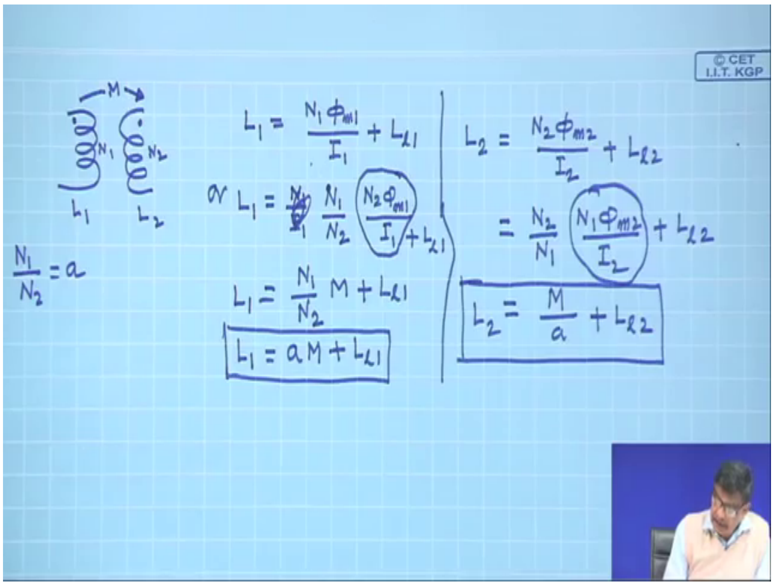
Electrical Machines - II
Prof. Tapas Kumar Bhattacharya
Department of Electrical Engineering
Indian Institute of Technology, Kharagpur

Lecture – 03
Equivalent Circuit from Circuit KVL Equations

(Refer Slide Time: 00:29)



Welcome to the 3rd unit of this lecture, you recall that we were discussing about the coupled coils, which I will draw simply like this. And this coupled coils have self-inductance L 1, L 2, number of turns N 1, N 2. And the mutual inductance between them is M, this is the thing.

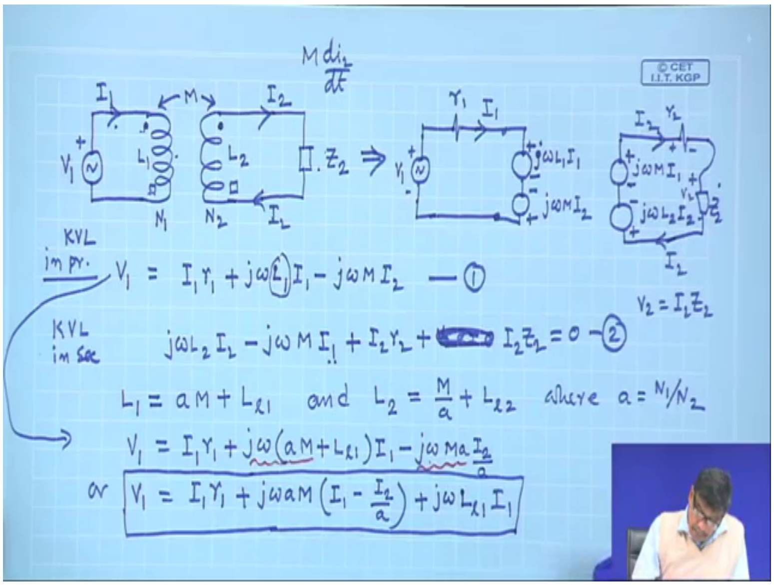
And the magnetic circuit, I am not drawing that means the core I am not drawing. You recall that the expression of the self-inductance, we got as N 1 phi m 1 by I 1, in our 1st unit we have discussed about this, and this is the leakage inductance that is the 1st lecture with this expression I am writing. And similarly L 2, we got as N 2 phi m 2 by I 2 plus L l 2, these are leakage inductances ok.

Now, this self-inductance can be expressed in terms of leakage inductance as well as this term. But, this term can be nicely simplified in terms of mutual inductance term. How to do it? This or I can write L 1 is equal to N 1 and then what I will do N 1 by I 1 I will write and I am sorry what I will do is this, I will multiply and divide by N 2. So, this can be as N 2 phi m 1 by I 1 plus L l 1 multiply and divide by N 2, you get this.

Now, you can certainly recognize this term. This is what coil 1 is energized with I 1 current, phi m 1 is the mutual flux created by coil one. So, N 2 into phi m 1 by I 1 is nothing but mutual inductance. So, this is N 1 by N 2 into M plus L l 1, this is the self-inductance. And if we say that the turns ratio let us define this to be small a, then L 1 is nothing but a into M plus L l 1. This is in terms of mutual inductance and leakage inductance, self-inductance will look like.

In the same way, if you look at L 2 what you do is this, you divide by N 1 and multiply with N 1, then we can recognize this to be once again mutual inductance. Since, there is no difference between m 1 2 and m 2 1 so, this can be then written as M by a this N 1 by N 2 is 1 by N N 2 by N 1 plus L l 2. And mind you this I 1 and I 2 are the currents considered I considered DC current one excited at a one time, but in any case this is the thing. Now, these things, we will use to derive the equivalent circuit.

(Refer Slide Time: 05:15)



And to do that in my last class, I was telling that you consider these two couple coils and consider that you have applied an AC voltage, which is whose RMS value is V 1 and this is I 1 is the AC current drawn by this coil having turns N 1. And these are having mutual inductance M and self-inductance L 1 and self-inductance L 2. And this side on the secondary side, we will connect some impedance Z 2. These are all complex number Z 2, V 1, I 1 and this current is suppose I 2.

Now, this circuit this is a very important step, it can be drawn like this V 1 applied voltage. And let us assume the winding is having a resistance that is r 1. And then you have two sources of EMF here, because both the coils are carrying current one is and polarity is important. L 1 d i 1 d t in time domain terms, which comes out to be j omega L 1, I 1. Then you have another voltage drop here whose polarity will be like this, because I 2 is entering here.

These two are also can be parallely thought of a dot. So, current entering here will have polarity of the induced voltage in the other coil as this plus, this minus. And the value of these in time domain is M d i 2 d t, but d d t if you substitute j omega, small i 2 have a phasor capital I 2, so this will be j omega M I 2, this is this side.

Similarly, the other side what will be the things, other side between these two terminals, there will be two sources of EMF. One is I 2 is entering through this, therefore the self-inductance drop j omega L 2 I 2. And then the mutual inductance, because primary winding 2 is carrying a current of I 1 this is dot, this is dot. Current is entering through the dot, so other dot is plus.

So, the polarity of this voltage will be this mind you this is I 1 I 1, and this is minus and this is j omega M I 1. And then of course if winding is having a resistance, you are having r 2. And then you have this Z 2 current, I think I move it a bit this side this is the thing that is two voltages are there because of self-inductance and because of mutual inductance r 2 and Z 2 and this is how I have said the current direction I 2.

And on the side there is no voltage source connected, but anyway this voltage let me call is to V 2 terminal voltage, where V 2 is obviously equal to I 2 into Z 2 ok. So, when both the coils are carrying current, this is the scenario I get it. Now, what I will do is this, I will write down the as I told you simply write down the KVL equations in both the loops. So, first this side KVL equation is see applied voltage V 1 will be equal to I 1 into r 1 plus j omega L 1 I 1 and then minus j omega M into I 2, and you are back here. So, this is the 1st equation, similarly so KVL in primary.

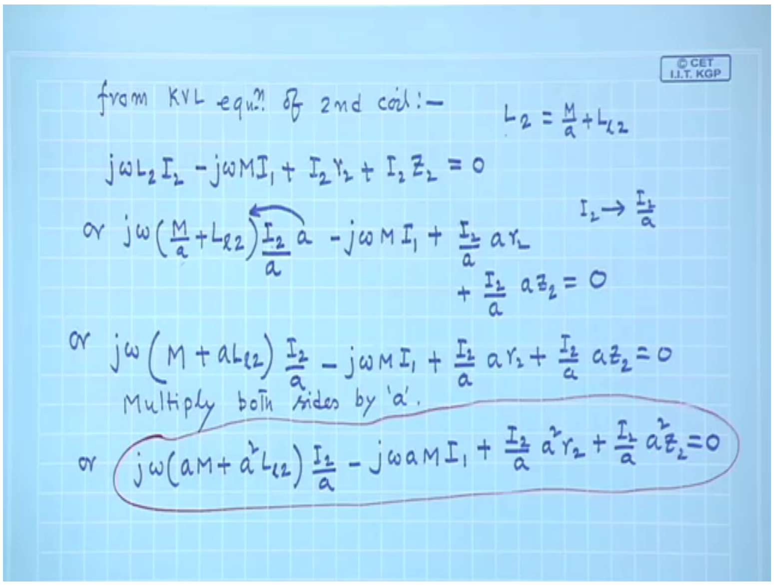
Similarly, similarly let us write down KVL equation in secondary, mind you all these quantities are phasors. KVL in secondary will be there is no external voltage connected. So, we start from any point we like and sum of all the voltages. So, suppose from plus to minus this voltage is j omega L 2 I 2, so plus to minus I am writing plus. So, then I must write j omega M I 1 and this drop is also plus to minus that I am writing plus plus I 2 r 2, and this plus to minus so once again V 2 is that correct.

So, plus to minus j omega plus to minus if I am going and this is equal to 0 let us write plus there. So, j omega L 2, then minus it is minus to plus j omega M I 1 because of this coil current, there is additional voltage here. Then plus I 2, r 2 once again plus to minus and plus to minus v 2 and this V 2 is of course I 2 into Z 2 I can write like that Z into 0 that is all. So, this is the thing. So, these are the two basic equations 1 and 2.

Now, we will use this knowledge that L 1 in our last lecture, we have found out that L 1 is equal to a M plus L l 1. And L 2 is equal to M by a plus L l 2, where a is equal to N 1 by N 2 is not it, this is the thing. So, let us take the 1st equation and for this L 1, we will now put this expression that is V 1 applied voltage is equal to resistance drop I 1 r 1 plus j omega for capital L 1, I will write a M plus L l 1 into I 1 and minus j omega M.

Now, j omega M I 2 is there I 2. What I will do this, I will just divide by small a and multiply by a, so it remains j omega M I 2. This is the trick I have applied here, division will be clear, why it is done. Then or V 1 is equal to I 1 into r 1 plus j omega a M, you see j omega a M occurs here and also occurs here. So, I can combine these two terms, and write j omega a M into I 1 and minus I 2 by a. This I can do, these two I can do, and then plus j omega L l 1 into I 1 you know this is what it comes out that is all.

(Refer Slide Time: 15:59)



Now, let us similarly play with the 2nd equation 2nd KVL equation that is the KVL equation of the 2nd coil, which we have already got. So, from KVL equation of 2nd coil. We have got j omega L 2 I 2 that is this equation, I wrote in the previous page, this equation I am now taking j omega L 2 I 2 minus j omega M I 1, hopefully I have written the polarities correctly plus I 2 Z 2 is equal to 0.

Then what I am going to do is recall that L 2 is equal to M by a plus L l 2 L 2 is equal to M by a plus L l 2, so that thing I will put plug in for L 2 here that is M by a plus L l 2 this into I 2. And wherever I 2 is there, you do this thing I 2 by a into a. So, this term is this term. Minus j omega M I 1 and then plus I 2 once again do like this, I 2 by a in to a r 2, wherever I 2 is there try to modify it to I 2 by a. But, you cannot just divide by a, if you divide you have to multiply by a that is what exactly I am doing it. So, this term is this term and plus I 2 by a into a Z 2 and this is equal to 0, this is the thing is not.

Now, or j omega this small a you push it inside, so that it will be M plus a L l 2 into I 2 by a minus j omega M, wherever I 1 is there do not disturb it plus this I have not done anything I 2 by a a into r 2 plus I 2 by a into a into Z 2 is equal to 0, mind you are all complex numbers. Now, what I will be doing is I will be multiply both sides by turns ratio a. So, if you multiply, then it will become or j omega this a you push it inside, it will be a M plus a square L l 2 into I 2 by a minus j omega a M I 1 plus I 2 by a into a square into r 2 plus I 2 by a into a square into Z 2 and this is equal to 0.

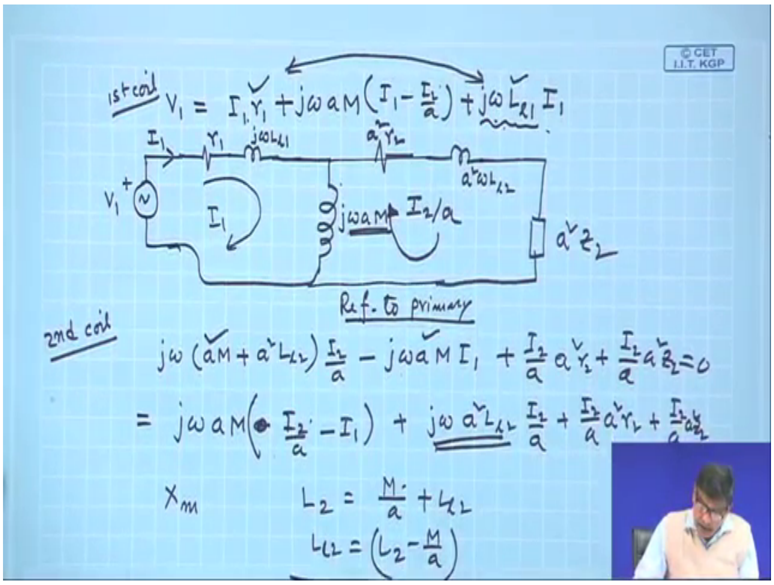
So, what are the things I have done, secondary coil KVL equation I have written, wherever so and then I have substituted for L 2; L 2 I have expressed in terms of mutual inductance and leakage inductance. What is the reason for that, because in the transformer equivalent circuit I know there is no term representing capital L 1 self-inductance is absent.

It is expressed in terms of mutual inductance terms and L l 2, so that is the reason L 2 I have replaced by expressed it in terms of mutual inductance and leakage inductance and put it here. And wherever I 2 is there, I am simply telling that you divide by a multiply by a, so that I 2 is written minus j omega M I 1 do not disturb and then plus I 2 by a into a r 2 plus I 2 by a into a Z 2 is equal to 0.

And then in the next step what I am telling the say you have put it inside, so it has become a a cancels. So, M plus this, then you multiply both sides by small a. So, j omega this small a you put it inside, so a M plus square L l 2 etcetera minus j omega a M I 1 plus I 2 by a a square r 2 plus I 2 by a a square Z 2. So, this is the 2nd equation. So, these two equations are now like this is one and this is one.

And we are done and in equivalent circuit now can be drawn, let us see how. So, these two equations we will keep in mind. So, these are the two equations. This is the primary side equation this is the primary side equation, this is the secondary side equation. Let us see, how it helps us to write down.

(Refer Slide Time: 23:29)



Suppose, I want to draw a circuit diagram or let me write here. So, the first KVL equation, we have got re-write V 1 is equal to I 1 r 1 plus j omega a M into I 1 minus I 2 by a and plus j omega L l 1 leakage inductance I 1. Now, this can be drawn like this, because I know the terms this is my V 1 and this is my I 1 earlier it was coupled coils, now I am telling ok, there is a resistance here I 1 r 1 draw, then there is I 1 into j omega l 1.

These two terms, you combine. So, r 1 plus j omega L l 1 leakage inductance drop so, this drop plus this drop. Then there is another drop, which is occurring in a inductance coil, it is not transformer coil. Now, I am trying to translate this in terms of circuit and I will tell it is a reactance of value j omega a M and I close it. So, this is I 1 is it correct.

If somebody says it means this, no because of the fact so I have taken this term this term and j mega a M into I 1 that is fine. But, there is a term minus I 2 by a into j omega a M, so how to do it.

So, what I will do, I will write I 2 by a here. And in this loop what is there I do not know, suppose a current source I 2 by a here, then of course everything is fine so far as primary coil is concerned, because V 1 is equal to I 1 r 1 plus j omega l 1 I 1 plus j omega M I 1, then mesh analysis I know there is another loop here existing, which is providing you this current I 2 by a flowing in the opposite direction, so that will be minus j omega a M into I 2 by a, then it is fine I mean primary coil.

Similarly, now the question is what is here in this side ok. 2nd equation will help me to do that. The 2nd equation if you recall, it was this was the first loop 1st coil KVL. 2nd coil KVL, we have already got. 2nd coil KVL was j omega a M plus a square L l 2 into I 2 by a minus j omega a M I 1 plus I 2 by a square r 2 I 2 by a a square Z 2 is equal to 0.

Therefore, the 2nd KVL equation is telling me that this voltage drop that is j omega a M into I 2 by a minus j omega a M into this so this can be written as j omega a M minus again I 2 by a this first term I am taken in minus I 1 plus j omega a square L l 2 I 2 by a and then you write this I 2 by a a square r 2 and then plus I 2 by a a square Z 2 and that is equal to 0.

So, you see this voltage drop I 2 by a into j omega M is taking place here. And then minus j omega a M into I 1 is in the opposite sense. So, I 2 by a minus I 1 mesh analysis, then what you have got. Then you have got the resistance from r 2 whose value is a square r 2 a square r 2 in to I 2 by a plus the leakage reactance drop, whose value is I 2 by a is this term a square omega L l 2 a square omega L l 2 into I 2 by a is this term.

And then this drop, whose value is a square into Z 2 is it not. So, I have played with these two KVL equations and that I did this circuit this is your mesh current, you must be familiar with mesh current equations. So, you now see both the loop equations are very nicely satisfied. And the equivalent circuit of a transformer, referred to the primary is obtained.

And this circuit was obtained by considering several physical things this way that way. But, I now get if the turns ratio is there, the secondary impedances must be multiplied by a square that thing has come this thing. Generally, now also it gives me a better physical inside. For example, the magnetizing reactance X m what about is drawing, I find they refer to the primary. It is not M it is a into M multiplied by j omega.

The leakage inductance L l 1, whatever is coming that we have found out how it is related that is you recall L 2 is equal to M by a plus L l 2. So, L l 2 is nothing but the difference of these two numbers. Are you getting? So, today, I am stopping here, but please go through it, it is a circuit approach to arrive at the equivalent circuit of a transformer in a very decent and nice ways.

Thank you.