Analysis and Design Principles of Microwave Antennas Prof. Amitabha Bhattacharya Department of Electronics and Electrical Communication Engineering Indian Institute of Technology, Kharagpur

Lecture – 34 Open Ended Waveguide Antenna and Microstrip Antenna

Welcome to this lecture, we were in the last few lectures discussing the analysis and also we are finding various aperture antennas. So, in the last one we have seen a slot type of antenna. Now, we will see an Open Ended Waveguide Antenna.

(Refer Slide Time: 00:39)

So, the open ended waveguide antenna means I have that on a ground plane so, this is the ground plane. And here only thing is instead of uniform sorry let me write the thing this is my x-axis this is my y-axis, this is the center. Now, this dimension is as before a by 2, this one is b by 2 and here the field will be something like this a cosine variation thing.

So, this is a t 1 0 modes excitation inside the waveguide; that means, suppose from z less than 0, the it was a transmission waveguide having t 1 0 mode. So, that was the field pattern we assume that the aperture field E a is y directed and having that t 1 0 mode pattern cos pi x dashed by a for minus a by 2 x dashed a by 2 minus b by 2 y dashed b by 2.

So, we can easily write what will be the M x. So, M will be I think to understand that this is y and n is z directed so, M will be x directed. So, I can writing M x is 2 E naught cos pi x dashed by a again in this zone, I am not writing. And we can actually in a previous example also we have seen there is a constant part. Now, since it is coming regularly, so we can put that constant as is constant C which is j a b k E naught e to the power minus j k r by 2 pi r. This was always coming so, we are putting it as a constant C.

(Refer Slide Time: 03:30)

$$
E_{\theta} = -\frac{K}{2}C \sin \theta \frac{C_{\theta}X}{X^2(E)} = \frac{S_{\theta}Y}{Y}
$$
\n
$$
E_{\theta} = -\frac{K}{2}C \cos \theta \sin \theta \frac{C_{\theta}X}{X^2(E)} = \frac{S_{\theta}Y}{Y}
$$
\n
$$
H_{\theta} = -E_{\theta}|_{\eta}
$$
\n
$$
H_{\theta} = +\frac{E_{\theta}}{T}
$$

So, in terms of this we can with the help of those formulas and already I am that time said. So, E theta can be easily written as minus pi by 2 C sin phi only change here cos X by X square minus pi by 2 whole square into sin Y by Y; this X Y definitions already from that uniform illumination case the same definition. So, you see only here instead of a sin x by X that means, sine function this is the new thing due to the illumination. Also E phi will be minus pi by 2 C cos theta cos phi cos X by X square minus pi by 2 whole square sin Y by Y. And H theta would be as before minus E phi by eta and G phi is plus E theta phi.

(Refer Slide Time: 04:50)

So, now your what will be your E-plane thing. So, E-plane you have that same thing and this since we have in the E-plane the when y is equal to pi, we will get that null. So, now the null will be theta in sin inverse lambda by b. And you can find out that theta dB 3 dB that will be 57.3 sin inverse 0.443 by b by lambda in degrees. Theta s will be sin inverse 2 into 4.5 by 2 pi b by lambda and the side lobe level is again the same minus 13.26 dB etcetera.

So, I am not gone into details because by the Fourier transform method also we have seen this pattern. You will see that here also you are getting the same pattern. And in the previous example, we have seen various those values that. So, sinc values also important sinc values that will required to have here, please note that sinc of Y is 707, when Y is equal to 1.391.

Similarly, the sinc becomes minus 0.271, when y is equal to 4.494. So, these values we will require actually remember that if you take the modulus then these becomes the next one, the second maximum that is why this value we will require etcetera. Now, you can say that in another example then let me write the H-plane things

(Refer Slide Time: 07:30)

 $\begin{array}{|c|}\n\hline\n\text{O CET} \\
\text{I.I.T. KGP}\n\end{array}$ $H - H = 277$
FNBW (deg) = $\frac{171.9}{9/2}$ $HPBN(deg) = \frac{68.8}{9/a}$ FSLL = -23 dB.
 $P_{\pi} = \frac{1}{2} \int_{-q/2}^{q/2} f_{\omega} |E_{0}|^{2} e^{-\frac{L_{\pi}}{a}} d\mu ds$.
 $= \frac{1}{q/2} \int_{-\frac{1}{q}}^{\frac{1}{q}} f_{\omega} |E_{0}|^{2} e^{-\frac{L_{\pi}}{a}} d\mu ds$.

The H-plane is a greater than lambda. So, first null beam width in degrees it will be 171.9 by a by lambda. So, you see almost in previous case for Fourier transform thing we say that this first null beam width that is very broad, it is almost like 180 degree. Similarly, half power beam width in degree it will be 68.8 by a by lambda first side lobe level is minus 23 dB here.

So, you see that this thing we have claimed in case of reflector antennas. Now, it is being proved you see that in since we have a amplitude taper side lobe level is coming down from minus 13 db, I am getting 23 db. So, this is the advantage always that if you have a amplitude taper, in this case you have an amplitude taper that is why you are getting it.

Similarly directivity again this needs to be done. So, again we can say that how to find P r so, P r will be again from the aperture field minus wave impedance E naught square cos square pi x by a dx dy. So, this if you do, you will get ab by 4 wave impedance of the T 1 0 mode field into E naught square this is coming because if you write the magnetic field for the T 1 0 mode, you will get this.

(Refer Slide Time: 09:44)

 $\begin{array}{|c|}\n\hline\n\text{O CET} \\
\text{I.I.T. KGP}\n\end{array}$ $\begin{array}{ccc}\n & \rightarrow & \times = 0, \times = 0 \\
 & \rightarrow & \times = 0, \times = 0 \\
 & \downarrow & \downarrow & \downarrow \\
 & \downarrow & \downarrow & \downarrow \\
 & \downarrow & \downarrow & \downarrow\n\end{array}$ $45 \frac{dP}{dx}|_{Q=0}$ $rac{64}{3^3}$ β

Again that this is P r, then dP d omega at theta is equal to 0 so, there again your X is equal to 0, Y capital Y 0. And E theta is here sin phi and E phi is proportional to cos phi in this case so, again E theta square that E phi square it will be no phi variation. So, you can write that dP d omega theta 0 that will give you r square by 2 eta E theta square. So, if you do that, it becomes 2 E naught square by eta pi square lambda square a square b square. So, directivity will be 4 pi dP d omega theta 0 by P r, so that if you put you will get 64 by lambda cube beta 1 0 ab.

(Refer Slide Time: 11:14)

 $dP|_{Q=0}$ C CET $4\pi \frac{dP}{dx}|_{0.00} = \frac{64}{\lambda^{3} \beta_{10}} a b.$ $Y_w = \frac{\beta_0}{k n}$
 $\beta_{10} = (k^2 - \frac{1}{4k})^{1/2} = \pi (\frac{4}{\lambda^2} - \frac{1}{\alpha})^{1/2}$

So, where I can for you I am telling that what is Y wave impedance of T 1 0 mode that is beta 1 0 by k eta. You will see your you can see our NPTEL lecture on transmission lines wave guides, there we have already found these terms. And also your beta 1 0, what is the value of beta 1 0, beta 1 0 is k square minus pi square by a dash square or a square to the power half, so that becomes pi by 4 by lambda square minus 1 by a square to the power half.

(Refer Slide Time: 12:19)

O CET $a \nabla^2$, $\beta_0 z \frac{e^x}{\lambda}$. $D \approx \frac{3e}{\hbar r^2}$ ab = $\frac{8}{\pi} \left(\frac{4\pi}{\lambda^2} \right)$ ab
 $A_p = ab$
 $A_{p} = ab$
 $A_{e_n} = \frac{2}{\hbar^2} (ab)$.

So, again if we have a is much greater than lambda, then beta 1 0 is almost 2 pi by lambda and then D becomes 32 by pi lambda square ab, so that I can write as 8 by pi square 4 by 4 pi by lambda square ab. So, now you see I can easily tell that what is my physical area? Physical area is ab. But what is my A em, just look at here so, A em is 8 by pi square into ab.

So, in this case, I have multiplied by a factor the A em, I am multiplied by physical area into 8 by pi square means 0.8, so that is obvious because I am not doing illumination uniformly. So, I need to give the penalty that maximum effective area cannot be more than the physical area it is 80 percent of these. So, here you can say that aperture efficiency will be basically this 8 by pi square and that is 0.81 ok. So, this so this close now we can also analyze please remember that if we do not have the ground plane, if we do not have the ground plane then also we can analyze the obvious open ended waveguide or any antenna.

(Refer Slide Time: 14:15)

 $CCET$
I.I.T. KGP $M_5 = -\hat{n} \times \vec{E_a}$
 $\vec{J_5} = \hat{n} \times \vec{H_a}$
 $E_6 \propto (1 + a)$
 $E_7 \propto (1 + a)$.

Only there we would not have the help because here we are having only magnetic current, there we will have to consider both the magnetic current and the electric current. That means, M s is there, M s is minus n cross E a and also we will have to take J J s is n cross H a. So, H a we know, we will have to find J so, we will have to find the fields due to this and then we will have to sum both of them, that means, all those n theta n phi l theta l phi will be present that is the only thing.

So, you can do there, you will repeat the problem you will see that if the same problem that means, this is open ended waveguide without the ground plane you do, ultimately you get E theta will be proportional to 1 plus cos theta and E phi also will be proportional to 1 plus cos theta, so that you can do ok.

(Refer Slide Time: 15:44)

So, I can show you some a things this field pattern of a rectangular waveguide mounted on an infinite ground plane; a is 3 lambda; b is 3 lambda. So, you see this is the a thing if you compare, already we will see that the planes.

(Refer Slide Time: 15:58)

So, this is the a first side lobe relative maximum magnitude for this. So, here you will see that half power beam width, this is the half power beam width, how it is varying. Then what else a first null beam width first side lobe beam width, these are shown here,

similarly, H-plane beam widths are also shown here. And beam efficiency versus half cone angle, for a square aperture etcetera.

(Refer Slide Time: 16:21)

(Refer Slide Time: 16:25)

(Refer Slide Time: 16:32)

This is for circular aperture also your circular waveguide if you know you can find that dominant field is T 1 0 mode ok. This is a constant field so, constant for that this becomes the thing.

(Refer Slide Time: 16:48)

Then in case of a circular waveguide T 1 1 mode is the dominant mode. So, if you have an open ended waveguide circular, then this becomes the pattern. So, this is the beam width H-plane beam width, this is beam efficiency etcetera.

Now, we will see the last example that will be microstrip antenna. Now, it is quite popular because of its conformal type of thing. If I have a planar structure they are putting you can put waveguides etcetera. But, you can also have that the waveguides etcetera they are a heavy and they are metallic things instead on a simple thing, you can have a microstrip type of antenna. So, you have only dielectric and they both are planar structures.

But, actually antenna wise microstrip antenna is a very very bad antenna, it is a very inefficient antenna also it cannot handle power, it is power handling capability is very less. But, those cases where you are not having high power you are not having sizable power suppose our communication and all those things the personal communication all those, there you have small mobile phone smaller there we use microstrip antenna. Now, since this is the age where all these things are very applications are very popular that is why a microstrip antenna. Otherwise, if you see the performance of this antenna this is very, very poor efficiency antenna.

So, but we will now see if you look at this microstrip antenna actually this top layer is called patch this is a metallic thing, also you have the ground plane. So, between the two these metallic structures, you have some substrate that is a dielectric. And so here what happens actually you see this is a slot, actually we can consider this is a rectangular type of aperture. This is an aperture you can say actually we I can analyze it this is a simply the top metal and bottom metal field with dielectric that is like parallel plate waveguide. So, it is there is no radiation from this, this is a parallel plate waveguide. Only the fields radiate at these because here I have a that parallel plate waveguide suddenly has an opening.

So, aperture is this, this is one aperture that side there is a another aperture. So, I have now simply two apertures; this is one and this is another they these waveguides dimension l that is cleverly put as lambda g by 2. Lambda g is the guided wavelength that concept we have seen in case of any waveguide. So, if I take lambda g by 2, basically they are the fields that are present here and here the aperture fields are opposite phase.

So, I can consider that I have a slot here I or I have an aperture here, I have an aperture here and they are forming a two element array where the inter element separation is lambda g by 2. So, I know how to analyze this we will have to assume what is the field here now since this slot is very small. So, we can assume an uniform field here, here also an uniform field and you see the this is the field lines are shown. So, by there it is seen that they are 180 degree out of phase. So, we will find the field for a one that procedure today we have seen. Then we will simply multiply that with the array factor for these two elements, so that is the analysis.

(Refer Slide Time: 21:27)

So, what we will do, we will now see that analysis because the this is the actually side view here you can see again that field thing is here. But this is a side view so, over the whole cross, if you go along the its previous, so if you go along this direction, so there always this if you take a cross section the same field is there. So, it is an uniform field throughout this slot.

(Refer Slide Time: 21:58)

So, this is the coordinate system we will use. It is the same coordinate that in the first micro where is the previous you see, the slot is here slot is made by it is thickness is h and width is w. And I can say that is in the x z plane ok. So, I will have to have this, you see the slot is in the z and x, x z plane so; the corresponding h and w are here. So, now I will assume that since this h which is the thickness that is very small so, it is same as our slot we consider. So, we can say that the field that will be uniform and so that will now discuss that what is the slot field?

(Refer Slide Time: 23:18)

 $\begin{array}{|c|}\n\hline\n\text{O CET} \\
\text{I.I.T. KGP}\n\end{array}$ $E_{\alpha} = \hat{a}_{\alpha} E_{o}$ $\begin{cases} -\frac{1}{2} \leq \kappa \leq +\frac{1}{2} \\ -\frac{\omega}{2} \leq \frac{\omega}{2} \leq \frac{\omega}{2} \end{cases}$ $\vec{M}_{5} = \begin{cases} -2 \hat{n} \times \vec{E_{A}} = -2 \hat{a}_{y} \times \hat{a}_{u} E_{0} \\ 0 & i = \hat{a}_{z} \end{cases}$ $\overrightarrow{J}_{s}=0$

So, we will write that E a that is x directed you see from here it is the x directed field, so that means, this side the fields lines are here just like our aperture a slot antenna E naught. So, minus h by 2 less than x dash less than plus h by 2 and minus w by 2 less than y dash less z dash less than plus w by 2 so, equivalence principle says that there will be since this slot I can say is backed by ground plane. So, there will be J s is 0 and M s will be minus 2 n cross E a, so that is 2 minus 2 a y, here you see that n is what n is a y cross E a is a x E naught over that minus h by 2 z dashed w by 2. So, this becomes basically a y x so, it is a z 2 E naught 0 elsewhere etcetera, J s is 0.

(Refer Slide Time: 25:25)

$$
E_{\phi} \approx 0
$$
\n
$$
E_{\phi} \approx -j \frac{h \omega k E_{0} e^{-j k n}}{\pi n} \left\{ s : \phi \left[\frac{s \times x}{x} \right] \left(\frac{s \times z}{z} \right) \right\} G_{n} \left(\frac{k!}{z} s \phi s y \right)
$$
\n
$$
F_{\phi} \approx -j \frac{h \omega k E_{0} e^{-j k n}}{\pi n} \left\{ s : \phi \left[\frac{s \times x}{x} \right] \left(\frac{s \times z}{z} \right) \right\} G_{n} \left(\frac{k!}{z} s \phi s y \right)
$$
\n
$$
F_{\phi} \approx -j \frac{h \omega k E_{0} e^{-j k n}}{\pi n}
$$
\n
$$
F_{\phi} \approx 0
$$

So, so now, you can apply the same procedure, you see so the field can be written as E r E theta will be almost 0 and E phi will exist that will be minus j h w k E naught e to the power minus j k r by pi r then sin theta sin X by X sin Z by Z then the array factor will be cos kl by 2 sin theta sin phi. This you check from the, this is I will say array factor this is the element factor, element factor or something you say. And what is X, X is k h by 2 sin theta cos phi and y y not, $Z Z$ is kw by 2 cos theta. Now, usually the substrate thickness h is very small.

(Refer Slide Time: 27:15)

So, if we have that that means for h much larger smaller than lambda, we have E phi becomes minus $i \geq 0$ e to the power minus $j \leq r$ by pi r sin theta sin k w by 2 cos theta by cos theta cos of k l by 2 sin theta sin phi. Now, here I have introduced v naught it is the I can say voltage across the slot. So, voltage is what I know the e field that I need to multiply by the thickness so, h into E naught is v naught, it is a voltage across the slot ok.

So, from this you can since you have E phi also you will have then h theta. So, now, you can find the power radiated etcetera. And from there actually you need to have if you wants to radiate, it should have radiation resistance which generally slot people this microstrip antenna people they call the inverse of that conductance. So, what is the conductance? It is the P rad or P r into 2 by V naught square, so that people can find out if you know the field you can find the P r etcetera, so that you can will be able to do.

Let me see the graphs, you see this graph this is a slot conductance as a function of slot width. So, for a wide slot, if we have w is large, let us say w is equal to 10 lambda, in that case the conductance is roughly 0.5so, radiation resistance will be 2. Whereas, if we have a small slot, then the conductance is very small you say 10 to the power minus 4 or 10 to the power minus 6, suppose w by lambda is 0.1, in that case conductance is 10 to the power minus 4. So, apply the radiation resistance will be quite high 10 to the power 4 ohm.

But one thing should be remember that these in these cases the loss resistance and the radiation resistance they are not exactly in series as we have seen for simple antennas, because, here there is a nearby ground plane or a lossy ground plane. So, that but still it shows that at small width because, slots if it is small width then we have sizable radiation taking place. But, if the width is quite large, then the radiation becomes quite poor, so that is why people try to make the width quite small, so that you get the benefit of good radiation.

(Refer Slide Time: 31:47)

$$
W << \lambda, G \approx \frac{1}{70} \left(\frac{W}{\lambda}\right)^{2}
$$

\n
$$
W \gg \lambda; G \approx \frac{1}{120} \left(\frac{W}{\lambda}\right)^{2}
$$

\n
$$
W \gg \lambda; G \approx \frac{1}{120} \left(\frac{W}{\lambda}\right)^{2}
$$

\n
$$
W \gg \lambda; G \approx \frac{1}{120} \left(\frac{W}{\lambda}\right)^{2}
$$

\n
$$
W \gg \lambda, D \approx 3 = 4.7748.
$$

\n
$$
W \gg \lambda, D \approx 1 \left(\frac{W}{\lambda}\right)
$$

Now, people have come out with various formulas as can be seen where the width is much larger than a smaller than lambda, G is of the order of 1 by 90 w by lambda square. And the other way round if you have width more, then G is 1 by 120 w,oh no, this time not square this is also people have found out that what is a reactive part by more advanced thing. And directivity what is directivity of each slot comes out to be that 4 pi into that dP d ohm max by P r.

So, by that people have found 2 pi w by lambda square into 1 by I, where I is a thing defined by some integration things. So, that I is 0 to pi sin k w by 2 cos theta by cos theta square sin cube theta d theta; sin cube theta d theta so this. So, basically ultimately it reduces to that if you have again w is less than lambda, then D is something like 3, so that means, 3 means 4.77 dB. And if w is greater than this, then D is almost 3 into w by lambda. So, as w high you get a things, but then your conductance is smaller so your radiation resistance which will be small. So, this is a dichotomy that shows that it is not a good antenna, but where power is sufficiently available you can use these as an antenna.

Also a major limitation of the this microstrip antenna is its narrow frequency bandwidth, because this bandwidth is primarily controlled by the characteristic of the transmission line the parallel plate things. So, it is a very small amount, so that is why it is not much and there are methods by which people use using higher dielectric constant substrate or increasing the thickness of the a substrate or cutting holes or slots into the or adding reactive components etcetera, this various people make various thing in better and better etcetera.

But basically I say that if you want to do the analysis this generalized method helps you even in these cases of microstrip antennas. So, with that we conclude this aperture antennas things and we will see that there are certain things particularly the measurement of any antennas various characteristic that is an important thing. Because always we cannot do analysis with all the antennas, many times it has been shown that antenna is first invented and then it is analysis comes. You see many times because antenna actually this thing is a lot of I will say arts is involved here. The with experience people can design antennas without analysis because, by their experience they understand how it radiates etcetera.

So, antenna design is something like arts as well as science. So, we are trying to say the science thing, but there are also a lot of arts which cannot be said like this, but with experience people get those word. So, that is why always it is not analyzable people try to have better and better models for that that is true for anyway I think. So, that is why measurement plays a very important role in antennas I think that you can always determine those things that what is the directivity, what is the beam width, what is the radiation pattern because these things ultimately matters.

So, how to do that, briefly we will touch upon that also we will see some new antennas in my array antenna while studying my said that there are there is one log periodic antenna which is broadband antenna, but it has a dispersive nature. So, how to overcome that thing and make a truly wideband broadband antenna that we will see that means, basically impulse radiating type of antenna we will see one day.

Similarly, other such things we will see and hope by now actually all the methods by which the analysis of antenna is done we have almost covered. But, obviously, in an any engineering stream there are certain other techniques by which you can perform this analysis in a much better way. Like, some integrations how to evaluate that numerically how to evaluate various integrations when analytic integration becomes tougher, those are part of engineering and those are part of antenna engineering higher courses.

So, but you have got a good amount of exposure by which you can start analyzing the simple antennas and also yourself start designing various antennas. The obviously, with experience you will rectify those mistakes that you commit in your earlier things, but this will I hope will give you a good starting point for your antenna design or analysis carrier.

Thank you.