Analysis and Design Principles of Microwave Antennas
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Lecture – 36
Solution of Intregral Equation by Moment Method

Welcome to this lecture NPTEL lecture.

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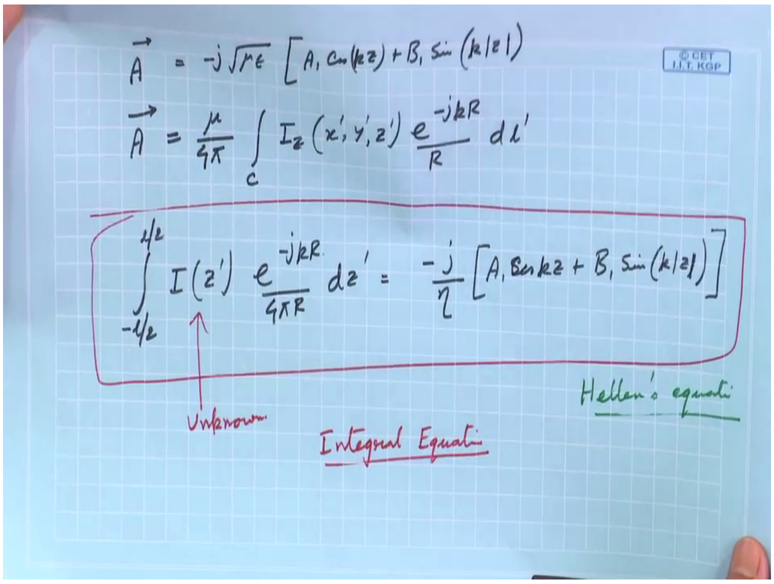

So, today we will discuss the Moment Method integral equation solution by moment method. We have already found out Hellen's formulation, it is an integral equation I z dashed is unknown this is for Pocklington's formulation.

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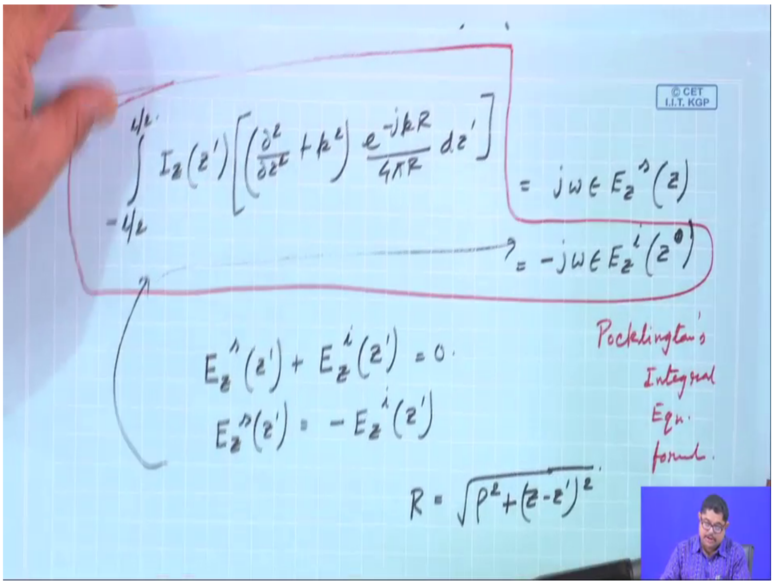

Again you say I z dashed is unknown there is an operator. This side right hand side is known. So, we can say both of these or any of this thing any of these integral equations.

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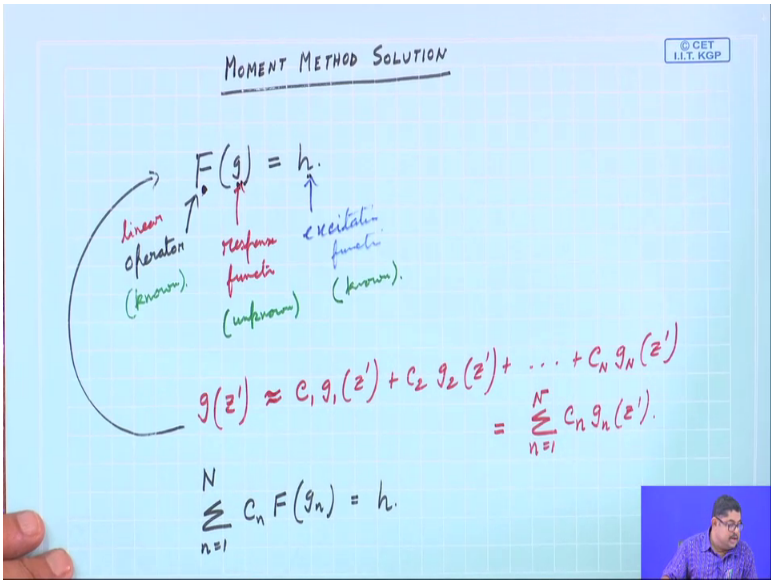

A class of integral equations can be written as F g is equal to h where F is a non-linear operator. In our case integral operator that integration is an operation that is a linear operation. So, F is a linear operator it is a non-linear operator, h is a non excitation function. In our case it is the whether we give spark gap excitation or any other feed type of thing so excitation is known. The response is g this is on your structure, but this is unknown. So, g is the response function I can say this is a response function. This is an excitation function. Now I can say this is known, this is unknown and what is this, this is an operator and this is known.

Because what operation exactly we know the objective here is to determine g once F and h are specified. Now doing these actually you can say this is an inverse problem that I need to find g as a operation of h. If I could do that then I need not have this method, but that is not so easy because you see here. If I want to do this I will have to perform this integration. But how do I perform or I will have to invert this I will take I z z dash is equal to something.

Now, I cannot write that by some operation here either by differentiation or integration something or multiplying it with something it is not so easy to write it. That is why we are solving this numerically. So, basically the linearity of this operator F; so, I should say that it is a very important thing that this operation should be linear. So, this linearity actually will help me to make the numerical solution that we will see.

So, what we do this unknown response function g we expand in a basis set. You know that in engineering many times we do that the best example is the Fourier series. Also there are other series Legendre series, Hermite series etcetera. Where actually we do that any were actually Fourier prove that any electrical signal can be expanded in a sinusoidal basis or exponential basis etcetera.

So, now in various other cases we have the Bessel basis, we have the Legendre basis for various types of functions. So, here we also say that we expand these in a series. So, c 1 g 1 z dashed plus c 2 g 2 z dashed plus plus C N g N z dash. Actually if it goes to infinity these can be exact, but; obviously, we have to truncate. So, here thinking that up to N number of terms we are expanding.

So, I can write it has n is equal to 1 to capital N C N g N z dashed. So, the unknown response function is expanded as a linear combination of n terms. Now each C N is an unknown constant, but each g N z dashed is a known function. Now basis said is a complete set that in that basis if I represent always I can make it the two functions left hand side and right hand side get exact.

So, the domain of this g1 g 2 g N etcetera is same as the domain of this response function g. Now, if I substitute this thing into this main equation, then what we get. So, if I substitute it here I get that n is equal to 1 to N C n F of g n is equal to h. This g n, this g n is chosen this choice is with you. So, that this F g n can be evaluated conveniently preferably in closed form, if not at least numerically that should be easily evaluated.

So, once that is there; that means, now this unknown has been boiled down to that I have N number of capital N number of unknown constants C n which needs to be determined. But the problem is you see that we have created that there was only one function which needs to be found out. Now, we have said this operator with the help of that we have extended it here, but we have created n unknowns, but we have only one equation.

So, that we now need to change; so, before going there let us talk a bit about these basis functions g 1 z dash g 2 z dash g n z dash. They are called how to choose this basis function these are called basis function; that means, the expansion is made in terms of this basis function. So, basis function now in general people choose basis functions as the set which has the ability to accurately represent and resemble the anticipated unknown function. So, if I know suppose in this case a current distribution. So, you see that we have an idea a priori that I can have a in a dipole.

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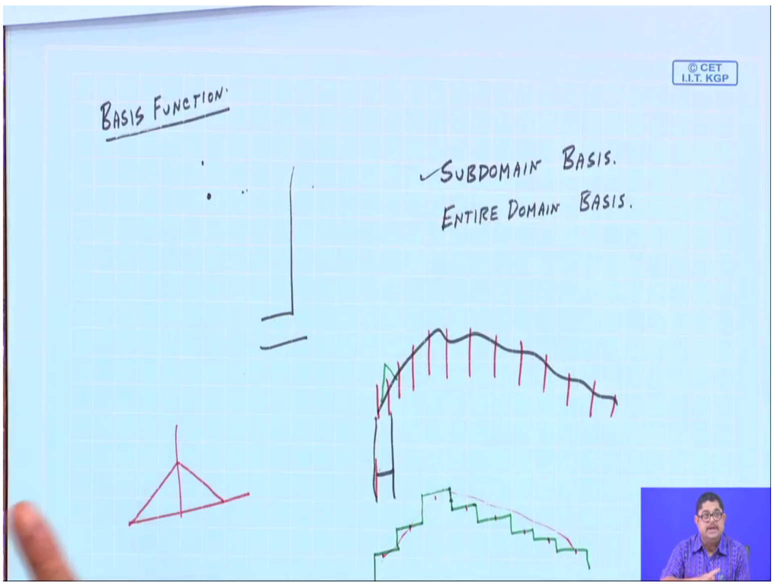

Suppose this is the field. So, something it will be high here. So, some sort of exponential. So, it can be various cos cosine various orders of cosine functions or various orders of sin function or combination of both etcetera. Suppose in a rectangular waveguide I want the field distribution I know that at the ns there are metal. So, there will be what the field should go there 0. So, again there we take cosine function etcetera etcetera, but it may also so, happen that I do not have that a priori information.

So, these two cases can be treated separately. So, actually there are two types of basis functions; one is that called sub domain basis, sub domain basis function, another is called entire domain function. Actually what happens when I do not know anything; we take first sub domain basis we find out what is the more or less functional behavior. Then people try to go for entire domain because in a sub domain the computational complexity is more whereas, entire domain reduces the complexion of complexity.

So, what is thing that basis function should be chosen one thing it should be able to accurately resemble the actual unknown function, but also it should be computationally less intensive or computational burden is less. So, these are the two things. So, people mix these two. Now, what is first let us say sub domain basis. What is sub domain basis? Most commonly use things suppose I have some distribute suppose I have some function here. Now a very clever choice is this whole structure suppose this is a structure on which this function is there.

Now break it into various parts, you can make it equal, you can make it unequal; that is your choice because where you see there are not much variation, you can increase the gap also. But to start with people generally equally do because I do not know that this behavior. And here you assume pulses. So, basis function is here so from here to here only a pulse. So, domain of this pulse is existing only over this, that is why it is a name subdomain basis. That means, it is not over the whole domain of the unknown function or the response function that is why it is subdomain. So, this is existing here this is existing here a.

So, once you put this basis finally, we will see that this thing can be you will gets the current distribution something like these that something like this we will get. So, if you add the midpoints you get more or less that shape you can come ok. So, this is one choice also people choose piecewise linear. So, piecewise linear means something like this so; that means, over two actually piecewise linear basis function is these now this is a good thing actually piecewise linear means over two adjacent domain it has over lapping two or three people make that from a one.

So, that you get a good feel that how the at the intersection point how they are behaving and this is called also triangle function. Also it is very easy to map any surface with triangle functions you do not put any gaps etcetera ok. So, this is about subdomain; now entire domain we already saw that suppose one example of a entire domain as I said the sinusoidal.

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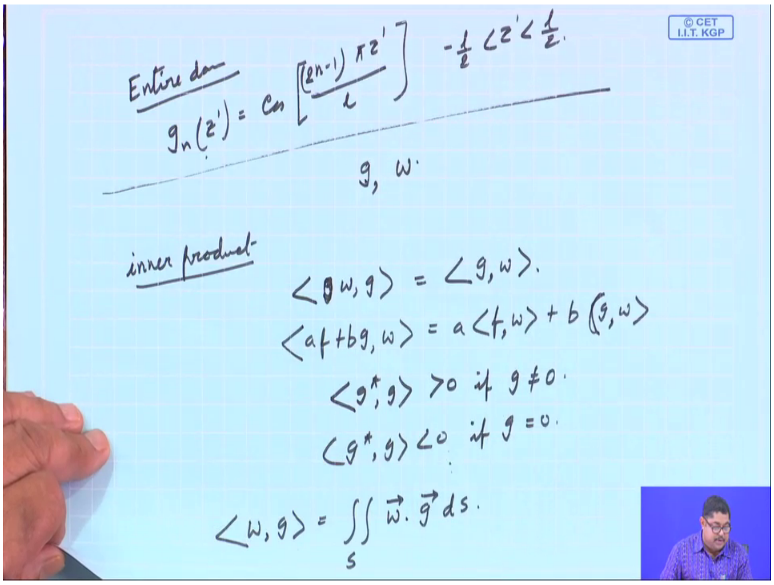

So, in our case maybe in entire domain we can say that, suppose I take that cos 2 n minus 1 pi z dashed by l minus l by 2 to z dashed to l by 2. So, this is a cosine, but it is at different n. So, this is an example of an entire domain function that, if I know that typically it is cosine then people do that, but that comes with experience etcetera. So, that is all about these basis functions.

Now the question is that what the question I said here, that I had created that one equation, but I have made unknowns I have created to be n numbers capital N number of unknown. So, how to solve that? The recourse is let us solve this or enforce this condition. Let us take capital N number of boundary conditions so; that means, this thing this is an for our case it is an that equation.

Now, let us enforce that equation on n number of points. So, boundary condition will be n. So, to do that what people do there is a thing actually you see the moment I am saying that I will solve it on capital N number; that means obviously, at all the points I am not enforcing it. So, there will be some errors because at other points it is not enforced. So, there will be errors now the, our idea is that can we make this error the average of this error go to 0 over the whole structure.

So, that is why we need to take a moment you see moment was you have come across this term that first moment second moment when that I have a distribution discrete distribution suppose your mean. So, I am taking that what is the first moment means the first time of average. Then the second moment then I am having a square type of average like that higher order things. So, here also we take a inner product it is something like the moment. So, inner product is formed.

Now what is inner product there are definitions, but suppose I have a g function already I have seen that g function I will also have. So, g function is expanded with a basis function I also have another function w their inner product is if I have w g in of this 2 brackets is inner product it is same as g w. Then there will be a f plus b g w is so linearity sort of thing a f w plus b g w. And others are for complex conjugation that g greater than 0; if g not equal to 0 g less than 0, if g is equal to 0.

If you get this two are that two operator. So, if type is you can have simple multiplication of two functions, but more generally a very good choice for electromagnetic things is. This is over the surface of the structure on which I am interested you take the dot product because these are generally these functions are also vector functions.

So, I can say w dot g ds. So, this is a inner product. So, by these choice, the solution satisfy the electromagnetic boundary condition general typically in electromagnetics the boundary condition is vanishing tangential electric fields on the surface of an electric conductor or for magnetic conductor you see. So, only at discrete points we are making that on n number of points will satisfy these between these points the boundary conditions may not be satisfied. And so we define as a residual.

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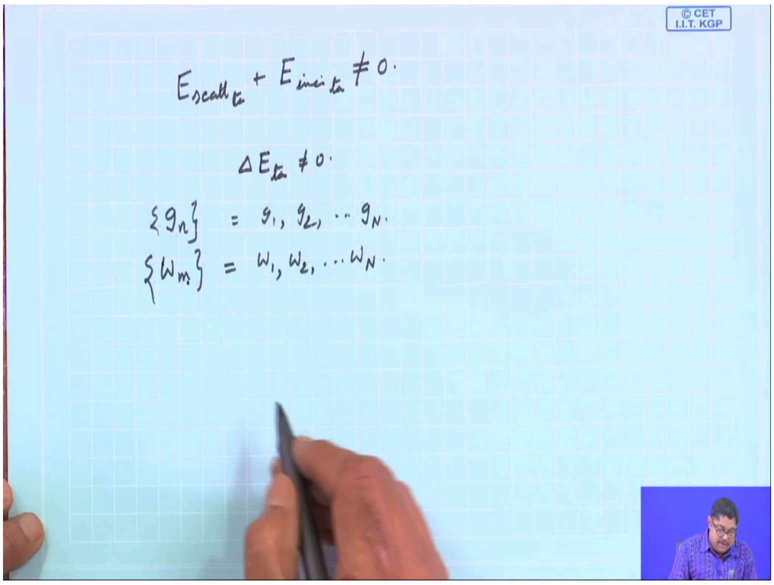

That means on the surface actually we want that E scattered tangential plus E incident tangential; that means, that is not becoming 0. So; that means, there is a delta E tan which is not equal to 0. Our job is for this producing this inner product because you see inner product we are putting over the whole surface our objective is to minimize the residual in such a way that is overall average over the inters structure approach is 0.

So, the method of weighted residuals is utilized in conjunction with the inner product this technique does not lead to a vanishing residual at every point on the surface of a conductor, but forces the boundary conditions to be satisfied in an average sense over the entire surface. Now, people that is why when this method came actually this moment method is a technique in mathematics (Refer Time: 21:23) first took this method to and applied it to electromagnetic problems.

That time mainly people said that due to these that everywhere it is not getting satisfied rather than analytical solution satisfied it everywhere. But it has been shown that the difference can be by proper choice of this inner product. You can make these differences inconsequential to the practical applications. And so moment method has now become an authentic method. Now let us come to our point that so we now define this w again as a set of testing function. So, w since it is a different one I am using a subscript m different, but it is again is made up that w 1, w 2 up to w N. So, same order you see basis function we have taken capital N order this is also N order. Only to distinguish that time we said g n it was expanded as g 1, g 2 etcetera etcetera g N. So, that is why n and m are thing.

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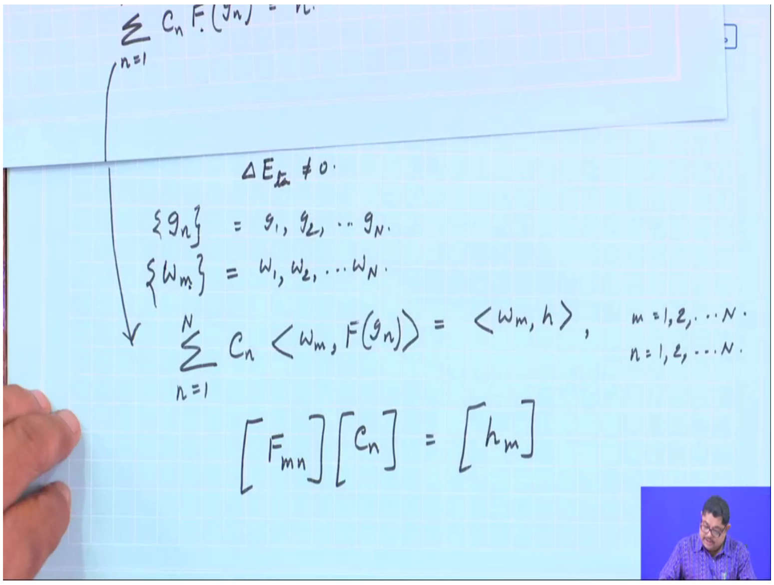

So, for a if we from this inner product. Now; that means, this equation we will this F will force as an that F will take it is inner product with the testing function w. So, this equation becomes, I can say like this that n is equal to 1 to N C n w m F g n is equal to w m h m is equal to 1,2 up to N. Obviously, n is also 1,2 up to N.

Now, this set of matrix equation this set of n equation now I have got n number of equations I have n unknown C n I can solve it. So, this can be written in metric form as F m n C n is equal to h m.

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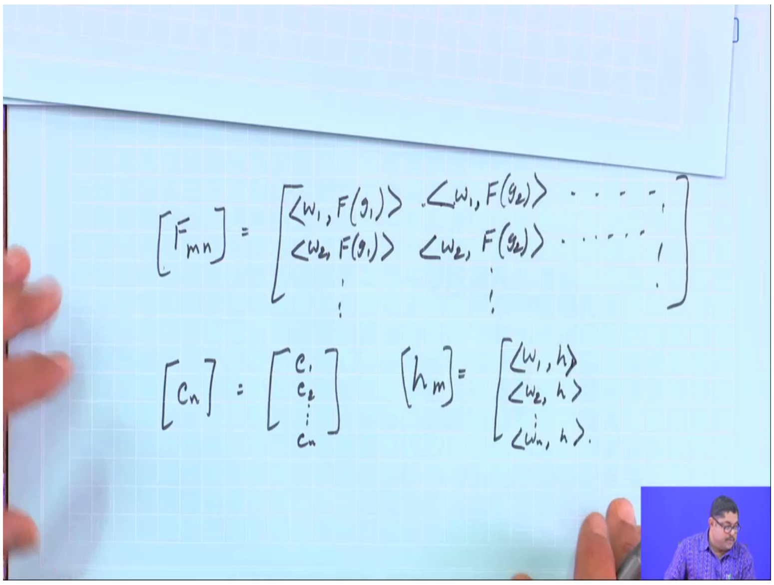

Where what is F m n F m n matrix is w 1 inner product F of g 1 next one you can write also w 1 F g 2 etcetera etcetera here next one you can w 2 F of g 1 (Refer Time: 25:03) w two F of g 2 etcetera. This is F n, C n is C n no problem, C 1, C 2 C n. And h m is so this equation basically my interest is C n.

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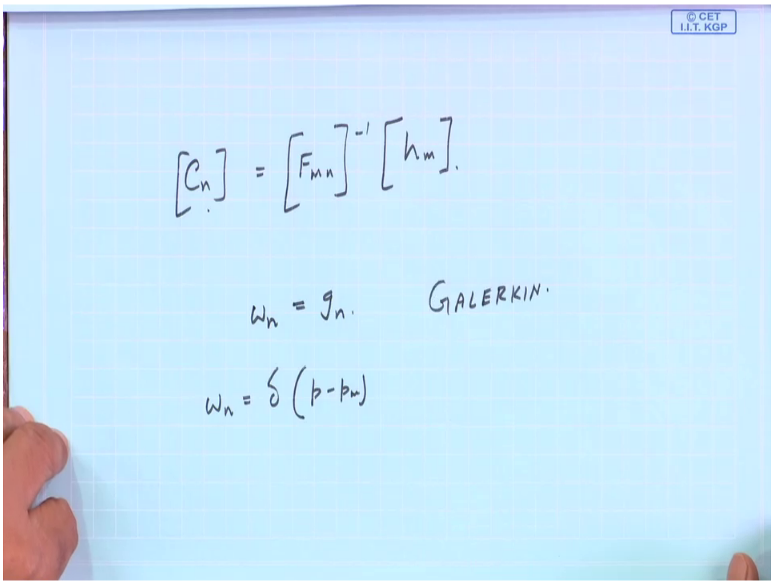

So, I can write C n as F m n inverse h m ok. So, this can be computed and once I know C n, I know the integral equation can be solved. Now, here the question is; How to choose the this w is called waiting function or testing function? Now the choice of weighting function should be such that the elements of w n must be linearly independent. So, that the n equations that we have formed a linearly independent, also it will be advantageous to choose weighting functions that minimize the computational burden.

Now, think the same was true for the basis functions also basis functions always a linearly independent set and also I said that basis function should be chosen to have computational lathing. So, the motivation or the guideline for choosing the basis function and weighting function they are same. So, a very standard procedure is called Galerkin procedure, where the we take the weighting function same as the basis function this is called Galerkin procedure.

So, now I am not going here actually in this F m n there is an integration in many times, that integration can be solved. If it cannot be solved then there is another way that instead of taking these Galerkin things you take the omega n to be a delta function at various points etcetera. Why because if you take delta function that inner product there the one order of integration gets reduced.

So, if you find propagating that you can do that, but; obviously, it puts the burden on the computational load. Whereas, in the many cases the this Galerkin stings the integration can be solvable. So, that is people done and by doing that people have found that I will again draw those two graphs. So, I am not further proceeding. So, you can easily now solve for getting.

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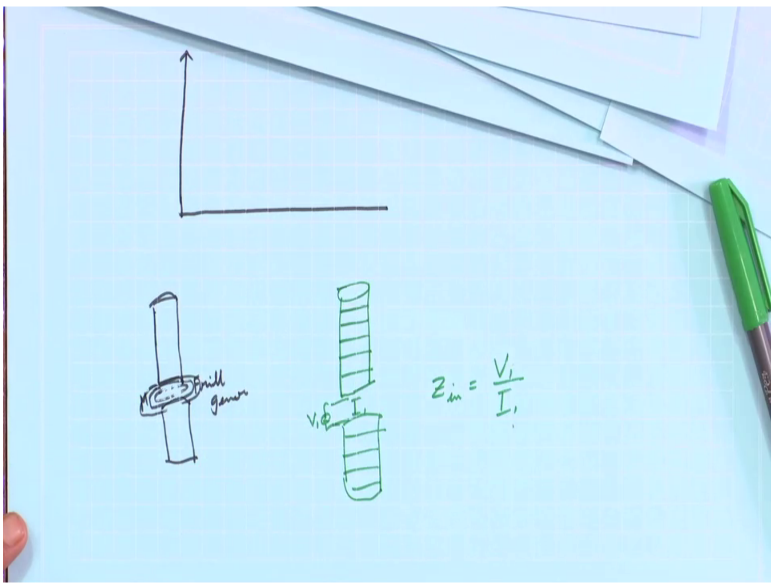

This is also you can see that this whole thing is given Harrington has written a book on the field computation of a moment method. So, there all these procedures are given meeting. So, the graph today I have drawn over like graphs are there. So, I will just these graphs.

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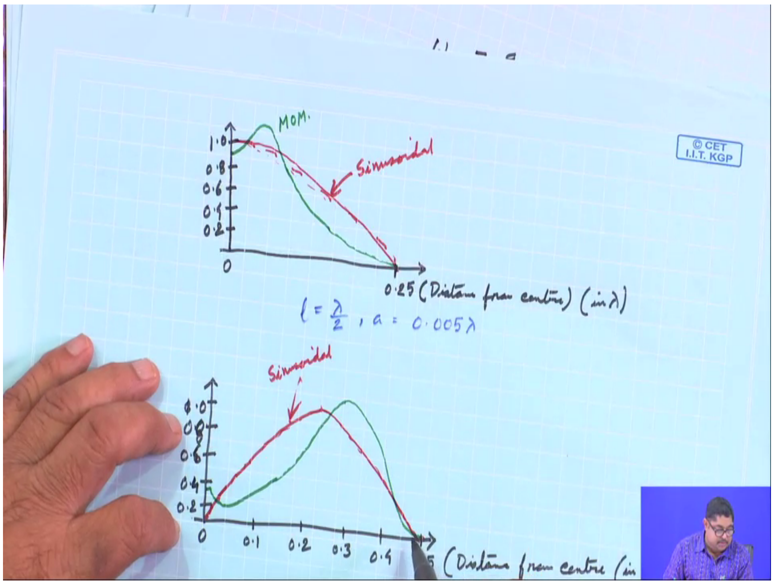

Now people have found that more or less this green graph. That the moment method graph and previous the not numerical techniques the iterative techniques graph that was by King Harrison etcetera so that almost gives this. So, based on this is also is more similar. So, you get the input impedance correctly here and so, now, it is very easy to the put the this input impedance or actually you can find the whole reactive impedance. Because, if you know this current distribution at various points; that means, you are dividing the whole dipole structure in various points and there you can find out what is the current distribution.

So, if you very finally, do this then you know the currents. So, you know the current here also you can here also put a element and from here if you know this. So, you can easily write z in is equal to V 1 by I 1 where I 1 is current here and V 1 is the excitation given here. So, that will give you the input impedance very correctly remember this V 1 and I 1 are complex numbers. So, you get the actual input impedance; that means, the complex input impedance you can get that is one thing.

Another thing is the source also you can model that the instead of a delta gap source you can also have a people have found one field generator that is here. So, let me draw again that. So, if I have a field here people have from that an equivalent magnetic current like these that will be also that is called Field. So, field generator this is called this is the Magnetic Current.

So, by that also you can model the source magnetic ring current. Actually you have a ring magnetic current thus, the electric field required can be found using any of these models. The field generator usually gives better results particularly for impedances and actually the delta gap instead of delta gap in this model the, is narrow strips of equivalent magnetic current density ok.

So, you have these then you have another strip, another strip. So, by that the delta gap can be more easily (Refer Time: 33:13). Actually the field generator model was introduced to calculate the near zone or far zone both fields can be done with the same formulation from co axial apertures. So, this shows that in modern days the actually this 20-30 years back this type of things came. And actually with you can say roughly in mid 80's this thing started.

And now these things all the things were revisited with this moment method thing. So, current distribution both the for where antennas or aperture antennas moment methods were used and found out. Similarly various discontinuity diaphragm near the various field then wall thickness of various a things wave guide things.

Previously wall thickness was not considered, but now wall thickness was accounted for slot radiation, slot thickness all those are now accounted for by this moment method thing because, by this you get accurate current distribution and from thus impedance. So, ultimately you get all the informations that are given by the field analysis, analytic analysis. But now almost any type of antenna can be analyzed with this.

Thank you.