Electrical Machines - II
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Lecture – 77
Expression for Induced Voltage and O. C. Phasor Diagram

Welcome. So, we were discussing about synchronous machines. It has got a 3 phase all method winding, it has got a field winding generally on the rotor through slip ring and brushes, your free DC current; Direct Current to the field winding, to establish the flux and in the 3 phase armature winding, it is also called armature winding. It induces voltage then it is the 3 phase armature winding which handles the main power.

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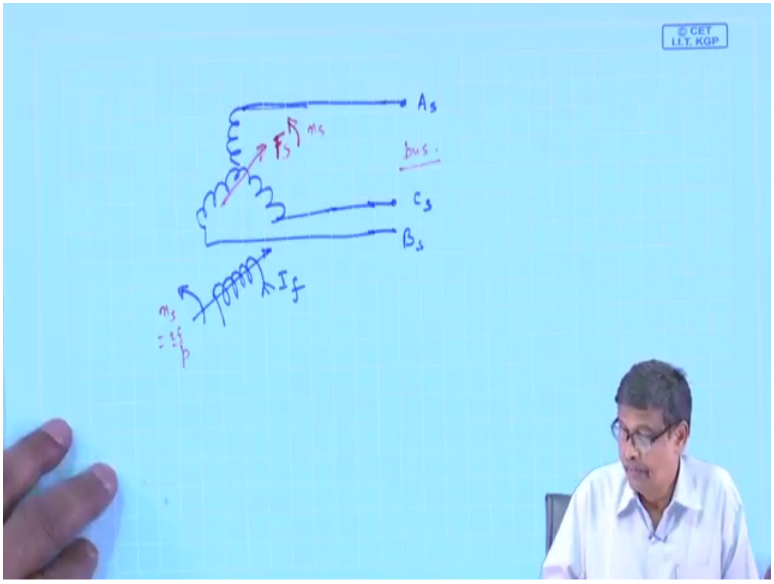

And as so hence forth, what I will assume? This is the stator 3 phase winding and this is the field winding and you are causing some exciting current, whose value can be will be in my control and this machine I as I was telling last time. That will be connected to supply A s, I will not even write A s, B s, C s and the this is the rotor field and it will be moving like that and so on. But if this 3 terminals are connected to the bus this voltages, line voltages is fixed.

And if this 3 phase armature winding carries balance 3 phase current, then also I know it will produce a rotating magnetic field, is not? If this windings carry balance 3 phase current. And everything is now in place, this is the stator field which will be moving with a speed and the everything of frequency F, this speed is synchronous speed 2 f by p etcetera. Therefore, both the fields this field as well as this field they will be moving with synchronous speed ns. So, machine will be able to develop torque and machine is said to be synchronized with the bus, this is bus that is there.

So, we will be handling this problem; now, the question is whether it is acting as a motor, generated these things I will tell subsequently. Now, before that how to model this synchronous machine? In case of induction motor I told you, this excitation current; exciting current which creates the magnetic field is drawn from the 3 phase supply itself, that is a singly excited machine. However, in synchronous machine the flux is established primarily by this dc current which is a separate source.

So, it does not put any burden to the bus and we will see that this is one of the very important virtues of synchronous machine, you can operated at various power factors by adjusting perhaps the field current of the machine; we will see. In synchronous machine you cannot do anything. The exciting current is drawn from the bus itself, that is all; magnetizing current and magnetizing current causes lower power factor particularly at loads, at lighter load conditions.

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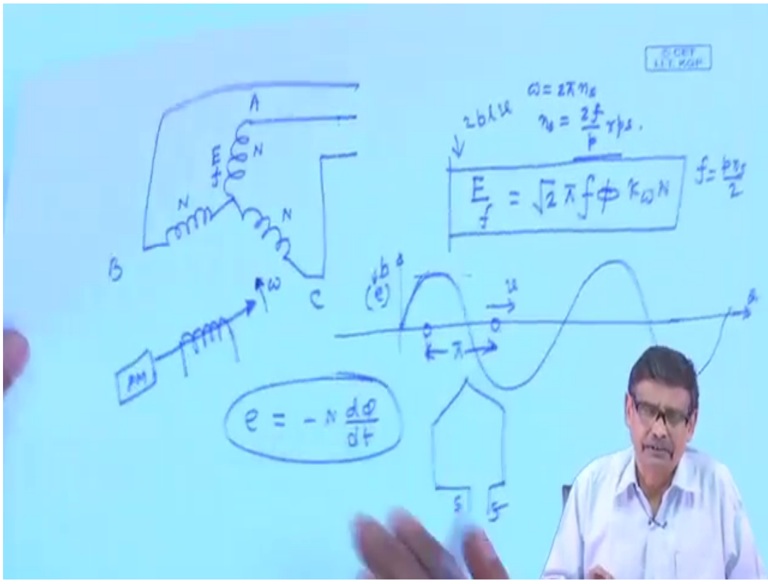

Now, I will now try to understand about the induced voltage in the synchronous machine. See you first consider, I want to know how these are the stator coil of the machine A, B and C each of N terns each of N terns ok. And you have this field winding, I will just indicate like that and it is moving with a speed say omega, what is omega? Omega is 2 pi ns. What is ns? Is the synchronous speed 2 f by p so much of rps is not?

So, I will run this with the help of a prime over here; it may be steam turbine, diesel, engine or maybe a dc motor in your lab anyway so, this is omega. Now, this flux will be moving and let us assume that, this thing this nothing is connected here, I want to examine. So, what will be the induced voltage? Induced voltage per phase which I called E f, it will be this result is known, root 2 pi f flux per pole k w into N this will be the and that what frequency f equal to pns pi 2 at that frequency this voltage is induced. How did I get this voltage? I got this expression of the voltage which is rms voltages by telling that your B distribution is sinusoidal, who makes its sinusoidal distribution of the coil if you neglect harmonic.

So, some B max sin theta is there, then if you move a coil; a single coil is like this with respect to this suppose this coil is moving with a velocity v meter per second, then blv, blv it is a coil single coil, 2 blv from that we got ultimately this expression of the voltage. Therefore, if this is v, nature of the voltage will be also like this 2 blv, whatever is the b distribution, this is the space angle theta it will be like that.

Now, I will in this case of synchronous machine, I will also try to look at this equation from another point of view that is I will try to find out or rather apply the Faraday's law of electromagnetic induction that is N d phi dt I will do. Of course, I will get same expression and why I will do it, it will be clear.

Also it is necessary to know that, I could get the same expression at least think in that way that this expression can was obtained from 2 blv, then I calculated the rms values in my earlier lectures if you see in the induction motor, I dependent on this one. So, b or e induced voltage there one and the same thing this. So far as the nature of the plot is concerned, variation of them with respect to time is concerned.

Now, what I will do is? I will go by Faraday's law that is induced voltage I also knew it is equal to minus N d phi by dt. So, I start with b, I will calculate what is the flux linkage with the coil at a given time t and I will differentiate it take negative of that, to get the instantaneous value of the induced voltage across the coil, what is this coil? This coil is like this you recall, this is the start, this is the finish of the coil, this is pi is not and suppose the relative speed between them is v. Now, I will try to use this formula this is necessary to understand and also it will give you another alternative way to calculate the induced voltage in a coil.

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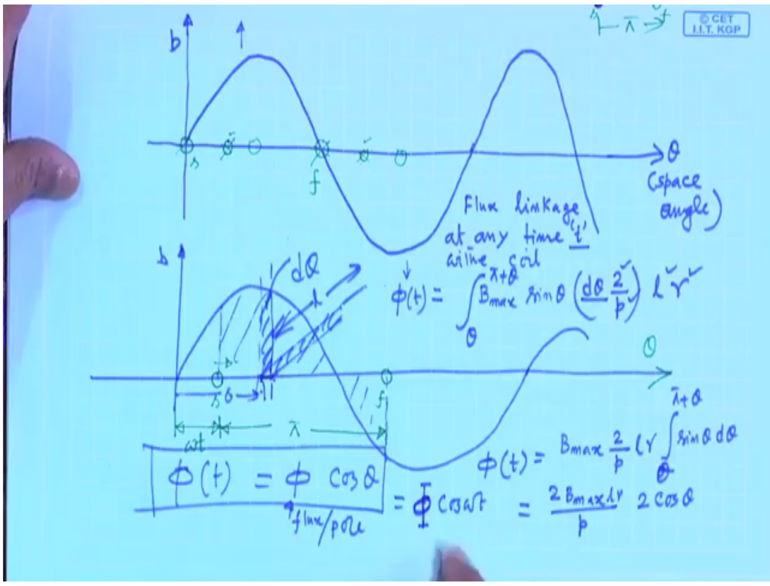

So, I do I said the problem like this. This is suppose your b distribution fine, what I will do is this, this is suppose theta space angle, all electrical space angle, this is b distribution. And positive means flux lines are this way, negative means flux lines are this way that is how we have assumed. Now, suppose a single coil is moving because you know in any machines all coils are identical. So, if I know what is happening in a single coil, I am I can always tell what is happening to some other coil, only thing I have to find out what is the displacement of the second coil with respect to first.

So, with that spirit we would like to find out the induced voltage in a single coil having N turns ok. So, what will be the thing and there must be some relative velocity between this field and that conductor. In this case, I will assume this field is stationary and conductor is made a coil is moving. What is a coil? Coil I will represent it like this pi, this is start, this is finish of the coil and it is moving underneath say from left to right.

So, I will start counting my time when this was when this start of this coil was at 0 of this b that is when this coil was here, it is moving. Let us start counting time from there and let the speed in radian per second of the rotor be relative speed be omega. Therefore, at any instant of time this coil [FL], first tell let us do like this, let us I will take another pen. So, I am telling this coil is here now, this is start this is finish. You see the maximum flux linkage is taking place, but this coil is moving, after sometime this coil will come here in newer position this is the second position, it is not here now.

Then you can see the total flux crossing this coil now has decreased because there is a negative portion of flux lines it goes like this, then when it will that is it further moves it comes here peak here, then what will be the flux linkage, 0 and once again if it moves further flux linkage will be on the negative side. Therefore with time, the flux crossing the plane of the coil is a time varying quantity and the induced voltage will be changing like this.

So, what are we are planning to do is this at any instant of time. So, I am drawing this once again and it is better I draw it, it is like this. Suppose the coil is any arbitrary position here and here, start and finish with I have started counting time when s was here, this axis is b, b here. Therefore, this angle must be omega t must be omega t some time has elapsed and let us assume full pitched coil this is pi and this axis is theta this poles are there.

So, the coil relative speed of this coil is omega, omega can be related with v as we know; so, anyways this is the thing. What is the flux linkage with the coil so, what I will do? This area minus this area, I mean I have to do; therefore, I am flux linkage, I write it flux linkage at any time t at any time t, what is this any time t? I have started counting my time when s was here at the origin, after that it has got and I have assumed coil is moving.

Ok It moves it comes here therefore, flux linkage at any time t with the coil the coil will be equal to what? It will be equal to B max sin theta, at any theta, consider d theta here etcetera this we have done d theta. So, area under this tree lrd theta. So, B max sin theta d theta, but it is electrical, area is mechanic d theta into 2 by p; d theta is electrical mechanical will be less and into length of the machine into radius, at the theta into l is this area here this area I am talking about that.

So, r d theta into l.

Student: Into theta 2 pi l.

No, this is rd theta, this is rd theta into l. So, l is the length of the machine this is l, we know that I will go faster so, this is like this. And this is the flux linkage with a single turn coil, it is like this B max sin theta. So, flux linkage will be you have to calculate this whole area from what; theta 2 pi plus theta. So, if you integrate this, it will be B max, 2 by p into lr, 2 by p lr I have taken and it will be theta 2 pi plus theta and sin theta d theta is not, that will be the thing.

And this if you integrate it will be minus cos theta. So, 2 cos theta it will become. So, it will become equal to 2 B max lr by p into 2 cos theta, this is theta sorry. And your phi t then will become, 4 B max lr code by p B max lr that is nothing but flux per pole, we know that flux per pole into cos theta. Therefore, induced I mean not induced voltage the flux linkage with a single coil at any given time and this theta is nothing but omega t is phi cos omega t, what is this phi? Phi is the flux per pole, should I write a capital phi like this. So, that with this phi is there this is phi not phi max in a transformer.

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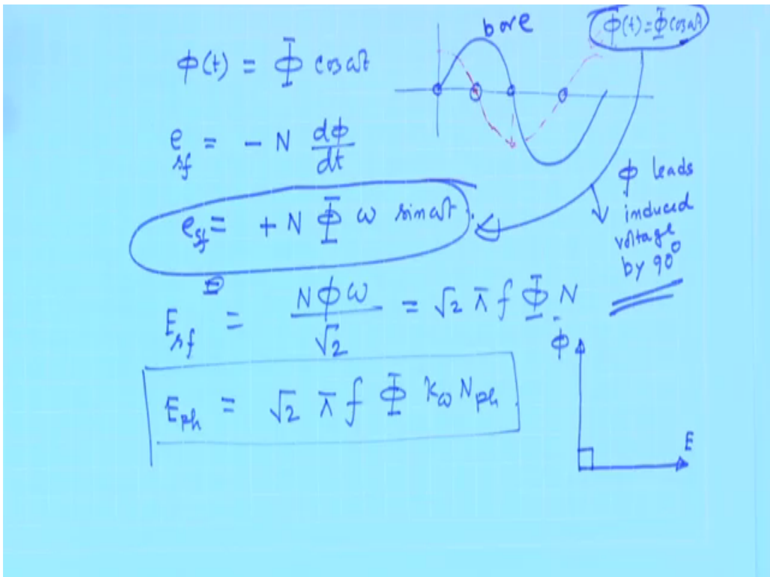

Therefore, flux linkage at a given time is becoming this phi cos omega t. Once I get that, then I will apply the Faraday's law, that is induced voltage in the coil, start finish potential of face with respect to f will be minus number of turns of the coil, if it is not a single turn coil N d phi dt. And if you do this minus N this phi and this will become omega sin omega t, there will be a negative sin, is not; cos I am differentiating. So, this will become plus here and that is it this will be the induced voltage time varying sinusoidal voltage nicely it will come. And the rms value of this will be nothing but N phi omega by root 2 and this will be equal to root 2 and mind you if this is for a single turn coil, single coil I have to taken root 2 pi f flux per pole N single turn.

Now, if this coil there are several coils connected in series, distributed, then I will say for this machine E phase per phase voltage will be root 2 pi f flux per pole k w into N that I have to do then, is not; for a single coil if you know you know. So, this is the induced voltage that is fine.

What in fact, this expression is same as I have got from flux density. Now, why I am doing this? This alternate approach also is exploited you will arrive at the same result that is fine. But one thing you try to understand out of this discussion is the point, what I am trying to make ok. This coil at omega t equal to 0 maximum flux linkage takes place and or rather I will just straight away go here, this is suppose your flux linkage was cos theta, [FL] phi t was cos theta, this phi t.

So, this was my b. I started with and phi t I got flux linkage turn, I got some flux per pole cos omega t. So, it was flux per pole, flux linkage if you sketch it was like this. This was flux, this was b or e sin omega t c induced voltage is also sin omega t, b or e are same, blv that is why people prefer b. So, this is this red one is the flux linkage and physically also it will be understood at t equal to 0, then the coil is here flux linkage is maximum, that is what? This ordinate rate curve, ordinate gives you flux linkage, when omega t is equal to 90 degree it is 0 because coil will be here, as many positive lines of force goes as many, so many negative goes 0 and so on.

Therefore, you can easily see that, this phi and e can be expressed in terms of phasor with a 90 degree phase displacement. This is phi t and this and this together this 2, what is the conclusion? Conclusion is this, phi t is leading induced voltage phi, this phasor phi leads induced voltage by 90 degree; That is I could say that, if this is phi phasor, your induced voltage must be here. What is b, b will be also here getting so and, this is 90 degree. Therefore, in synchronous machine, I will try to draw the phasor diagram with this flux linkage phasor and d phasor because that will explain things in a much better way as we as we proceed. So, this is the thing, this induced voltage lacks the flux per pole phasor by 90 degree.

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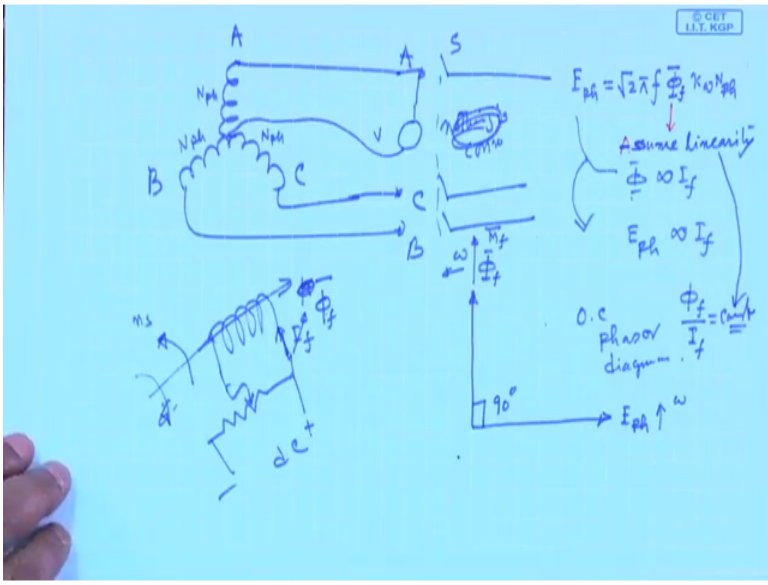

Now, let us come to this problem. That suppose let us assume, now I have a synchronous machine this is A, this is B, this is C terminal. I will assume ABC phase sequence, this is the field winding and I will try to say this field winding create some flux linkage, flux per pole it creates some flux per pole and then it links ABC winding with time changes, this is N phase, N phase, N phase.

And of course, this flux per pole I have a control, some dc voltage is there, I will adjust this to adjust the field current hence, to adjust the flux per pole that is what I will do. So, it is like this and suppose this terminals are open circuited nothing is connected here; nothing is connected down or you put it like this put a switch this switch is open, it is like that. I will connect a volt meter here, it is a per phase voltage.

Now, if you increase this value and I f I can change by controlling the field rheostat in the circuit and if field current is 0. You do not expect any voltage because no flux per pole, because induced voltage per phase is root 2 pi f, where is the prime over speed, p ns by 2; that ns is the prime mover speed, root 2 pi f flux per pole kw into N phase is the thing. And this phi I will assume linearity one important assumption I am making assume linearity.

So, that flux per pole is directly proportional to I f and this is the thing therefore therefore, I can with this two things I can say that, E phase is directly proportional to field current, nothing is connected here; no synchronization nothing. A synchronous machine driving it by a prime over this way at a speed synchronous speed. So, that 50 Hertz say if it is a 2 pole machine I must turn it at 3000 rpm, to generate 50 Hertz if it is a 6 pole machine generator I must trun it at 1000 rpm to generate 50 Hertz and so on. Therefore, it will go and this voltage will be induced.

And in the phasor diagram I will say, this is your flux per pole. Sometimes people write f produced by field current, I think those things will not bother you much I can write it phi f, this is phi f. And the induced voltage is here E phase and this angle is 90 degree and both of them are moving with omega speed because synchronous machine will only work when it runs with speed omega. So, this is time phasor. This phi f is also is M f; MM f, MM f and flux are in phase.

Mind you, B and phi are not in phase, B is here flux density phasor flux linkage is there. So, this is either field MM f or phi f flux per pole and induced voltage will be lagging this by 90 degree. This angle is 90 degree and this is called open circuit phasor diagram. Why open circuit? Nothing is connected here, open circuit phasor diagram. What is open circuit phasor diagram? Run this machine it is running as a generator with the help of a prime over run it, excite the field winding to create phi f and this is what it happens and when this phi and I f is constant means phi f and I f is equal to constant because of linearity we will continue with this.

Thank you.