Electrical Machines – II
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Lecture – 37
Torque Angle and Torque Expression

Welcome to this new lecture on Electrical Machine course. Last time we were discussing about one very important thing that is, if there is a stator field which is sinusoidally distributed and if there is a rotor field which is also sinusoidally distributed. And I am not bothered from where that B S and B R, this stator field and rotor field as the resultant, ultimately it is two sets of magnetic poles created on the stator. And we told you that how to calculate then the torque, which will be acting on the rotor.

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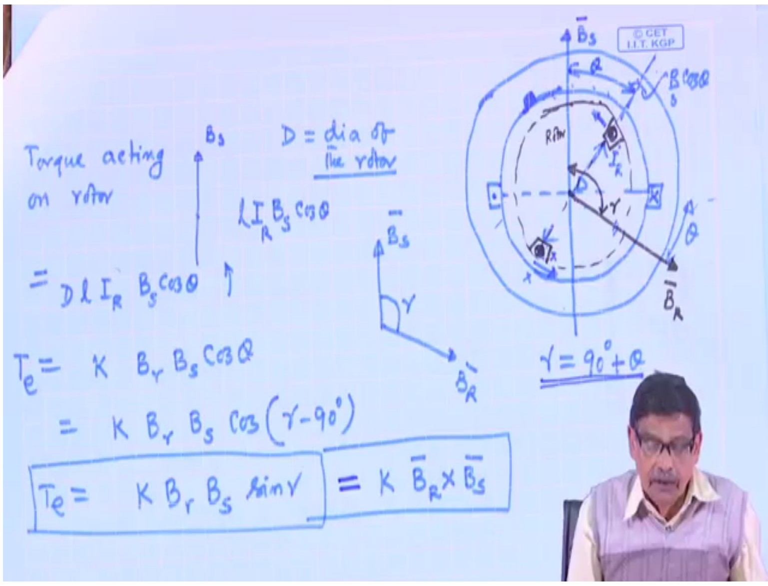

So, the last slide, in our last lecture was like this that is suppose to understand in a better way, I have assumed this a DC current here cross dot on the stator produces B S, which is actually B S cos theta if you measure theta from this reference ok.

Similarly, if you have a on the rotor a pair of slots and this side current is cross and this side current is dot also assume DC it does not matter. And the field created will be perpendicular to this line is B R and suppose this rotor is held fixed. Let us try to understand this point, I want to calculate under this condition what is the torque experienced by the rotor. Now, to do this I will assume this angle gamma between B S and B R that is B S and B R and B S can be represented as a vector where the peak occurs you can understand, because at any angle theta its value will be B S cos theta the way we resolve forces. Similarly, similar thing happens here also. So, B S is sinusoidally distributed and also B R is sinusoidally distributed. Let us assume that.

Under this condition and this current on the rotor is suppose I R then the torque on this individual conductors we calculated which is D l I R B S cos theta the value of B here. And the direction I found out by left hand rule, on both the conductors and then torque will be or the couple will be multiply with D like that and it can be written K B r B s cos theta ok.

Then we note that this angle is theta means this angle is theta. So, it cos theta can be replaced by from this relationship gamma is 90 plus theta, therefore, gamma minus 90 degree which is equal to sin gamma K B r B s sin gamma. B r B s are the peak values of the B distribution of rotor and peak value of the stator field and K is a constant of the machine, D l and other constants which may have relative permeability things like that.

So, anyway, but the essence of the thing is it is like this B r B s sin gamma. Then I told that this can be expressed as a cross product of this two space vectors B R and BS, because B R cross B S is nothing but magnitude of B r magnitude of B s into sign of the angle to be measured from B r to B s. And this angle I told gamma so it will like this, so, we start from that.

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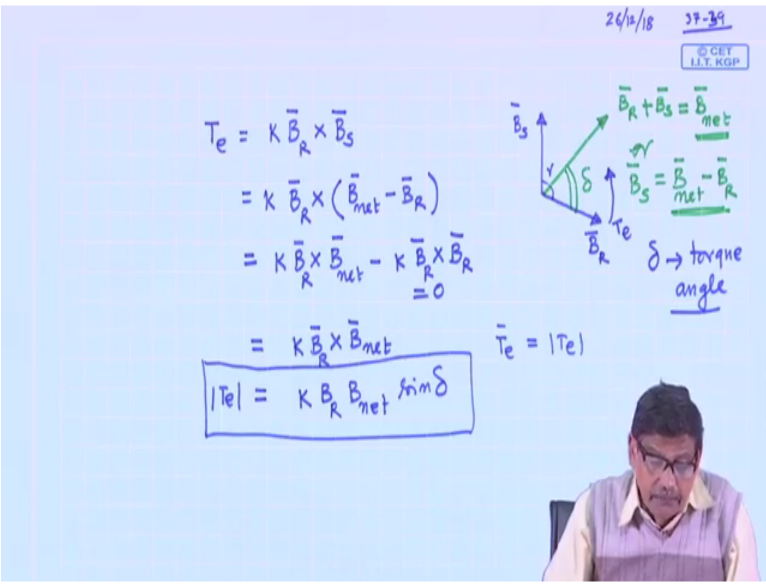

So it can be told that the electromagnetic torque is equal to some K B R cross B S that is the thing [FL]. Now so what we have here is this is your B S, I will not draw this stator rotor no point in drawing now, we have understood what we are doing. This is B R, and this is gamma, this is the thing. Now, you see this expression of the torque can be also expressed in terms of the resultant field. What is resultant field? Resultant field of B S and B R will be somewhere here B R plus B S.

And let this resultant be called B net B. In the air gap you can imagine that stator field is there, rotor field is their or you can say there is one field which is B net this is one and the same thing. And I will calculate the resultant because this two are expressed in terms of vector. So, they can be vectorically added and you get the B net is this one. And this, this angle; this angle I will call delta, this must be very clearly understood, the angle between B R and B net is called B delta.

Now, because so what we want to do is I want to express the electromagnetic torque developed or acting on the rotor in terms of rotor field and the net field ok. So, we note or from this B R plus B S is equal to B net, I can get B S as B net minus B R and I will put it in place of B S this thing. So, this will be then K B R cross vector multiplication cross multiplication B S, B S is nothing but B net minus B R.

Then if you open this bracket, you will be getting K B R cross B net minus K B R cross B R, I will get this two terms of which this second term is always 0 A cross A is 0, we know from vector algebra. Therefore, this essentially becomes then K B R cross B net only.

And if I define this angle to be delta, I will simply write this cross product is essentially means what magnitude of B R, magnitude of the torque will be K B R B net into sin delta A cross B is magnitude of A magnitude of B into sin of the angle between A and B. So, this is this thing. Of course torque is a vector and that vector if you write like this, it will be the magnitude of T that is this term into a unit vector perpendicular to the plane of the paper indicating, the torque is acting in the anticlockwise direction anyway, but what we want is the magnitude of the torque.

So, henceforth what we will do is this in any machines, where there is a rotor, where there is a stator, both of them are carrying current, then you identify where is your rotor field, you identify where is your stator field at a given point of time. And then at that time, you find out the resultant of this two that is B net. If this angle is delta, then I will say ok, magnitude of the torque is strength of B R, strength of B net into sin delta. And the direction is from B R to B net and that is anticlockwise direction in this case.

So, torque will be acting always in this direction. So, we drop that unit vector perpendicular to the plane of the paper. Let us not go to that complexity. Because it is clear from this equation, I now know that rotor we will try to move from vector B R to B net that is it. So, this is a result which may look like a very unique problem we are dealing with that is there is a DC currents, there is BR, there is B S, B net, but it is it can be generalized why not.

This now tells me that no matter whether B R is created by a DC current or not, B S is created by DC current or not, whether they are moving or not. But at a given instant of time if you can position correctly BR, B S and B net, you can be rest assured at that instant torque will be strength of B R strength of B net into sin of this angle between this two that is why delta is called torque angle, which is the angle between not B R and B S that is gamma B R and B net. Now, after knowing this, now we will once again come back to induction motor and try to apply this to understand what to do.

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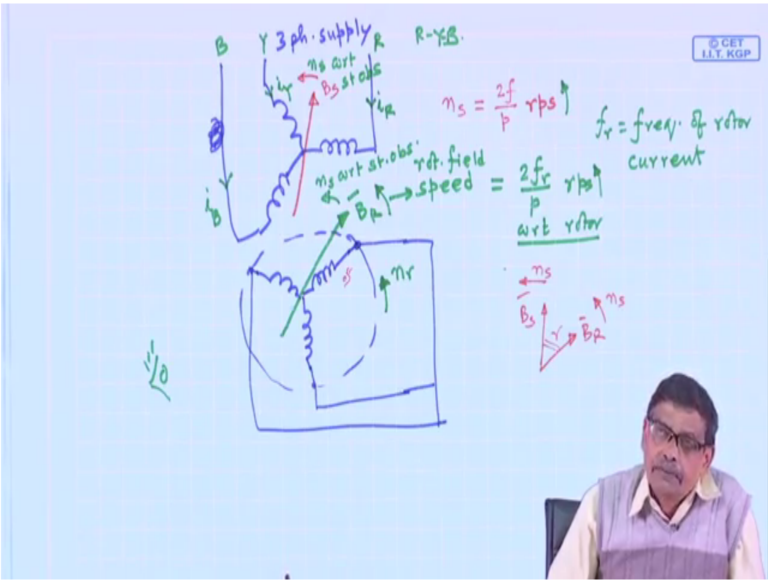

Because last time I told you in a 3 phase induction motor if you recall that three this is the discussion way, you are doing suppose, you have a balance 3 phase coil here, where this terminals are connected to 3 phase supply; 3 phase supply. And I will energized this three the windings of the machine from a balance 3 phase supply, so, it will draw current i R, suppose this is i Y and this is i B ok.

And supply R is also connected here, supply Y, supply B and phase sequence is R Y B. If you do like that, then we have discussed at length result will be a stator field B S, but which is not now stationary it is moving from leading to lagging phase that is in this case in the anticlockwise direction B S ok.

And then I told you that there is also a rotor has got a 3 phase winding just like the stator had. And this rotating field moves stator field moves with a speed n s, which is equal to 2 f by p rps mechanical speed and there is a rotor as well. And rotor terminals if it is open circuited forget about any torque, because there is no rotor field, because there is no rotor current, but under the scenario, when this rotor terminals are shorted ok.

And then of course, there will be rotor currents and rotor then two will be producing a rotating magnetic field, which I am drawing with a green vector B R and it will also move along this direction. What is the speed of this rotor rotating field? It will be speed is rotor rotating field speed rotor field speed will be equal to 2 f r by p. What is f r? F r is the frequency of the rotor current f r is the frequency of rotor current and so much rps.

And if stator field is moving in this direction, rotor field two will be moving in this direction, we have also got that result from our last time discussions. Therefore and rotor we will also moving with a speed n r all speeds are with respect to a stationary observer all speeds. So, I am not writing, but it is better you write that. So, all speeds with respect to stationary observer and it is like that. Now, B S is there, B R is there and both of them this; this; this one 2 f r by p I am sorry rotor field speed 2 f r by p if you write it is with respect to rotor.

This structure which houses the widening, but with respect to stator this speed will be once again n s. So, with respect this speed, if it is n s, this is also n s with respect to stationary observer, this is with respect to stationary observer. Therefore, we have got two fields B S and B R wherever it is I am not sure where it will be, but it will be something like this. And both of them are rotating with a speed n s with respect to a stationary observer. Therefore, the angle between this two which I was calling gamma will remain constant is not that is how the things will go. And we would expect the machine motor to develop some torque because torque after all depends upon K B r B s sin gamma.

And gamma is there which is not time varying and B S B R is there, but leaving with B S and B R alone is not a problem, but it is always somewhat complicated in the sense that suppose, I am sitting on a stator coil what will be the induced voltage under this scenario, when the rotor is closed rotor is carrying current, stator is also carrying current then induced voltage in this coil, he will see two fields are moving B S, B R.

So, for B S you have to calculate induced voltage; for B R, you have to calculate induced voltage and this two will not be in phase then you have to add those two phases of course, they will be of same frequency f no doubt, but you have to do those things. But instead what is suggested this B S and B R, I will add, because I can add them now it, it is meaningful, because the angle between them is constant, I will add them.

And B net I will get resultant field in the air gap and due to B net you calculate the induced voltage in one stroke you get everything. Similarly, the rotor induced voltage mind you it will be, because of B S as well as because of B R, because if you are sitting on the rotor what you will observe B S is moving with a speed n s minus n r is not. And rotor it is moving with a speed also n s minus n r is a an observer sitting on the rotor what he will concluded. He will conclude that sitting on the rotor; rotor itself is moving with n r, he will say that the stator field with respect to him, the rotor field is moving with a speed ns minus nr and stator field n s minus n r.

Therefore, there he has to calculate the induced voltage both for B R and B S separately then add them up and so on, but I will add this two and get the net field. And I know B R and B net, I will concentrate upon and I will involve delta to indicate how much torque it will be produced so that is the whole idea is that point clear [FL]. If that is clear then let us see, what is going to happen [FL].

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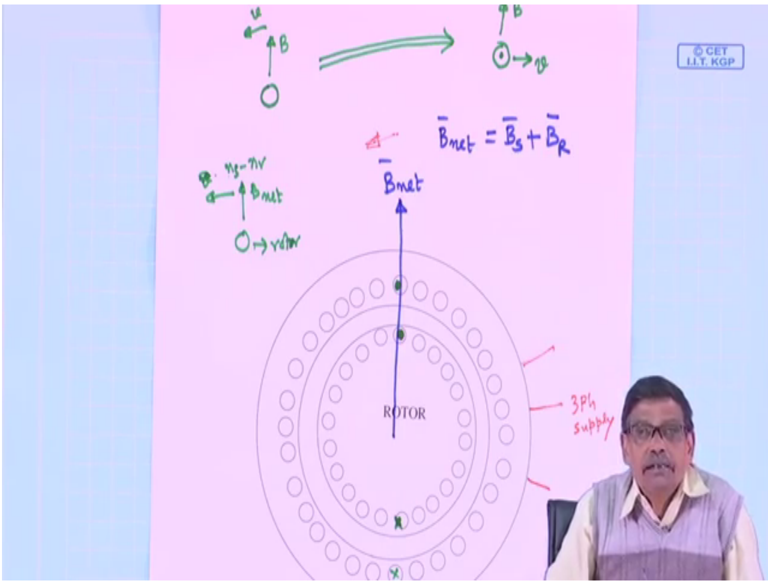

I have made one template so that, I do not take too much time of drawing. Now, let us come to the actual machine, now here is a machine, this part is stator you know and it houses a 3 phase windings I have not shown which one is R 1, Y 1, B 1 I have followed that rule and got these windings. But what I know is this these are the slots; slots also I have not drawn just to indicate the conductors are important. These are coil sites or whatever way you call it. Similarly, here is rotor it also houses a 3 phase widening and this is the air gap. And here also it is a 3 phase widening, but symbolically I have shown what is there after all whether it is 3-phase, 2 phase in the slots there are conductors.

Now, look at the argument suppose. So, from this machine three terminals comes out from R 1, Y 1, B 1, you connect 3 phase supply 3 phase supply, then the result is there is a rotating field produced by stator. And similarly this rotating field will induced voltage in the rotor and suppose rotor is closed, rotor two will produce a rotating magnetic field that is the idea ok.

And both of them are rotating with same speed with respect to any observer you like with respect to rotor of course speed will appear difference, if you are sitting on the rotor you will tell stator field is moving with a speed n s minus n r rotor field is also moving with a speed n s minus n r.

If you are standing outside stationary observer, you will conclude stator field is moving with a speed n s rotor field is also moving with speed n s. Therefore, the angle between them is constant. And therefore, this two fields can be added vectorially and the resultant field is B net, which is equal to whatever will be the stator field and rotor field I am not bothered, but I know this much B S and B R if it if they are added, there will be a net field and that two will be moving with synchronous speed.

So, we start our discussion from B net listen carefully what I am telling is suppose at a given instant of time this is my B net, it is moving with a speed n s suppose, it is positioned here clear. So, this is my B net. And I am certain about this points that at this instant when B net; B net is moving like this no doubt when it is vertical like this at that instant what is the direction of B net is the peak value of B net.

Therefore at that instant I am sure about one, one thing these conductors, which are facing this B net magnitude they must have maximum voltage induced, because (Refer Time: 24:50) is this story, no matter whether they belong to R phase, Y phase, I do not care. But I am sure about one thing that when B net is here maximum voltage will be induced in these conductors as well as in these conductors it as to be.

Now, the question is what will be the polarity of this voltages. So, here is a situation where you have a stationary conductors stator coil if you look at. And you have a field like this which is moving. And I want to know what will be the induced voltage in this conductor, but do not try to apply right hand rule. Now, because the rule says that if a conductor velocity, if it is v and if it is B like this, then apply right hand rule and get the polarity of the voltage that is this way.

If your B is there, if conductor is moving from left to right v, then you apply this right hand rule to get this dot. But here situation is different in which way it different conductor is not moving. And B is moving with a velocity say v tangential velocity and tangential velocity no rpm n s etcetera v, I have to calculate. But fortunately what happens is this, the relative speed is v to know the correct polarity of the induced voltage, you translate this problem to this.

You imagine that your B is stationary and conductor is moving in the opposite direction, conductor was stationary earlier B was moving like this, now to get the correct voltage and polarity it is the relative speed which matters. Therefore, you translate this problem to this what is this assume B stationary and v is moving to rights then apply right hand rule to get dot that is the whole idea, just do not apply right hand rule, here and say it is cross, because conductor here is stationary stator conductor how it can.

Therefore, there will be induced voltage whose polarity will be this will be dot whichever conductor is there R phase, Y phase, I do not matter bother, it will be this. Similarly, whichever rotor conductor is here that will also dot, because ns with respect to rotor conductor only change is rotor conductor if it is rotor. Only thing is, if you are sitting on it, you will say B net is moving with some other v corresponding to n s minus n r phi d n is the tangential velocity.

So, relative speed here it is n s minus n r. So, corresponding to that whatever is the tangential velocity and then since n s is greater than n r I told you. So, its polarity of the induced voltage will be also this is dot, this is dot. And for obvious it is in the other side whichever conductor is here this will be cross, cross like this. So, this will be the polarity of the induced voltage in that. So, please be with me in this discussion, which holds the key to understand, what happens to the resultant field, which I will discuss in the next class.

Thank you.