Electrical Machines - II
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Lecture - 36
General Expression of Torque in Terms Stator and Rotor Fields

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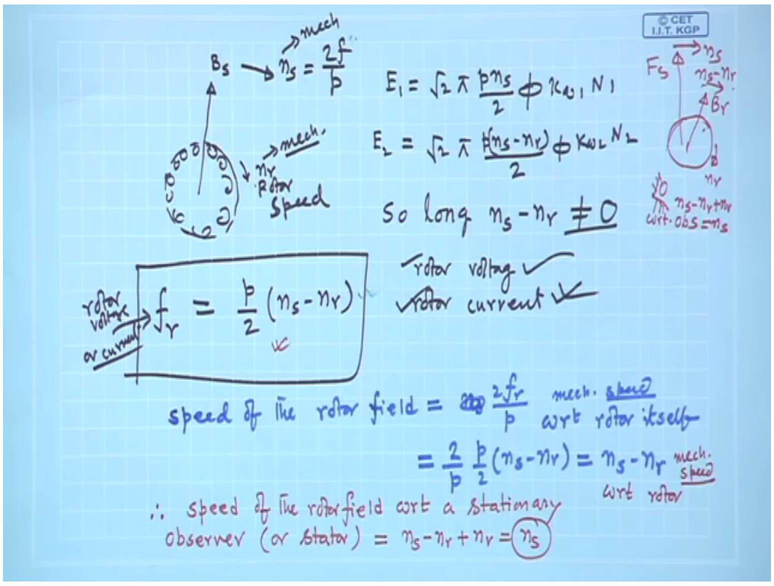

Welcome to this lecture on Machines, Electrical Machines.

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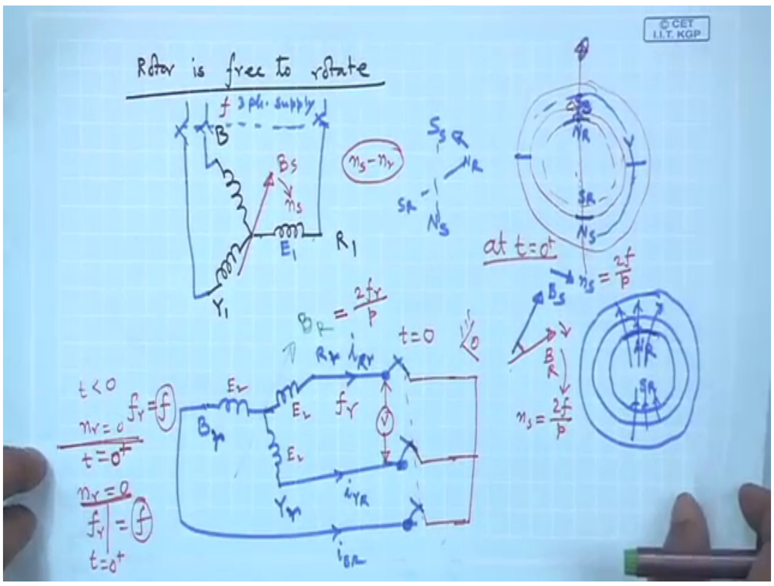

And in our last class, we came up to this that is you recall that at t equal to 0 we first observed what is going to happen and we told you that frequency of the rotor current are one and the same. Rotor is yet to rotate, but it is carrying current at t equal to 0 plus it still remains 0 because of its inertia and rotor frequency at 0 plus is p by 2 into n s minus n r which is p n s by 2 and then, it is supply frequency f.

At least one thing we have assured, we assure about that the stator field and rotor field with respect to a stationary observer. He will conclude that both of them are rotating with same speed, synchronous speed so-called at t equal to 0 plus and therefore, it is excepted there will be a constant angle between these two and perhaps the constant torque will persist and rotor will accelerate. And, rotor will accelerate in the same direction as that of this stator field because it is only in this direction the cause for which the rotor currents or rotor voltages are due will reduce. That is what Langer's law tells us.

So, rotor cannot but rotate in the same direction as that of this stator field, then I am telling that that t equal to 0 plus things is now over time has elapsed quite a bit. So, that rotor as started rotating in the same direction at any speed n r; n r nr could be 2 RPM 5 RPM whatever it is. Suppose at some time the rotor speed is n r stator field speed will of course not changed because, it is connected to a supply whose frequency is f. So, stator field speed with respect to stator structure remains same and rotor is now moving and in my we know that the RMS voltage induced depends on n s minus n r which decreases.

Now, from the instant at t equal to 0 plus at t equal to 0 plus n r was 0, but it will decrease because n s minus n r is there and so long the n s minus n r is not equal to 0, there will be rotor voltage and there will be rotor current. And if there is rotor current, there is bound to be rotating field created by the rotor. Now, the question is what is the frequency of the rotor? Current rotor is wound for same number of holes and frequency of the rotor current will therefore be p by 2 into relative speed between these two. So, this will be rotor frequency.

So, if this is the case frequency of the rotor current is this one, then speed of the rotor field rotor resultant field always it means that rotor resultant field rotor field, I will simply write. You understand now what is what speed of the rotor field will be equal to how much, it will be 2 f r by p so much mechanical speed.

If rotor current is f 2 f by p that is the speed of the rotor field is 2 f r by p because frequency of the balance three phase current is f r for stator frequency of the current is f. So, stator field speed was 2 f by p. So, now it must be 2 f r by p. It is no longer f because machine has pick up some speed. Now, the question is as I am telling the moment I write this. I must attach a rider to this with respect to whom? With respect to whom? With respect to the structure which houses the three phase widening, which houses the three phase widening is the rotor body.

So, with respect to rotor itself this speed is 2 f r by p with respect to rotor itself. So, that will be the speed of the rotating field, ok. Now that is there; so, but it is 2 by p and f r is once again p by 2 into n s minus n r is not and this will be equal to n s minus n r perhaps I could write straight away, but anyway it is good to do like this. So, this is n s minus n r and do not forget to write that this is and this is mechanical speed whether you remain in electrical or mechanical do not get mixed up one electrical speed with another mechanical either remain in electrical or mechanical. So, I am always remaining in mechanical speed.

So, this is n s minus n r mechanical speed and with respect to rotor. So, here is a rotor where there is a rotating field B r and it is moving with a speed n s minus n r with respect to rotor itself and a stationary observer also sees a rotor speed n r with respect to this stationary observer. So, rotor is also moving in with a speed n r with respect to a stationary observer and with respect to the rotor the field is moving with a speed n s minus n r. Therefore, the conclusion is therefore speed of the rotor field; with respect to a stationary observer or in this case stationary observer and this stator, structure stator are one and the same thing.

Both of them are not moving will then be equal to n s minus n r plus n r and that will be n s. Therefore, a to a stationary observer he will always conclude whether the rotor is stationary or it is moving in with any speed n r this speed of the stator and rotor speed are ns. With same speed both of them will be moving like this. Therefore, the angle between them will remain time invariant and therefore, we accept some torques will be developed. I am repeating once again these steps rather fast.

See speed of the rotor field is 2 by p into frequency of the rotor currents and frequency of the rotor current is f r we have calculated earlier. So, 2 f r by p will be the mechanical speed of the rotating field and with respect to whom, with respect to the coils, rotor coils or rotor structure. So, 2 f r by p with respect to rotor itself and then, you put the value of f r which I have already calculated put it here, it comes out to be n s minus n r straight mechanical relative speed.

So, speed of the rotor field with respect to a stationary observer will then be that rotor is also moving. So, stationary observer will say n s minus n r plus n r and is equal to n s and a stator field in any case it is moving with n s that we have already concluded and decided because the frequency of this stator current is always f. It cannot be other than f. Therefore, it is excepted machine will develop a torque as you have been successfully created two fields and a observer, fixed observer. No matter wherever it is seated, the observer may be moving with some speed 1000 RPM, but he also should conclude that these two fields are moving at same speed. So, we have taken conveniently a stationary observer and the conclusion is it will move with speed ns.

Therefore, rotor will pick up perhaps speed, therefore this if rotor is allowed to move, then no lock no rotor locking rotor is allowed to move and the rotor circuit is closed, then only of course rotor current can exist and then only question of movement comes in and this is how speed grows. So, this part in this part I have shown that rotor will create your field and rotor field speed is same as the stator speed, field speed with respect to a certain observer and they are same and its excepted conclusion is to develop some torque [FL]. Now, the big question remains now interesting thing happens. See I will just touch upon first, then we will come back to this point.

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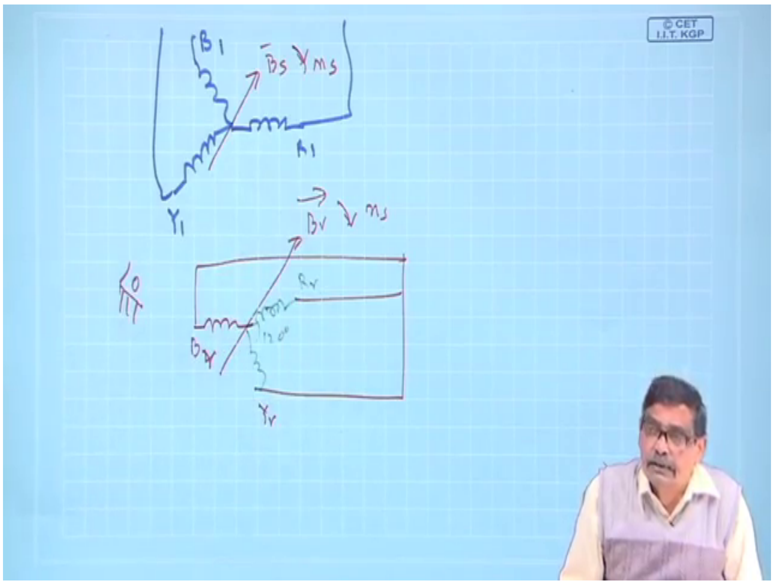

So, what is the thing I have told there is a stator coil. We ask ourselves this is what is this. This is R1 this is Y1 this is B1, and there is rotor terminals which is r r which is Y r. This is 120 degree please bear with me. So, and this is r r Y r and B r and these things I have kept short circuits like that. Rotor current is filling now and it is moving in this direction. Now, there is some B s moving. When there was no rotor current, it was their. Now, there is rotor current, B r is also moving with same speed n s, n s and this n s with respect to a stationary observer they conclude if they are moving in the same direction when this fellow is energised that is there.

But now I will I am slightly this not disturbed, but I now feel that feel like that now in the air gap of the machine, there are two rotating fields and both of them to respect to a stationary observer is moving with a speed ns. Therefore, induced voltage in any coil either on this stator or on the rotor, both B s and B r is going to induced voltage B r. Why it is going to induced voltage in this coil? Because of the fact there is a relative speed n s minus n r between the coil and B r is going to induce voltage in the stator coils, yes because stator coil is stationary and B r is moving with speed n s with respect to the stationary coil apart from B s is inducing voltage here. So, these are the issues.

Now we must be knowing very clearly what is then going to happen [FL] before that. So, this I just raise this question, so that you also feel like something is to be done about this. What is then going to happen?

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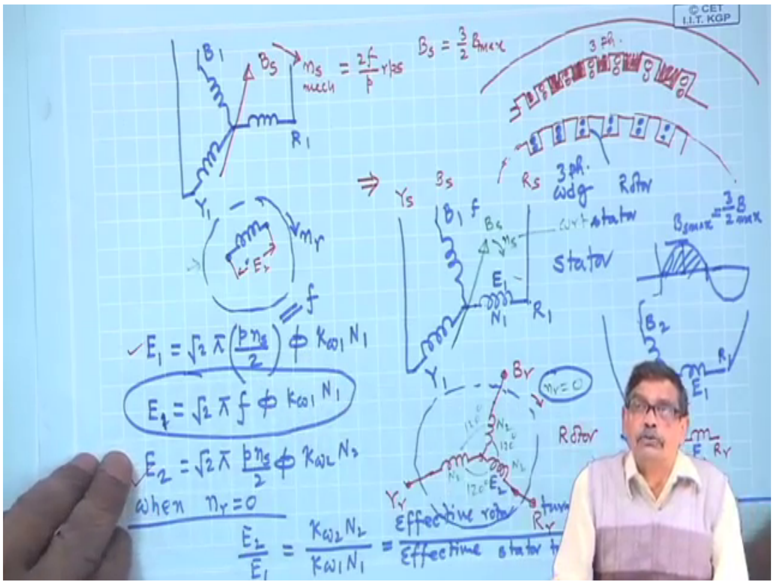

For example, what I am trying to tell that in this expression flux per pole phi what should be used because in this stator coil e1 root 2 pi p n s by 2. Both of them are moving no doubt with synchronous speed. This term remains same, but what is flux per pole? Which flux per pole should I use? Understood this is the issues and this can be very nicely explained and this will come slightly later.

But I just leave this question to you to to think over it over and over again and that is then is worth [FL] before that and also, I am not sure I told you B s B r are produced and there must be some difference of angle. Both of them are rotating with same omega s, no doubt ns, but how that angle is decided things like that now becomes important. Although we have concluded machine is expected to run, what is the torque how to relate the torque in terms of B s and B r [FL]?

At this point at least you must agree with me that one thing can be done. This B s is there, B r is there, I will take the resultant of these two fields, these two space vectors can be added in space and resultant field I will get and with respect to that resultant field I should calculate this phi flux per pole. What is the point because if you take the resultant field, then a lot of things you will now get simplified because of the fact you can assume B s is present, B r is present or you can assume B s plus B r resultant is present and let us see and because of that resultant field because both of them are going to induce voltage in these coils. I will take that method I will I am giving you some hints, so that you can further think over it [FL].

Now, whatever time is left I will do another important thing because the issue of the angle between B s B r things like that is to be resolved. So, to do this what I will take? I will take a very simple example [FL]. This thing whatever I will be doing now I have taken from I have although, I have not told you any name of the books, I will tell you the name of the book in the next class ok, but let us try to understand very simple [FL].

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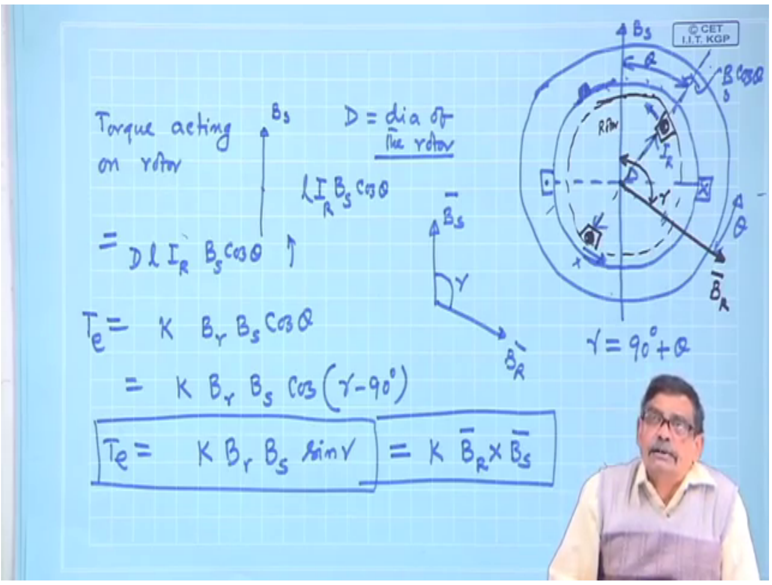

Suppose you have a stator field B s and you can imagine this stator field it has nothing to do with induction motor, but this problem if you understand you can extend these two induction motors, synchronous motors. What not this is suppose your stator lots stator coils etcetera and suppose for simplicity let us assume there is a pair of slots, you are passing some DC current and there is a field which is created B s. Let me call it B s and these B s of course can be created by a distributed widening carrying ac currents and this B s will move.

But the problem statement you try try to understand. Suppose there is B s which is stationaries. Stationary is a special case of moving thing. Now, you must understand with n s equal to 0, there is a field which does not move, ok. That is possible if you energised the stator coils. If DC current is not f equal to 0, N s will be 0. Anyway DC current suppose let us take a simple of course this can be disturbed coil dot here dot here, but I am using a single coil B s and fundamental component I am sketching, ok. This is B s [FL].

Now, suppose there is a rotor here. This exercise everybody must do to understand the torque production and how the angle between B s and B r etcetera comes in. Suppose you have the rotor, rotor iron and here also you have a pair of slots. This I will do slowly, so that you understand two pole structures and this is the rotor current cross dot and therefore, the rotor field will be perpendicular to this line. It was stator field was perpendicular to this line axis of the coil.

So, rotor field will be also perpendicular to this line. This is rotor iron and let this be B r [FL]. I have put an arrow, what does that mean? It means the peak of the fundamental component of this B s because I can distribute this widening and make it more sinusoidal or only consider it will be rectangular, but fundamental component I am talking about, so its peak is here and what is this arrow means? It is the lines of force direction. No issues.

So, this is also lines of force direction and peak value of this current here is B r, Ok. That is fine. Now, as I told you study this topic not considering in do not keep induction motor in your mind, ok. This is B s produced, B r produced I want to find out. What will be the expression of the torque in terms of the angle between these two field say gamma? I want to find out what should I do [FL]? You see this is the cross current, this is the dot current. Let us assume that this angle is theta and this rotor I have locked it. So, in this position what torque it experiences that I want to find out.

Imagine rotor is not allowed to rotate. Stator we have energised with this current. It produces B s rotor, you energise with this cross, this dot, it produces B r, it is excepted that you see stator poles, this is south pole ss. I am not writing and rotor this is north pole. So, there will be an attraction between N r and ss here and it is excepted torque will be in this direction. I can physically tell that.

But none the less I will apply left hand rule. For example, the magnitude of this current is I R rotor current and you know this is the direction, what will be value of B s? Here if B s is the peak value, so B s cos theta is not along this line like this. So, this B strength is B s cos theta. So, B s cos theta and I r is this current and you can easily see the conductor will experience a force like this and what will be the magnitude of this force? It will be B s cos theta l I R B s is the rotor current I R into l l is the effective length of the conductor perpendicular to this length we know. So, it will be just like this.

And similarly this is just same place. It will also experience a force in this direction. You can apply this current is cross apply left hand rule and magnitude of the force will be same. So, here are two forces and the distance between them is the diameter of the rotor. Therefore, the torque this coil torque or couple acting on torque acting on rotor will be this magnitude of the force l I R B s cos theta into this D D equal to dia of the rotor.

This will be the torque acting and which appears to be in the anti-clockwise direction like this. So, this is the force. Now, you see this I R this can be written as some constant k into B r into B s into cos theta because I know the fundamental component of B r peak value of it is directly related with I R. is not therefore with some constant k. This dl also is getting absorbed in this constant apart from whatever other h to b main 0 this that. So, ultimately the torque, electromagnetic torque is this one cos theta B r B s cos theta. What is B r? B r is the peak value of this sinusoidally distributed rotor flux density. What is B s? Peak value of this one now and this angle is gamma I have adjust defined and what is theta? Theta is this.

If theta is this angle you note that theta is the angle between this line and this line. So, theta will be also be the angle between their perpendiculars. So, this is perpendicular to this line and this is perpendicular to this line. Therefore, this angle 2 will be theta is not therefore this is the thing. My goal is to find out this expression of the torque in terms of gamma say. So, gamma is nothing, but as you can see 90 plus theta is not. Therefore, this expression of the torque is B r B s and cos of theta is nothing, but gamma minus 90 degree which is nothing, but K B r B s sin gamma because cosine 90 minus theta is same as cosine gamma minus 90 that is why cosine, fine very good function.

So, these is the expression of the torque and see now I will just draw it here. It is B s a vector and it is B r at any arbitrary position br. This angle is gamma is not. So, can you not be written as k into cross product of this two vector, a cross b is magnitude of a magnitude of b and the angle between B r and B s and these also gives you see B r B s. If you multiply thumb give you the direction of the torque direction of the torque means anti-clockwise.

So, this is the important result that electromagnetic torque developed by a machine. Essentially if you know the position of B s field produced by stator coil and position of the rotor field at any instant of time, this is the torque experienced. What I told you in the derivation rotor is not allowed to move. I have hold held it to understand really how much torque is developed in this position, ok.

If you release your hand, what you will observe? The rotor will move and in this particular example if this is a DC current, B r and B s will finally get aligned and there will be no torque that game will be over, but in which way it is connected with induction motor, ok. We are now matured enough that B s is created by some three phase current or whatever it is and it moving its position that is also fine. B r is also produced by a valence three phase coil that is also moving angle between them is remaining constant is not in case of rotating field we have seen.

Therefore, I can apply this same formula to get the electromagnetic torque. There it will give me a clearer idea about how torque is produced particularly as a motor operation or as a generator operation. So, we will continue with this, this today's lectures that is the last 3 lectures. Please go through it very carefully, so that we can proceed further. After we have done this, it will be all nice.

Thank you.