Electrical Machines - II
Prof. Tapas Kumar Bhattacharya
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture - 87
Expression for Load Angle & Expression for Power

Welcome to this study of salient pole synchronous machines. And, in the salient pole synchronous machine similarly can be operated both as motor and generator mode. In case of motor mode and phasor diagram is very important in synchronous machines. In case of cylindrical synchronous machine, the we drew the equivalent circuit; E f Z s and terminal voltage. In salient pole synchronous machine it is not done, it is difficult to obtain some equivalent circuit of that sort because, of the presence of two reactances.

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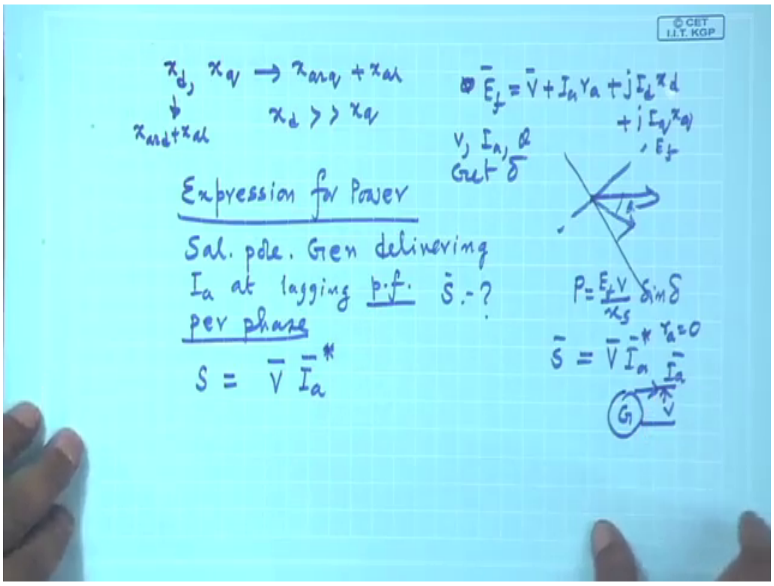

Synchronous reactances one is along d axis. These are the keywords in x d and x q. What is x d? x d is its armature reaction reactance plus x l, x q is x arq plus x al anyway this we have discussed. So, and x d will be much higher than x q and r a sometimes is neglected also we know how to draw take into account the effect of r a ok. So, this is the thing and in our last classes, we found out the; we were trained to draw the phasor diagram correctly that is essentially we have to determine q. What to draw in the phasor diagram? I must be provided with this information that motor is a tell me motor is operating whether in generator mode or motor mode that is one information, whether that generator is operating at lagging power factor and how much current it is supplying.

So, I a theta and terminal voltage will be known and then you can draw the phasor diagram. In case of generator, E f is equal to V plus I a r a plus j I d x d plus j I q x q, since I d I q are different so, they cannot be put in a single circuit and say that x d x q in series or they are in parallel. No, it is not possible separately we have to do. In case of generator mode, this point must be very carefully handled that is V is equal to I a r a plus j I d x d plus j I q x q and I d I q will be different because armature current is broken up into d. So, they cannot be represented in series or in parallel no.

So, the important point is from the knowledge of V, I a theta get delta and then generator mode V there, delta will E f will be above it which will be along q axis, then draw d axis. Then I a theta is known, break it up I d I q go on adding this drops you will correctly land up with the phasor diagram and we have extensively discussed about this. Now, today another important thing I will do that is called the expression for power, expression for power. Like in cylindrical machine, we found that the real power was equal to E f V by x s sin delta where, delta is the angle.

So, similar expression, we will try to derive for a salient pole machine and in this expression recall r a was assumed to be 0. And of course, I could take r a into account another term will come and the essential for deriving this type of equation; we started with the fact that V I a star we did. And, we got the in one stroke, we can obtain the expression for both real power kilo Watt and the reactive power how much the machine is handling. So, first we what we will do is this expression for power, a salient pole generator say delivering lagging power factor current; lagging delivering I a at lagging power factor. What is the expression of S? Both the real and reactive can be found out ok.

So, we will start and we will first find out per phase power, then at the end we will multiply with 3 to get the total power expression or torque because, the moment you get power expression, you can easily find out torque because speed is constant. So, in synchronous what the torque is also derived in one stroke; let us see. So, I will say that S so, it is a generator at a per phase it is delivering a current I a that is the thing. So, S will be V I a star per phase; this is the thing, but we know that or I will draw the phasor diagram. It will be so, this is the starting point.

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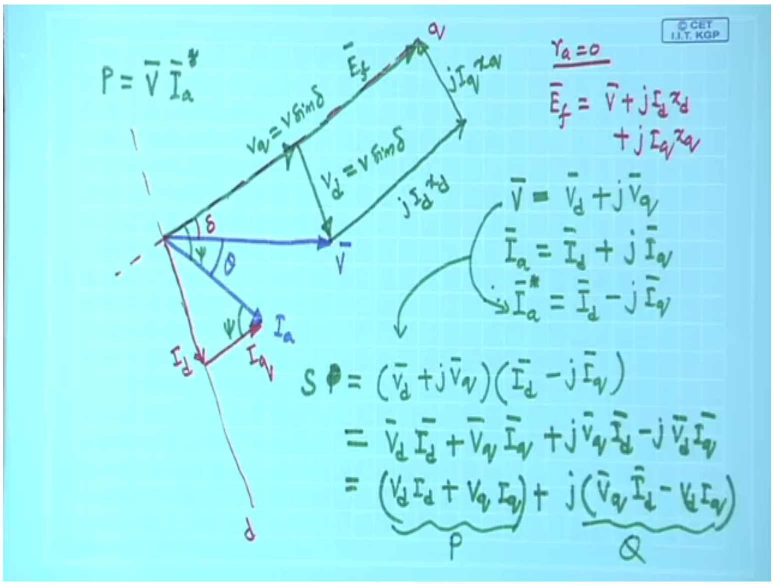

And rest of the things are routine works except that you have to draw the phasor diagram with r a equal to 0, you start with V, you draw I a you know theta. Then delta is known tan delta expression and I will draw then, q axis and I will draw then d axis. Then I will break up this as I d and I q and this is delta tan delta is known and then if r a is neglected r a is 0, then E f this is true is equal to V plus j I d x d plus j I q x q. So, V plus I d x d it will be like this, this is j I d x d and you know to this you add this one j I q x q. And, this length will be your E f and this whole angle is also psi and this is psi, we have got this.

Now, this voltage phasor V it can be written as this voltage can be also thought of consisting of this and this is not. And, this is V d and this is V q; V d is V sin delta and V q is V sin delta. What I am telling is this we can be written as V d plus j V q what else, this phasor

plus this phasor; similarly I a can be written as vector I d phasor plus j I q what else. Now, power I am planning to do this operation V I a star therefore, I a is this. So, I a star complex conjugate will be I d minus j I q is not, this will be the thing then I have to multiply these two to get power.

So, let us multiply so, voltage is V d plus j V q and complex conjugate of the current is I d minus j I q. So, open the brackets so, that so, I am sorry this I must write S total complex power and this will be equal to real part. So, V d into I d and there will be another this j V q I q j j minus minus minus plus. So, this will be V q I q is not and then the imaginary part will be plus j V q I d this and this and minus j V d I q. Now, V d and I d are in same phase therefore, it will be simply multiplication of V d and the into I d. V q I q in same phase it will be this one, V q I q is not and then plus.

So, this is the thing plus j whatever it is there V q I d minus V d I q this will be the thing and this will be only magnitudes you know this will be magnitudes V q into I d and this. Now, the question is that I mean what I am trying to tell, that once you get this expression this will give you the real power and this part will give you the reactive power Q. If I simplify this expressions and from that I will get the expression of power delivered to the bus or to the load by the salient pole synchronous generator. So, I have assumed generator mode of operation lagging power factor is that clear. So, that is the thing.

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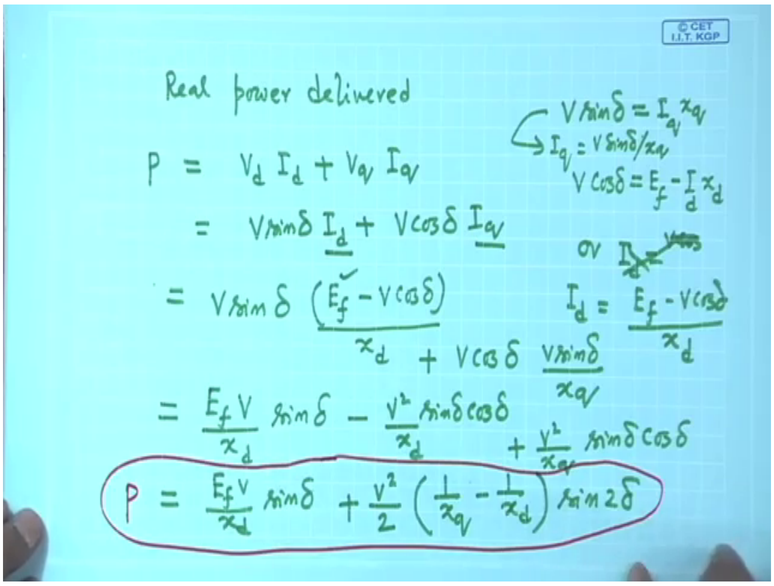

Now therefore, real power delivered is P is nothing, but V d I d plus V q I q that is all.

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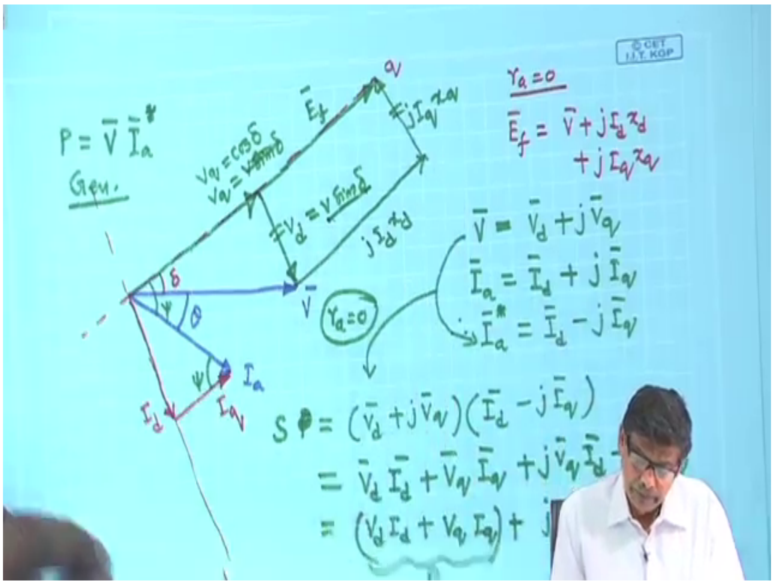

Now, recall the V d value is nothing, but V sin delta and V q is V oh my god this is V t sin delta and V q is V cos delta correct that V q is cos delta V cos delta V sin delta. So, I will put those value. So, I will write this to be V d is equal to V sin delta into I d plus V q is V cos delta into I q. What is my target? My target is to get an expression of power in terms of excitation voltage, terminal voltage and load angle and x d x q etcetera. So, that is the target, but here you know unfortunately I d is present, I q is present. So, I must express this I d I q in terms of I a x d x q etcetera.

So, how this can be done? It is very simple you see this length V d is equal to V sin delta, mind you this is r a equal to 0 yes. So, V sin delta will be nothing, but I q into x q this length and this lengths are equal. So, so note that V sin delta is equal to nothing, but I q into x q is not, that is there and V cos delta is equal to E f minus this length; this total thing is magnitude of E f minus I d x d. So, V cos delta is E f minus minus of I d x d this will be the thing

So, we have to put the value of I q and I d from these two expressions here so, let us do that. So, V sin delta I d I will write from this I will write it is equal to V cos delta oh sorry I d is equal to E f minus V cos delta divided by x d. And, I q from this I will write, I q is equal to V sin delta by x q; these are the substitution I have to do to get rid of I d and I q ok. So, V sin delta into I d that is that will be equal to E f minus V cos delta divided by x d plus V cos delta into I q. And what is I q? I q is V sin delta by x q that is all.

So, now in the right hand side everything is known, how do I know delta? Because, I know at what power factor it is operating which mode it is operating this that we discuss so, much E f is known excitation voltage x d x q are known b terminal voltage is of course, known bus voltage. So, this is the thing now this this if you simplify you will get the first term that is this will give is E f V by x d into sin delta, put it inside you will get this. Then the second term from this fellow will be V square sin delta cos delta by x d and this term will be simply V square by x q. So, once again sin delta cos delta it will be like this.

Now, this one is do not disturb it, I have already got it sin delta nice form then these two you divide by 2 multiplied by 2. So, that it will become V square by 2 and then 1 by x q this minus 1 by x d into sin of 2 delta and this is the expression for power; you have to multiply with 3 to get the total power. So, this is the expression we are looking for. So, this is the real power expression; if I wish I can also calculate how much reactive power it is delivering by calculating this put the value of V q I d. And, you can get the expression for q which I am not doing because, I will concentrate on this it tells you many thing.

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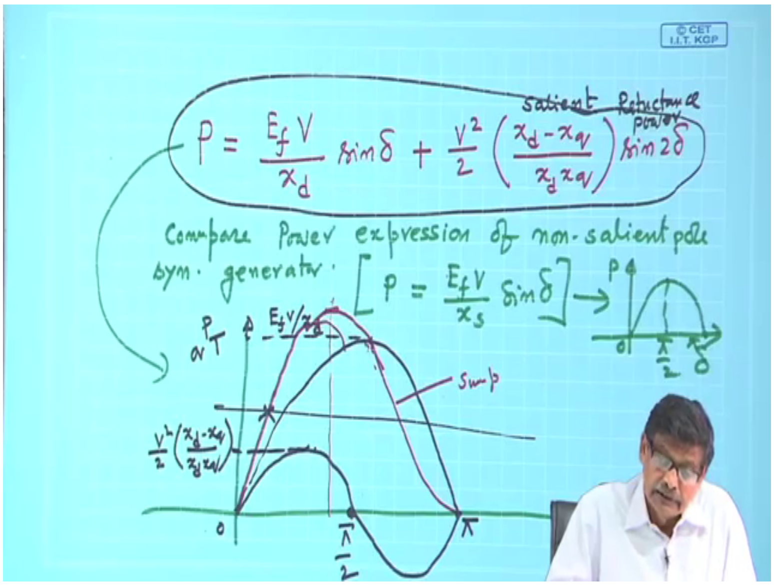

Recall that so, we have got the power as E f V by x d sin delta minus V square not minus plus; this is can be written also in this form x d minus x q by x q by x d x d x q into sin 2 delta. This compare it, compare power expression power or torque this is synonymous with torque because. So, compare power expression of a non-salient pole synchronous generator.

How it look like? It was P is equal to per phase E f V these are all phase voltages by x s sin delta. So, first thing you notice that in case of salient pole machines there are two terms: one is sin delta another is sin 2 delta. So, in this case the P delta curve looked like this is 0, this is pi. Maximum value of delta for stable operating point is pi by 2 that is all, in case of these it will be if you sketch it will have two terms. One is this term and another is a sin term, it is 2 delta. Therefore, one complete cycle will be present between 0 to pi, this is pi by 2; this will be the and these two if you add point by point you will get the expression for power or torque on the y axis ok.

Now, the question is what is this maximum value? This maximum value depends upon E f V by x d. And what is this maximum value of the sin 2 delta terms? It will be V square by 2 into x d minus x q by x d x q is not, this will be the maximum value. Now, recall one thing that this is the expression of a salient pole machine. If I say in a machine x d equal to x q, what does it mean? It means it is a cylindrical machine, there is no difference between direct axis and quadrature axis, reactances and this term will become vanish and only sin delta terms will be present, if x d is equal to x q.

However, in a salient pole machine x d is not equal to x q. There will be difference, but not much difference, there will be difference. And, V is the terminal voltage, this is a large number no doubt; here also E f V by x d is a large number. But, this factor is not so, larger number difference of x d and x q, that is how much salient the pole configuration is, is will decide what is this peak value may be 10, 15 percent of this little, but anyway this will be the thing. So, and if you add these two point by point so, peak will not occur here. This curve will be somewhere here, this curve if you add a roughly I am drawing; it will be somewhat skew sort of thing it will be there.

And, it must pass through this point because this plus this is this and it will become something like this. This is the sum that is the total power will be that red curve. You see the stable region will then occur slightly below this one, you cannot go up to 90 degree before that maximum, operating point will be decided by this thing. Whereas, this load demand you gate and this will be the operating delta of the synchronous machine. But one interesting thing is that this equation tells me now, the second term is called sometimes salient, because of saliency present this term is present.

It is called reluctance power, reluctance power or torque it is because salient x d x q are different that is why you are getting this term. And, this term this power does not depend on excitation E f, no E f is there V is terminal voltage bus voltage. Therefore, it looks like this point is very important you must understand that a salient pole synchronous machine connected to bus may produce torque or power even when E f is 0. The E f is what, field current of the synchronous machine. So, suppose field current with 0 this equation tells me that that even steady state operating point it was running, you gradually make the field current of the your synchronous machine down to 0; still a salient pole machine will produce a torque or power.

And, then the machine is set to be operating as a reluctance motor because, it will still operate at synchronous speed, but the level of power it can handle becomes less. Because, difference of x d and x q will decide that how much will be the maximum power. But, nonetheless what I am trying to stress upon is the fact that a salient pole machine even with field current 0 can produce torque and power. It will still be running at synchronous speed, but the level of power will be reduced no doubt. But, it will be able to pump some power into the bus or take power from the bus as a motor as well. So, this expression is very important ok.

One should memorise these of course, just not that you memorise just out of nothing, but this is worth remembering for solving problems or explaining many other things; that is this term there will be no effect on excitation of this term. So, a salient pole synchronous machine then I know that how to draw the phasor diagram correctly, how to fix up the q axis, d axis because, I a is to be resolved into I d x q I d and I q. And, their effects are to be taken into account by two reactances x d x q and then this is important how do I find out the power. So, we started from the V I a star and that is there from terminal voltage can be written as V d plus j V q; similarly I a can be written as I d plus I j q and so on. Put the values of I j star get rid of I d I q from the phasor diagram and you can get the power delivered by the machine.

I will leave as an exercise to you to find out the expression for the power after drawing the phasor diagram of a synchronous motor and solve many problems. In the last next time I will take maximum another three lectures where I will conclude and draw the phasor diagram of synchronous motor. But, nonetheless you on your own try because, in case of synchronous motor you have to start from; it is better that is what I do that is translate the problem in terms of addition of vectors. It becomes easy draw first E f I a, then E F plus I a r a plus j a I d x d plus j I q x q will give you terminal voltage and E f will be below P. So, with this note I finish today and we will take up in the next class some interesting informations about salient pole machines.

Thank you.