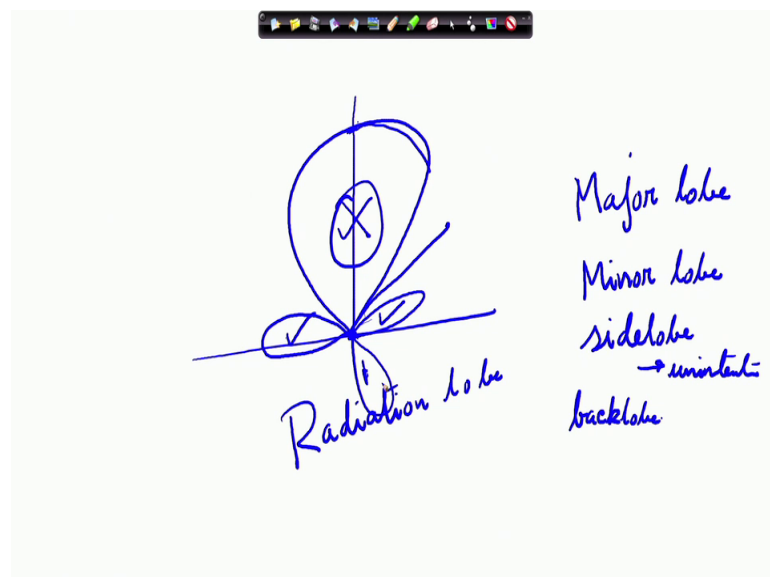


Analysis and Design Principles of Microwave Antennas
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Lecture – 06
Directivity and Gain of an Antenna

Welcome to this lecture on basic antenna parameters. In last class we have seen the radiation pattern. I again tell you that radiation pattern in the far field only the radiation pattern get stable. So, always radiation pattern should be taken only at the far field. So, you can measure actually if you put a power probe and at the far field distance, take any convenient distance and they are at angularly vary either azimuth or elevation and plot the power, you see power plots, that will be the 2 D power pattern in that plane. Change the angle that means, if you have already taken elevation variation, make azimuth variation put it according to the definition characterize it whether it is e plane, h plane or pi plane, theta plane, any of that.

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Now, always you will see that the, if I have a pattern generally, we do it in a polar pattern. And the intensity of radiation, it is clear that here radiation is taking place, the maximum is taking place here, it is gradually reducing etcetera. But again you see here is an comparatively very non radiating zone; that means, in this angle there is no radiation.

Again radiation is kicking up, again there are no radiation. So, the radiation pattern is generally broken into various lobes.

So, we define radiation lobe as the portion of the radiation pattern bounded by region of relatively weak radiation intensity, portion of the radiation pattern bounded by region of relatively weak radiation intensity. So, this is a radiation lobe, this is a radiation lobe, this is a radiation lobe and in between zones there are region of weak radiation intensity.

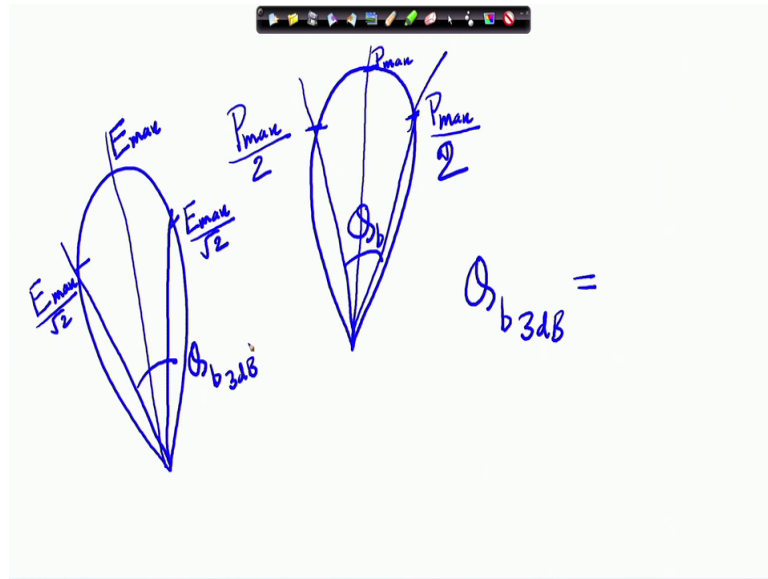
Now, out of that, there is a major lobe. Amongst radiation lobes, there is a major lobe. What is a major lobe? Major lobe is radiation lobe that contains the maximum radiation direction. In this case you see this is the maximum radiation direction. So, this one is the major lobe. The others are not containing maximum direction of radiation. So, only generally one major lobe is there. Now what is minor lobe? Any lobe except major lobe are called minor lobe. So, any lobe which is not containing the maximum direction of radiation is called minor lobe.

Now, there are side lobes. Some of this are side lobes like this one is side lobe. What is a side lobe? Generally, the lobe in an unintended direction, I want that my antenna should radiate only in these zone, but obviously, it is also radiating here. So, any interference that will come from that is why it will receive that also. So, it is an unintentional one. So, where I do not want to put my radiation those are called side lobes.

So, this is a maybe a side lobe, this may be a side lobe may, may not be. If you want to put here, because nowadays those applications are coming that some radiation I want in this direction; also I want to cover that direction. So, in that case this would not be a side lobe. But if it is an unintentional direction, then this is a side lobe similarly there is. So, I will write side lobe is one which is unintentional radiation. Similarly, there is another term called back lobe.

Now, here I should say that, in side lobes generally we call side lobes. The lobes which are unintentional and also in the same hemisphere as the main lobe, so, these one, but suppose this one is generally not called this one is not called side lobe this is called back lobe. Back lobes are lobes which are in the opposite hemisphere to the main lobe, opposite hemisphere occupying the, opposite hemisphere of the main lobe that is called back lobe. So, from radiation pattern we give a visual estimate that how the radiation in which direction radiation is taking place, but we want some quantifying measure.

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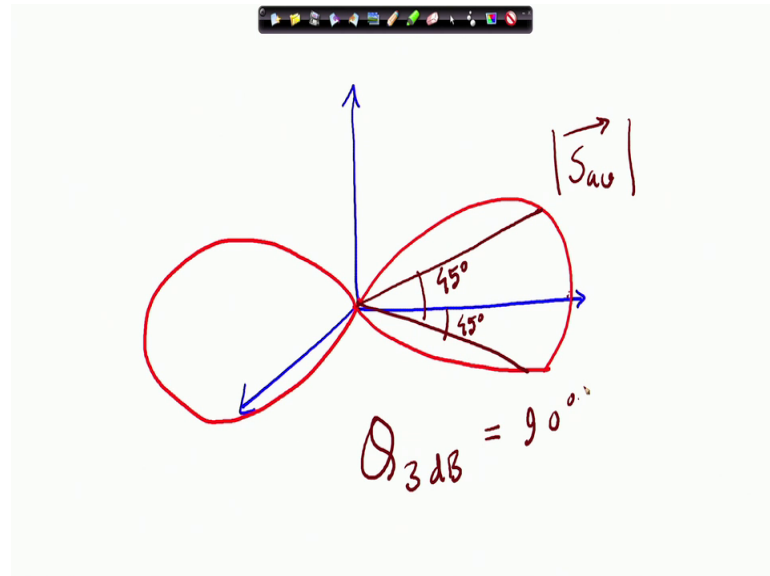


So, if this is a beam and you see that if this is a main lobe, then this is the direction of maximum radiation. Radiation is gradually diminishing and here it is coming to 0. Now, what is the beam width because that is a quantification. So, what we do we find out that what is the maximum power here, let us say P_{max} and find out, if this is a symmetric one like this, then what is the P_{max} by sorry by 2 and point where P_{max} has come power is half and find the angular distance between this two directions. So, these are called beam width. This angle θ_b , so θ_b generally, you see p if power is half that is actually 3 dB.

So, generally we say 3 dB beam width will be this. From the pattern, we can find out that this is the 3 dB beam width if instead of power, we can plot fields. So, if I plot a field and it has got a main lobe like this, then in that case, this is the maximum field and then we locate two points; one is E_{max} by root 2, you know this that for fields we have this root 2. So, this angle is the 3 dB beam width.

So, either from the field pattern or from the power pattern, you can quantify the 3 dB beam width.

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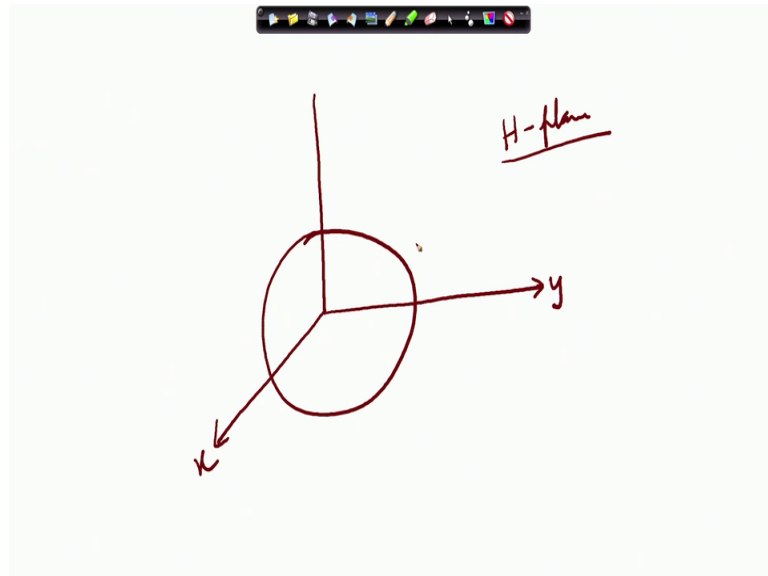


So, we can go back and, if we look at the E plane pattern of the current element that we have already seen, as I already said the current element means that in the theta plane how it is varying. So, we have already seen that it was a sin square theta pattern.

So, this is maximum though my drawing may not show like that and here, this is maximum that was a S here we are plotting, the radiation intensity vector. So, here I locate what is the S av by 2, here also this. So, we know this is a sin square theta variation. So, at which angle sin square theta will be half of this value; that means, sin square theta, half means sin theta 1 by root 2.

So, this is 45 degree, this will be also 45 degree. So, I can say that for current element theta 3 dB be will be 90 degree. So, in the E plane, the beam width is 90 degree and in the H plane of the current element, what will be the bandwidth?

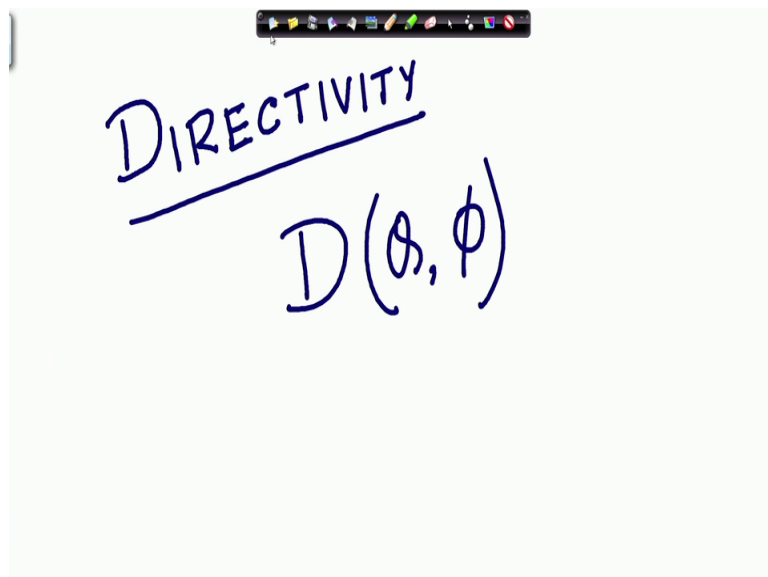
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H plane means, this was the pattern the H plane pattern of a current element. So, actually there is no maximum or all points are maximum. So, there is no beam width cannot be defined here. So, if you have some variation, then only you can define beam width.

So, this is radiation pattern we have said. So, the next concept from here, next we will see another parameter.

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Very important parameter of antenna called Directivity; all of you may have heard this. This is actually from the radiation pattern what we get visually, but if we want to

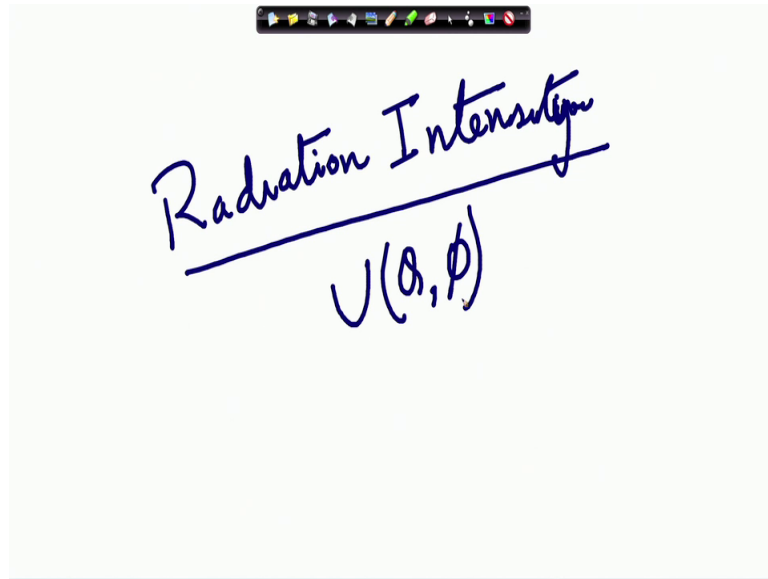
quantify that at each direction, at each direction how much power is flowing. Beam width gives us an idea in this zone there are good amount of radiation, but if want to find out in each direction how much power is going; that means, the angular distribution of power throughout not only in the main lobe, in the side lobe everywhere. So, that we can get from this concept Directivity.

Actually, this is the characterization of directive nature of the antenna as I am saying from the beginning, antenna has two purposes; one is it's power transfer impedance matching with the free space, another is directing the radiation make it focused in a particular direction. So, that focusing part is measured by this parameter directivity. So, actually the variation of the intensity, the variation of the intensity with direction in space is described by a directivity.

So; obviously, this directivity D , this should be a function of the angular coordinates; that means, θ and ϕ . So, directivity is a function it is a function of θ and ϕ . Now, before so obviously, you know any source of radiation; that means, a current distribution that is radiating. So, how it is radiating in different angular sectors it is radiating. So, I will have to find the power that it is giving in a particular angular direction.

Now, already we have seen E and H fields that radiation fields in the far field, the E and H fields and we have found the pointing vector and from that the power density that is taking place in a area which is perpendicular to that flow of power, radial flow of power. So, that is on a planar area, power density is defined. But, now we want to find out what is the angular distribution of that. So, we just need to define another quantity that quantity is called that is a new quantity it is called Radiation Intensity.

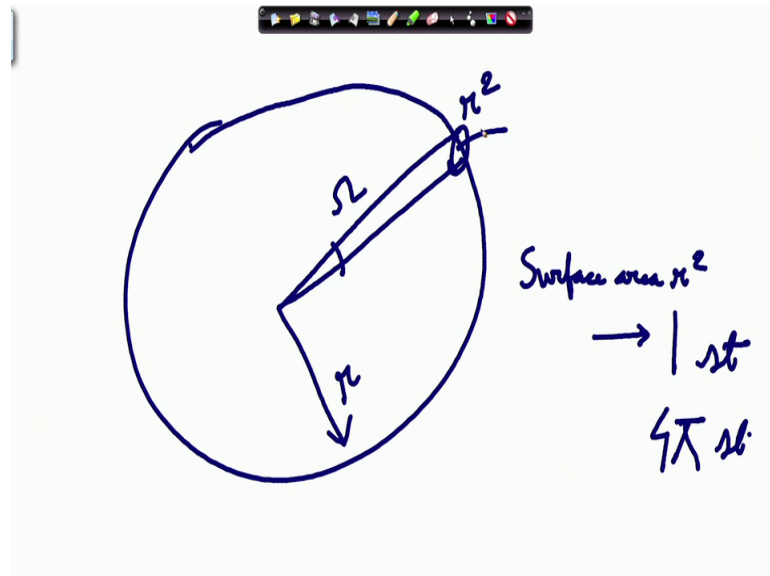
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Radiation intensity actually the intensity the name does not say much the thing is it is a function of angles. So, U , θ and ϕ ; that means it is the power that is radiating in θ ϕ direction. So, if you tell me what is the power in 10 degree θ and 35 degree ϕ , this function value I can give. So, we will have to understand the and here you understand that actually antenna is a 3D structure. So, the angle it is creating that is a solid angle. So, we need to have a relation between the solid angle and our planar visualisation thing. That is why I want you to get revisited to the concept of solid angle to understand that I will come to here, what is the concept of a solid angle I think you remember what is a planar angle that is a radian.

So, what is one radian? The angle subtended at the centre of a circle by an arc whose length is equal to the radius of the circle. Similarly, the unit of solid angle is, so solid angle suppose in this sector this area.

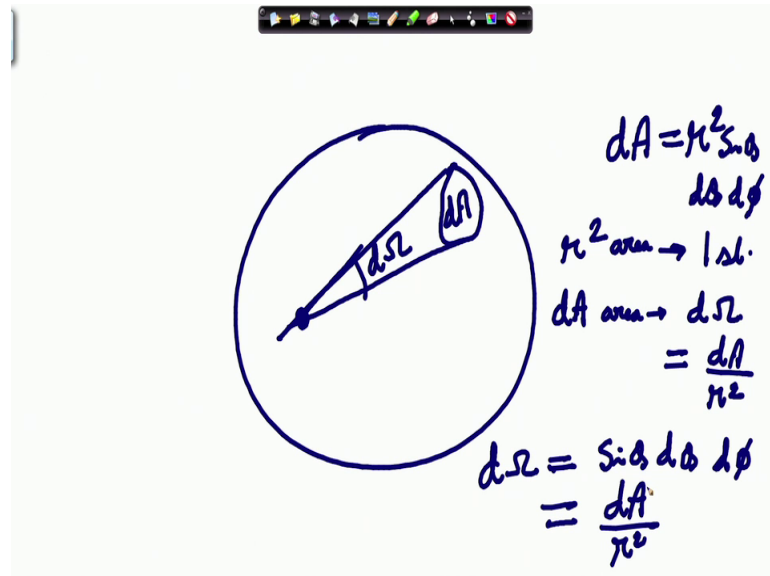
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So this area subtends an angle solid angle Ω . So, unit of this is Steradian you know. What is the definition of Steradian? So, if this is a spherical surface of radius r , then an on the surface an area r square the angle it subtends at the centre that is called one Steradian, that is the definition.

So, that means, an surface area r square subtends one Steradian solid angle. So, the whole spherical surface whole close surface that will subtend how much area; simple unitary method will show you that 4π is the total solid angle that is subtended by a closed surface, a closed sphere will subtend at the centre 4π Steradian.

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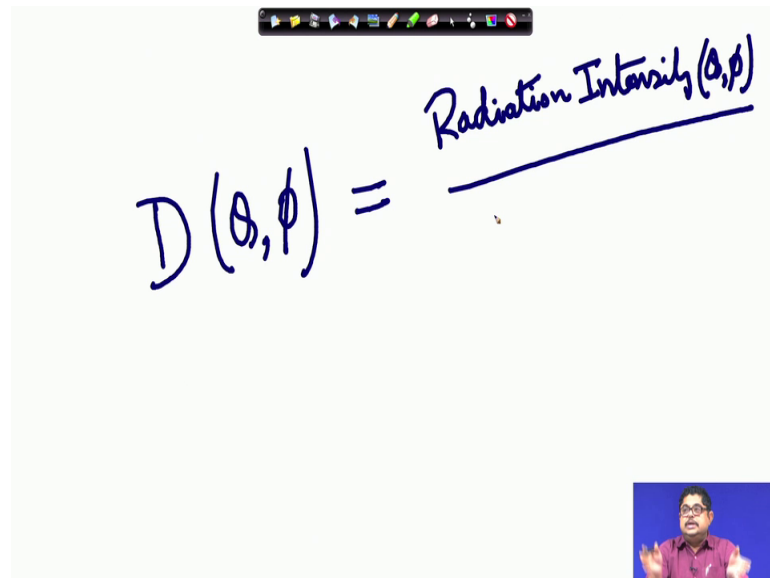


Also, if I call suppose now I have a spherical surface, on that I have an area dA , a elemental area dA . So, sorry this is let us called the centre.

So, how much angle it is solid angle it is putting at the centre. That is $d\Omega$. So, what is the relation between these two? For that I will remind you that what is the value of dA in the spherical co-ordinate it is $r^2 \sin \theta d\theta d\phi$. So, I can write that a dA area subtends an angle of $d\Omega$ or I can start that an r^2 area subtends an angle of one Steradian.

So, dA area will subtend how much angle, this will be dA by r^2 . And that is nothing but $d\Omega$. So, what is the value of $d\Omega$ from here it comes, it is simply $\sin \theta d\theta d\phi$. So, $d\Omega$ is in this also a area dA elemental area dA will subtend an elemental solid angle $d\Omega$ at the centre and their relation is $d\Omega$ is equal to this is also equal to dA by r^2 , this much I need.

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The image shows a whiteboard with a handwritten equation in blue ink. The equation is $D(\theta, \phi) = \frac{\text{Radiation Intensity}(\theta, \phi)}{P_{\text{rad}}}$. The denominator is partially obscured by a small video inset of a person in the bottom right corner. The whiteboard also has a toolbar at the top with various drawing tools.

Then let us come to the concept of directivity of any general antenna. So, directivity function is defined as [FL] before that I need to already, so what is or what is this directivity function?

So, I want that what is the radiation intensity at theta phi direction. So, I can say, radiation divided by what is the if instead of angular nature, I had an antenna which was fictitious antenna because no antenna can do that, a fictitious antenna which can radiate equally in all direction, then I could have taken that equally it is radiating all with respect to that I am finding the radiation intensity of this.

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$U(\theta, \phi) \rightarrow$ power radiated per unit solid angle.
 $P_r =$ Total power radiated.
 $U(\theta, \phi) = \frac{dP_r}{d\Omega} = \frac{dP_r}{dA/r^2} = r^2 \frac{dP_r}{dA}$
 $= r^2 S_{av}$
 $= r^2 \frac{1}{2} \text{Re} [\vec{E} \times \vec{H}^*] \cdot \hat{a}_n$
 ← time average power density.

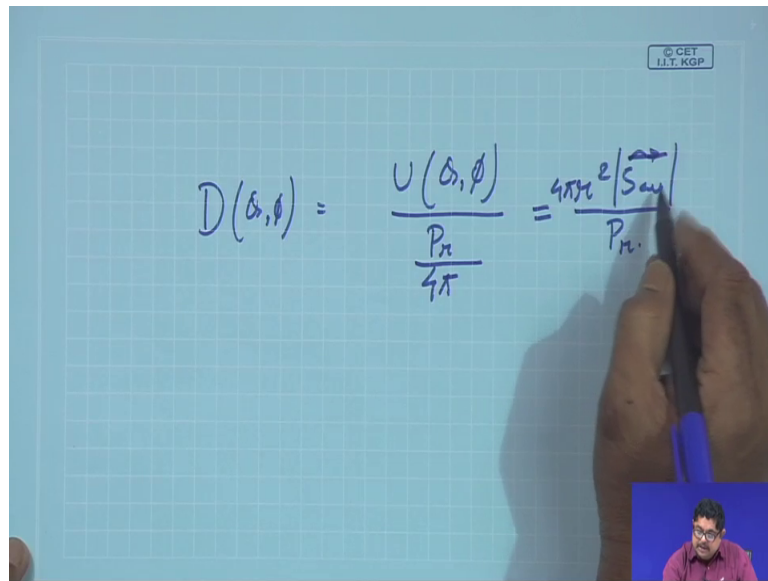
So, the radiation intensity is the power radiated per unit solid angle. And already we know that P_r is the total power radiated by the antenna, now $U_{\theta, \phi}$ that I can call that instantaneously, it is basically at every solid angle what is the $dP_r/d\Omega$, that is the radiation intensity. Now, we do not know its expression, but we already know that this we can write as dP_r this, we have already found this value of $d\Omega$ in terms of area that is dA by r^2 .

So, this is nothing but $r^2 dP_r/dA$. So, what is the relation? This is the radiation intensity is r^2 into dP_r/dA that means per surface area how that is varying and this is actually the variation along the solid angle. So, this is solid angle variation and this is the on the surface area how the power is varying. And we can say now that what is our this $dP_r/d\Omega$, now how we will evaluate this.

So, we know what is dP_r/dA . dP_r/dA is nothing but our already it is nothing but power intensity, power per unit area. So, this is same as our S_{av} average vector. So, already we have found S_{av} average vector. What was S_{av} average vector for clarity I am writing r^2 , then it was nothing but half, real, the pointing vector in the radial direction how much going this.

Now, you see what we will do, we will define directivity. What is directivity?

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$$D(\theta, \phi) = \frac{U(\theta, \phi)}{\frac{P_r}{4\pi}} = \frac{4\pi r^2 |\vec{S}_{avg}|}{P_r}$$

Directivity function is basically, this U theta phi; that means radiation intensity in theta phi direction divided by radiation intensity of an average antenna or omnidirectional antenna which radiates equally in all direction. Now, any antenna which equally radiates means equally in 4π angle solid angle how much power it radiates, that is simply P_r by 4π because total solid angle is 4π .

So, if anything radiates equally in here, this will be this. So, already we have seen that U theta phi is $r^2 S_{avg}$. So, this is divided by P_r and this 4π goes here. So, this is the expression. So, $4\pi r^2 S_{avg}$ obviously, this S_{avg} will have to take the magnitude because S_{avg} is a vector. So, for S_{avg} vector but we are taking the power ratio, power is a scalar quantity. So, we need to take the magnitude. So, directivity is these.

So, at different direction I can calculate that that what is the power density, this is nothing our power density S_{avg} if you remember we used to call it, the time average this is nothing but time average power density. So, these gives an idea that at this solid angle how much better my antenna is compared to an omni. If it was omni, if my antenna is directive performance is same as omni, then this D theta phi in that particular angle will be 1. If it is more than 1, then only I will be able to say that I have a better directed antenna.

So, this is general for any antenna, so for any antenna we can find that, but for current element let us find out this that.

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Current Element:

$$\vec{S}_w = \frac{\eta_0}{8} \left(\frac{dl}{\lambda_0}\right)^2 |\tilde{I}|^2 \frac{\sin^2 \theta}{r^2} \hat{r}$$

$$P_r = \frac{2\pi \eta_0}{3} \left(\frac{dl}{\lambda_0}\right)^2 \frac{|\tilde{I}|^2}{2}$$

$$D(\theta, \phi) = 1.5 \sin^2 \theta.$$

$$D = D(\theta, \phi)_{max} = 1.5.$$

Already because for current element, this two things, we know we have already calculated the total power radiated we have already found out the power density vector. So, we know this expression. So, we can easily find the directivity of the current element.

So, current element we have already found what was our $S_a v$ if you see your notes that our $S_a v$ was η_0 not by $8 d l$ by λ_0 not square $\sin^2 \theta$ by r^2 . Then it was actually a r directed. In actual expression, so when I am taking magnitude, so I can put it there also we have seen what was our P_r , P_r was $2 \pi \eta_0$ not by $3 d l$ by λ_0 not square I^2 by 2 . Now, if you put that you will get that directivity function; that means $4 \pi r^2$ into $S_a v$ by P_r that will be $1.5 \sin^2 \theta$.

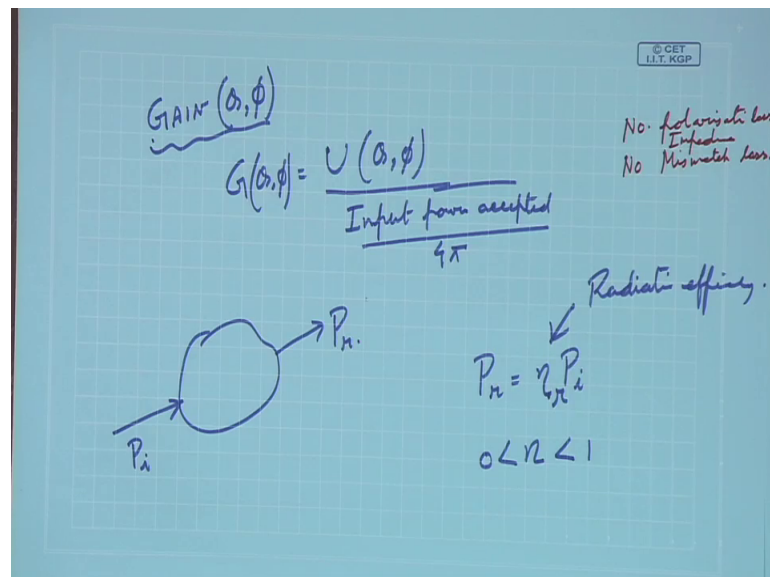
So, it says that you give any angle, so at suppose 30 degree θ , θ is 30 degree. So, $\sin \theta$ is half, so one fourth; that means 0.25 . So, you can find out it is 1.5 by 0.25 . So, it is 1.5 by 0.25 that is 6 , like that you can get those values. So, what is the maximum value you can get? Obviously, when $\sin \theta$ is 1 so that will be 1.5 .

So, maximum directivity generally instead of directivity function many times only d is said the implicit thing is it is actually $d_{\theta, \phi}$ is maximum value. So, in this case, this

maximum value is 1.5. So, the maximum directivity of a current element is not very good, but it is not an omni. It has we have seen that it focuses the power in the broad side direction. So, how much more than a omni? It is 1.5 times omni and said theta is equal to pi by 2 direction. So, maximum directivity is often referred as simply directivity.

So, from that you can find out that whether that antenna is good or bad at your directed one. And here we want to say that what is isotropic radiator? It is a fictitious antenna sorry that radiates uniformly in all directions and is commonly used as a reference. Now, a related quantity that is the another parameter of antenna, that is called gain.

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Now, gain also is an angular function. So, we can find out that how much gain we are having. So, it is defined as same that what is the radiation intensity but only thing is its denominator is a bit different because in directivity, you do not find out that whether that is really radiating or not, you see anything, any in efficient antenna also radiates.

So, suppose I do not have a good directed sorry, I do not have an antenna suppose I have a actually an power absorber, but that me also have a power distribution because some power will always be radiated from any metallic structure. So, that will have some directive nature. So, by looking at its radiation pattern and looking at its directivity, I may conclude that no it is an antenna, it has a directivity. But Gain actually will find out that it is not, that that is why the denominator of Gain unlike directivities.

So, Gain is we can define gain as the. So, Gain function $U_{\theta\phi}$ is $U_{\theta\phi}$ by input power accepted by 4π . So, here input power is there it was radiation total power radiated. Here instead of total power whatever power has been given. So obviously, there will be some losses in the antenna, that will be in the Ohmic losses, that may be in the impedance mismatch losses that may be in the mismatch losses.

So, after that loss, whatever is remaining whether it is able to radiate that that will come from this Gain function. So, we will discuss these because this is actually will bring that if you look at the two expressions, that actually the relation between gain and directivity is that you see that change is taking place here in case of directivity here we have written total power radiated, here we are writing input power accepted.

So, if this is an antenna, the input power is p_i and its output is a radiated power p_r . So, basically the, what is p_r it is some factor η into p_i . This η is a constant it is always less than 1. It is between 0 to 1 because antenna is a passive device. It cannot produce power. So, this η , do not confuse it with α . So, sometimes we write it as η_r do not confuse if the intrinsic impedance, it is radiation efficiency this I called radiation efficiency.

So, whatever input power I am giving, if radiation efficiency is substantial I know that power is also getting radiated because I am giving input power. If it is an absorber type of thing, then radiation intensity will be very low but for microwave antennas the radiation intensity should be 70 percent, 80 percent, 90 percent like that, we were 99 percent also it can be because Ohmic loss is very small. If you have a good conductor, Ohmic loss is very very small.

So, the main loss is in the there is there may be the impedance mismatch or polarization mismatch or in the near field, there may be storage of energy, so the radiation is not taking place etcetera. So, that is why the gain gives you much more information, apart from the directive nature it also says that what is the radiation efficiency of the antenna. So, obviously, this is please remember that this is subject to no polarisation loss. If it is there, simply at those loss factors; no mismatch means impedance mismatch loss, mismatch loss you know if it is there, if there impedance mismatch, then there will be the reflected power. So, that you take into account.

If polarization change, then there will be polarization loss. So, you take those losses this is this formula is valid, if you have all these losses are not there, then this relationship. So, again you find out from the radiation pattern you can find out or if you derive the expressions of a say (Refer Time: 36:18) radiation intensity, this is radiation intensity, this is input power that you have given you know.

So, you can find the gain function. There is some other few parameters so that will see like there is an effective isotropic radiated power, then the impedance of the antenna, these are all other quantities in effective area of antenna, those will see in the next class.