Analysis and Design Principles of Microwave Antennas
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Lecture – 35
Numerical Evaluation of Wire Antenna Currents

Welcome to this NPTEL lecture. Today, we will see the Numerical Evaluation of Antenna Currents. Actually, earlier while discussing wire antenna for long dipoles we discuss that the current distribution is approximately or that time we said that it was sinusoidal, because if we are feeding the dipole at the center, and the currents conduction current should go to 0 at the two extremes, and at the feed point that should go to maximum. So, it was said that it will be sinusoidal.

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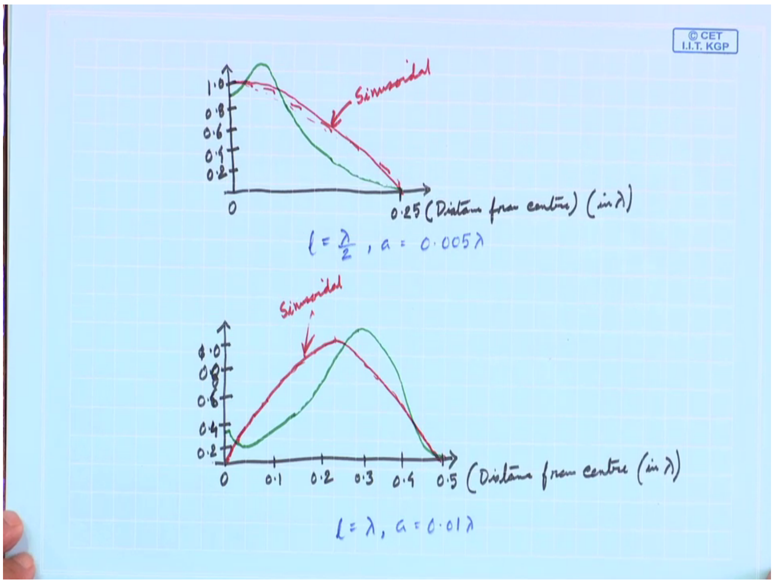

But people have found out experimentally also. And also by other techniques that it is not exactly sinusoidal. Look at the top graph it is for a lambda by 2 dipoles. It is a thin dipole, because the usually dipoles radius is quite small in terms of lambda. And here in the x-axis is plotted the distance from the center, so that means this is the central point, and this is the top point. And the current distribution is symmetric in both the poles.

So, you see that we have plotted the sinusoidal one by the red color, but actual current is not exactly. It is the maximum is not at the center first thing, because peak is somewhere else. And also you see that typically it can be said exponential, but it is not exponential there are deviations. And, now if this is have a profound impact, if we try to find out the input impedance of the dipole, because at the center you see that by sinusoidal thing whatever we will expect the current that is not same.

So, if we, because what will be the input impedance at the feed point, we will find out the applied voltage divided by the current. So, if the current is different then, the input impedance will be difficult. Basically, the near field effect is different for if we assume a sinusoidal distribution, and a actual distribution like this. The second graph is for a lambda dipole that means full wave dipole. Again you see it is a thin one.

Here also you see that sinusoidal is as expected, but the peak is at a different point. And most importantly you see that here the center fed at the centre the sinusoidal current distributions is 0 so that should give you input impedance should go to infinity which is not the case. No is in no dipole gives me infinite input impedance. We have seen the input impedance is something like some complex term, but the radiation resistance is something like 73 Ohms so near about that etcetera.

So, you see that, but that actual distribution that is not zero. So, you can easily get the exact one. Now, how to get this type of exact current distribution; that we will discuss actually; so we will today's topic is Numerical Evaluation of the Dipole Current.

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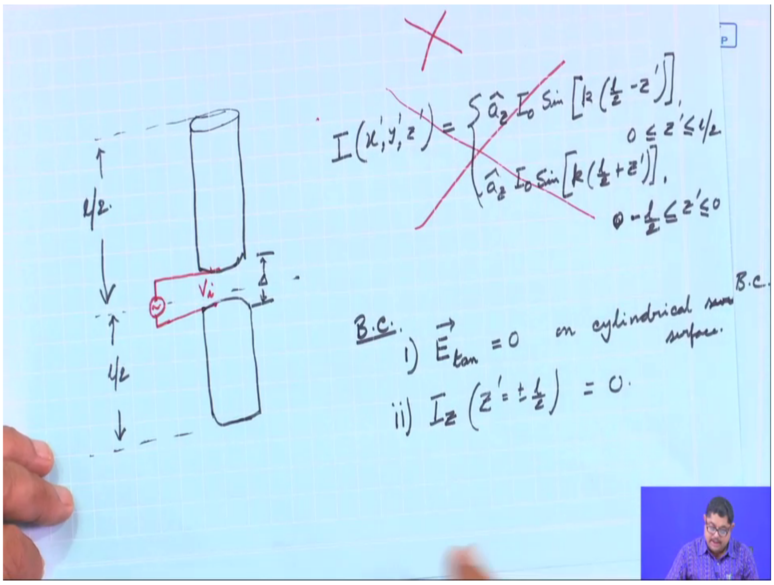

So, let us say that we have a dipole. Now, actually in the earlier discussions we always neglected the finite radius of the dipole, but actually if I want to make a dipole, it will be a cylinder. So, any practical cylinder will have a finite radius I can keep it small, but that means on the surface there will be currents. So, currents will be distributed. If the dipole is made from a perfect conducting elements or any practical elements, then there will be currents in the circumference.

And so, it is not always that the through center the current is going, so that is one thing. Another is we are a having a finite gap at the feed point, now that gap we do not consider, because we consider that the dipoles centre is also at the center, but actually dipole center is a delta up, and the delta down so that is not model. Today we will take that also into account. And so, a dipole is something like this, and there is this gap in this gap we put let us say a some feed whose job is to put the voltage generator. So, I can say that this voltage let us say in phrase voltage.

And let us say this gap is something like delta let me call. So, I can say that from the central point to the top this is l by 2, similarly so that means, this length of the top pole that is l by 2, the length of the bottom pole that is also l by 2. Now, previously I help you to recall that we have taken the current distributions. While we discuss sinusoidal we have taken that this is a z I naught sin k l by 2 minus z dashed for 0 less than equal to z dashed less that equal to l by 2, and in the bottom half it is I naught sin k l by 2 plus z dashed sorry minus l by 2 z dashed l 0 that was our earlier thing.

Today, we will see that for a center fed and finite diameter, where this is not exactly true. Though we will say that as we have seen that sinusoidal typically we can say, but it is not exactly true. So, if you want the radiation pattern, if we want the input impedance etcetera to be exactly determined, because that is very necessary for circuit matching etcetera so, this is true.

So, there are two techniques more or less similar, but there is a big difference. So, those techniques we will see. The first one scientist Hallen he first found out these, so that is why this formulation of the analysis that is called Hallen’s formulation which we will see. So, this is not our current distribution we, now say that we do not know what is the current. So, we will determine the current.

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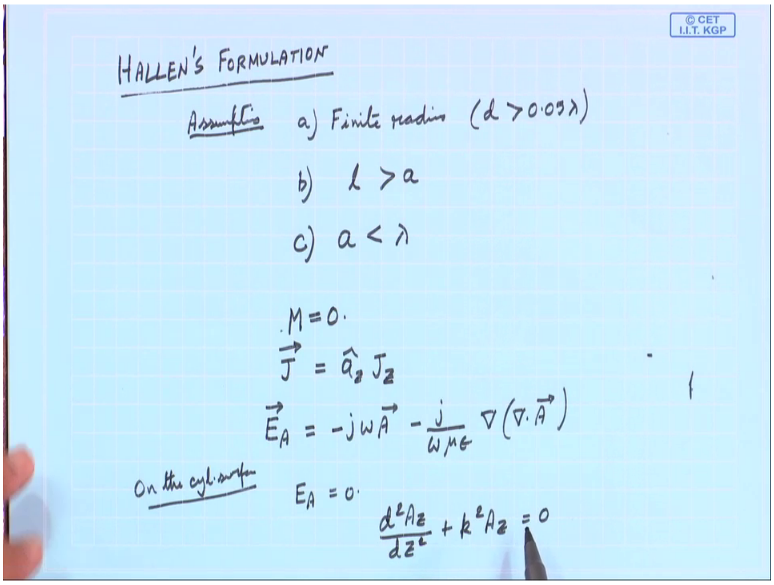

So, we will write the Hallen's formulation is a very well known technique very much used. Actually previously people used to have some iterative techniques to solve that, but today with the advancement of various numerical techniques that also we will discuss probably in the next lecture, that there is a technique which is very much appropriate for this type of current distributions problem, or in case of determining currents for scattering problems or any radiating problem that is called moment method that is an integral equation method that also we will see.

So, this Hallen's formulation has assumptions. So, the there are first one is that the radius is finite radius of the radiating structure. So, we will say usually they call that the diameter that is greater than 0.05 lambda. Then the dipole is long. So, l is greater than a, and I am not writing much greater, because this is applicable for all these cases, but l should be greater. Actually and also we required that a should be much less than lambda.

Now, these actually what happens if I have a practical cylinder like this; so there will be currents in the sides also in these top caps of the cylinder top and bottom caps there will be current, but by this assumption actually we are neglecting that cap. So, we will say that only the current is on the cylindrical surface not on this planar caps that is why these assumptions. Now, on this we will put the boundary condition.

What are the boundary condition? You see always these boundary conditions that one is since this is made of perfect conductors, so we will say the first boundary condition is total tangential electric field will be 0 on cylindrical surface sorry cylindrical surface. This is always 2 plus please remember here it is not true in the gap it is not true, but on the cylindrical surface. The second boundary condition is obvious that our current we do not know the current here, but at the endpoints this will go to 0. These are the 2 boundary conditions sorry we will enforce.

So, what is Hallen's formulation, as we have seen that since this is a perfect conductor. So, we assume that there is only electric current. So, we can say that M is equal to 0 here, and our current density J that we will write as J z. Now, this is an assumption actually that I will come that. So, Hallen's formulation says that current is typically going like this actually it is an equivalent thing there are currents here. So, he assumes that the since it is a symmetrical structure. So, summing that I can have a equivalent current on the axis of the a things.

So, if I have this, then we can write that what is the electric in terms of the applied electric field E A. Please remember those A is come from that magnetic vector potential. While we discuss the general solution that thing so we can write E A is equal to minus J omega A minus j by omega mu epsilon del cross A. On the cylinder on the cylindrical surface I can say applied electric field is 0, because we are not applying any electric field there. So, this boils down to that I would not go these boils down to that vector potential satisfies this differential equation. This we have seen many times.

So, we will have to solve this equation.

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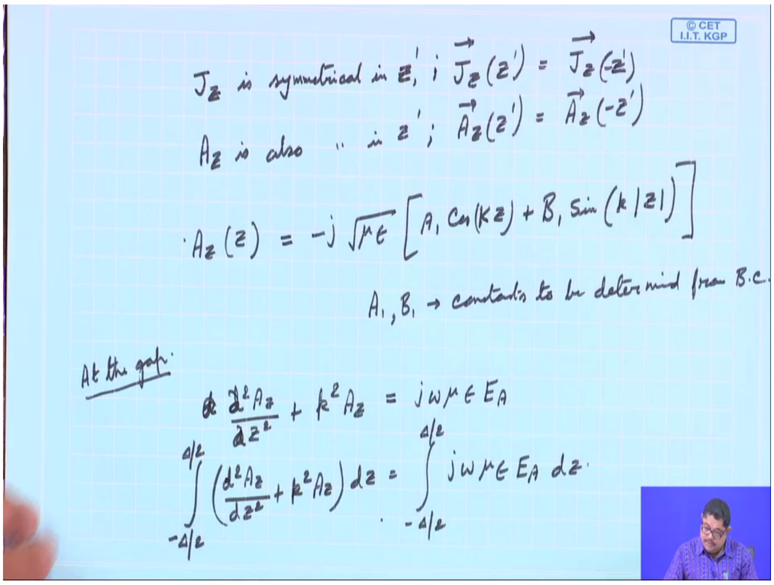

Once we know the vector potential we can easily find J so that will be. Now, we can say that. J z is symmetrical in z or z dash that that means j z z dash is equal to j z minus z dashed, because of the structure the excitation is symmetrical, so bipolar excitation, so these. So, this tells me that A z is also symmetrical in z dashed that means A z z dashed is also A z minus z dash.

So one solution, because A z is satisfying this differential equation: one solution is A z A z z is equal to minus j mu epsilon A 1 one cosine function plus to have this symmetry, and this you can put it here, and see that it satisfies. This term is coming due to this k presence of k you are getting this term. So, A1, B1 are unknowns so, A 1 and B1 are constants to be determined form boundary conditions. I have 2 unknowns I have 2 boundary conditions. So, I will be able to do that.

Now, I will come to this solution a bit later. First let me were, also let me evaluate this. So, now, let us look at the gap now, at the gap at the gap. Let me see this is the structure. So, at the gap the applied voltage is v i. So, there we can write the differential equation del square A z plus del z square plus k square A z is equal to or I can write d, because there is only one k square A z is what, if you put it j omega mu epsilon E A. This is at the gap.

Now, let us integrate this whole thing over the gap. So, we are integrating it minus delta by 2 to delta by 2 square A z d z square plus k square A z d z is equal to minus delta by 2 plus delta by 2 j omega mu epsilon E A d z ok. Now, I can easily integrate this, these are all constants. So, it is basically let me write it that.

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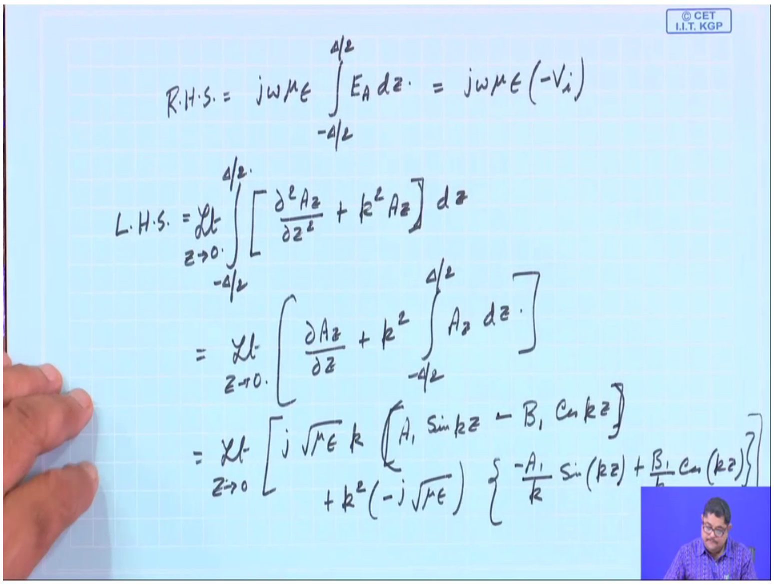

Right hand side is j omega mu epsilon minus delta by 2 to delta by 2 E A d z what is that j omega mu epsilon into what is E dot d l this a line integral minus V i. Now, let us look at LHS. So, LHS is I can say that if I integrate it, then del square a z del z square plus k square a z sorry plus k a z d z minus delta by 2 to delta by 2. Now, this I should take that this is happening at limit z tending to 0. So, this will be limit z tending to 0.

The first one will be that del a z del z one integration plus k square minus delta by 2 to delta by 2 A z d z for whole limit. Now, I have the expression for a z this is the expression. So, del a z is you can put and also this one you can integrate, so that will be limit z to 0 j mu epsilon k A 1 sin k z minus B 1 cos k z of limited plus k square minus j mu epsilon minus A1 by k sin k z plus B 1 by k cos k z is equal to j k epsilon these are simple integration sign functions.

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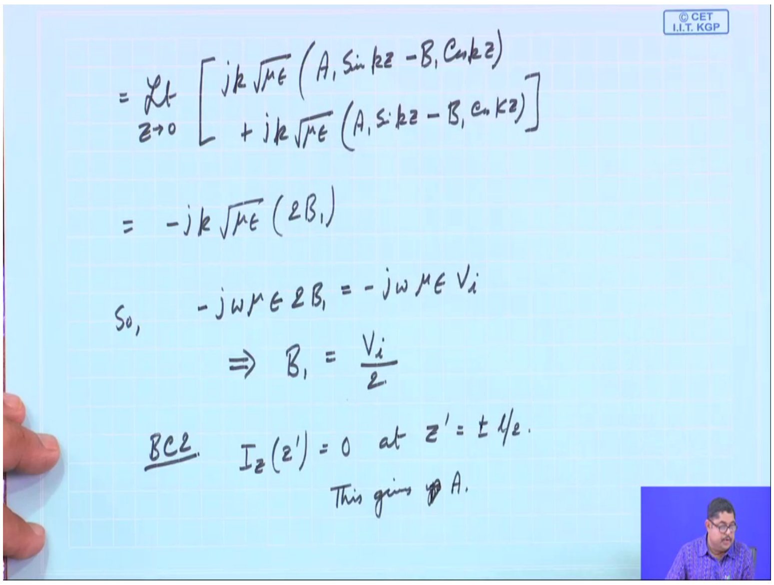

So, you can do yourself, but still for helping you I am writing it. Now, you enforce the limit. So, it will be minus j k mu epsilon 2 B 1. So, you can this is your R H, LHS, RHS we have found equate these 2. So, we get minus j omega mu epsilon 2 B 1 minus j omega mu epsilon vi that implies that B 1 value I got as V i by 2. So, B 1 I got, A 1.

Now, knowing this I know the second. So, this is the first boundary condition the second boundary condition is that you put it there. BC2, so that will that I z z dashed is equal to 0 at z dashed is equal to plus minus l by 2. So, from this A you now find out I would that so that will give you. This you can take as an exercise this gives you A 1 so that means, now we know A 1, B 1 so that means I know the solution. So, from there we have so that means A we know.

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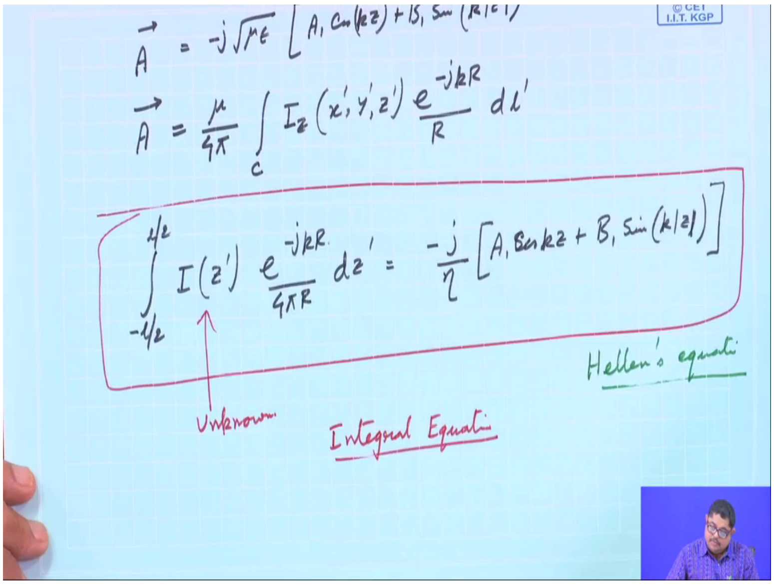

And already previously this expression we have found that, if we have that in terms of what is the relation between A and I z that is an integral relation in the previous we founded it. If I have a one way linear current, then it is a contour integral I z x dashed y dashed z dashed e to the power minus j k R by R d l dashed.

So, also I have the solution of A that A is in our case is minus j root over mu epsilon A 1 cos k z plus B 1 sin k z. So, this two are equated. So, this one I can put the thing minus l by 2 to plus l by 2 I z dashed e to the power minus j k R by R 4 pi r d z dashed is equal to minus j by eta A 1 cos k z plus B 1 sin k z ok. This mu and this gives this impedance.

Now, these equations now you see the right hand side is fully known A 1 B 1 I know. So, I will have to find I z dashed, but unfortunately I z dashed is under or it is an integrand of this equation that is why these equations are called integral equations. So, this formulation is an integral equation formulation, because this is unknown. And this is under an integration or this is a integrand to an integration. So, that is why it is an integral equation. So, this is the definition of any integral equation that if the unknown is under in the integrand of an integration, then that is called an integral equation.

So, this needs to be solved actually this equation is called Hallen's equation. This is the famous equation. Now, this equation if you can solve that means integral equation solving, then we can find the current distribution. Actually we will see later that how to solve this equation ok. So, this we will see that another formulation is also very much used and that has certain advantages.

The advantages that actually the will finally, see if I want to solve it by moment method, then I need to inverse actually this one to find this I need to inverse something. So, one order less, because here this A 1 also is an unknown which needs to be solved. Because, we are seeing the vector potentially actually this A 1 is a confusing thing, because this is a vector potentially. And this is the unknown A 1.

Now, order of the inversion matrix inversion we will see in class one that we will discuss. So that and also the generator that is a voltage generator, but other types of various speeds that cannot be analyzed by this formulation, because it always assumed that there is a voltage generator or spark gap generator there. But another general field thing that is called Pocklington's formulation that is also integral equation formulation.

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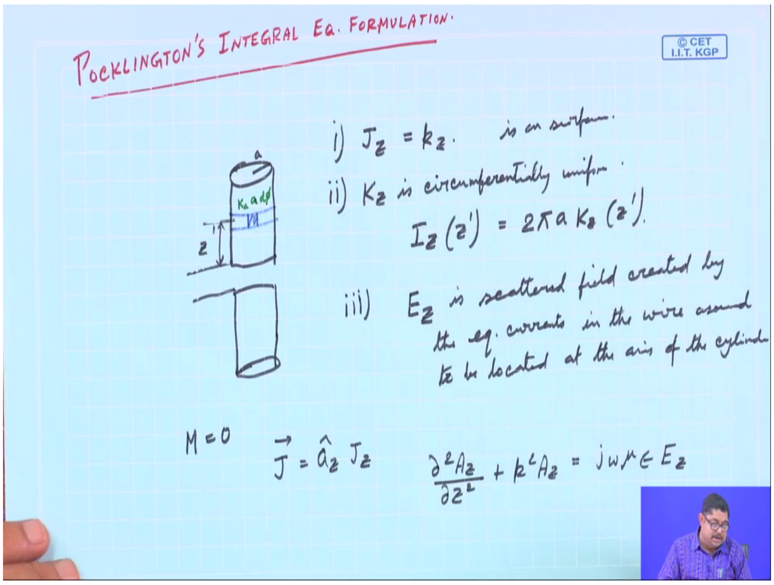

So, we will see now that that Pocklington's integral equation formulation, again we have that cylinders. Here the current density is on the surface. So, the first assumption of this is the current density J j is on surface. So, it is called k z that, now the second thing is this surface current density k z is circumferentially uniform.

So, we can write that I z at a point z dash that will be 2 pi a k z z dashed. So, here it says that it is uniform circumferentially. And the third thing is important it is refinement from

Hallen said that the. There is a scattered field, so the thing is E z is a scattered field created by the equivalent currents in the wire. Actually J z is on surface here, but Hallen says that these circumferential current I can finally, take an axial current equivalent to that. And that will scatter the field that field we will call E z thing created by the equivalence current in the wire assumed to be to be located at the axis of the cylinder.

So, remember we are not saying that the current is that current is circumferentially distributed, but equivalent thing that we take. We can easily take this uniform thing. What is the equivalent current that is that the axis, and because it is a symmetrical structure and that the scattered thing is this. Now, again we can say that M is 0 the magnetic current and current density is given by this a z, J z, so same as Hallen's thing. So, we will have to solve this equation del square A z plus del z square plus k square A z is equal to j omega mu epsilon E z.

So, you see Hallen's equation we said this is 0 everywhere except the gap, but here we are saying no there is also a scattered field, because of these. Obviously, there is an applied field at the gap. So, that is why a more than general. And so, we can now solve this from the earlier thing. So, I can directly go to this Pocklington's formulation that.

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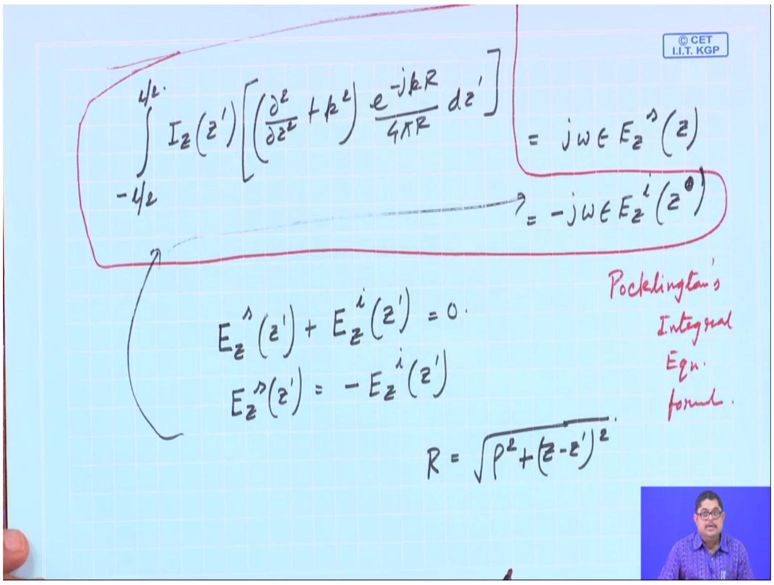

So, we can ultimately write this l by 2 I z z dashed.

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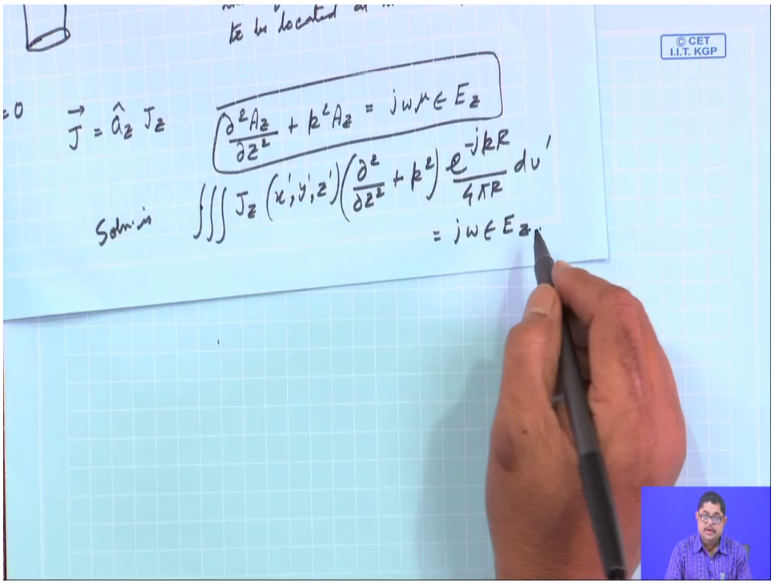

Or, let me say that what will be the solution of this thing its solution will be obviously in general, this we have already seen earlier. This differential equation its solution is this. And, now specializing these for our all those assumptions, we can write that this is del square del z square plus k square e to the power minus j k R by 4 pi R d z dashed is equal to j omega epsilon E z s from Z directed Z by s.

Now, what is E z s, we know at the cylinder surface the total tangential electric field should be 0. So, they are since both are same, I can write that E z s Z dash plus E what I will write E z incident. Incident means what is applied they should go to 0; that means I can always write E z s Z dashed is equal to minus E z i Z dashed.

So, this if I put here I get or I can write here, that this is minus j omega epsilon E z i Z dashed Z ok. And also what is R, R is nothing but if we go to cylindrical coordinates rho square plus j minus z dashed whole square. So, this formulation is called Pocklington's integral equation formulation. You see it is a similar structure integral equation, because this is unknown. This part is known. Actually from the given excitation spark gap, if you say I know what is E z i, I will calculate. If any other type of feed, I can find that. So, E z is in my hand, but it is known.

So, this side is known, this side is some operations, this function is known. So, I need to solve it; so this integral equation for both Hallen's and Pocklington's, people have solved. And then will, with there are various techniques one of that I said most efficient one most easy for computer implementation is moment method that we will discuss a bit of moment method in the next class. And then we will see how to solve this equation in the next class.

Thank you.