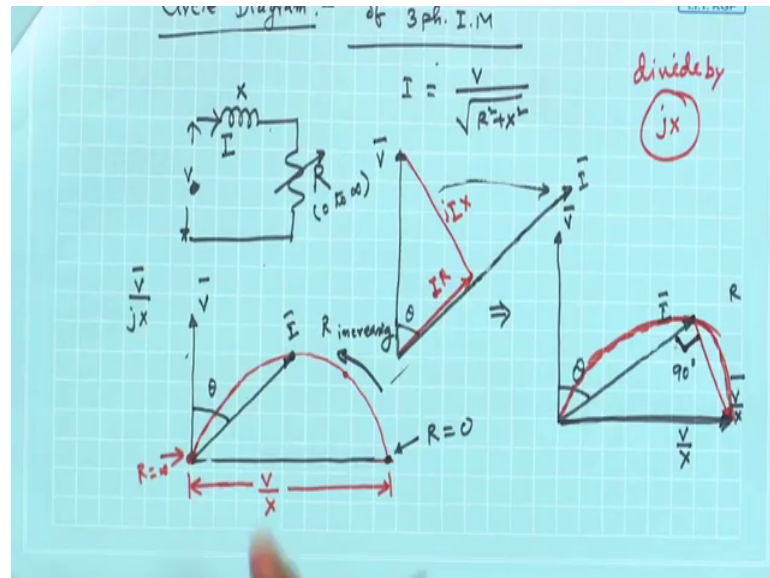


Electrical Machines - II
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Lecture - 52
Circle Diagram (Contd.)

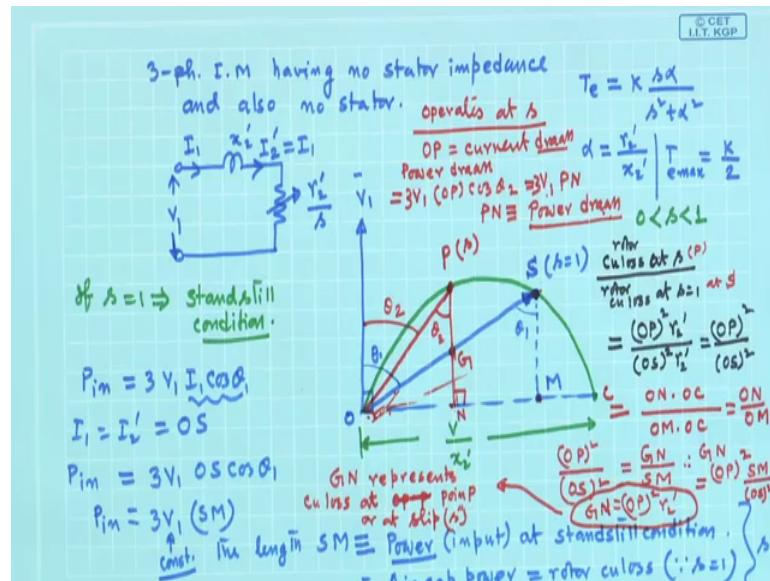
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Welcome to this course on new lectures. And last time, we just started a circle diagram of a 3-phase induction motor. And as a prelude to this we just verified that in case of any R L circuit ok, and if it is excited by a fixed voltage at fixed frequency, and if suppose the resistance is varied from say 0 to infinity, then the locus of the tip of the current phasor will be a circle. And diameter of the circle will be V by X , where X is inductive reactance.

And also note that this point is R equal to 0, because if R equal to 0 that is this is shorted, current will be V by X lagging V by 90 degree, and this point is R equal to infinity. When impedance of the circuit will be infinitely large, current drawn will be 0 and so this will be locus. And this is the position of the tip of the current phasor for a finite value of R in between 0 to infinity, and this we have established.

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Now, this particular thing can be then applied to obtain the circle diagram of the induction motors. So, we will start first with a 3-phase induction motor having no stator impedance and also no stator loss, so that is the equivalent circuit is like this $\frac{r_2}{s}$ at any slip s . And this is the per phase applied voltage.

In fact, sometimes as I told you to get quick estimates of the performance of the induction motor, people may result to this equivalence circuit which is very simple. And for that we found out those things that electromagnetic torque at any slip is equal to $\frac{k \cdot s}{s^2 + \alpha^2}$. And the slip at which maximum torque occurs is $\frac{\alpha}{2}$. This results very important and interesting.

And also $T_{e \max}$ is equal to $\frac{k}{2}$, where k is a constant depends on the per phase voltage, reactance of the rotor, and synchronous speed ω_k . But, for this type of equivalence circuit, I want to record the current I_2 . And I_2 will happen to be then equal to I_1 itself, this is equal to I_1 itself, because I have neglected all the things on this stator side. So, here it looks like depending upon the value of this slip at which machine is operating, the tip of the current phasor will be located. And here it is the same case as that of this were R is varying, as slip will vary. Range of slip is between 0 to 1 we know that [FL].

Therefore, we expect that the circle diagram of the induction machine will be somewhat, suppose this is the voltage phasor V_1 supply voltage per phase. Then the diameter of this circle will be at 90 degree apart like this. And the locus of the tip of the current will be here. So, so this length is v by x_2 dashed. And it will and depending upon the value of this slip, corner phasors will be located ok.

Now, we have seen that for motor operation range of operation I mean range of s is between 0 to 1. Therefore, s equal to 1 corresponds to this starting condition stand still condition. Suppose s equal to 1, if s equal to 1 meaning that stand still condition.

The current drawn from the supply will be suppose located here, suppose this is the current drawn, let this point be called O, and this point let us called capital S. This operate this will be operating point on the circle diagram. And this is the power factor angle θ_1 say. Now, the this point therefore is corresponds to s equal to 1 stand still condition, mind you applied voltage is rated value ok. So, this is the thing. Now, what you do is this, you drop a perpendicular here on this diameter of this one. Suppose, it is S M ok.

Now, now as you can see the input power to the circuit P input from the s supply side will be $3 V_1 I_1 \cos \theta_1$. I_1 in this case is equal to I_2 dashed is the length O S O S. Therefore, $I_1 \cos \theta_1$ this is also θ_1 is your S M.

So, P in will be equal to $3 V_1$, and for $I_1 \cos \theta_1$ that is O S $\cos \theta_1$, it is nothing but this $\cos \theta_1$. And this length will be this will be the length S M S M, see it is a graphical method of solving because, if I know the parameters, I can use this equations to predict the performance for various values of slips. Now, we are trying to tell that you can lessen the burden of computation by once for all drawing a so called circle diagram of the induction motor that is the main point. So, it is that s equal to 1 this is the thing.

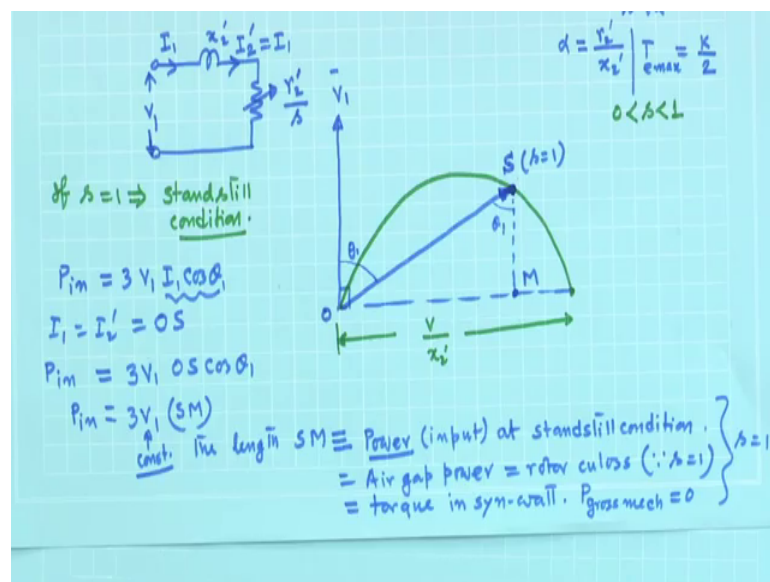
Therefore, as you can see this P in V_1 is constant per face applied voltage. Therefore, the length the length S M is will be also a measure of the power is not. Although, S M is current scale we have chosen, we have drawn this circle based on current. But, if I know what is a O S $\cos \theta_1$ is not that is S M, it will be into V_1 is gives you also power. So, in some other scale, I will chose. And say that the length S M represents the input power in this case. So, this is the power input, power input when at stand still condition at stand still condition fine.

Now, if the rotor is not allowed to move, this will be only the situation. So, this is the total input power, where this power goes this power in what edge will be consumed in r_2 by s only is not s equals to 1. So, at s equal to 1 slip equal to 1, this will be the air gap power, and this itself also will be the copper loss in the rotor. And there will be no output mechanical power, because rotor is not moving. So, everything is fine, input power that itself becomes your air gap power s into air gap power is loss. So, s equal to one, so any way that represents so $S M$ the length $S M$ is the air gap power also air gap power.

And also this is equal to rotor copper loss rotor copper loss, since small s equals to 1. And output mechanical power is 0. How much is the torque produced, torque developed in a induction motor is nothing but air gap power that divided by $2\pi n_s$, n_s is constant.

Therefore, $S M$ also represents, since it is air gap power also in some other scale will represent torque developed or torque in synchronous what is not. So, everything is at s equal to 1, this is the picture nothing else. And output mechanical power 0 in this case $P_{gross\ mechanical}$ is equal to 0 I am sorry $P_{gross\ mechanical}$ is equal to 0 at s equal to 1. This everything is at s equal to 1 ok.

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Now, motor will be operating at some full load slip say may be 4 percent, 5 percent of slip. Therefore, as slip value will be present which is a number less than 1, so whatever slip value, you assume you see that r_2 is increasing is not r_2 r_2 dashed let me write

correctly, r^2 dashed by s ; s is some finite value. And it will then be located somewhere here perhaps.

So, the operating point P is at slip s , this is the value of the slip. At that point what is happening, how much current is drawn from this supply with red color I will write this all this things associated with the general operating point P that is it will be and suppose this angle is θ , θ^2 .

Therefore, OP is the current drawn this length will represent the current drawn by the machine, when the machine operates at slip s general value of slip it operates OP is the current drawn, this length will represents the current drawn. You can see current drawn becomes lesser, because earlier it was OS as stand still condition now it is OP , it is expected to be because s has decreased impedance of the circuit has increased supply voltage fixed. So, this will be the thing.

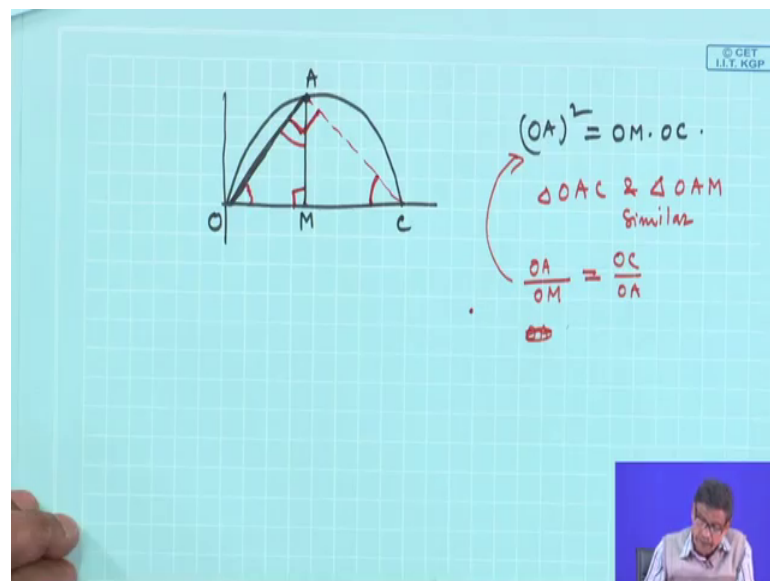
Now, what we will be doing, we drop a perpendicular on this once again on the diameter, and let this be called PN . Then the at slip s power drawn from the supply power drawn from the supply is $V_1 OP$ that is the current into $\cos \theta$ into $\cos \theta^2$. θ^2 is also this angle, because this two are parallel which is nothing but this is equal to $V_1 PN$. So, this vertical draws can be also can be used also to represent power, because it is multiplied by constant capital P_1 .

Therefore, PN in some other scale is equivalent to power drawn is not. And if you want to write total power multiply with 3 as we did here, V_1 is the per face voltage. So, PN is the total power drawn from the supply, so that is also fine. So, SM is the power drawn at s equal to 1, and this is the power drawn, when it is operating at any slip s . And we are talking about an induction motor, whose stated impedance is have been neglected very simplified situation, but nonetheless it will bring out the most important things that we will considering.

And only resistance present is in the rotor, there is not stator resistance. So, after you get this point of intersection, let me call to be G here this point of intersection. In earlier case, I showed that SM was equal to the rotor copper loss. In this case, total input power is this [FL]. Consider this triangle, and this triangle, this triangle, and bigger triangle, and this smaller triangle. These two triangles are I mean similar is that clear [FL].

Now, copper loss let this diagram be present, otherwise it will be difficult [FL]. I will take this ratio copper loss, copper loss at s copper loss means rotor copper loss divided by rotor copper loss at s equal to 1 that is corresponding to capital S. This ratio will nothing but be equal to this length O P square, because that is the current into r 2 dashed divided by O S square copper loss at s equal to 1 into r 2 dashed, this will be the thing that is it will be equal to O P square, this length square by this length square is not it will be like that [FL].

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Now, we will use one result of geometry which we have done in school days, I will just very quickly tell that is if you have a circle like this, take any point A on the circle. And drop a perpendicular on the diameter, and this is an arc, this O A square this length square can be shown to be equal to suppose this point is O is equal to this point C, it can be shown to be equal to O M into O C.

And the proof is very easy to prove. Suppose you join this two points, then this big triangle, and this small triangle O A C, and triangle O A M, they are similar because of the fact this is 90 degree. And this is a common angle in both of them it is there. So, the third angle will be also equals, so they are similar they are similar, where is the third angle; third angle is here this and this.

So, ratios of the opposite sides of the like angle will be equal that is what similar triangle means, therefore take the bigger triangle first. So, opposite side of this is O A O A

divided by this opposite side is OM , these two angles are same OM . And this must be equal to a I have taken first the bigger triangle $O A$ by $O M$. So, then opposite sides of this right angle in both the triangles are $O C$ and $O A$. So, $O C$ by $O A$, and therefore it gives you this results this results $O A$ square is equal to $O M$ into $O C$. So, this is a standard result nonetheless for ready reference (Refer Time: 21:33).

So, after you get this, we were here we were taking the ratio of the rotor copper loss at operating point P that is with any slip s that is at P , and this is at S capital S point. And we found that r^2 dash cancels out, so it will be the ratios of $O P$ square by $O S$ square. Now, the thing is that this $O P$ square, if you call this point to be C . $O P$ square will then be because this is a perpendicular drawn from P upon the diameter, so $O P$ square will be $O N$ into $O C$ divided by similarly $O S$ square, this is also a chord from this I have drawn a perpendicular, it will be equal to $O M$ into $O C$ here. So, this ratio will be $O N$ $O M$. So, $O N$ by $O M$ that is fine.

Now, this $O N$ by $O M$ is nothing but $G N$ by $S M$, because these two triangles these two triangle and this bigger triangle, there similar for obvious reasons these are parallel and things like that. So, in a triangle you know this is to this will be equal to what $G N$, because this is the common angle $G N$ by $S M$ simple, so this is the thing. Therefore, what we have got, we have got here is $O P$ square by $O S$ square is nothing but $G N$ by $S M$. So, what is $G N$, therefore $G N$ this length this line segment $G N$ will be equal to $O P$ square into $S M$ divided by $O S$ square is not, it will be like this.

So, $G N$ is nothing but $O P$ whole square into capital $S M$ by $O S$ square. Can you tell me what this quaintly is $S M$ by $O S$ square? $S M$ is what, $S M$ is copper loss rotor copper loss, and that divided by current square is nothing but r^2 dashed. So, so this is nothing but $O P$ whole square into r^2 dashed. Copper loss divided by magnitude of the current square, so it will be like this, therefore this is equal to $G N$.

Therefore, capital $S M$ this length represents the rotor copper loss at s equal to 1 slip equal to 1, capital $G N$ is $O P$ square into r^2 dashed, then represents the copper loss at slip s . So, so from this we conclude $G N$ represents copper loss at operating point P , at point P or at slip s .

So, the interesting point is, I have drawn this circle diagram, at stand still condition there is no mechanical power output. And all power that has been drawn is equivalent to air

gap power as well as that air gap power value, where it will go there is no mechanical power output everything will be lost in the rotor copper loss that is sM (Refer Time: 26:40) square into r^2 dashed.

Now, when it is really operating at some slip s , current drawn is OP that is what I get from the circle diagram. Then from the operating point P I drop a perpendicular PN , it cuts this slip equal to 1 line at point G . Then I find that when the machine is operating with some slip s , GN is nothing but the copper loss at the slip in the rotor circuit that is the only place where copper loss takes place. But, what is PN ? PN is the total power drawn from the supply total power drawn of which GN is the rotor copper loss.

Therefore, I must conclude that PG length PG must then give me the gross mechanical power output that is the conclusion that is when it is operating at this point any slip s drop a perpendicular, it cuts it here. And this I know represent rotor copper loss, and total power drawn from the supply is represented by PN , therefore the input power minus the rotor copper loss must give me the mechanical power output is not.

So, this is the very basic thing, and based on that we will next develop the circle diagram of a induction motor, where all the parameters are not neglected stator, rotor, those parameters, I will take into account. And draw the complete circle diagram of the induction motor in the next lecture.

Thank you.