Electrical Machines - II

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Lecture - 18
Induced Voltage in a Coil in a Rotating Machine (Contd.)

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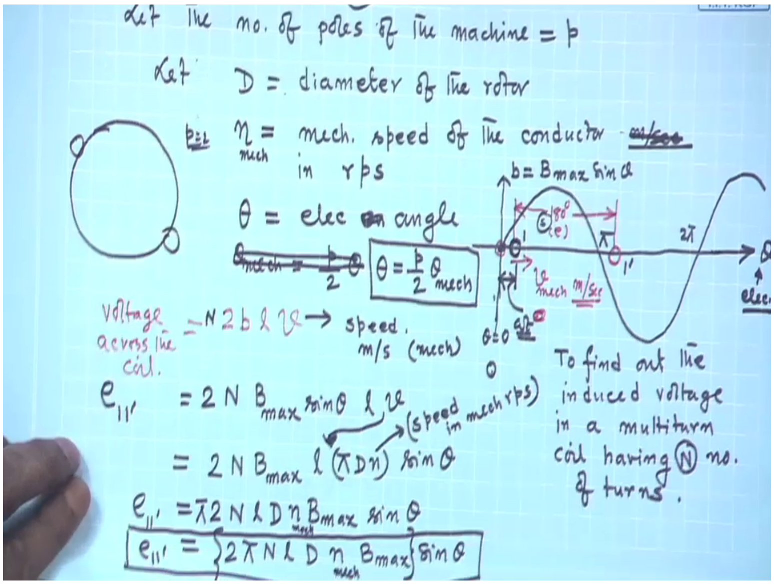

Welcome to this course. And we were discussing in my last class that we found out the difference between electrical and mechanical degree, and their relationship. I have assumed the b to be sinusoidally distributed. And when I am not writing any suffix, it means electrical electrical radian this angle. So, my goal is now very important goal that if a coil moves in a magnetic field, which is sinusoidally distributed and the relative speed between this b and the coil is known, then what will be the expression of the AC voltage, AC voltage it will induce.

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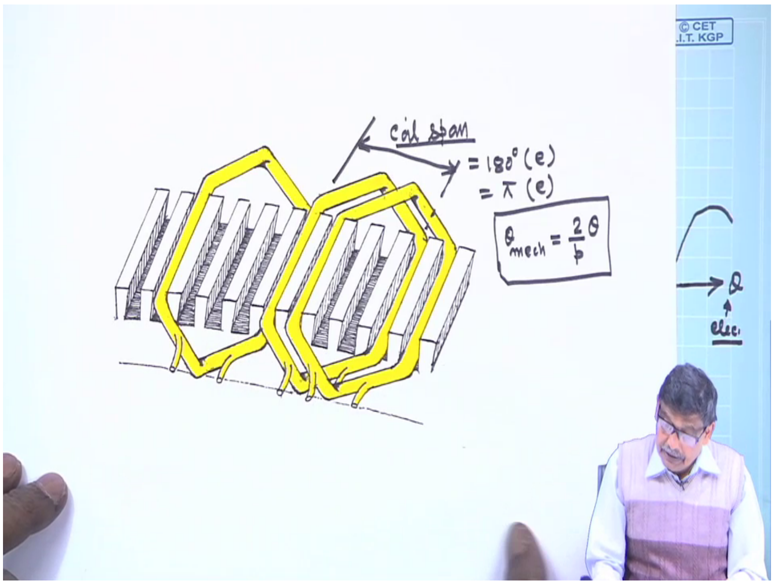

And recall that a coil means like this, it will have a multi-turned thing. You take a piece of wire; go around it several times like this.

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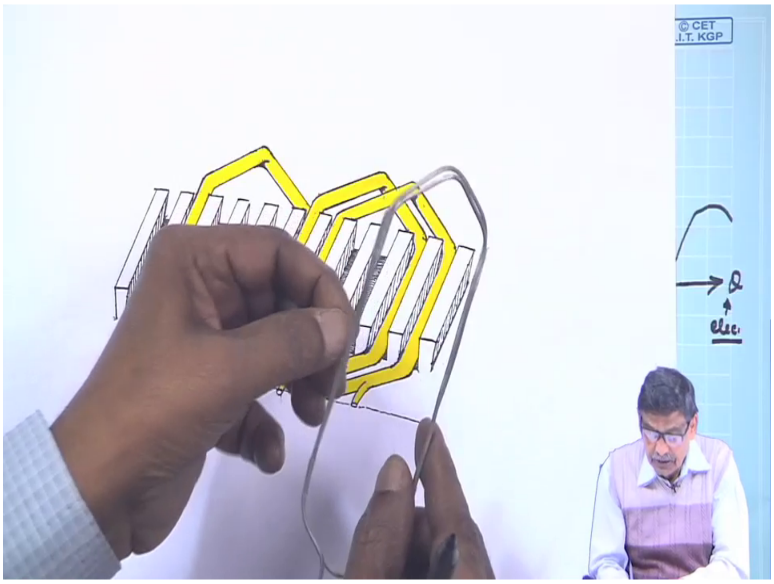

Here I have moved twice. So, induced voltage in this conductor all the EMF's will be added up. Provided you have seen that if this is called one coil side, if one coil side at any time, if it is under center of North Pole, the other coil side will be under the centre of the South Pole.

Then only the velocity of these two coils, they cannot be different, because the rotor will be moving with some speed, and they are fixed in space, therefore they will also move with same velocity that is there. So, it is a two-turned coil, and with two terminals available. But, only thing you have to see is this distance, this to this distance that is called the coil span, this is coil span.

In general, it should be equal to 180 degree electrical or pi electrical. And that will ensure that if because b distribution, I will always sketch in terms of electrical angle. Therefore, if I ensure that one coil side is so this is one coil side, this is other coil side. If one coil side is under the North Pole center, then the other coil side its return thing should be under the center of the South Pole. And this should be 180 degree electrical.

What is the mechanical angle between these two? The mechanical angle will be theta mechanical, it should be theta electrical into 2 by p ok, this is the thing electrical angle that is if you move traverse at certain angle in mechanical terms 30 degree, if it is a four pole machine, it means that you have moved by 60 degree ok. So, this picture you keep me in your mind.

So, I was trying to derive an expression of the induced voltage in a coil in a single coil ok, where the distance of the coil is in terms of angle is 180 degree electrical coil sides. So, if you take a two pole structure, it should be diametrically opposite that is all for two pole. Anyway, so this will be the thing, but here there was some mistake here you see, you should be very clear theta electrical angle should be p by 2 theta mechanical that is very important.

So, what will be the induced voltage 2 b l v, what is v? v is speed mechanical speed meter per second, it is mechanical speed that is what that formula says b l v velocity of the conductor is mechanical meter per second. Now, only thing if it is a multi-turned coil, so that in each coil sides there will be several conductors, it is to be multiplied by n.

So, we want to find out the induced voltage to find out the induced voltage in a multi-turned coil having n number of turns, you do not confused it with North Pole ok, it is a with the respect to context you have to understand. Coil having N number of turns that is what I would like to do.

Therefore, voltage across the coil should be multiplied by then N. So, it will be equal to twice N, b you can put it like this B max sin theta, and l what is l, l is this length active length of the coil, where induced voltage will be there this side, these are called overhang of the coil. They are outside the machine length active length, so this l is this active length of the conductor 2 N B max sin theta l, and then v the velocity v it is like this.

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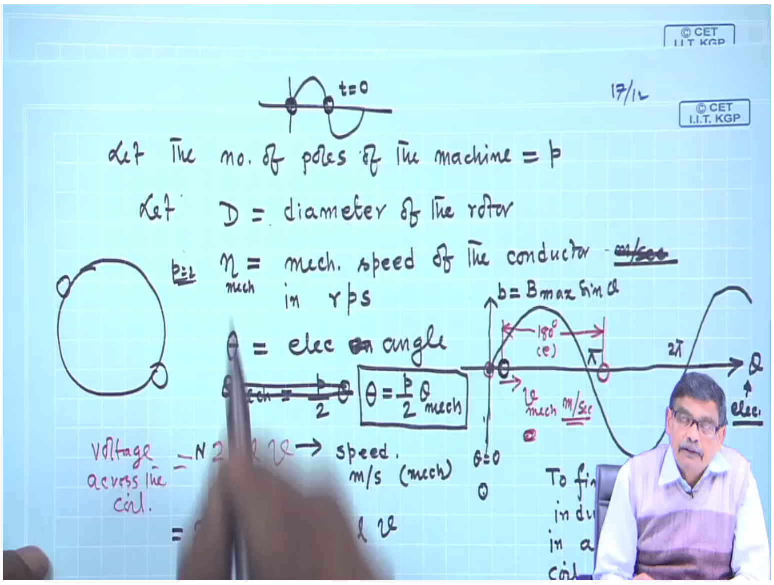

Now, suppose the one coil side was at t equal to 0 occupying this position that is theta equal to 0 is with respect to I will draw it is so that you do not miss any point. This is a the position of one coil side, and the other coil side at t equal to 0 suppose, this is these are the position of the coil. Then after sometime, it has moved by some angle. By what angle it has moved, it has moved by this is electrical degree omega t, it has moved by an angle of omega t.

And in this position, I want to find out what is the induced voltage in the coil. So, I have taken the things correctly no problem. Now, what I am going to do, this let me indicate it this coil let us indicate it by e 1 1 dashed. This is one other side is 1 dashed, this is 1, this is 1 dashed, and it is moving like this. So, this is South Pole that way the polarity will become it will be.

Now, this 2 N B max into l next I will write down the v meter per second, which is equal to pi D n for velocity into sin theta, this is the thing. Where n is speed in rps mechanical rps this is like this. So, let us bring the constant all the constant 2 N l is constant capital

D is constant pi is constant this pi, I hope I have taken everything B max sin theta and then n small n all the terms I have taken ok. So, this is the velocity here.

So, so let me further write it 2 pi N l D small n B max sin theta, theta is in electrical radian. So, this will be the speed this is theta n mechanical here. If since I have written there n mechanical n mechanical n mechanical ok, [FL] I leaved this expression at this stage in this way.

So, in terms of machine constant and this one and theta is equal to omega t if you substitute, you can easily see this voltage will be a nice sin wave. And what will be the rms value of this, peak value will be this thing that I will simply divide by root 2 to get the rms value this one divided by root 2 that is pi. But, generally what happens is this, people express this expression of the induced voltage not in terms of diameter length of the machine things like that or B max, they try to express this induced voltage in terms of flux per pole.

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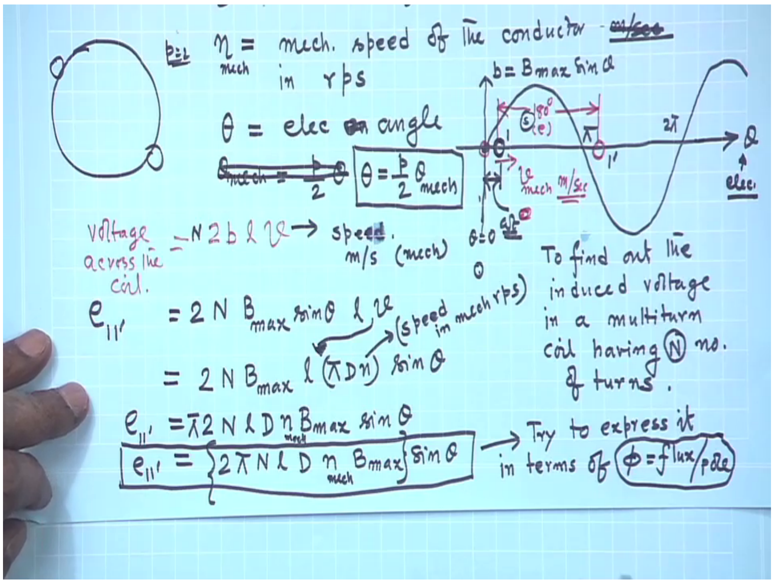

So, my goal will be try to express it express it in terms of a turn in terms of phi, which is equal to flux per pole, which will get rid of the terms B max l D etcetera as we will rightly see now. And the expression also will be similar to that of a transformer that is very interesting thing. So, let us see how to do it.

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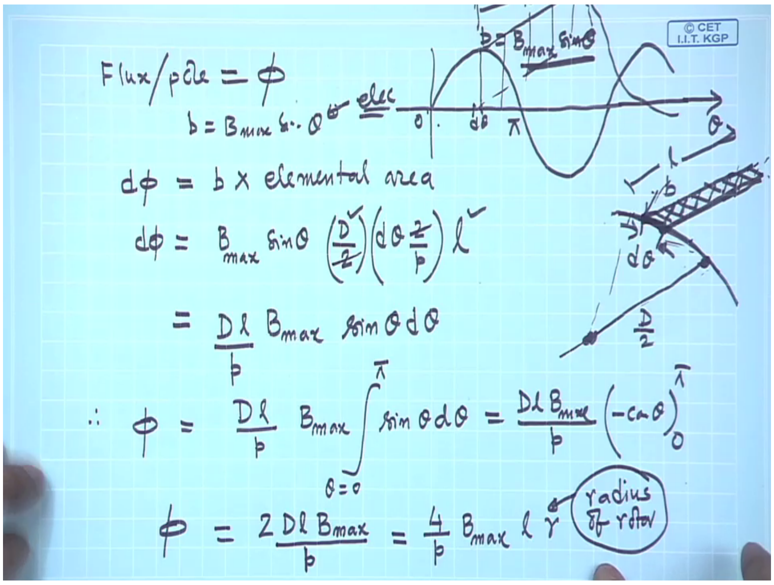

Now, what is flux per pole? So, flux per pole is denoted by this phi, and as it indicates flux is flux density into b. So, this is 0, this is pi, this is your electrical angle theta, and this is b is equal to B max sin theta, this is the thing.

Now, how to calculate flux per pole? It simply tells you that this b is to be multiplied by with some area, what is that area. Suppose, this is the rotor surface here is some b. And this b value is varying as you move from one point to the other, because it is sinusoidally distributed. So, it cannot be just like some instantaneous b and the whole area, it cannot be. So, we will first calculate d phi at a distance theta, I will consider an angle d theta. And this is the length of the machine mind you active length. This b distribution, it is the sectional view; it is like a wave you know spanning over the length l b is like this like your water waves in seas.

So, all along the length at a particular position, it will have either trough or crest or any instantaneous values. So, it is sinusoidally distributed like in three-dimension, it will look likely this here this is the length here, you have got the point. So, at a particular position b here over the length l is constant b max sin theta here like that.

So, what you have to do, you have to take a small strip here calculate this area. And multiply with the instantaneous value of b at that position to get d phi. So, d phi will be small b into elemental area. What is the instantaneous value of b there, it is b max sin theta. And what is this area that is the big question this area. If d is the diameter of the rotor if d is the diameter of the rotor, so its radius is D by 2 capital D you know I have assumed D is the diameter of the rotor. So, it is radius is D by 2 center somewhere here there, it goes like this. So, D by 2 is the area.

So, it is d r d theta is this arc length, so D by 2 into d theta is this length, but is it correct to say that D by 2 into d theta is this length in meter no why, because this theta I have I am measuring in terms of electrical degree. So, therefore r d theta is the elemental length on this arc on the rotor elemental arc, this must be converted to mechanical degree that is d theta into 2 by p. So, r d theta, so this theta theta I have already told, it is b is equal to B max sin theta theta in electrical. So, so take the mechanical angle corresponding to d theta, we have moved by d theta here electrical angle phi. But, to calculate the flux there elemental flux d phi, we have to multiply with this elemental area.

Now, what is it is a small rectangle spanning over the length l, but this distance is r d theta, where D by 2 is the radius of the rotor. D by 2 distance is measured this distance is are mechanical. So, D by 2 into this d theta, but this d theta is in electrical angle, because theta d theta whatever you show here is electrical. So, convert it to mechanical d theta into 2 by p, then this is r d theta. And then multiplied by l, and that will give you the elemental flux.

So, these two goes ok, hopefully it is correct let us see, so it will be B max. Then this D and l let me write, D is the diameter of the rotor, l I have written d theta, and there was a p here p, and then this is sin theta d theta. Now, this theta is in electrical, d theta is also in electrical. Therefore, to calculate flux per pole d l by p B max, I have to integrate this from theta equal to 0 to pi, because everything is in electrical degree, so it will be sin theta d theta is it this will be the thing.

And this integration is of course very simple D l B max by p, and integration is minus cos theta, and pi to 0. So, it will be 1 plus 1, so 2 this integral value. So, this will be twice D l B max by p. So, flux per pole is like this, but we must understand what is its term D is diameter, l is length active length of the motor along this that is this length over which b is like this not, a single curve it is a wave there. And sometimes people also write it like this 4 by p B max l into r, where r is radius of rotor in terms of radius, you can write in either of the way.

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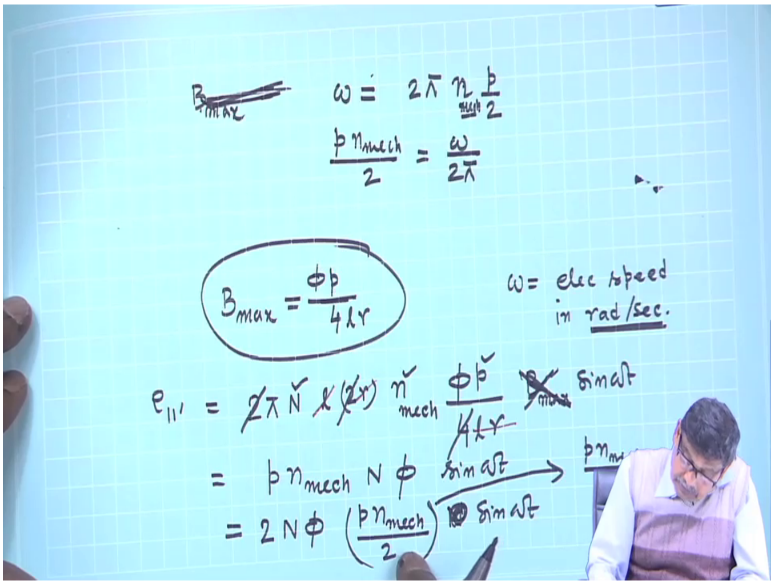

Now, the plan is that I will try to get rid of this B max from this. So, I can say that I cannot go. So,. So, I can say that B max will be B max let me write here, B max will be p by 4 this B max will be phi p by 4 l r. And this I will substitute in this expression of the induced voltage here. So, e 1 1 dashed will be equal to twice pi capital N, what is this capital N - number of turns of the coil then l into capital D.

So, capital D is 2 into radius of the rotor this capital D, and then n mechanical, what is the small n mechanical the speed of rotation of the rotor or the coil which can be measured with a tachometer mechanical speed it indicates, and then B max. And then for B max what I am telling write phi p by 4 l r from this. And then B max into sin omega t. What is omega? Omega is the electrical speed in radian per second ok. So, so this is the thing.

And if you look at it carefully this 4 goes; 2, 2, 4 goes. Then also l r goes. And you will be left with is equal to p n mechanical that is this term this term into capital N this term below nothing is there phi, I should not write B max because I have substituted B max by this you know. So, I extra B max I wrote that must not be there in this line of derivation. So, so this is the thing and then sin omega t.

Now, what I will do is this I will write it like this N phi and this is p n mechanical. And I will multiply by 2 and divide by 2 n sin omega t this is the thing this then I have written. Now, the question is what is this p n mechanical by 2? It is the electrical speed. So, so p n mechanical by 2 is your speed in electrical rotation per second that is the thing. So, this p is this.

Now, p n mechanical can be expressed n see it is like this. Omega can be related with the mechanical speed as 2 pi 2 pi n into p by 2 you know it radian per second p by 2. Therefore, p n mechanical by 2, so p n mechanical by 2 is nothing but what omega by 2 pi is it, p n mechanical by 2 is omega by 2 pi.

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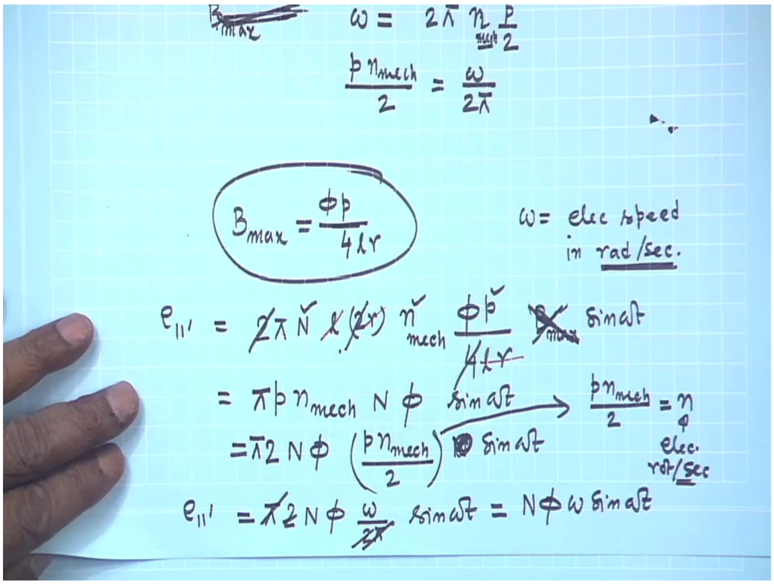

So, so this will be then equal to so this will then be equal to 2 n phi and p n mechanical by 2 is omega by 2 pi into sin omega t, is it correct. So, this is e 1 1 dashed, I have not missed anything hopefully. So, omega electrical radian per second is omega. So, n mechanical came here b l v p n mechanical by 2, no pi I have left just carefully see ok. One pi is coming extra here.

Student: Second line.

Second line here?

Student: Yes

This?

Student: After that, one point missing.

Student: p n by pi is missing.

There was a pi here is it? I missed one pi here; there was a pi here, correct now. So, this is pi. There was a pi here and there was a pi here. Now, I am happy ok. So, I miss some pi as you can see from this step to this step. So, this pi will go off and this 2 will go off. So, your induced voltage will be n phi omega sin omega t.

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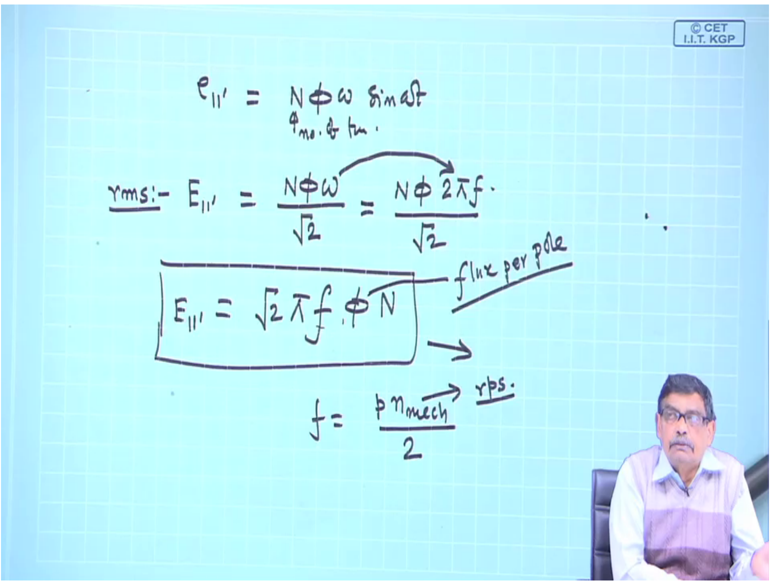

So, the induced this is very nice. So, 1 1 dash this induced voltage let me write it, it is equal to n phi omega sin omega t. Therefore, here omega is the speed in radian per second all electrical speed phi is the flux per pole you see and the dimensions of the rotor d l etcetera goes radius etcetera goes and what is n number of turns.

So, what will be the r m s voltage e 1 1 dash r m s voltage, r m s value of this voltage will be equal to simply this divided by root 2. And this omega N phi can be written as 2 pi f where f is nothing but n r p s expressed in electrical degree number of turns, so that is root 2. So, it will be equal to root 2 pi f phi into N. And expression which is similar to that of a transformer why I am telling similar because in transformer it was phi max, but in case of rotating machine it is flux per pole.

Mind you going from omega equal to 2 pi f, what is omega, omega is the electrical electrical speed in radian per second and that decides what is the frequency of the ac voltage generated? We have told you earlier. Therefore, this is the most fundamental expression of induced voltage; you want to generate ac voltage. So, so having n number of turns means this coil is having n turns and they are placed in appropriate slots, so that you maximize the induced voltage that means, that 1 coil side if it is under the centre of north pole other must be placed under south pole. And they will be moving with constant velocity varies that. So, inputs of a generator are flux per pole and the speed. Now, speed is in this expression where it is, it is here f equal to p n mechanical by 2 n mechanical is speed tachometer speed n rps.

So, thank you go by this through this lecture carefully and this will be the starting point. So, if you have understood what has happened to a coil induced voltage, then in a rotating machine there will be number of coils I will not be (Refer Time: 32:39) by these because I know what is happening to a single coil.

In other coils, they will be simply displaced from this coil by some angle either in the forward direction or in the backward direction. And if you connect those coils in series, this, that, and I know in each of the coils there are ac voltages induced. So, they will be only displaced in time phase by some angle. So, phasor notations can be incorporated very efficiently there.

Thank you very much. So, go through this.