Electrical Machines - II
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Lecture –19
Induced Voltage Due to Fundamental and Harmonic Components of Flux Density Distribution

Welcome to this 19th lecture on this machine course and in our last class we did one very important thing that important thing is that basic building block of any rotating machine apart from stator iron, rotor iron, slots and slots wheel house windings. So, ultimately the basic building block is a coil side ok. Then from that we went further ahead and told that basic building blocks will be a single coil, a coil single coil if you recall it has got 2 coil sides.

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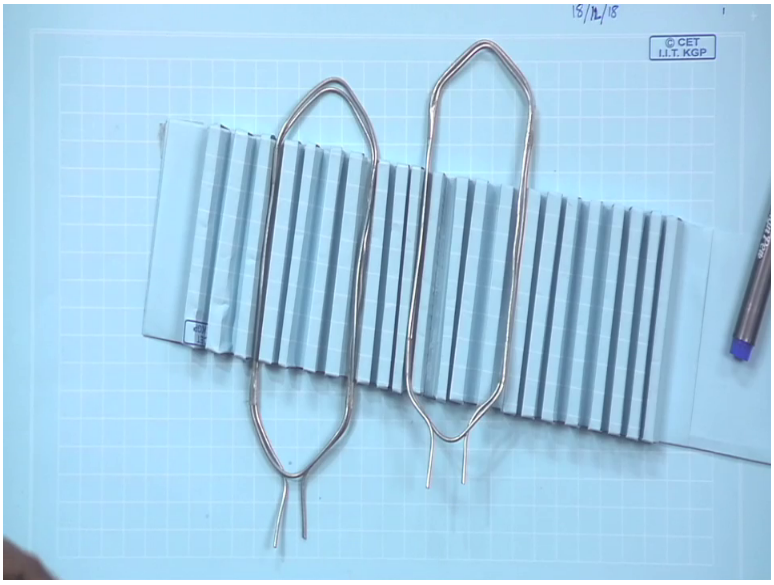

And they will be placed in slots, it is better to repeat that this is how and it could be a multi turn coil. So, hence forth this will be my basic building block of any rotating machines.

There maybe number of coils present, but each coils will be identical. But, only thing they will be placed in some other slot. This coil and this coil these two coils are suppose identical and they will be placed in some other slots. And, when this things will rotate move relative to a sinusoidally distributed magnetic field there will be induced voltage across the two terminals of the coils. And the terminals you should note that this point is 1 and this point is 1 dashed. This is second coil it should be also named 2, 2 dashed and things like that.

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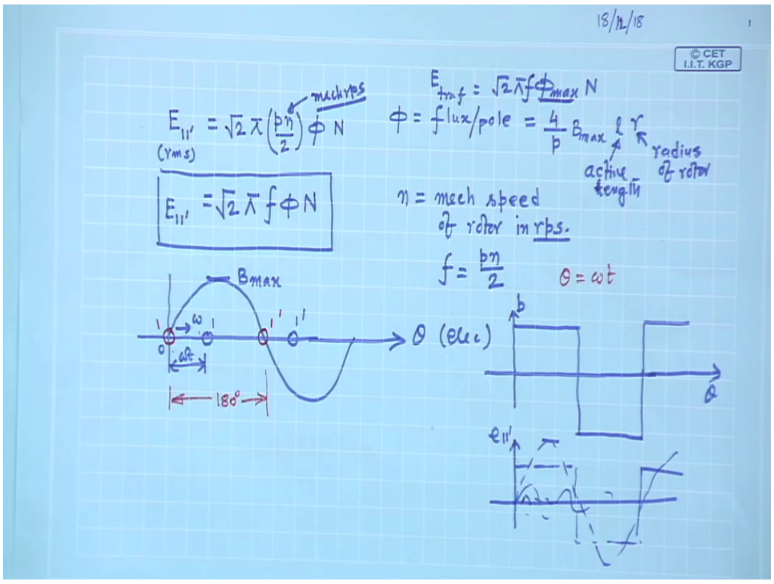

So, in our last class what we did that we found out the expression of the induced voltage in a single coil as E 1 1 dashed rms value. This rms I will not try it henceforth, but for the first time let me write root 2 pi pn by 2 root 2 pi. Then I wrote phi and then I wrote capital N. What are these things? This things are phi is flux per pole; flux per pole in wave bar wave bar per pole. And each expression for a multipolar machine we derived as 4 by p B max l into r, l is the length of the machine that is active length of the conductor active length; active length and r is the radius of the rotor radius of rotor. So, this is in terms of flux per pole.

And what is this? N is the mechanical speed of rotor in rps and this f is nothing, but the pn by 2; pn by 2. So, for a generator input should be the speed. So, speed appears here, but we will write it like this route 2 pi f phi into N. So, that is the rms voltage induced in a particular coil; this equation similar to that of a transformer. But, in case of transformer remember the rms voltage in transformer as root 2 pi f phi max into N. So, it is phi max there, but it is phi for different reasons. So, it is easy to remember a formula.

Now, and the idea was that this was I assumed the flux density was sinusoidally distributed in space. This is space angle without anything written, it means it is electrical

And, this is B max. And if you imagine your coils are staying here at t equal to 0 1 1 dashed, coil span should be 180 degree electrical. So, this is always 180 degree electrical this angles. So, this is how B is distributed and if this coil moves there will be AC induced voltage in that, the rms value of that AC voltage is this one.

Now, this tangential velocity B was converted to the angular speed like this by noting that position theta is nothing, but omega t. At anytime t the coil new position of the coil will be say here as it is moving with a speed omega radian per second. So, this was at any instant t this was omega t and 1 will come here 1 dashed will go there and relative to this it is moving. So, all these things we did last time.

Now, only one thing I would like to tell, I was telling you many a times that after all if you have a single coil just listen. If you have a single coil the flux density distribution will be rectangular in shape, is not that is what I was telling. South, north they will go like this and this is your space angle theta. Now, if you have a rectangular distribution then what will be the induced voltage ok? Induced voltage will be similar to this because, it follows the same path same pattern as that of b. If this is b this will be your induced voltage e 11 dashed instead of sinusoidal if you are having.

But, the another way of looking at it. So, you find out this sinusoidal voltage and then you suppose want to examine how much of it is fundamental component and what are the harmonic components of the voltage. One way of doing is you this is rectangular, you find out voltage pattern which is also periodic simply Fourier analyse this. Of course, in this case the rms value voltage should not be calculated by this formula; obviously, because it is not sin wave. But, I know given a periodic waveform how to calculate the rms voltage etcetera.

But anyway this periodic wave can be broken up into fundamental component and third harmonic, fifth harmonic all odd harmonics will come. That is this voltage waveform can be found out fundamental, then third harmonic, then fifth harmonic. And, for each one of them this rms voltage formula will be valid because, each one of them is sinusoidally distributed. Another way of doing it then it will be clear what exactly I am trying to tell that you can suppose your b distribution is rectangular single coil, multi turn etcetera.

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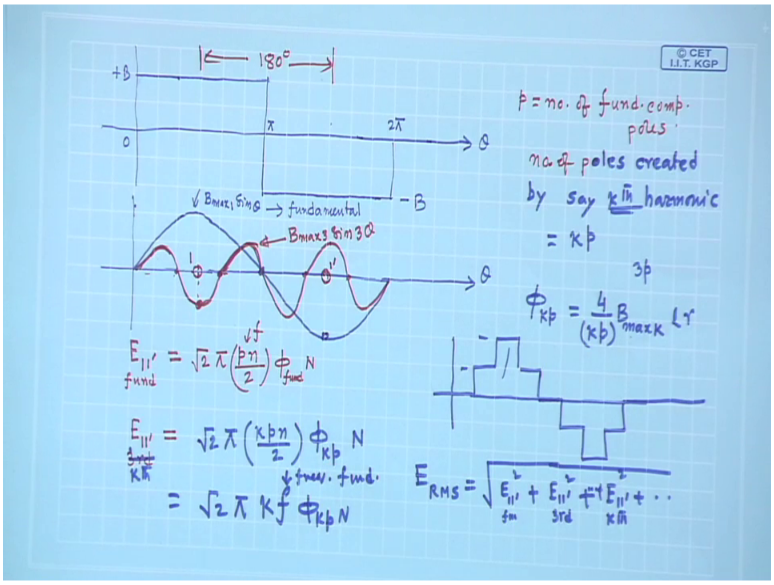

So, this is suppose only one cycle I am drawing, this is the plus B and this is minus B and this is all electrical theta and this is 0, this is pi, this is 2 pi. Now, if you break it up into a fundamental component this is theta and this is your pi. So, your fundamental will be this one and this is the B max sin theta fundamental component. Say let me write B max 1 sin theta, this is the fundamental component.

Now, there will be next higher harmonics which will be present will be third harmonic. Only for that 3rd harmonic I will show you how to calculate the voltage and then we will be able to do it for other higher order harmonics. For example, fifth seventh, but I am sure about one thing that this amplitude of this harmonic components will progressively decrease. No point in taking thirteenth harmonic component to calculate the voltage because, they will become negligibly small. Because, those we have done Fourier analysis know that higher order components amplitude will go on decreasing.

But, once you break it up into sinusoidal distribution each of the component then you will be able to apply these formulas, why not because this is valid for sinusoidal thing. So, this is thing now, the 3rd harmonic component. What is a 3rd harmonic component? 3rd harmonic component of B max 3 will be also sinusoidally distributed. But, when the fundamental max one complete cycle your 3rd harmonic will may 3 complete cycles that is why it is called third harmonic.

So, let me divide this things with 60 degree intervals each one is 60 degree this red marking, this is 60; 60. So, this the B max 3 will have 3 cycles; 1 cycle will be like this. Next cycle will be like this one sorry 1 cycle 120 degree, next cycle next 120 degree and the 3rd cycle is here. So, 3 cycles of this is this can be its equation is B max 3 sin 3 theta. Similarly if you can sketch fifth harmonic, in one cycle of fundamental there will fifth 5 such cycles. But, I am not going to draw those things if I understand how to tackle one harmonic components you can easily figure out what to do. So, this is the thing.

Now, this B max 1 B max 3 their magnitudes I will get after Fourier analysing. This if I know this B value I will be able to find out some expression and this amplitudes unknown. Now, if I write it the voltage 1 1 dashed fundamental component it will be root 2 pi pn by 2 that is f fundamental frequency f 2 pi f flux per pole of fundamental into number of turns that will be the rms voltage. Now, the question is what will be voltage across 1 1 dashed due to the 3rd harmonic, I want to find out. So, to find out that I must know what is the flux per pole due to be 3rd harmonic component. Now, if you look at it, if you concentrate your attention on the 3rd harmonic component alone fundamental we have done. So, idea is find out 3rd harmonic component, fifth harmonic component add them to get the total voltage.

Now, if you look at it 3rd harmonic component how many poles this machine has got; ep is the number of fundamental poles fundamental component poles. Number of poles sorry number of poles created by say kth harmonic will be simply kp. Suppose, your original number of poles p equal to 2, if it is 3rd harmonics 6 poles are created therefore, 3rd harmonic component. So, this is south, this is north of the fundamental you see there is south north, south north, south north. So, 6 poles will be created because of third harmonic. So, 3 p for example, it will be like this. Therefore, that is the thing and N is the mechanical speed. Therefore, wherever p is there it should be kp now. So, it will be so to calculate flux per pole for kp any order harmonic, it will be 4 by p B max of that component into lr.

But p in this case is kp number of poles is increased therefore, 4 by kp into B max k into lr. Therefore, E 1 1 dashed 3rd harmonic component will be root 2 pi wherever, p is there it should be kp. In fact, it is kth harmonic in general kth harmonic root 2 pi kpn because, number of poles for this particular kth harmonic has increased by kp. So, kpn by 2 into phi flux per pole due to that component which should be calculated by using this B max lr are fixed. I mean B max k you have to take and 4 by kp because, number of poles of that machine has increased. So, root 2 pi this thing in to phi kp into N number of turns of course, remain same.

Now as I told you so this is how rms value of 1 1 dashed across the same coil because, of 3rd harmonic component, fifth harmonic component can be calculated by putting appropriate value of k. But, do not forget to calculate phi 3 p phi; phi p in this formula depending upon which harmonic voltage you are calculating; fifth harmony harmonic component its amplitude will be further less. So, this should be remembered and another thing you see the span of the coil is 180 degree electrical is always with respect to the fundamental component. As I told you if you place the 1 here this 1 dash should be place such that it is 180 degree is not electrical. So, 1 dashed will be placed here at the centre of the south pole 1 1 dashed.

Now, what about and this 3rd harmonic component has been created, it will be there I cannot help. If this is the way then due to the 3rd harmonic component once again is it true that when 1 is under one of the pole centre 1 dashed will be under the opposite pole centre. Yes, it is true, because it is now you see under this pole centre and this is opposite pole centre. Therefore, once again emf will be added up. So, odd harmonics fifth also so, 1 2 3 4 then that thing comes. Therefore, you can easily verify that all harmonics odd harmonics will induce a little bit of voltages depending upon their strength B max k. And, they will be added up of course, there may be plus and minus sign before the Fourier coefficients that is that can be easily taken care of.

So, to use this formula to calculate the flux per pole for kth harmonic you have to do like this, use the same formula 4 by p B max lr. But, if it is kth harmonic of B distribution then the number of poles are really k into p times. If it is originally 4 pole machine because, of fundamental it is it will become 12 poles is not. So, that is very interesting, but in any case as I told you if the if you are using multiple coils your B distribution may be like this. If your B distribution is like this, then once again do the Fourier analysis of this waveform get the fundamental, this amplitudes will be known. If they you are using distributed coil not a single coil, there are 2 coils connected in series in order to achieve more sinusoidal distribution more nearer to a sinusoidal flux density distribution.

Then also I can find out the this is a slightly involved waveform, but nonetheless it can be easily done. I can find out phi k p put k equal to 1 get the fundamental flux per pole and so on. And, get the rms value of each one of them and what will be the rms value of this waveform. This can be also calculated by squaring each of the rms voltages then summing them up and finally, taking one 1 square. For example, E RMS of this voltage waveform in generally if harmonics are present you are not neglecting, it should be done like this, E 1 1 dashed only fundamental square plus E 1 1 dashed only 3rd harmonic plus E 1 1 dashed only square kth harmonic make dot dot dot. Add all this one take under root get the RMS and you will get the same expression, if you know B waveform this RMS you calculate this square wave you will get that.

So, just to point out that the odd harmonics if it is present all the odd harmonics will once again you can see the one conductor is placed under the centre of a particular pole other coil side, which I have already placed based on my requirement fundamental thing that will be also under the opposite pole and you can calculate the voltages. So, this is the thing you should remember to conclude this part. Therefore, this expression is the most important one, n is the mechanical rps mechanical rps speed, phi is the flux per pole. And, we will generally neglect the harmonics because of the fact we will see I will try my best to make the B distributions as close as sinusoidal so, that harmonics can be neglected.

So, in this lecture what I have done have told and other important thing to conclude this that ok, if even if B distribution is not is squared step type like this or this. And if you are using a single coil it will be like this only we have seen then we know it can be broken up into fundamental third, fifth and so on. So, I will do Fourier analysis and this is suppose the fundamental component and based on this fundamental component coil span has already been fixed. And this I cannot change, I have fixed up the coil sides into identified 2 slots where this angle is 180 degree electrical. This I cannot do anything that is fixed.

So, now what is happening fundamental component is there, then 3rd harmonic little amplitude 3rd harmonic component will be there and this fellow is rotating. So, far as 3rd harmonic is concerned it looks like it has become a 3 p pole machine and if it is 3 pole machine, will there be also voltage across 1 1 dashed. Yes, there will be simply because if one conductor wherever you have placed under centre of one says north pole, the other one I will find it is under the centre south pole. And therefore, there will be voltages, this is also true for fifth and seventh harmonic.

And, can I use the formula we have already derived? Yes, you can use provided you are careful. Suppose I want to find out the rms voltage contributed by the kth harmonic component of B distribution, then E 1 1 dashed kth should be root 2 pi and this is about the frequency term on the speed term n. And, it has number of poles of this machine is now kp so far as kth harmonic is concerned. So, p should be replaced by kp wherever it is there n by 2.

And this one therefore, looks like root 2 pi k into f where, f is the frequency of the induced voltage due to fundamental and this is phi kp. What is phi kp? Phi kp is we know 4 by p B max into lr, for a original machine p pole for kth harmonic number of poles has increased. It has become kp poles because, over same degree of mechanical separation so many poles are present. So, 4 by kp B max B max k kth harmonics amplitude B max k is this one into l into r; lr of course, will not change, length of the machine is fixed, r is fixed. We will continue next time.

Thank you.