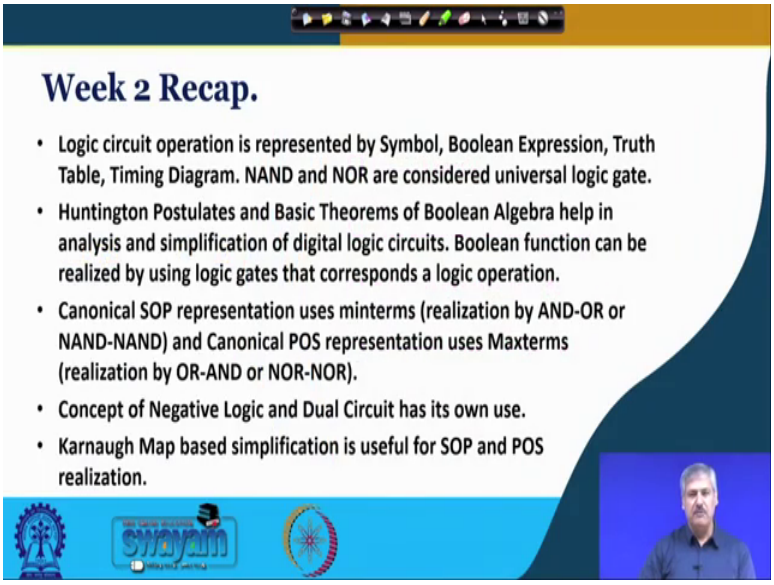

Very quickly in week 2, we have seen that logic circuit operation can be represented by a equivalent symbol, a Boolean expression like if it is a AND circuit, then Y is equal to A B, truth table the different combinations of the input generating a specific output and also through timing diagram. So, different kind of you know timing input combinations of the digital waveform high and low will come, and accordingly output will be generated. And we also saw that NAND and NOR can be considered for universal logic circuit as a universal gate for different kind of logic circuit generation.

Then we looked into Boolean algebra. The basic premise of Huntington postulates and certain theorems, and then we found that using that we can simplify Boolean expressions and can have more efficient realization of digital logic circuits. And given a Boolean function, we can convert it to a particular logic circuit for each operation we have a specific logic gate associated, and we just implement one by one, and then the final circuit is obtained.

We also saw that canonical SOP sum of product representation using minterm, and canonical product of sum representation using max term can be realized easily by two level circuits. Two level circuits for SOP was AND and OR alternately NAND-NAND circuit can also be used, and for POS the basic circuit is OR and AND direct from the equation we can get, and then also we can we found that it with little bit of manipulation, we can realise that by NOR-NOR circuit as well.

We also looked at the concept of negative logic and dual circuit and found it's so use. And finally, we ended the last week with a discussion on Karnaugh map based simplification ok. So, any truth table how it can be mapped into a Karnaugh map, and then that Karnaugh map can be used for simplifying SOP and POS representation. And also you looked at do not care input, and how that can be used for minimizing the circuit.

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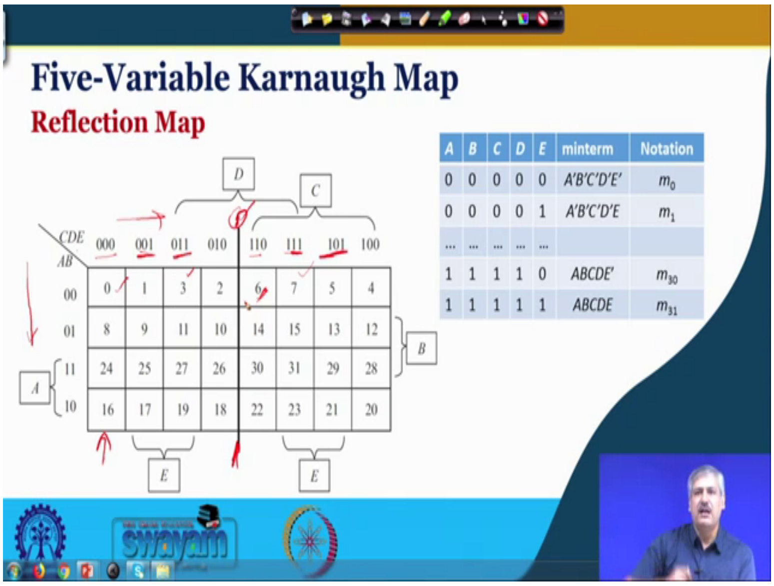

So, in today's class before into discussing entered variable map, let us see if the number of variables under consideration is five ok, we have discussed Karnaugh map up to four. So, now we are looking at these things these Karnaugh map for five-variable representation ok.

So, now we are in a bit you know difficulty, because for five-variable representation up to four-variable we had a two-dimensional plane, we had considered 0 0 0 1 1 1 1 0 on one side like this, and this side also 0 0 0 1 1 1 1 and 1 0. And then we had considered logical adjacency even at the edges and so on and so forth. But, when it becomes five-variable, so number of the sales this number of this square boxes is 2 to the power 5 ok, so that is 32.

So, now, this 32 the finding logical adjacency, this is a bit difficult. And one perhaps required a 3D visualisation to understand how the is adjacency is established. So, what we show here is a reflection map based five-variable simplification. So, in the reflection map base these side these side remains what it was earlier for four-variable, and the other side here we are representing the remaining three-variable out of five-variable.

So, CDE three-variables are there. So, C is 0 0 C D E is 0 0 0 for this particular column the first column ok. Next one is 0 0 1 and so on, then next one is 0 1 1, and after that 0 1 0. And this vertical line over here the vertical line over here. So, this represents the axis against, which a reflection is done ok.

For example, so these 0 1 0, it is reflection against this means, this 0 becomes 1 for the value C. 0 1 1, it is reflection is 1 1 1 reflection means 0 and 1 that this is getting a reflected I mean the 1 1 remaining the same 0 and 1 is. So, between then this cell 3 and 7 there is a adjacency, because only one variable is changing that is C. Similarly, this is 0 0 1 and this is 1 0 1. So, this 1 and 5 is also adjacent.

So, wherever you see a reflection along a about this axis ok, and the corresponding positions are logically adjacent. So, this is the way we have to visualise this one ok. So, there can be another method popular method which is called overlay method, where here this one this is 0 0 0 the first cell what here 0 0 0 0 0 all values are 0. So, in this case the C value will be only changing to 1. So, this will be 0 0 1 0 0 this particular position this particular square box.

So, as if this I mean both are just lying one above another ok, and the adjacency is drawn across the overlay or vertical direction. So, in this present example the present method, it is about the reflection about a particular axis in the overlay method just one above another. So, we shall discuss only one of this method, which is the reflection map based method.

The other one is similar just the adjacency first needs to be considered through just one line above the another and from there, so because of this the numbering the minterm numbering in the specific cell is what you see over here 0 1 2 3 is the more more significant bit here, and E is the least significant in terms of the binary representation that we are showing. And then this is 4, 5, 6, 7 8, 9 8, 9, 10, 11, then this is 12, 13, 14, 15.

Then there is a jump, because there is a 1 0. So, 16, 17, 17, 18, 18, 19 and this is 20, 21, 22, 23 is it fine ok. So, this is the way this minterms the 32 minterms will be represented here ok. And the positions here is showing that D is remaining one over this four cells ok, this four columns that means, total 4 into 4 16 a is remaining one for these two rows these two rows, this row and this row, so 2 into 8 16, so that is the thing that you see for other variables ok.

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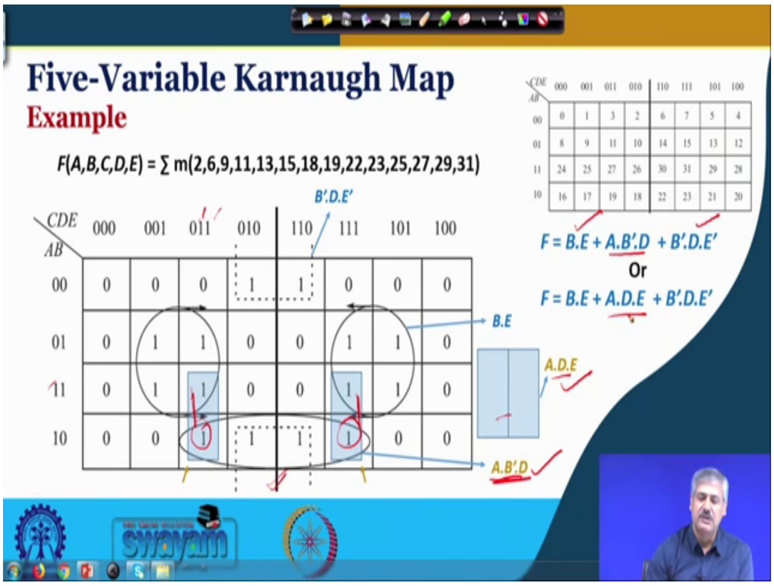

So, we take up one example to help us understand how this five-variable Karnaugh map can be used for simplification of Boolean expression. So, in this example we are taking up minterms 2, 6, 9, 11, 13, 15, 18, 19, 22, 23, 25, 27, 29, and 31 ok. So, what is the first job, since we already know since we already know from the previous discussion that how this minterms are placed.

So, accordingly wherever those minterms are there, we are placing the 1's ok. So, 2 0 0 0 0 1 0, so 2 is the minterm so 1 is placed here ok. Next is 6, so 0 0 1 1 0, so 6 is over here you can see 6 is over here, so corresponding 1 is placed. So, accordingly all the minterms the specific positions the 1's are placed in the five-variable Karnaugh map ok.

Now, after that we need to do the grouping. And in doing the grouping, we need to consider logical adjacency. And logical adjacency in the neighbourhood region, which is easily clearly visible as well as D 1 that is coming through the reflection D 1 that we just discussed. So, the one that we can see, which is a group of four we can see over here, it is a group of four this there is a group of four.

Now, about this vertical axis, you can see this or just you know reflection of one another. So, they remain they are logically adjacent. So, this four and this four that this total eight, they form one particular group. And they that will contribute to one product term ok. And in this particular group, which variable is remaining constant, you can see B is remaining constant ok. And the other variable that is remaining constant over here is you see E, E again here E and E. So, this is also remaining constant. So, for this 8 minterms B and E are remaining constant. So, the corresponding product term that will get generated is B, E is it clear.

Next we look at remaining, I mean all the 1's we need to cover through the largest sized, which is size is integer power of two number of groups ok. So, next is this one, and this one let us consider this one. So, these two 1, and the other two 1, they are because of the reflection they two are neighbour and can be grouped.

Again we have already seen before that this at the edge, and this is at these two are at the edge, they are also neighbour. So, these 1, this two 1, and this two 1 they can also be considered as one group. And in the in that particular group, we can see what are remaining constant B is remaining constant with a value 0 that will contribute to B prime. And D is remaining constant with one, this is one in both the places. And E is remaining constant with 0. So, B prime D, E prime is the corresponding product term ok.

Now, what is left? So, this 1, 2, 3, 4 and the other 4 plus 4, 8 are covered. So, what is left is this one and this one. These two 1's are not yet covered. Now, to cover that we can see that we can I mean one thing that is clearly visible is that a group like this can be found with you know logically adjacent four 1's. And this particular generates a term A, B prime. And D is remaining constant, so this product term will it will get generated.

The other thing which is not very apparent; but, again if we consider the reflection, we can see that this two are logically adjacent easily understood ok. But, again against this particular axis, they are one is reflection of the other ok, so that way also this group up to this group of two, they are logically adjacent. So, accordingly a group of four can also be formed, which is of similar size of the as the previous one what we considered over here and this group of four, if we just put them together right. So, this is this four members together, you can see that A, D, and E, these three are remaining constant over here. So, this one and this one can be covered by either of these two term ok. So, accordingly you have got B E right B prime, D prime E, and A prime, A B prime D or A D E. One of these two as representation of the minimized expression of this five-variable function through

Karnaugh map is it clear. So, similar thing we can do for previous presentation as well.

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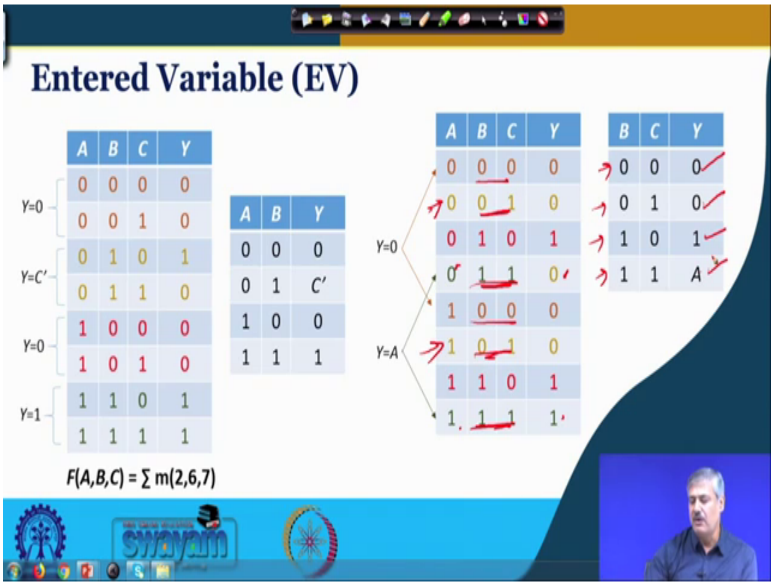

Now, what we look at we shall come back to this problem little later. What we understand by entered variable map, before which we shall discuss what is entered variable ok. So, entered variable is if you look at a truth table of three-variable as you see here A B C, and the corresponding function output is Y, so A truth table is generated out of it ok, where the output is retained as a function of one of the variable, which is the entered variable ok.

So, in this case we look at two examples for the same truth table. In one case we see C as the entered variable ok. So, (Refer Time: 14:34) C is C as the entered variable, we look at how the output function changes with respect to change in C, when other variable other variables remain constant. So, other variables here are A and B, so A and B remaining constant with value say 0 0.

So, what are the two positions in the truth table? These two positions are in the truth table 0 0 0 0 for A and B, and C is changing from 0 to 1 over these two rows ok. So, we pair it in this manner. So, this is for A B 0 1, this pair is for A B 1 0, and this pair is for A B 1 1 ok. So, for each of these cases for each of these cases, we get one entry into the truth table, where the variable is entered the variable is entered.

Now, we consider at this point that for this particular pair, how the output is changing with respect to C. We see that output is remaining constant 0 irrespective of change in C input variable, output is not changing ok. So, Y is equal to 0 in this case. In this case, what is happening? If C is 0, Y is 1; if C is 1, Y is equal to 0 ok, so that means, it is just compliment Y is output is compliment of C. So, Y is equal to C prime for case cases, where A B is 0 1.

And next case again we see that output is not changing, remaining value remaining constant with the value 0, so Y is equal to 0 and this is Y is equal to 1. So, accordingly this truth table is found, where a variable you see C prime has entered ok. So, if such is the case in another place C can be there or another place, I mean depending on the truth table the. So, there is a possibility of a variable entering into the mapped entered variable entered truth table ok.

So, instead of C, if we had a different variable say for example A entering into the truth table ok, so A is the entered variable. In that case, so what would have been the way the pair would have been formed. So, in this case since A is entering, so B and C remaining constant that is what you have to see so, B and C remaining constant four possibilities 0 0 0 1, 0 0 and 1 1 ok.

So, in for these cases four cases, we see the order the corresponding pairs, where A is changing from 0 to 1. So, A is B C 0 0 is this one, so the colour is matched here. So, these two cases the B C is 0 0, corresponding value A is 0 here one is A is 1 here. And what is the value of Y at that time, Y is remaining 0. So, Y is not changing remaining constant with 0, so Y is 0, so that way if B C is 0 1 B C is 0 1 over here, so this will be pairing with this one. And you see that A is changing from 0 to 1, what is happening to Y, Y is remaining 0 so Y is remaining 0.

So, similarly for next case is equal to 1. And last one for B C is equal to 1 1, this case, these two the these two rows. So, A is changing from 0 to 1 and at the time Y is also changing from 0 to 1. So, Y is just following the A, so at that time we are writing Y is equal to A is it clear, how we are entering a variable similarly. We could have entered B, we could have you know instead three-variable, four-variable and all the process would have been similar. So, forming a pair and finding a relationship and accordingly coming up with the truth table ok.

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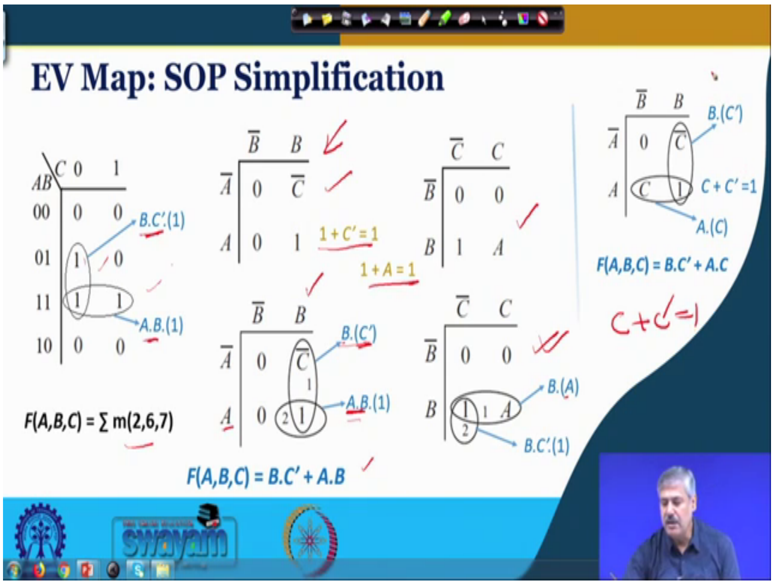

Now, how it is useful in how it can be used in simplification alright entered variable. So, now entered variable map that we are talking about in this the variable will be part of the Karnaugh map ok, it will be included in the Karnaugh map. So, before that let us look at the normal simple Karnaugh map based you know simplification that we have we could have done for the same example. So, this is the example right, the one that we have taken up. And these are the two you know groups that is possible right. And the corresponding product term that is available is B C prime and A B.

Now, we can also think of this as B C prime, the product term that is generated multiplied by 1, ended with 1 any minterm that is there. So, the corresponding value ended with 1. And if the minterm is not present, the corresponding input variable combination ended with 0, so that is why it is 0. So, this is the way also it can be visualised with no you know loss of meaning. So, if we look at this way, then it will be clearer how we are getting a SOP entered variable map based simplification. So, the truth table that we are just seen, we are simply mapping it to a that was a three-variable Karnaugh map, I mean there are eight locations eight cells.

Now, since one variable has entered A and B are the variable against, which the map is form. So, this is a four cells are there. And in the cell we will be having entry of 0 1 and the variable ok. So, these are the likely members variable and it is compliment form, these are the possible members ok. So, for the same problem the present problem, we have this as the entered variable map ok.

Now, for the simplification, so we have to form the largest group, where all the 1's and the variables need to be covered, all the 1's and the variable need to be covered. So, what we see over here? So, there is a one this is the one ok, so there is no other one. So, this one is covered ok. So, this one is covered how, this A is there with a value 1, and B is there with a value B 1. So, A B is the term, and we are considered it is just it is ended with 1. So, A B is one term that will get generated ok.

Now, we see there is a term C prime C C bar ok. Now, this C bar we can take this as A bar, B ended with C bar that is also 5 that is also is Boolean algebra wise expression wise it is correct, but it is not the minimised expression. So, to get the minimised expression, we can consider this one as 1 plus C prime also right. So, there is a hidden ok. Implicit C prime here without any making any difference in the meaning of the entered value. So, this C prime and this C prime, they can be combined together, and for which B is remaining constant with a value B. And the variable that is entering, we are simply ending it with the value that is remaining constant, which is B here.

So, B C prime is the corresponding the term that is getting generated. So, same truth table with entered variable map considering A, we shall see this is the corresponding map. And for minimization again with one we are getting the B and C prime ended with 1. And for A, we can consider one as 1 plus A without any difference in meaning, and that will generate B and entered variable is A, so A B so B C prime A B. Here this is the way B C prime, and A B and we get the same expression. Now, we take into note one interesting point here that the one that we have seen here, which we are writing as 1 plus C prime or 1 plus A to get a minimised expression ok. The same one can be written as C plus C prime also right.

So, if we have a hypothetical truth table like this for the for which the entered variable is what you see ok. Then your this one this C part can be combined with this, and C prime both can be combined over here. And we do not need a separate coverage of one, because the components that are making one C and C prime, they are individually covered by these two groups ok. And that will generate you A, B, C prime plus A C, which is a minimized expression and no separate coverage of one required is it ok, so that is what we take a note of in minimization using entered variable map.

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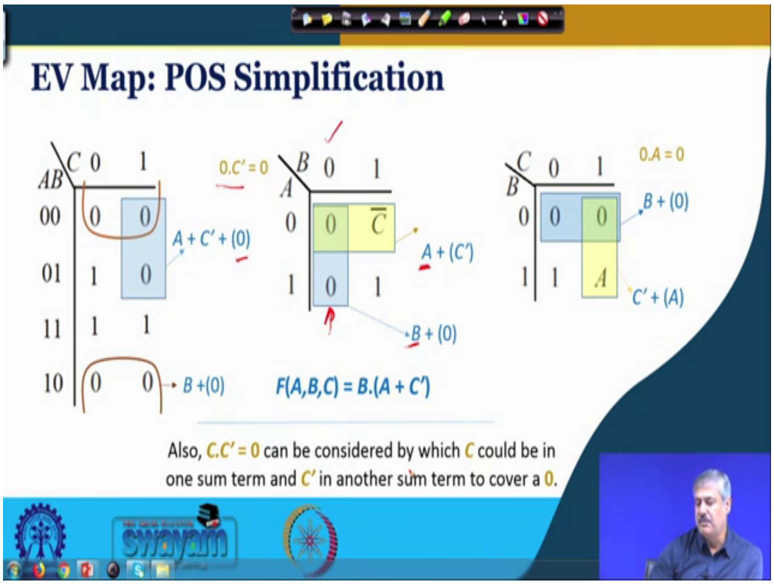

The same thing can be used for POS simplification product of sum simplification, the same truth table. In this previous case, we consider that it is getting wherever there is a minterm present, it is getting ended with 1. Minterm absents means, it is the corresponding input combination is getting ended with 0, so that is why it is not there.

So, in this case the sum term whenever is present ok, we can consider that it is getting added summed with the 0 ok. And if it is not present, then it is getting summed with one ok, so that is the way so one means just it the corresponding max term is not available ok. So, the corresponding POS simplification so, again earlier we considered 1 as 1 plus C or 1 plus C prime right or C plus C prime, so this is the way. So, here 0 can be considered as 0 and C prime 0 and C or C C prime ok. And accordingly, we can use it for forming a larger group, if it is so possible.

So, in this case what we see these 0's can be combined together, these two 0's for which B is remaining a constant with a value 0 ok. So, B remaining constant with a value 0 means B, uncomplemented will be coming as the sum term right. And here the other group that can be form 0, so 0 can be considered 0 C prime. So, A is remaining constant with 0, so that is A prime will be there and B C prime is already there, so that is the way you get the corresponding term; so, similarly for the other case ok.

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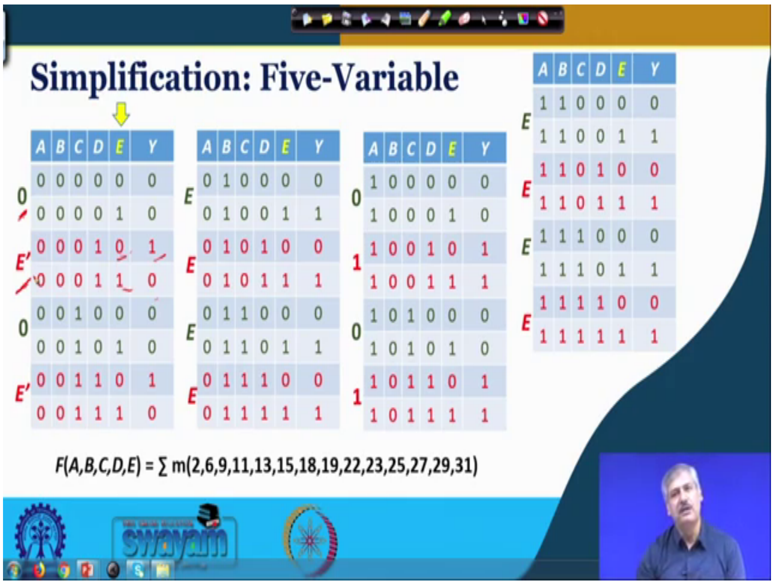

Now, we to end we look at the example with which we started the five-variable example. So, the five-variable example, now we would want one of the variable to be entered. So, we will consider the variable E is equal to entering into the map. So, when variable E enters, so then we will be forming pairs the way we have seen, so this is the adjacent columns ok.

So, 0 and 1 is changing the rest of the values are remaining constant A B C D 0 0 0 0. And accordingly the corresponding output how Y changes with E that is mentioned here. So, Y is not changing with E for the first case Y is remaining 0, Y is just opposite of G compliment, you know the compliment of so 0 1 1 0. So, this is E bar, and accordingly we have all the positions covered ok.

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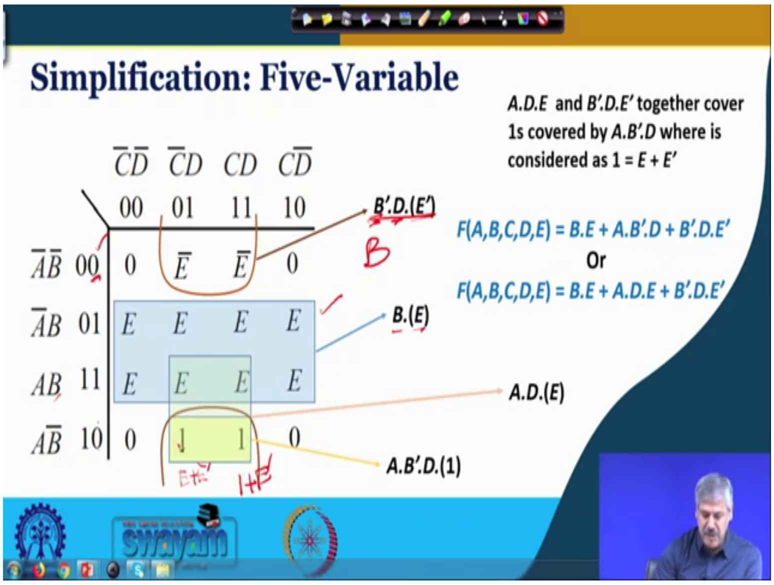

So, with that we look at the five-variable representation of the this truth table that we had just seen ok, where E is one of the entered variable. So, with E entered variable in the respective places, now the five-variable now has become 40-variable problem ok, which is now which is more convenient. So, this is one of the advantage that it reduces by a the number of cells reduces by a factor of two I mean it is divided by 2. So, 32 now becomes 16 ok. So, 16 is a more manageable proposition four variable Karnaugh map ok.

So, in that case these are the way the variables are entering here right. And then this is the largest group with E contain for which B is remaining constant, so B ended with E the variable that has entered ok. Then what is remaining to the covered this E prime and 1 1 ok. So, if this 1 1 can be considered as E plus E prime, so with that one group can be found that is this 1 1 can be considered in this manner right, so that is one group. And this E prime will go there that is another group that is one possibility, another is only it can be considered as 1 plus E prime right. So, each will each of these cases is considering four possible I mean three literals in the expression.

So, in one case this is B prime, D prime, and this remaining with 1. And E is remaining with E is variable that is entering, so B D E is it fine this is SOP. So, the one that is 0 will be corresponding prime term will be there right. And one if you consider this particular block, so this is coming as A and D right, D is remaining constant, A is remaining constant, and E is the entered variable that will cover.

And if this E is not included only one is considered, then A B prime and D ended with 1 right. So, this is the these two 1's can be covered this two 1's can be covered as 1 plus E or E plus this one can be covered as 1 plus E or E plus E prime right, so that way you have got two possibilities the way you had seen before ok.

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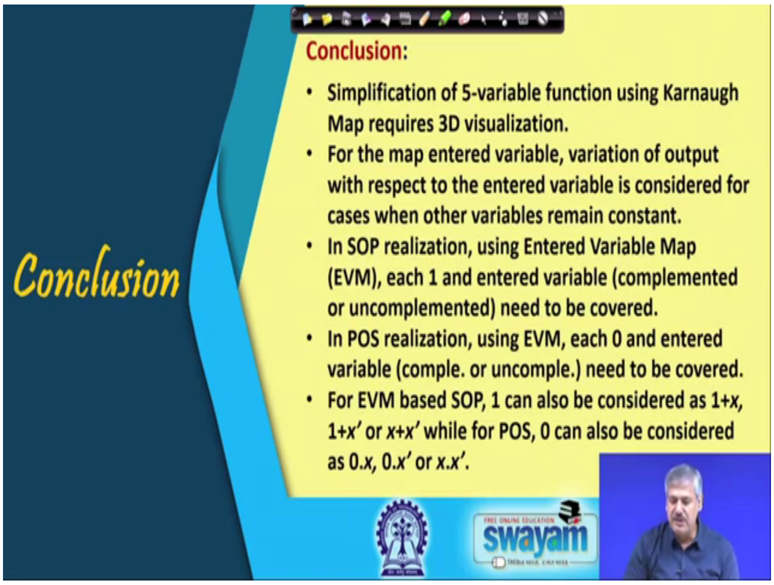

So, with this we come to the conclusion of today's class that this particular class. So, simplification of 5-variable function using Karnaugh map requires that 3D visualisation, whether it is reflection map or overlay for by which the adjacency can be found. For the map entered variable, variation of output with respect to the entered variable is considered for cases when other variables of the function is remaining constant. In SOP realization each one and the entered variable complemented or uncomplemented need to be covered.

And POS, it is 0 and those entered variables need to be covered. And for EVM entered variable map based SOP minimization 1 can be considered as 1 plus the variable 1 plus x, 1 plus x prime, 1 x plus x prime. And for POS, 0 can be considered as 0 ended with x, 0 ended with x prime or x x prime by which the minimization can be made more efficient.

Thank you.