Electrical Machines - II
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Lecture - 51
Torque Expression From & Simplified Equivalent Circuit introduction to Circle Diagram

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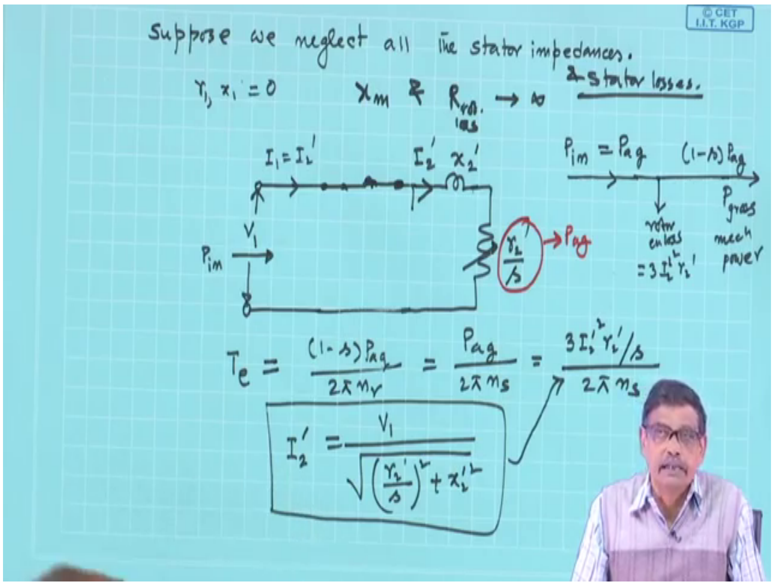

Welcome. So, we were discussing about somewhat simplified equivalent circuit of 3-phase induction motor, which are very much used I mean and so that you can quickly get the idea about the performance of the induction motor. So, what I told is that you find out the equivalent circuit.

Assuming that stator impedance is very small that is r 1 x 1 series part of this stator impedance is small r 1 x 1 is vanishingly small 0. And the parallel branch x m and loss representing rotation of the machine because of the rotation only eddy current loss will take place. And therefore, unless there is rotation eddy current loss will it take place, at stand still condition it will take place, but anyway that will be there all the time with respect to stator iron, because with respect to stator iron body the frequency is f induced voltage on this stator winding.

But, anyway we are assuming that is to be high and x m is high, so the equivalent circuit comprises of r 2 dashed by s and x 2 dashed, then what happens? Then obviously, the input power that is no stator loss that power itself will be the power that is V 1, I 1 cos theta whatever is the input power factor must be equal to I 2 dash squared r 2 dashed by s that is the power in the r 2 dashed by s which is nothing but P a g. And therefore, electromagnetic torque is P a g by 2 pi n s and I 2 dashed is this one very simple.

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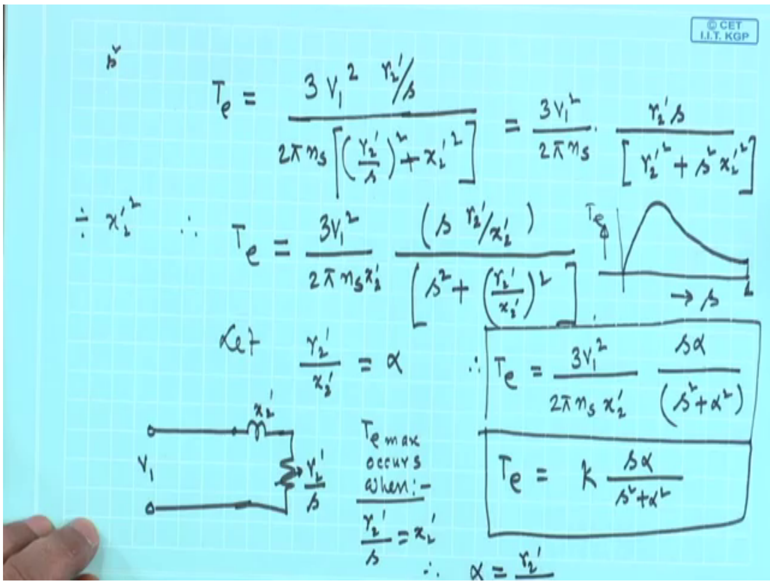

So, we proceed from this, therefore once I 2 dash is known, then I substitute this I 2 dashed here. So, to get the torque developed by the machine as T e is equal to 3 I 2 dash square that is V 1 square by r 2 dashed by s whole square plus x 2 dashed square. And by 2 pi n s is there into in the numerator r 2 dashed by s, this is the expression of the torque developed by the machine [FL].

Now, what we do, we do a little bit of simplification, so that we can have a very nice formula to remember. What we will do is this; we will first this 3 V 1 square by 2 pi n s is sort of constant. And then I will multiply both numerator and denominator by this factor s square numerator and denominator, so it will become r 2 dashed into s. And divided by this will become r 2 dashed square plus s square x 2 dashed squared, this it will become one s goes in the numerator.

Then what you do you divide by divide both numerator and denominator by x 2 dashed square ok, therefore T u electromagnetic torque developed by the machine will be 3 V 1 squared by 2 pi n s. If you divide like that, it will become s square plus r 2 dashed by x 2 dashed whole square s square I am dividing by x 2 dash square, so s square plus this.

And on the top it will be s into r 2 dashed by x 2 dashed squared and that is x 2 dashed. And another x 2 dashed, I will write here clear this is the thing.

So, x 2 dashed squared one x 2 dashed I skip it. And I say let r 2 dashed by x 2 dashed to be equal to alpha some alpha this ratio is. Then therefore, torque will be equal to 3 V 1 square by 2 pi n s into x 2 dashed, this is the constant part. And this can be written as s alpha by s square plus alpha square.

Now, this is the formula which can be very easily remembered and people write T e is equal to some constant, it is independent of slip etcetera synchronous bit constant x 2 dashed stand still rotor leakage reactance refer to stator constant, so some k into s alpha. And now formula is simpler than this to remember product of s into alpha divided by s square plus alpha square.

Now, in this machine where I have neglected, this stator impedance that is the equivalence circuit, I have assumed all are not there applied voltage is V 1. And here it is only x 2 dashed and this is r 2 dashed by s is not. Now, here once again suppose we ask our self, so at any slip I will be able to calculate the torque.

And if you plot it s versus T e, it will be same shaped like this, this is T e and this is s. And s varies from 0 to 1, it will be of this nature only similar nature. And let ask ourselves that at what slip maximum torque occurs, once again it is a circuit, where this resistance is being varied as slip changes. So, and power when it will be maximum in r 2 dashed by s, then also torque will be maximum. Therefore, T e max occurs T e max occurs, when by applying maximum power transfer theorem, when r 2 dashed by s is equal to x 2 dashed.

Therefore, this slip at which maximum torque occurs is r 2 dashed by x 2 dashed, it is consistent with that that is why I assume this to be alpha, because I earlier defined this slip at which maximum torque occurs is r 2 dashed by x 2 dashed in this case. Therefore, this is a formula worth remembering for some gate exams or some competitive exams, problems are often set assuming stator impedance is neglected as well as stator losses neglected.

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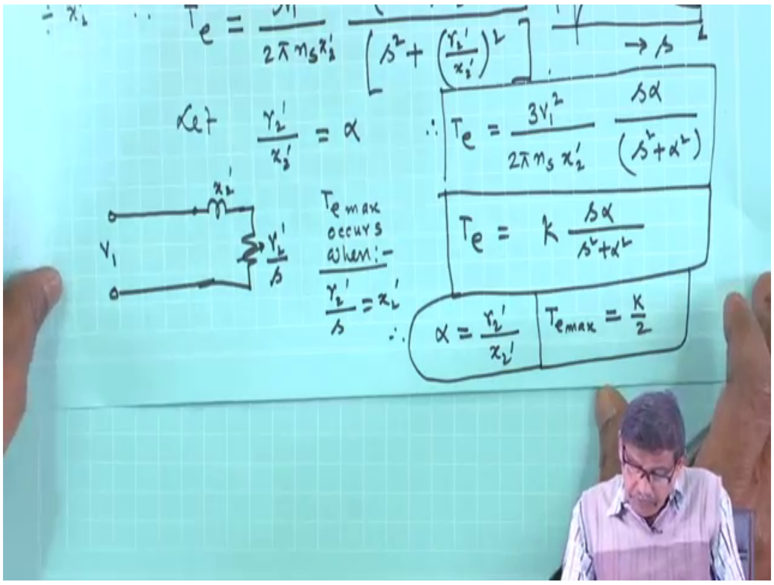

So, with that in mind we have got this. And here once again you see, the value of T e max maximum value of the torque will be for x write alpha if you do that, it will be k by 2. So, these are the three things, maximum torque is this k divided by 2.

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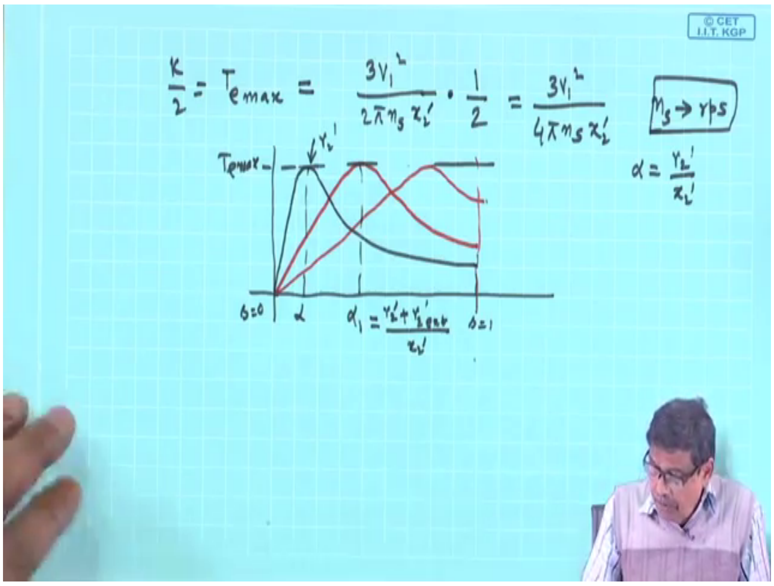

And what is the expression of T e max; let us write that T e max in terms of equivalence circuit parameter. In this case, it is only r 2 dashed and x 2 dashed, it will be 3 V 1 square by 2 pi n s into x 2 dashed that is this k into half, because s equal to alpha, it gives you half. Therefore, it will be 3 V 1 square by 4 pi n s 4 pi n s x 2 dashed, mind you n s is in r p s, do not put r p m, then it will be wrong, so that is the thing.

Therefore, you can use this equivalence circuit as a first approximation to find out the machine performance. And you can easily show that the torque slip characteristics, what I got earlier can be just drawn. So, s equal to 0, this is with r 2 dashed. Similarly, the sleep at which maximum torque occurs is r 2 dashed by x 2 dashed. And T e max is k by 2, which is this level, it will remain same T e max. So, if we vary rotor resistance, newer torque slip characteristics will be T e max will remain same. And it will be all the things which I got earlier is obtained, in this same fashion.

So, this is the original alpha. And this will be some alpha 1; alpha 1, which will be something like r 2 dashed plus r 2 external dashed by x 2 dashed and so on, I thing you have got, so starting torque can be improved. And of course, it can be done only for slippering induction motor k j induction motor, addition of external resistance is out of question that is it ok.

So, remember that in every simplified form the induction motor performance can be evaluated, where is have neglected stator impedance, just rotor impedance are kept in this circuit. And you can conclude everything which I conclude earlier. And this is often referred to particularly to understand the problem of starting, what is the point of considering, how much starting current the machine is going to draw. When full supply voltage is applied across its terminal, to get the idea of that it is only state rotor impedance, you considered consider only because r 1 x 1 is already small.

Therefore, that will be the largely the that will decide, what will be the starting current drawn by the induction motor, when full voltage is applied across it and machine is stationary, I will discuss that. But, to study those things, I will only refer to this equivalent circuit, although it does not mean that I cannot incorporate the other equivalent circuit ok. Starting these things I will do a little later. And use this formulas extensively that is why I told how it is to be derived [FL].

Now, I will tell you another interesting thing that is this current of induction motor with all its stator impedance, magnetizing current etcetera. Can be see, what exactly we are doing here? Suppose, I say the problem will be like this, this is the motor, these are the parameters; find out how much torque it produces? So, I will calculate the value of this slip, and substitute that value of the slip in the appropriate formula to get the torque, how much mechanical power it produces the amount of torque into 2 pi n r all this things I can do at any value of slip, you want me to do them do that.

Earlier days people used to for a given message, people used to draw what is known as a circle diagram of a given induction motor. And there for every value of slip the things are already present there, you will give me the value of this slip from that circle diagram of the 3-phase induction motor. I will be able to tell how much is the rotor copper loss, how much is the torque developed, how much is the efficiency of the machine without calculating anything that is I will use this circuit parameters or some test data of a 3-phase induction motor to draw a circle diagram once for all.

And then you give me any value of this slip, I will go to that circle diagram and tell see this will be the at full load this will be the power factor, at full load this much will be the efficiency, you read something from that circle diagram.

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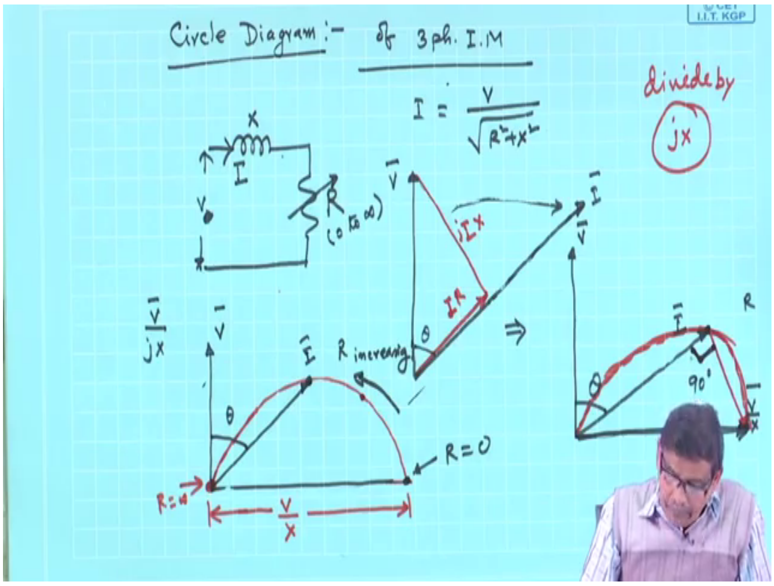

So, my next topic is about the circle diagram of a 3-phase induction motor circle diagram [FL]. Circle diagram of the 3-phase induction motor comes from this that after all you see in this simplified diagram itself. There is a series R L circuit in which as a slip changes you are changing the value of the resistance. So, it can be easily shown that tip of the current phasor drawn from the supply that is I 2 dashed, it will lie on a circle.

So, it can be easily shown for example, consider an R X circuit, which is true in general for any R X circuit R and X. And suppose, I say that resistance is being varied with supply voltage fixed V 1 why V 1 V and suppose the current is I. So, obviously for and this resistance I will vary from 0 to infinity, then obviously the current value I magnitude of the current is V over root over R square plus X square.

When I is 0, what are the things I know, when R is 0 current will be V by X correct, because impedance is X only. And this current phasor will lack this supply voltage by 90 degree. And when R is infinitely large, I can easily see impedance of the circuit will be infinitely large, so current drawn will be 0. So, this two points I am pretty sure, in between of course also I am sure if I calculate, I is equal to V by this Z that R, I will put X and I will say this is the current. And the current phasor, it will lack the supply voltage by an angle tan inverse X by R, but you have to do it for all the values of R.

So, how this can be elegantly represented in a circuit circle diagram that I am telling [FL]. Suppose, this is your supply voltage figure V, this the supply voltage figure. And suppose R is having some finite value, then the current I know it will be lagging this, this is V, it will lag this along this line the current phasor exists high is not this is the current. And this is the power factor angle by which it will lag. And I know this one that the drop in R and X will be added.

For example, you add I R, then you add I X, this is I R and this is I X j I X is not this way. And this is the power factor angle of the machine ok. Now, what you do in this one, you see that is a current phasor, but it is I have not made it integral part of this diagram. In this phasor diagram, what I will be doing is let us divide these are all vectors or phasors, you divide all these vectors in this triangle by j into X divide all the sides.

So, as you divide what it will be, it will be like this. This V when you divide by X, its length will be V by X and it will be 90 degree this clockwise. So, your this was your original V that I will draw V no doubt. But, this V by j X V by j X, it will now be here 90 degree, this is V by X j X, this length is V by X. So, this length is divided by this.

Similarly, this length will be divided by j X is that. So, if you divide this by j X this current length, it will rotate by 90 degree and it will be like this one will be simply this I that is all. So, this side will become this here I, how it is obtained j I X by j X, all this sides of this triangle I am dividing by this number j X.

So, everything will be scaled down appropriately by a factor X and rotated by 90 degree. Since, you are dividing clockwise rotation, so this is I. And another side is left that is this I R I R by j X will also rotate by 90 degree, it will become like this. And since all the sides I am dividing by X, so that is side will be has to be this one nothing doing, because after all the sides are given a twist by an angle of 90 degree that is this triangle has been rotated by 90 degree. So, this angle has to be 90 degree that is all.

Therefore, after dividing its side by j X, I get I here. And these I this angle has to be theta that is already known no doubt about that. And this side is V by X. Once you have got this two sides, the third-side has to be the transformation of this side and it will fit in there. With that side we have nothing to do, except that to say that this angle will be also 90 degree.

Now, this is for a particular value of R any arbitrary non-zero value of R is that. Now, the question is the for if you change the value of R, what is going to change? This side V by X is not going to change, this angle is not going to change, only the value of I will change. Therefore, as you vary R, this will be the diameter of the circle, because any on the diameter any to segments you draw, it has to cast an angle of 90 degree no matter where it is.

Therefore, the circle diagram for any other value of this, I can say it will lie on this circle I think it is badly drawn, but I think you have got the idea, it has to be laid. If I draw it nicely, it will be like this. This is your supply voltage, and this is the circle draw the circle first and tell that this is V by X, this length. And for any value of R, this is the thing this will be the tip of the current, it must lie on any points on this circle depending on the value of R. And this is the power factor angle theta that is all diameter is V by X ok.

Now, since I am bearing R and the value of R is from 0 to infinity I will vary, so where on this tip of the current phasor, which will lie on the perimeter of this circle is R equal to 0, it is at this point. This point is R equal to 0, because R equal to 0 means, it is shorted purely inductive V by X, this is the void, intermediate value here. And where is R equal to infinity, it is this point this point. Therefore, to this circle diagram I must attach this thing R increasing from 0 to infinity and the current phasor will be like this.

So, for example if you want to solve a circuit problem, I say that R will be varying between 5 to 10 calculate the current, you draw the circle diagram. And then identify the portion which is valid there, and say tell me any value of R, what you simple do is this, you calculate the power factor angle fix up this point and tell this is the angle. Instead of each time calculating V by root over R square plus X square and getting the results ok. So, this is called the circle diagram of an R X circuit.

Now, the circle diagram of a 3-phase induction motor in its simplest form can be obtained, when the circuit is like this. In this circuit, you see slip decides at what is the operating point ok. Slip value can change from 0 to 1, when slip is 0 that means, this resistance is infinity and when slip is 1, it will have a finite resistance. Therefore, for different values of slip, the locus of the tip of the current of I 2 dashed, we expect will also lie on s circle.

So, we will continue our discussion with the with the circle diagram of a 3-phase induction motor. Why it comes, it is because of the fact I would like to calculate operate, I mean calculate the performance of the induction motor at a given value of slip. Therefore, why not better draw a circle diagram, where for all values of possible values of slip things will be there, I will be able to calculate current, power drawn from this supply. And of course, I have to identify which of length of that circle diagram will represent output power, input power etcetera. And those things we will discuss in our next class.

Thank you.