Analysis and Design Principles of Microwave Antennas
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Lecture – 38
Array Pattern Synthesis (Contd.)

Welcome to this lecture on Array Pattern Synthesis. In the previous lecture, we have discussed that an uniform array a designer does not have sufficient control to specify the side lobe level and the directivity or the main lobe beam width. So, that is why the a more complicated thing is if we also change the individual excitation amplitudes they are varied or we will say that complex amplitude; that means, the individual excitation amplitude and phase if that is varied, then we have we will see that for better control can be exercised on the shape and width of the main beam and on the location and amplitudes of the side lobe.

So, thus it will be this topic that is why that we can so that we can synthesize a given specified pattern that we will do. We will completely restrict ourselves here in the u plane. So, array factor we will see we will take the a few and we will specify how to choose the excitations, individual element excitations to achieve at a different one. Now, there are various techniques actually, there are actually this is an active area of research even now also it is evolving because there are actually one thing we are not considering that mutual coupling between the array elements when they are placed closely.

So, that actually changes the whole game plan because here we are saying that there is no mutual coupling, but mutual coupling changes. So, how to choose the excitation coefficient that people are coming up with better and better methodologies etcetera; we would not go into detail of all those advanced topics, but we will just see some fundamental methods by which this array synthesis can be done. Actually, we will discuss only one dimensional array, but the method that we will discuss can be easily extended to the two dimensional arrays etcetera.

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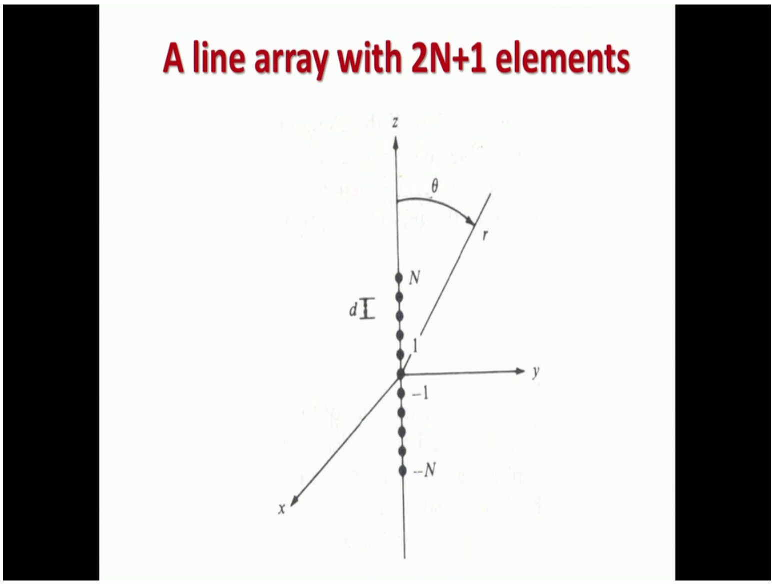

Now, first let us look at a line array with 2 N plus 1 elements. Let the excitation of each element so, elements are on the z axis and get the excitation of each element be proportional to C N for the Nth element; that means, C 1, C 2, etcetera. Here, we will see also we will have a C 0 because we are considering 2 N plus 1 element. So, there is one element at the center.

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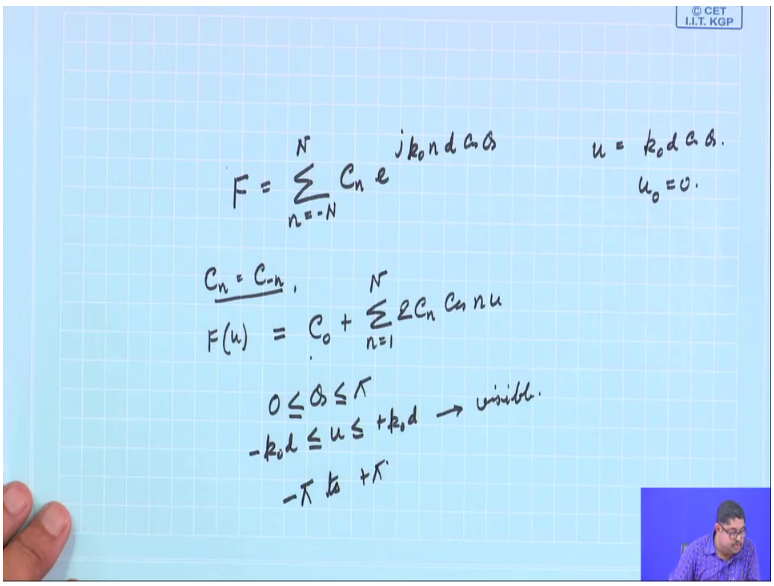

So, the array factor, we can write as n is equal to minus N to capital N C n e to the power j k naught n d cos theta. Now, if we choose C n is equal to C of minus n, then we get if becomes C 0 plus n is equal to 1 to capital N 2 C n cos n u. So, this side is if I call it F u where again now it has before u is nothing but our k naught d cos theta. So, again we assume here actually that means, here we have assumed that u naught is 0 in phase excitation.

So, C n is a complex quantity. Now, if we look at this is nothing but a Fourier series, cosine Fourier series with n plus 1 are known amplitude. So, if I know F u or if you probably specified, I can find out this C naught and this C n things. Note that our theta is varying from 0 to pi so, minus k naught d to plus k naught d is the range of u that corresponds to visible space.

However, so the pattern should be specified for a complete period in the u plane; that means, from minus pi to plus pi. So, if the pattern is specified, we can find it approximately so, what will be now C n value?

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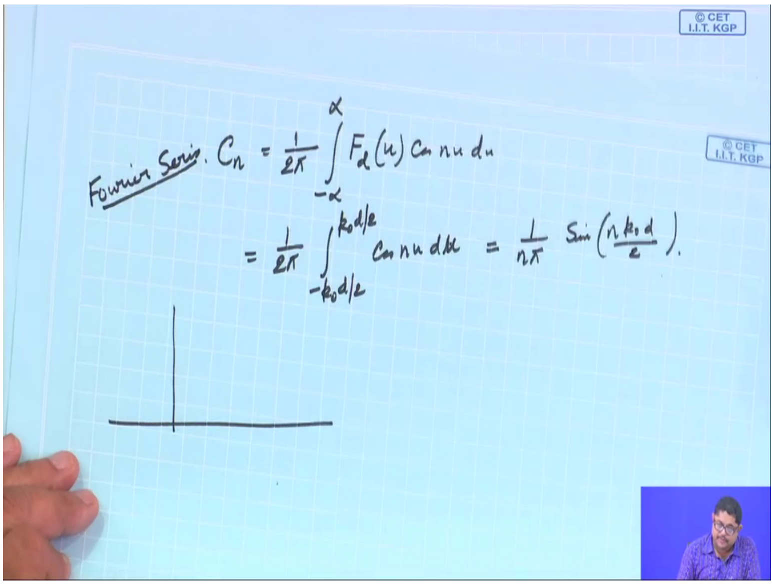

From the Fourier series knowledge, we can easily write that 1 by 2 pi minus pi minus infinity to plus infinity F I am writing F specified or a desired u cos n u d u and this is suppose I want that the I think in the next one it is given.

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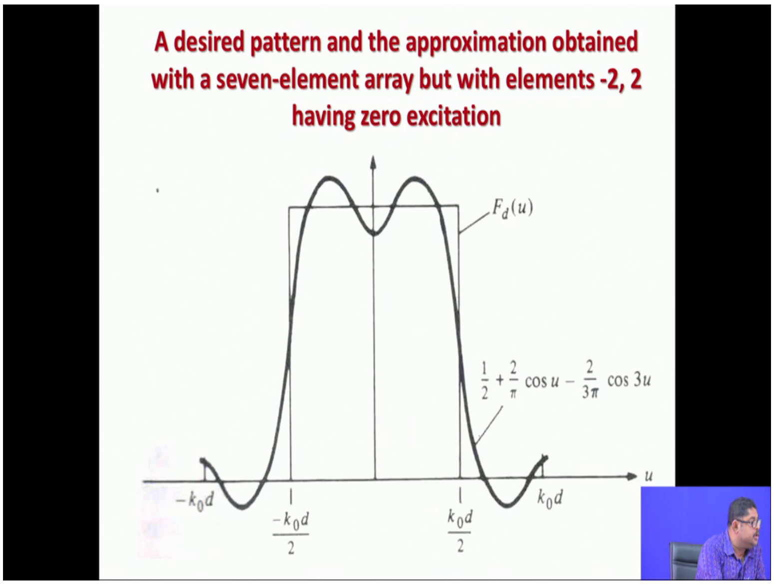

A this is F d u; that means, suppose the rectangular function from minus k naught d by 2 to plus k naught d by 2. The visible space is here minus k naught d to plus k naught d for a broadside array. So, this one so I can find C naught that will be a 1 by 2 pi. This is cos nu du because rectangular function; so, 1 here, so this becomes 1 by n pi sin n k naught d by 2. So, for a seven element array, you see the pattern is like this, the synthesized pattern.

Now obviously, seven element is small that is why rectangular is a like this. If you go on adding more number of elements, you can go up to the desired pattern. So, this is one approach it is very easy and actually people have proved that this is a it gives a least mean square error approximation to the desired pattern. But if you, but the problem is you see that here you have side lobes etcetera so, if another choice is that suppose I do not want any side lobe, so actually this method what I said that is called Fourier series method already you have seen.

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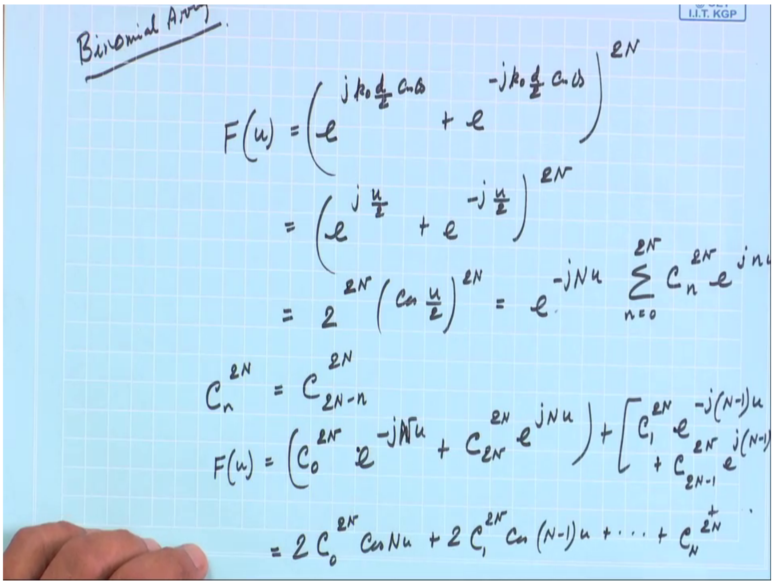

Now, the next another approach is let us see that we write the array factor as e to the power j k naught d by 2 cos theta plus e to the power minus j k naught d by 2 cos theta because, we have symmetrical those two into 2 to the power N let us write like this. So, what we get is e to the power j u by 2 plus e to the power minus j u by 2 whole to the power 2 n is equal to 2 to the power 2 n cos of u by 2 whole to the power 2 n is equal to e to the power minus j N u n is equal to 0 to 2 N C n 2 N e to the power j n u.

So, actually, this one if you expand you get this. Now this you see the excitations here because when I will take the array factor this factor will go. So, this is nothing but binomial series so, that is why this array is called binomial array. Now, we know the property that a binomial array C n 2 N, this is same as C 2 N minus n C n 2 N. So, F u now can be written as C 0 2 N C 0 e to the power minus j n u plus C. Since we again actually it is a sort of Fourier series, but the beauty of these that if we choose like this to see the pattern.

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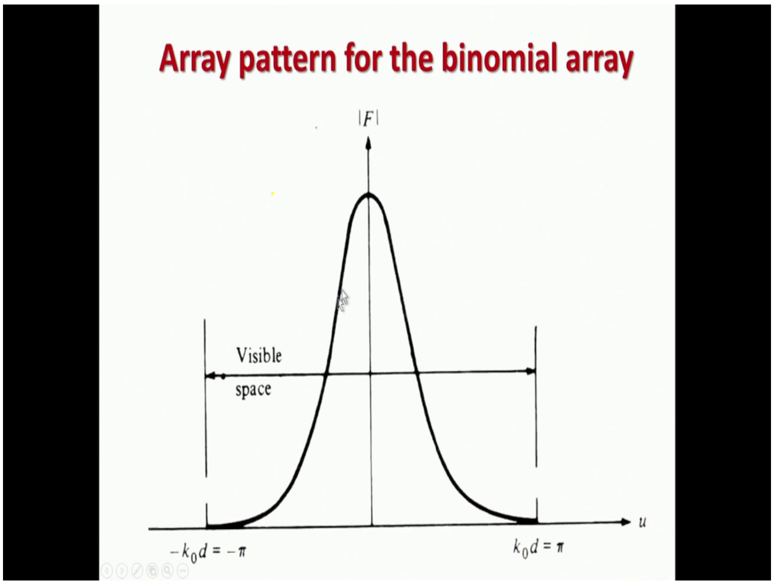

The pattern is like this, the visible space is same as before because we have taken the alpha to be 0 or u 0 to b 0. So, this you see there are no side lobes so, this is the other extreme that there are no side lobes, but what is the problem; obviously, this beam width is much more than uniform. That is obvious because the moment we are taking this binomial ones so, at the sides the excitation amplitude is quite less.

So, actually the array length is getting reduced because effectively, the interference or the radiations from the edges they are not contributing to the actual radiation that is why the half power beam width is broader. So, these are inefficient use of the available array length etcetera. Now, let us come to the point that we have seen. So, if you want to these are simple things that if you want to reduce side lobes you can go for these. If you do not care for gain etcetera, you can easily take binomial array that is a good choice.

Similarly, if you want to have an large number of elements if you want to take, you can use the Fourier series things. If space etcetera is not your constant, so you can approximate by taking Fourier series thing and that also you can do. Now, let us come to actual point and that the development came mainly from Schelkunoff. He actually we were discussing this what he said that let us do not have any negative things actually that is a numbering game. So, let us number so from one side let us start 0 so, go to N. So, do not have this so, the phase, suppose the phase now we consider u naught so, our u naught is alpha d.

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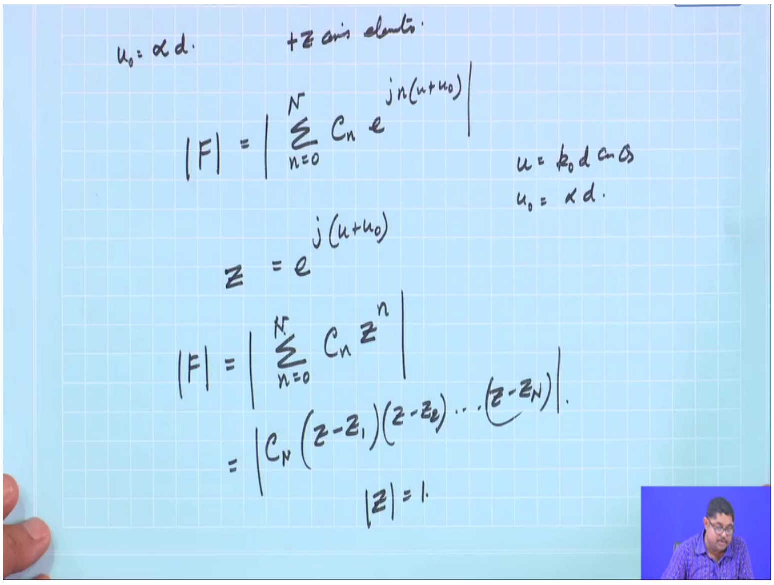

So, only we have in the plus z axis the elements this we can say without losing generality. So, array factor will be so, now our element length is still N plus one. So, n is equal to 0 to n C n e to the power j n u plus u naught, where u is equal to k naught d cos theta, u naught is alpha d. Now, let us define actually this Schelkunoff did this he defined a complex variable z that is e to the power j u plus u naught.

So, array factor is F is equal to n is equal to 0 to N C n z to the power n. So, this z to the power n or that means, this expression is nothing but a polynomial expression of order n that is his contribution that. So, now the mathematics has reach richly study this is various polynomials so, that polynomials properties will transport to the synthesis of the array patterns. Firstly, let us see this polynomial of order n has n number of zeros, this is a polynomial of degree N if I expand capital N.

So, it has n number of zeros so, F can be written as C N z minus z 1, z minus z 2, z minus z N and what is each factor z minus z 1? We can say, we can interpret it as the array factor for two element array. Thus we say that the array factor of an n plus 1 element array is the product of the array factor of capital N number of two element arrays superimposed to produce nulls at the zeroes of F. Also so, the total arrays pattern is the product of the patterns associated with each polynomial by itself.

So, now he said since you see this transformation this z is this so, z magnitude of z is equal to 1. So, for all real values of u and u naught, visible space will correspond to a portion of the unit circle extending from minus k naught d plus u naught to plus k naught d plus u naught.

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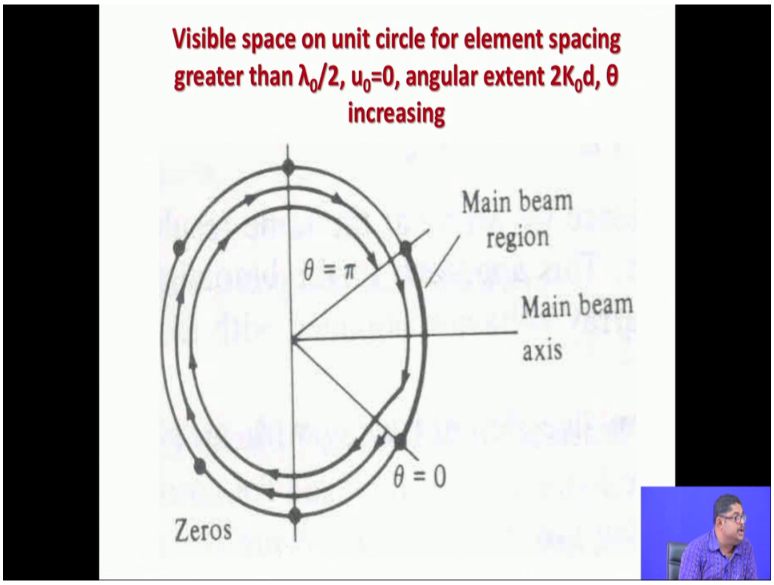

Now, let us see I think so, if I have an unit circle and now you see a designers choice, if I have in this case the d element spacing is greater than lambda naught by 2. So, here actually if you do this that, so this is the start of visible space and throughout the visible space, if you moves, it moves completes one circle and then come one circle once then again come here. So, many times it is revolving actually, it is just because it is a case of a broad side array u naught is 0 and we have taken d is greater than lambda naught by 2 so, we can move it so many times. Now, 0s of the polynomial that if I have more than more times I travel it so, I have multiple 0s at places.

Now, multiple 0 at a place means that it becomes more and more flat near the 0s. If you remember the Butterworth polynomial that it has n number of 0s any nth order Butterworth polynomial as n plus 1 0s. So, there it becomes maximally flatter and flatter. So, if I want about the nulls because these z 0s are nothing but nulls of the array pattern. So, there if I want a maximally flat type of pattern, then we put more and more zeros. Actually, I draw an analogy what happens? Consider that you have a rubber band and you are holding it above some place.

Now, if I at some point of the rubber band, if I pin it to the surface ok. So, near that surface the function is bound to go to small values. Now, I pin it again another point so, if these 2 points are nearby then I can make the level go down. So, 0 placing is in your hand where you will place the 0s and if I multiple pins if I put, then it will be that maximally flat means it is not going to a side lobe of very higher sharply. So, side lobes in effect will be kept below. So, this is the designers choice that where he will put actually what designers have that is the inter element spacing.

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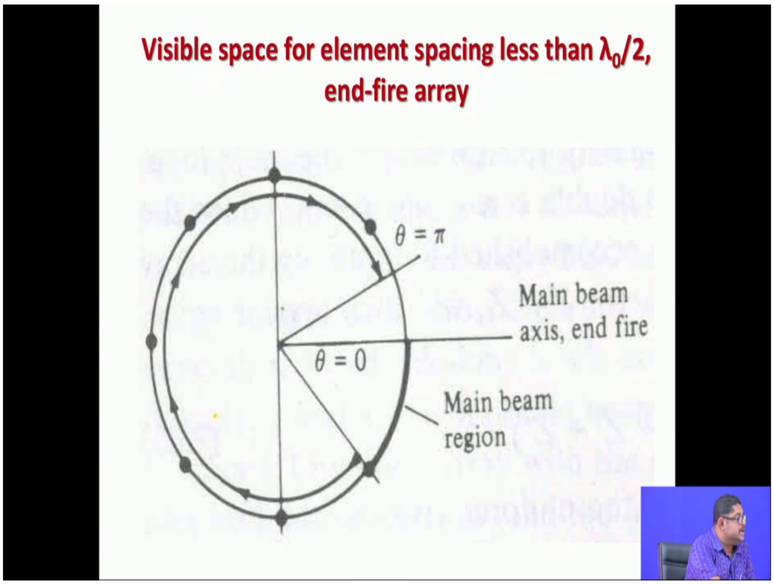

You see this instead if inter element spacing is lambda naught by 2, but here there is a phase shift it is not written it is an end fire array. So, phase shift is minus 90 degree you see that visible space is not even fully covered. You see this is one point theta is equal to 0 so, visible space starts from here visible space ends here. So, all the 0s need to be placed here in the nonvisible zone, no visible the 0s will be placed ok.

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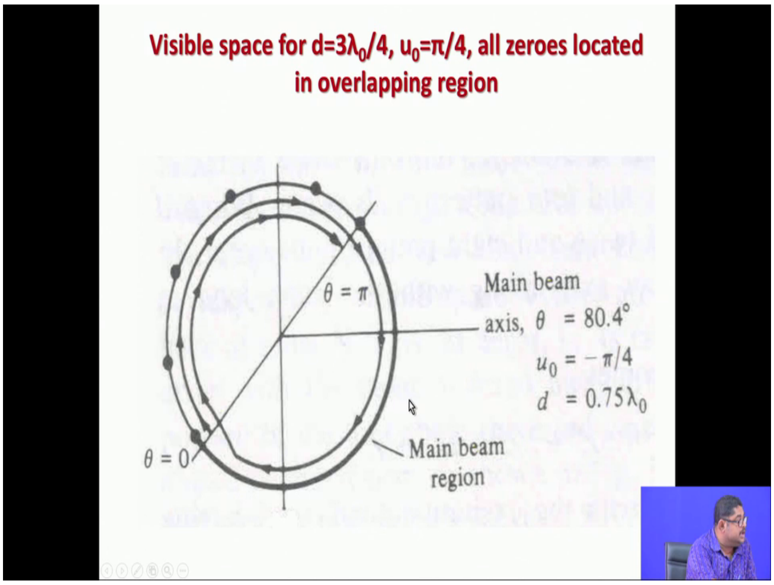

And let us say that the I one theta at this point, u naught is this it is minus pi by 4, d is in between 0.75 lambda naught. You see that it is starting from here going and one full circle then up to some distance. So, because where is your theta is equal to 0, that is important and then how much you are covering. So, here you see that no 0s are kept here so, the all the 0s you see the designer has placed in these points so that the second time the portion that is circling that is seeing a twice 0. So, more flattered things so, this type of things can be done so, let us see in more detail.

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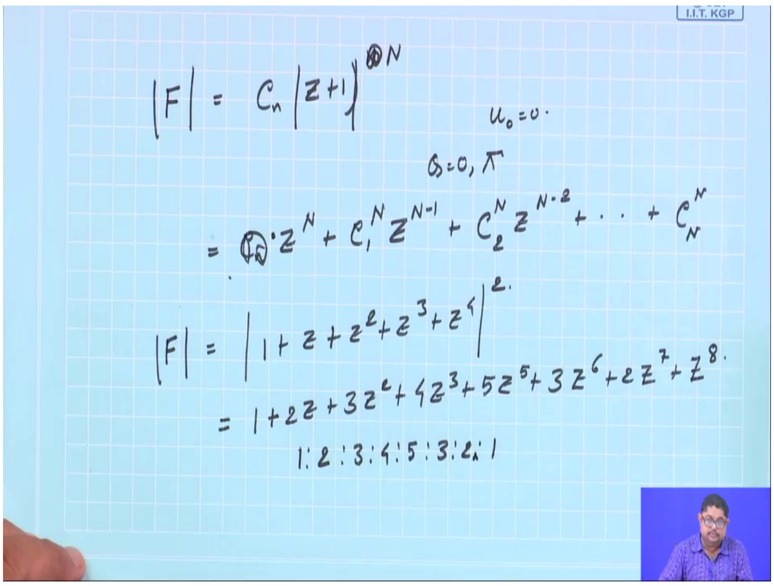

Suppose I choose the array factor to be that I will place let us say that all the 0s; that means, sorry I have an N plus 1 number. So, capital N number of 0s I am putting at z is equal to minus 1. What is z is equal to minus 1? Z is equal to minus 1 from here you can see that; that means, my let us say for u is equal u 0 is equal to 0. So, z is equal to minus 1 means theta is equal to 0 and pi; that means, at the at 0 point and pi point theta is equal to pi point, a pi point I will put the 0s. So, many 0s and if we expand these then we get that Butterworth in binomial thing so, it becomes z N plus z N minus 1 plus N C 2 z N minus 2 etcetera plus N C N.

So, this is actually Butterworth and you see that is why there are no side lobes. That is why the whole thing if I can do this, then there are no side lobes because you think a rubber band has been put the 0s are coming at that two ends of the visible spectrum. Actually, 0 and pi are the two ends of the visible spectrum so, that is why it does not have any a lobe. Now, consider that instead of these, I squared these so, that means, suppose let us say an array polynomial is like this 1 plus z plus z square plus z cube plus z 4 and I want double 0s at all these 0 points of this polynomial.

So, I can square this and that will become that 1 plus 2 z plus 3 z square plus 4 z cube plus 5 z 5th plus 3 z 6th plus 2 z 7 plus z 8. So, this becomes a 9 element array with the excitation functions, you see a triangular distribution 1 is to 2 is to 3 is to 4 is to 5 is to 3 is to 2 is to 1.

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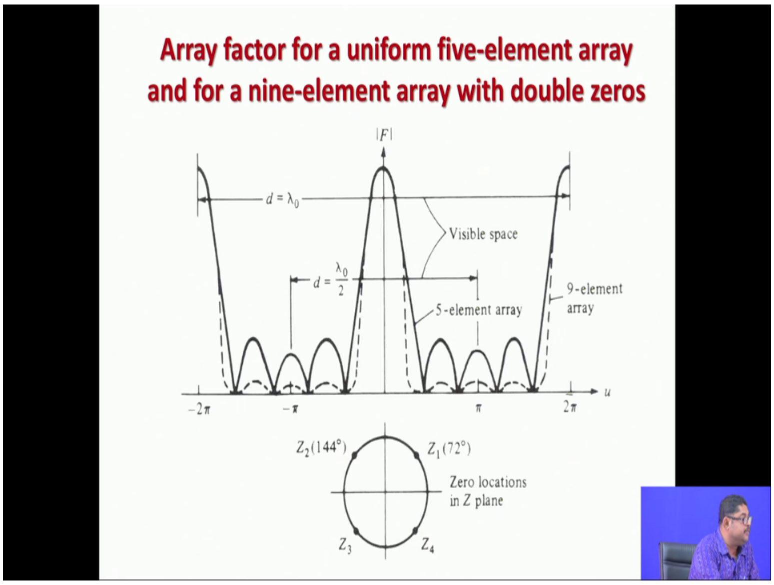

Now, you can see the array factor for this that the 5 element array is the black one and the dash one is the that squaring that gives you nine element array where it is becoming in near the 0 more flatter. So, I can reduce the side lobes; obviously, I will have to increase the rating. This is the placement of 0 that which point you want to put the 0s etcetera so, by this you can go on. Now, the question is which in any synthesis problem finally is asked that what is the optimum of these? Suppose, I am specifying side lobe levels what is the maximum directivity I can obtain or conversely if the directivity is specified what is the minimum side lobe level I should suffer.

So, that is an optimum problem and that problem was solved by Dolph. So, that is why it is called Dolphs array or actually you see that we for that we will look into the I recall actually I think I we have seen it you can see my NPTEL lecture on the impedance matching, basic tools of microwave engineering where we have discussed about Chebyshev polynomials.

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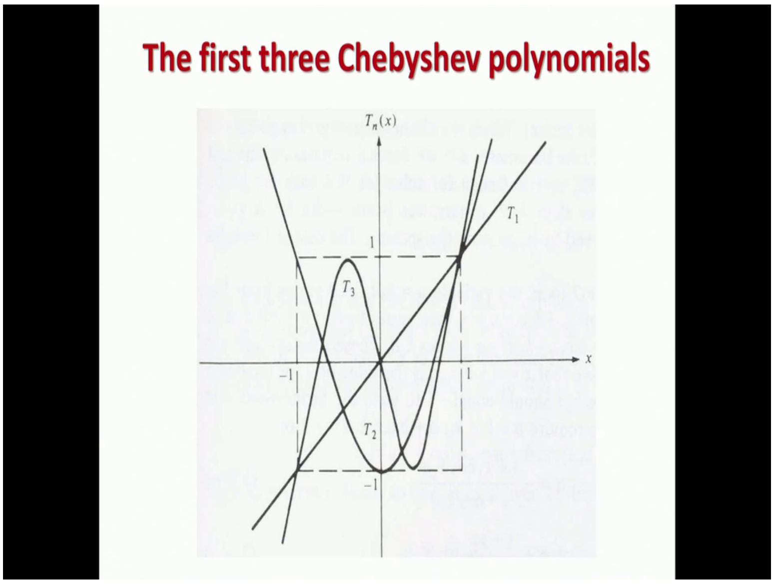

Now, Chebyshev polynomials have a very nice property actually, I am not again discussing Chebyshev polynomial, but we can see the properties of Chebyshev polynomial. Actually, this is various order of Chebyshev polynomial sorry here so, you see that it is a T n x. So, you see T 1, T 2, T 3 x etcetera all have a property that it oscillates between x is equal to minus 1 to plus 1 and the after that when it is more than x is equal to 1 or less than x is equal to minus 1, monotonically it increases. Actually, based on these we have in the impedance matching or filter designs we put pass bands here and stop bands here.

Now, here what we will do? We will you see that the oscillations also so, are all confined to plus 1 to minus 1. So, we will put all the side lobes in this zone and one side will pin with let us say this one another side because the main beam certain cases in end fire etcetera the main beam comes at the end so, there I can go up. So, certain portion I will cross x is equal to 1 because, if the main mean is there then there is no problem I am actually putting more directivity.

Actually this thing I mentioned in discussing end fire array that sometimes in Dolph Chebyshev arrays, the a portion in the invisible face is gone actually that comes from here. Actually, these arrays are called Dolph Chebyshev arrays so; here I will go a certain point greater than this. Based on what based on my the specified side lobe level requirement, SLR requirement.

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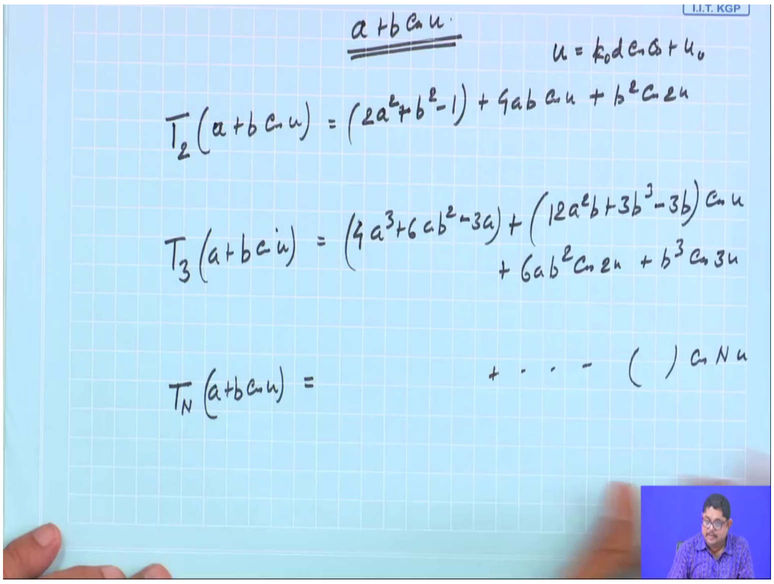

So, let us see that to this let us see a 2nd order Chebyshev polynomial and here you see our function is instead of a x, I will write it as a plus b cos u please remember because our actual function is or cos u or something you can say, but actually our u is what that k naught d cos theta plus u naught. So, this is the form is similar here, though this u may be confusing, but let us see a plus b cos u is equal to if you do this, there are formulas given in the book also in my NPTEL lecture, you can see that basic tools that is why we call it basic tools many times we are using it in microwave technology.

So, here also you see that this second order polynomial can be written as a Fourier series in u. If we go for T 3, then that becomes 4 a cube plus 6 a b square minus 3 a which is also a finite Fourier series. In general, T capital Nth order Chebyshev polynomial is a finite Fourier series up to the term cos n u. And so, I can say that it can be identified or we can say that this is the array factor for an array with 2 n plus 1 elements.

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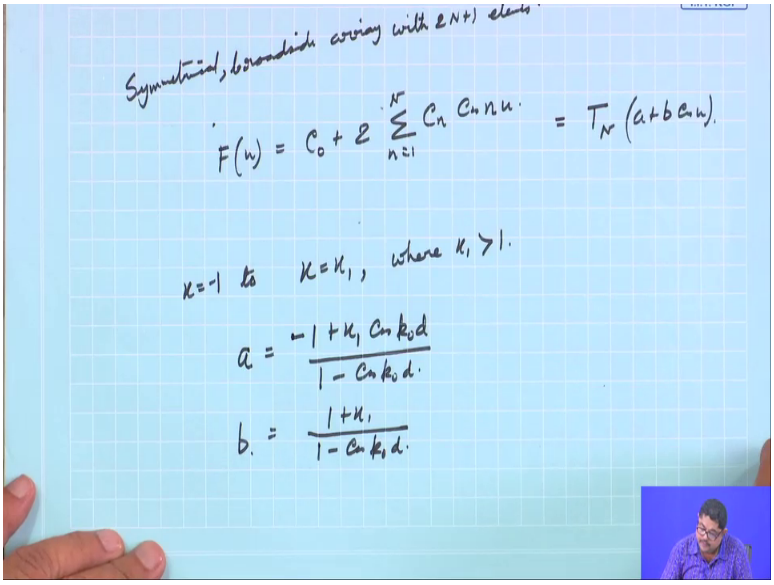

For a symmetrical broad side array with 2 N plus 1 elements, the array factor already we have shown that it can be written like the Fourier series time we have written this. This can be equated to the Chebyshev polynomial of degree N. The constants a and b can be chosen to make the visible range of u correspond to values of x in T N x that range from x is equal to minus 1 up to so we say that up to x is equal to x 1. So; that means, x is equal to minus 1 to x is equal to x 1 where x 1 is greater than 1.

So, people have done that and on what we will find the a let us say that this x 1 value. So, before that if you solve this, you can get some design formulas are given I am writing, but there are various cases various assumptions. So, something like to give an idea that you can find out the this a b constants as something like this and b is. So, see that once I can find x 1, a b can be determined. The moment a b can be determined, you can find out various thing.

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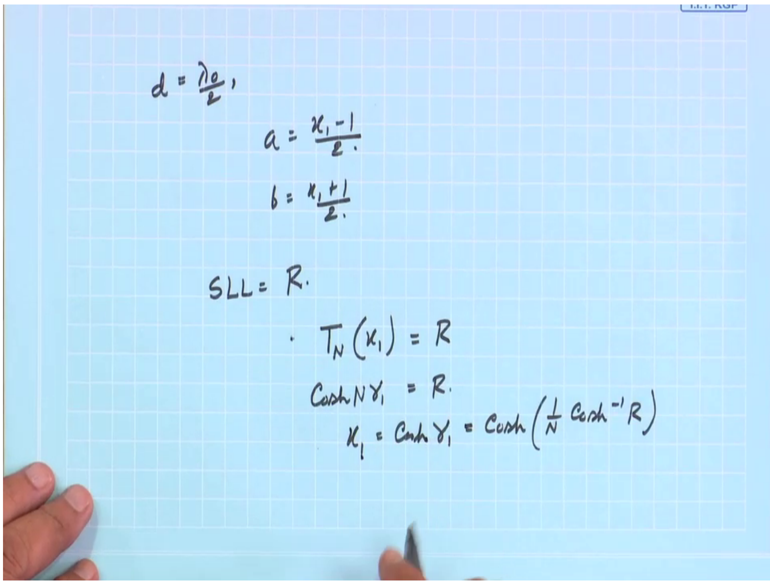

If this d if the d becomes lambda naught by 2, then a becomes x 1 by one by 2 b is equal to x 1 plus 1 by 2.

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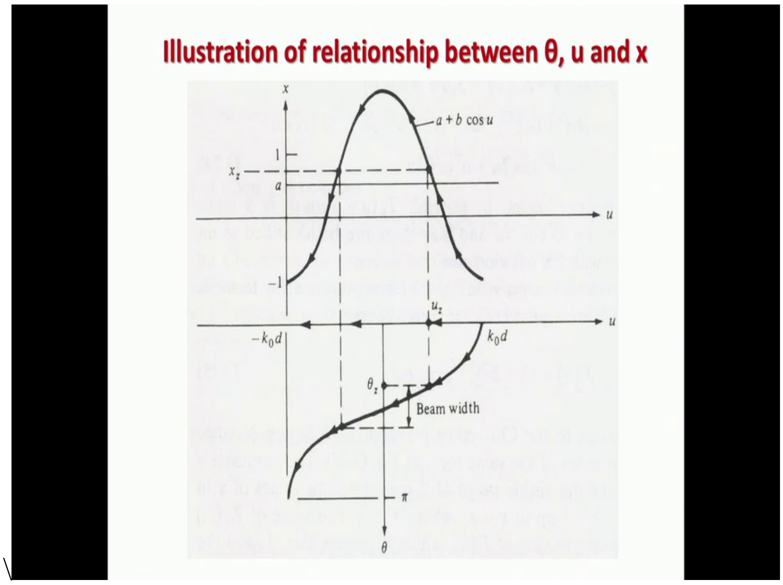

And how they vary? You see that in the u plane u versus x, this is the variance and maximum it goes to a plus b. You see it is initially increasing going up to a plus b, then coming down again to minus 1 and this is about the k naught thing so, theta and this their relation is this. Now, the question is how to select x 1? So, let the side lobe level S L L be specified as R. So, we require T N x 1 is equal to R because it is in the I require the main beam so, where the main beam this value is more than 1 so, I require it here.

So, at work point based on the S L R specification, so if S L R is this, then I will put it here and hence I can say what is T N x 1. If you look at it is expansion is cos hyperbolic N, gamma 1 is equal to R and so, this x 1 is cos hyperbolic gamma 1 is equal to cos 1 by n cos hyperbolic R. So, from R N I already know that number of elements I want to put, x 1 is found. The moment x 1 is found, a b is found then the constants a b found means you now know so, what is the beam width etcetera you can easily say that this is the optimum design that given a side lobe, I can find what is the maximum directivity that I can obtain.

In some designs the opposite, the beam width is given; that means, the directivity is given. So, the in that case the main beam extends from the last 0 of T n x before x reaches the value of 1; that means, last 0 is here suppose I am taking T 1 or let us say this is T 2 or T 3 so, T 3 is this is the 0 of T 3. So, from here, up to that let us say this point so, the main beam extends from the last 0 of T n x before x reaches the value of 1 up to x 1. If the beam null is placed at theta z, then the corresponding value of u you can find and then x z you can find.

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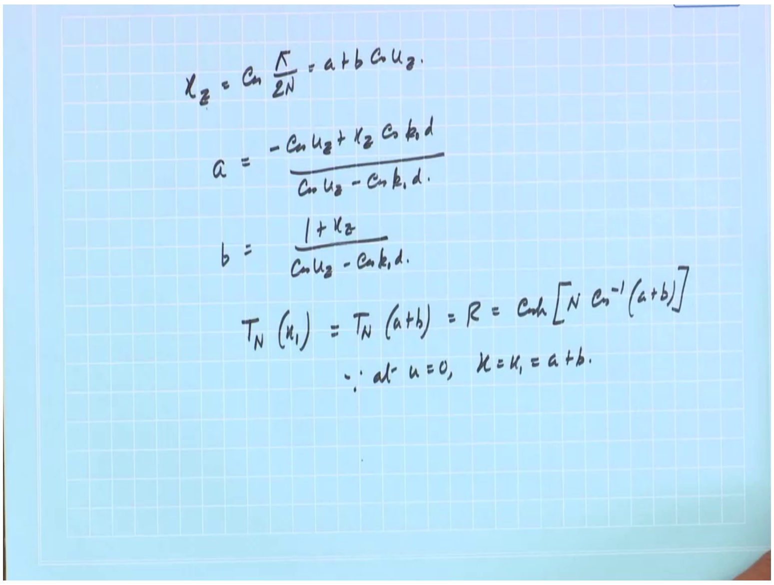

So, that value x z that will become this is the point of 0 before the last one so, cos pi by 2 N is equal to a plus b cos u z. So, finally, with the help of this a becomes minus cos u z plus x z cos k naught d by cos u z minus cos k naught d b becomes 1 plus x z by cos u z minus cos k naught d. The beam maximum to side lobe level ratio are can be found from T N x 1 is equal to T N a plus b is equal to R is equal to cos hyperbolic N cos hyperbolic a plus b.

Since, at u is equal to 0, since at u is equal to 0, we have x is equal to x 1 is equal to a plus b. So, we can also specify the side lobe ratio parameter R in which case the beam width is fixed and can be found from the known value of x 1 and the value of x. So, this is involved I am not going into details of derivation, but Dolph Chebyshev array you can easily see there are good references available.

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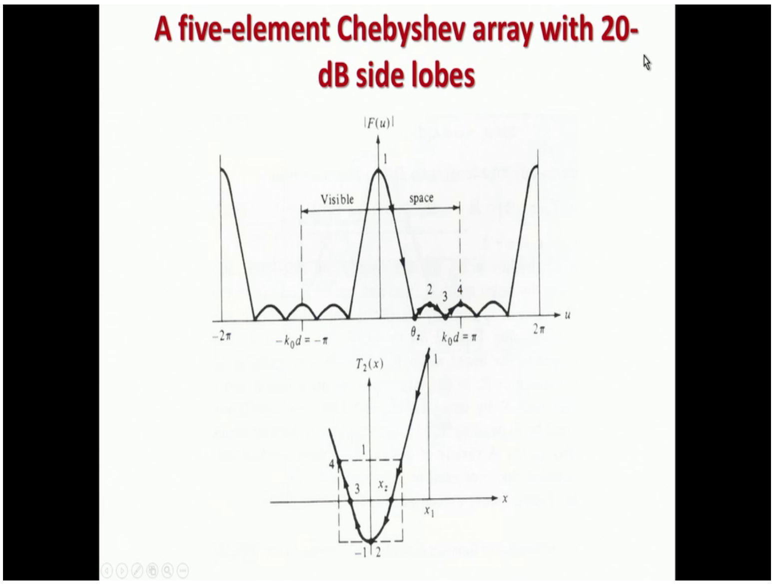

Now, let us see this is a design that in this case 20 dB side lobes are specified. So, you see that this can be obtained, this is the visible space and you can find out that there are actually 1, 2, 3, 4 so, 5 elements are used by that you can get 20 dB side lobes etcetera. Actually, theoretically you can with this not only with Chebyshev with others there are there is a term called super directive array in a thing. Actually anything which is a order of factor larger than the uniform arrays 13.5 dB side lobe level that is called super directive array.

So, it has been proved that you can have with this synthesis procedure you can achieve any directivity with the a limited or given array length, but those are not practical. Actually, if you do that you will see that they required one thing is very high currents excitations typically, excitations may be in the 10 to the power 6 Amperes etcetera. So, very high another thing is they require generally that the adjacent elements they should be oppositely faced.

So, what happens due to that high, one thing is it is very difficult that if you have a very high current excitation and nearby to have a 180 degree face opposite a and what happens, ultimately, there though theoretically it is possible but actually the loss is so high. In those structures, because of such high current that holmic losses are much larger than the radiation resistant. Since, we are not taking any losses into account holmic losses into account in this, so they show that the directivity is very high.

But, actually the practically those antennas the you know. So, these super directive arrays are no, but maybe in some day if some loss control mechanism is found then that can be a promising one maybe someday because maybe the excitation etcetera that can be done provided the loss can be controlled. So, it is not that researchers stopped in that direction so, super directive arrays with the new various materials etcetera there may be promising one day.

So, array designer antenna people should not lose sight of that it can be possible to design the very high directivity things given a limited day thing. With that this is an advanced topic so, maybe the material etcetera the formulas etcetera are not so important, but the concept that comes that how we can do that, how Schelkunoff made that polynomial thing. So, but there are various other polynomials which are also candidates very effective candidates Chebyshev maybe one but there are other polynomials also with are has various good characteristic which can be easily ported to this framework and then it we can be explored.

Thank you.